Homework & Solution

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1 HW Lecture1

1 Derive Euler-Lagrange eq. for $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$

Solution)

$$\begin{aligned} S &= \int d^4x \mathcal{L}(\phi, \partial_\mu\phi) \\ \delta S &= \int d^4x \left[\frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi)\right] \\ &= \int d^4x \left[\left\{\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right\}\delta\phi + \partial_\mu\left\{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta\phi\right\}\right] \end{aligned}$$

so, as the total derivative term vanishes on the boundaries with $\delta\phi = 0$

$$\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = 0$$

then, we can write

$$\begin{aligned} \partial_\mu\partial^\mu\phi + m^2\phi + \frac{1}{3!}\lambda\phi^3 &= 0 \\ (\Box + m^2)\phi + \frac{1}{3!}\lambda\phi^3 &= 0 \end{aligned}$$

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2 Where are the field from? [Research "Representation of Lorentz group"]

Solution)

The Lorentz algebra could be written as $SO(3,1)$ that is same with $SU(2)_\mathrm{L} \times SU(2)_\mathrm{R}$. It is a kind of Lie algebra. The Lorentz algebra is defined as

$$\mathbf{J}^\pm = \frac{\mathbf{J} \pm i\mathbf{K}}{2}$$

where $\mathbf{J}$ is a generator of rotation and $\mathbf{K}$ is a generator of boost. Their representation is labeled by angular momentum, j, where $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots$. And the dimension of the representation (j_-, j_+) is $(2j_- + 1)(2j_+ + 1)$. And if considering the scalar field, we get the values of j_- and j_+ could be both 0 since the dimension is 1 and spin is equal to zero. Thus that is labeled by $(0,0)$. In the same way, Dirac field is four-dimension and its spin is $\frac{1}{2}$ and its representation is a direct sum of $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$, which is called The Weyl spinor representation(the left-handed Weyl spinor and the right-handed Weyl spinor, respectively). Vector field having spin-1 belongs to the $(\frac{1}{2}, \frac{1}{2})$ representation, which is also called as the gauge field.

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3 Why "δS=0" gives the Largest Contribution when $\hbar \to$0 limit?

Solution)

From the path integral formulation, the probability amplitude is provided by the formula

$$\text{Prob} = \sum_{\text{all possible paths}} e^{\frac{i}{\hbar}\mathcal{S}}$$

This problem is highly related to stationary phase method. Here is the simple review of this.

Consider,

$$I = \lim_{\lambda \to \infty} \int_{-\infty}^{\infty} dx\ e^{-\lambda f(x)}$$

when $f(x)$ has a global minimum at $x = x_0$, i.e $f'(x_0) = 0$.

Then the dominant contributions to the above integral, as $\lambda \to \infty$ will come from the integration region around x_0, since it the largest value on the integral region, and the value exponentially decays on the other region.

Formally, we may expand $f(x)$ about this point:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2 + \cdots$$

Since $f'(x_0) = 0$, this becomes:

$$f(x) \approx f(x_0) + \frac{1}{2} f''(x_0)(x - x_0)^2$$

Inserting the expansion into the expression for I gives

$$\lim_{\lambda \to \infty} e^{-\lambda f(x_0)} \int_{-\infty}^{\infty} dx\ e^{-\frac{\lambda}{2} f''(x_0)(x - x_0)^2} = \lim_{\lambda \to \infty} \left[\frac{2\pi}{\lambda f''(x_0)} \right]^{1/2} e^{-\lambda f(x_0)}$$

This approximation is known as the stationary phase or saddle point approximation. This formula also holds for the imaginary case,

$$I = \int dx\ e^{i\lambda f(x)}$$

since when λ is very large, phases change very rapidly as the value of exponent is large, hence they will add incoherently, varying between constructive and destructive addition at different times.

Our case is a trivial application of this formula, which is $f(x)$ replaced with $\mathcal{S}$, and λ replaced with $1/\hbar$.

