Interstellar

Makers of the movie "Interstellar" boasted how much they tried to be faithful to physics of gravity. One interesting part of the plot is centered on the Miller planet entirely covered by ocean, orbiting just outside of a giant black hole called "Gargantua." Those who visited its surface, in a small landing ship, finds upon their return to the mother ship that 23 earth-years has passed during their few hour of absence due to the time-dilating effect of gravity. Some journalists criticized the movie for not making much sense, referring to "extremely strong gravitational force, necessary for the extreme time dilation." Give your own scientific opinions, physics-wise, on this part of movie that apparently caught attention of a lot of movie-goers. To do this, consider the following:

1) Motion of an object of mass m around a non-rotating black hole of mass M (Gargantua is supposed to be a fast-rotating black hole but, here, for simplicity we will assume otherwise) follows Lagrange equation of motion with the action

$$-mc^2\int ds\sqrt{\left(1-\frac{2GM}{c^2r}\right)\dot{t}^2-\left(1-\frac{2GM}{c^2r}\right)^{-1}\dot{r}^2/c^2-r^2\left(\dot{\theta}^2+(\sin\theta)^2\dot{\phi}^2\right)/c^2}$$

c is the speed of light and G is the Newton's gravitational constant. s is an arbitrary parametrization of the motion, $t(s), r(s), \ldots$, with respect to which the "time" derivative as in $\dot{t}, \dot{r}, \ldots$ is taken. You may wish to confine your attention to $r > r_H \equiv 2GM/c^2$, below which things become really weird. With these, classify possible orbits of the planet, which should be similar to Kepler ones when $r \gg r_H$ but qualitatively different when $1 < r/r_H < 10$ or so. What is the smallest possible circular orbit, for example?

2) The time lapse felt by different observers are different, depending on position and on velocity. If a static observer far away from the black hole feels time lapse of δt, the person moving near black hole feels, instead, a smaller time lapse of

$$\delta\tau = \delta t\times\sqrt{\left(1-\frac{r_H}{r}\right)-\left(1-\frac{r_H}{r}\right)^{-1}\dot{r}^2/(c^2\dot{t}^2)-r^2\left(\dot{\theta}^2+(\sin\theta)^2\dot{\phi}^2\right)/(c^2\dot{t}^2)}$$

Now, you are in position to address physics of the Miller planet and the plot around it. What is the maximum possible time-dilation can you imagine for a reasonable, in the context of the movie, orbit of the planet around Gargantua?

3) More questions: Was the criticism by journalists mentioned above valid? Why did the producers choose a very large black hole, do you think? What other serious flaws can you find with this part of movie, physics-wise?

4) Gargantua is actually a Kerr black hole, meaning that it rotates very fast. What qualitative differences does this make, relative to what you found above.