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14.2.4 Classical limit

As a first check on the path integral, we can take the classical limit. To do that, we need to put back $\hbar$, which can be done by dimensional analysis. Since $\hbar$ has dimensions of action, it appears as

$$\langle 0; t_f | 0; t_i \rangle = N \int \mathcal{D}\Phi(\vec{x}, t) e^{\frac{i}{\hbar} S[\Phi]}. \tag{14.31}$$

Using the method of stationary phase we see that, in the limit $\hbar \to 0$, this integral is dominated by the value of Φ for which $S[\Phi]$ has an extremum. But $\delta S = 0$ is precisely the condition that determines the Euler–Lagrange equations which a classical field satisfies. Therefore, the only configuration that contributes in the classical limit is the classical solution to the equations of motion.

In case you are not familiar with the **method of stationary phase** (also known as the **method of steepest descent**), it is easy to understand. The quickest way is to start with the same integral without the i:

$$\int \mathcal{D}\Phi(\vec{x}, t) e^{-\frac{1}{\hbar} S[\Phi]}. \tag{14.32}$$

In this case, the integral would clearly be dominated by the Φ_0 where $S[\Phi]$ has a minimum; everything else would give a bigger $S[\Phi]$ and be infinitely more suppressed as $\hbar \to 0$. Now, when we put the i back in, the same thing happens, not because the non-minimal terms are zero, but because away from the minimum you have to sum over phases swirling around infinitely fast. When you sum infinitely swirling phases, you also get something that goes to zero when compared to something with a constant phase. Another way to see it is to use the more intuitive case with $e^{-\frac{1}{\hbar} S[\Phi]}$. Since we expect the answer to be well defined, it should be an analytic function of Φ_0. So we can take $\hbar \to 0$ in the imaginary direction, showing that the integral is still dominated by $S[\Phi_0]$.

Figure 1: Homework 1.3 reference(Schwartz "Quantum field theory and Standard model")

2 HW Lecture2

1 Show

$$\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} dq_1 \dots dq_N e^{-\frac{1}{2} qAq + Jq} = \left(\frac{(2\pi)^N}{det(A)} \right)^{1/2} e^{\frac{1}{2} JA^{-1} J}$$

Solution)

Let's look at the 1 dimensional case, which is expressed as

$$\int_{-\infty}^{\infty} dx \; e^{-\frac{1}{2} ax^2 + Jx}$$

This is a simple Gaussian integral, which can be evaluated as

$$\int_{-\infty}^{\infty} dx \; e^{-\frac{1}{2} ax^2 + Jx} = \int_{-\infty}^{\infty} dx \; e^{-\frac{a}{2}\left(x - \frac{J}{a}\right)^2} e^{\frac{J^2}{2a}} = \left(\frac{2\pi}{a} \right)^{\frac{1}{2}} e^{\frac{J^2}{2a}}$$

Now, if we expand this to an $N \times N$ matrix, the exponent in the integral takes the form

$$-\frac{1}{2} q_i A_{ij} q_j + J_i q_i$$

Using the same analogy with the 1D case, we get

$$-\frac{1}{2}\left(q_i - \left(A^{-1}J\right)_i\right) A_{ij} \left(q_j - \left(A^{-1}J\right)_j\right) + \frac{1}{2}J_i A^{-1}_{ij} J_j$$

By introducing $\widetilde{\widetilde{q}} \equiv \widetilde{q} - A^{-1}q$, we can evaluate $-\frac{1}{2}\widetilde{q}_i A_{ij}\widetilde{q}_j$ as,

$$\begin{aligned}
-\frac{1}{2}\widetilde{q}^T A\widetilde{q} =& -\frac{1}{2}\widetilde{q}^T S^T \left(SAS^T\right) S\widetilde{q} \\
=& -\frac{1}{2}\widetilde{\widetilde{q}}^T D\widetilde{\widetilde{q}} \\
=& \frac{1}{2}\left(d_1\widetilde{\widetilde{q}}_1^2 + d_2\widetilde{\widetilde{q}}_2^2 + \cdots + d_N\widetilde{\widetilde{q}}_N^2\right)
\end{aligned}$$

Where S is an orthogonal transformation matrix, and D is the diagonalized matrix. As the matrix is diagonalized, we get the following relation

$$det(A) = d_1 \cdots d_N$$

Hence, we get

$$\begin{aligned}
\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} dq_1 \cdots dq_N e^{-\frac{1}{2}q^T Aq + Jq} =& \left[\frac{(2\pi)^N}{d_1 \cdots d_N}\right]^{\frac{1}{2}} e^{-\frac{1}{2}J^{-1}A^{-1}J} \\
=& \left[\frac{(2\pi)^N}{det(A)}\right]^{\frac{1}{2}} e^{-\frac{1}{2}J^{-1}A^{-1}J}
\end{aligned}$$

■

2 From

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3}(a_p e^{-ip\cdot x} + (a_p^\dagger e^{ip\cdot x})), \text{with } [a_{\vec{p}}, a_{\vec{p}}^\dagger] = (2\pi)^3\delta^3(\vec{p} - \vec{p'})$$

$$a_{\vec{p}}\,|0\rangle = 0, \langle 0\,| a_{\vec{p}}^\dagger = 0.$$

Show

$$\langle 0|T(\phi(x)\phi(y)|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2 - m^2 + i\epsilon}e^{-ip\cdot(x-y)}$$

Solution)

Free field operator

$$\phi_0(\vec{x}, t) = \int \frac{d^3}{(2\pi)^3}\frac{1}{\sqrt{2\omega_k}}[a_k(t)e^{-ikx} + a_k^\dagger(t)e^{ikx}]$$

$$\begin{aligned}
\langle 0|\phi_0(x_1)\phi_0(x_2)|0\rangle =& \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_1}}\frac{1}{\sqrt{2\omega_2}}\langle 0|a_{k_1}a_{k_2}^\dagger|0\rangle e^{i(k_2x_2 - k_1x_1)} \\
=& \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{1}{\sqrt{2\omega_1}}\frac{1}{\sqrt{2\omega_2}}(2\pi)^3\delta^3(\vec{k_1} - \vec{k_2})e^{i(k_2x_2 - k_1x_1)} \\
=& \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}e^{ik(x_2 - x_1)}
\end{aligned}$$

$$\begin{aligned}
\langle 0|T\{\phi_0(x_1)\phi_0(x_2)\}|0\rangle =& \langle 0|\phi_0(x_1)\phi_0(x_2)|0\rangle\theta(t_1 - t_2) + \langle 0|\phi_0(x_2)\phi_0(x_1)|0\rangle\theta(t_2 - t_1) \\
=& \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}[e^{ik(x_2 - x_1)}\theta(t_1 - t_2) + e^{ik(x_1 - x_2)}\theta(t_2 - t_1)] \\
=& \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}[e^{i\vec{k}(\vec{x_1} - \vec{x_2})}e^{-i\omega_k\tau}\theta(\tau) + e^{-i\vec{k}(\vec{x_1} - \vec{x_2})}e^{i\omega_k\tau}\theta(-\tau)] \\
=& \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}e^{-i\vec{k}(\vec{x_1} - \vec{x_2})}[e^{i\omega_k\tau}\theta(-\tau) + e^{-i\omega_k\tau}\theta(\tau)]
\end{aligned}$$

Lemma

$$e^{i\omega_k\tau}\theta(-\tau) + e^{-i\omega_k\tau}\theta(\tau) = \lim_{\epsilon\to 0}\frac{-2\omega_k}{2\pi i}\int_{-\infty}^{\infty}\frac{d\omega}{\omega^2-\omega_k^2+i\epsilon}e^{i\omega\tau}$$

Proof.

$$\begin{aligned}\frac{1}{\omega^2-\omega_k^2+i\epsilon} &= \frac{1}{[\omega-(\omega_k-i\epsilon)][\omega-(-\omega_k+i\epsilon)]}\\ &= \frac{1}{2\omega_k}\left[\frac{1}{\omega-(\omega_k-i\epsilon)}-\frac{1}{\omega-(-\omega_k+i\epsilon)}\right]\end{aligned}$$

$$\begin{aligned}\int_{-\infty}^{\infty}\frac{d\omega}{\omega-(\omega_k-i\epsilon)}e^{i\omega\tau} &= -2\pi i e^{i\omega_k\tau}\theta(-\tau)+O(\epsilon)\\ \int_{-\infty}^{\infty}\frac{d\omega}{\omega-(-\omega_k+i\epsilon)}e^{i\omega\tau} &= 2\pi i e^{-i\omega_k\tau}\theta(\tau)+O(\epsilon)\end{aligned}$$

□

Putting it together, we find

$$\lim_{\epsilon\to 0}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}e^{-\vec{k}(\vec{x_1}-\vec{x_2})}\frac{-2\omega_k}{2\pi i}\int_{-\infty}^{\infty}\frac{d\omega}{\omega^2-\omega_k^2+i\epsilon}e^{i\omega\tau}$$

■

Remark

. $k_0 \neq \sqrt{\vec{k}^2+m^2}$. The propagating field can be off-shell.

. This is a classical Green's function for the Klein-Gordon equation : $(\Box+m^2)D_F(x,y) = -i\delta^4(x-y)$

3 HW Lecture3

1 Show that the Dirac equation, $(i\gamma^\mu\partial_\mu - m)\,\psi = 0$ implies the Klein-Gordon equation, $(\Box + m^2)\,\psi = 0$

Solution)

$$(i\gamma^\nu\partial_\nu + m)(i\gamma^\mu\partial_\mu - m)\psi = -(\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu + m^2)\psi = 0$$

Note that

$$\gamma^\mu\gamma^\nu\partial_\mu\partial_\nu = \frac{1}{2}\gamma\{\gamma^\mu,\gamma^\nu\} = \frac{1}{2}2g^{\mu\nu}\partial_\mu\partial_\nu = \partial_\mu\partial^\mu = \Box$$

Therefore, each compoents of ψ solves the Klein-Gordon equation,

$$(\Box + m^2)\psi = 0$$

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2 Using the following relations $F^{0i} = -E^i$, $F^{ij} = -\varepsilon^{ijk}B_k$ construct the Maxwell equations (note that the Bianchi identity is $\partial_{[\mu}F_{\lambda\nu]} = 0$).

Solution)

By Bianchi identity,

$$\partial_i F_{jk} + \partial_j F_{ki} + \partial_k F_{ij} = 0$$

– $i = 0$)

$$\begin{aligned}
\partial_0 F_{jk} + \partial_j F_{k0} + \partial_k F_{0j} &= 0 \\
\partial^0 F^{jk} + \partial^j F^{k0} + \partial^k F^{0j} &= 0 \\
(using\ \eta^{\gamma\alpha} F_\alpha &= F^\gamma) \\
\partial^0 \epsilon^{jkm} B_m + \partial^j E^k - \partial^k E^j &= 0 \\
(using\ \epsilon_{abc}\epsilon^{abd} &= 2\delta_c^d) \\
2\partial^0 \delta_n^m B_m + \epsilon_{jkn}(\partial^j E^k - \partial^k E^j) &= 0
\end{aligned}$$

then, we can find

$$\begin{aligned}
2\partial^0 \delta_n^m B_m &= -\epsilon_{jkn}(\partial^j E^k - \partial^k E^j) \\
2\partial^0 \delta_n^m B_m &= 2\epsilon_{jkn}\partial^k E^j \\
\partial^0 B_n &= \epsilon_{jkn}\partial^k E^j
\end{aligned}$$

this equation means,

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} \quad : \text{Faraday's Law}$$

– $i, j, k \neq 0$)
let $i = 1, j = 2, k = 3$

$$\begin{aligned}
\partial_i F_{jk} + \partial_j F_{ki} + \partial_k F_{ij} &= 0 \\
\partial_i \epsilon_{jka} B_a + \partial_j \epsilon_{kia} B_a + \partial_k \epsilon_{ija} B_a &= 0 \\
\partial_1 \epsilon_{23a} B_a + \partial_2 \epsilon_{23a} B_a + \partial_3 \epsilon_{12a} B_a &= 0 \\
\partial_\alpha B_\alpha = \partial_\alpha B^\alpha &= 0
\end{aligned}$$

likewise, explicitly calculate other cases, then we can find,

$$\nabla \cdot \mathbf{B} = 0$$

■

There is similar problem in "An introduction to quantum field theory Ch.2 pb.2.(a) (Peskin Schroeder)"

3 Classical electromagnetism (with no sources) follows from the action

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right), \qquad \text{where } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Derive Maxwells equations as the Euler-Lagrange equations of this action, treat ing the components $A_\mu(x)$ as the dynamical variables. Write the equations in standard form by identifying $E^i = -F^{0i}$ and $\epsilon^{ijk} B_k = -F^{ij}$

Solution)

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
&= -\frac{1}{4} \partial_\mu A_\nu - \partial_\nu A_\mu \partial_\nu A_\mu - \partial_\mu A_\nu \\
&= -\frac{1}{2}(\partial_\mu A_\nu)(\partial^\mu A_\nu) + \frac{1}{2}(\partial^\mu A_\mu)^2
\end{aligned}$$

So, let's derive a equation of motion.

$$\frac{\partial \mathcal{L}}{\partial_\mu A_\nu} = 0$$

and,

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = -(\partial^\mu A^\nu) + (\partial^\alpha A^\alpha)\eta^{\mu\nu}$$

Lemma

$$\begin{aligned}\frac{\partial}{\partial(\partial_\mu A_\nu)}(\partial^\alpha A_\alpha) &= \frac{\partial}{\partial_\mu A_\nu}(\partial^\beta A_\alpha)\eta^{\alpha\beta} \\ &= \delta^\mu_\rho \delta^\nu_\alpha \eta^{\alpha\beta} \\ &= \eta^{\mu\nu}\end{aligned}$$

using this lemma, we can get

$$\begin{aligned}\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} &= -\partial_\mu \partial^\mu A^\nu + \partial_\mu \partial^\alpha A^\alpha \eta^{\mu\nu} \\ &= -\partial_\mu \partial^\mu A^\nu + \partial^\nu \partial^\alpha A^\alpha \\ &= -\partial_\mu \partial^\mu A^\nu + \partial^\nu \partial_\mu A^\mu \\ &= -\partial_\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ &= -\partial_\mu F^{\mu\nu}\end{aligned}$$

so,

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = \frac{\partial \mathcal{L}}{\partial_\mu A_\nu} \quad \longrightarrow \quad \partial_\mu F^{\mu\nu} = 0$$

1. $\mu = i,\ \nu = 0,$

$$\begin{aligned}\partial_i F^{i0} = 0 \quad &\longrightarrow \quad \partial_i E^i = 0 \\ &\longrightarrow \quad \nabla \cdot \mathbf{E} = 0\end{aligned}$$

2. $\mu = \mu,\ \nu = j,$

$$\begin{aligned}0 &= \partial_\mu F^{\mu i} \\ &= \partial_0 F^{0i} + \partial_j F^{ji} \\ &= -\frac{\partial E^i}{\partial t} - \partial_j \epsilon^{jik} B_k \\ &= -\frac{\partial E^i}{\partial t} + \partial_j \epsilon^{ijk} B_k \\ &= -\frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{B}\end{aligned}$$

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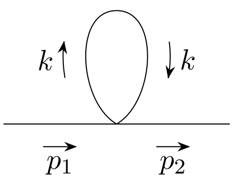
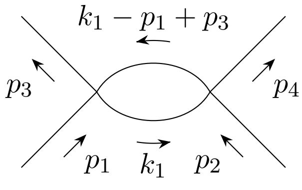
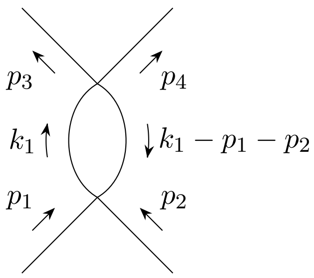
4 Calculate the amplitude $\mathcal{M}$ of the following diagrams using Feynman rules.



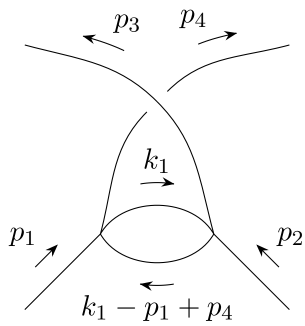
[Diagram 3-1]



[Diagram 3-2]



[Diagram 3-3]



[Diagram 3-4]

Solution)

Diagram 3-1)
The S-matrix in the momentum space is written as,

$$S = i\mathcal{M}(2\pi)^4\delta^{(4)}(p_1 - p_2)$$

Then using the Feynman rules, one can evaluate the matrix element $\mathcal{M}$, which is actually the amplitude.

$$i\mathcal{M} = \underbrace{\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2 - m^2}}_{\text{integral of large k part}}(-i\lambda)(2\pi)^4\delta^{(4)}(p_1 - p_2)$$
$$\sim \int k^3 dk \times \frac{1}{k^2} = k^2\Big|_0^\infty = \infty$$

Therefore the amplitude goes to infinity, the mass renormalization is needed.

Diagram 3-2)

$$\begin{aligned} i\mathcal{M} =& \frac{(-i\lambda)^2}{2}\int \frac{d^4k_2}{(2\pi)^4}\frac{d^4k_1}{(2\pi)^4}\frac{i}{k_1^2 - m^2 + i\epsilon}\frac{i}{k_2^2 - m^2 + i\epsilon} \\ & \times (2\pi)^4\delta^{(4)}(k_2 + p_4 - k_1 - p_2)(2\pi)^4\delta^{(4)}(p_3 + k_1 - p_1 - k_2) \\ =& \frac{(-i\lambda)^2}{2}\int \frac{d^4k_1}{(2\pi)^4}\frac{i}{k_1^2 - m^2 + i\epsilon}\frac{i}{(k_1 + p_3 - p_1)^2 - m^2 + i\epsilon}\delta^{(4)}(p_3 + p_4 - p_1 - p_2) \end{aligned}$$

By evaluating the large k_1 integral part, one can get

$$\begin{aligned} \mathcal{M} &\sim \int k_1^3 dk_1 \times \frac{1}{k_1^4} \\ &= \log(k)\Big|_{k=0}^\infty = \infty \end{aligned}$$

Diagram 3-3)

$$i\mathcal{M} = \frac{(-i\lambda)^2}{2}\int \frac{d^4k_1}{(2\pi)^4}\frac{i}{k_1^2 - m^2 + i\epsilon}\frac{i}{(k_1 - p_1 - p_2)^2 - m^2 + i\epsilon}\delta^{(4)}(p_3 + p_4 - p_1 - p_2)$$

Diagram 3-4)

$$i\mathcal{M} = \frac{(-i\lambda)^2}{2} \int \frac{d^4k_1}{(2\pi)^4} \frac{i}{k_1^2 - m^2 + i\epsilon} \frac{i}{(k_1 - p_1 - p_4)^2 - m^2 + i\epsilon} \delta^{(4)}(p_3 + p_4 - p_1 - p_2)$$

Clearly, this digerves. Therfore, we need to introduce the renormalization of quartic coupling lambda. Using the same method and summing up all contributions, we could get the total λ^2 ordered amplitude.

$\blacksquare$