Bayesian Naturalness and NMSSM Focus Points

Doyoun Kim
IBS-CTPU

arXiv:1709.07895 [hep-ph]
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Introduction

- History of endeavor to seek beautiful and well-performing theory recently raised Naturalness problem.

- For three decades, the problem of the physical Higgs mass has been rephrased as ‘Hierarchy’, ‘Naturalness’, and ‘Fine-tuning’ prob. etc.

- Various mechanisms (models) are found to solve the ‘Big Hierarchy’ prob. that stabilizes the EWSB scale from the radiative corrections.

- However, it may require another fine-tuning even for such models in order to reproduce the observed world.

- Here, the supersymmetric (SUSY) examples will be discussed.
  But the extension to other models are straight forward.

Focus Point Scenario

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- In SUSY models, electroweak symmetry breaking (EWSB) condition relates the low energy observables (Mz, tanb) to the model parameters:

$$\frac{M_Z^2}{2} = -\frac{\bar{m}_{H_u}^2 \tan^2\beta - \bar{m}_{H_d}^2}{\tan^2\beta - 1} - |\mu|^2 - \frac{1}{2}\mathrm{Re}\Pi_{ZZ}^T$$

Z pole mass

Tree + Coleman-Weinberg

Transverse part of Z boson self energy

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Tree + Coleman-Weinberg

- Fine-tuning problem asks how the new physics:
    1. Satisfies EWSB at a proper energy scale. (Little hierarchy prob.)
    2. Stability of 1.

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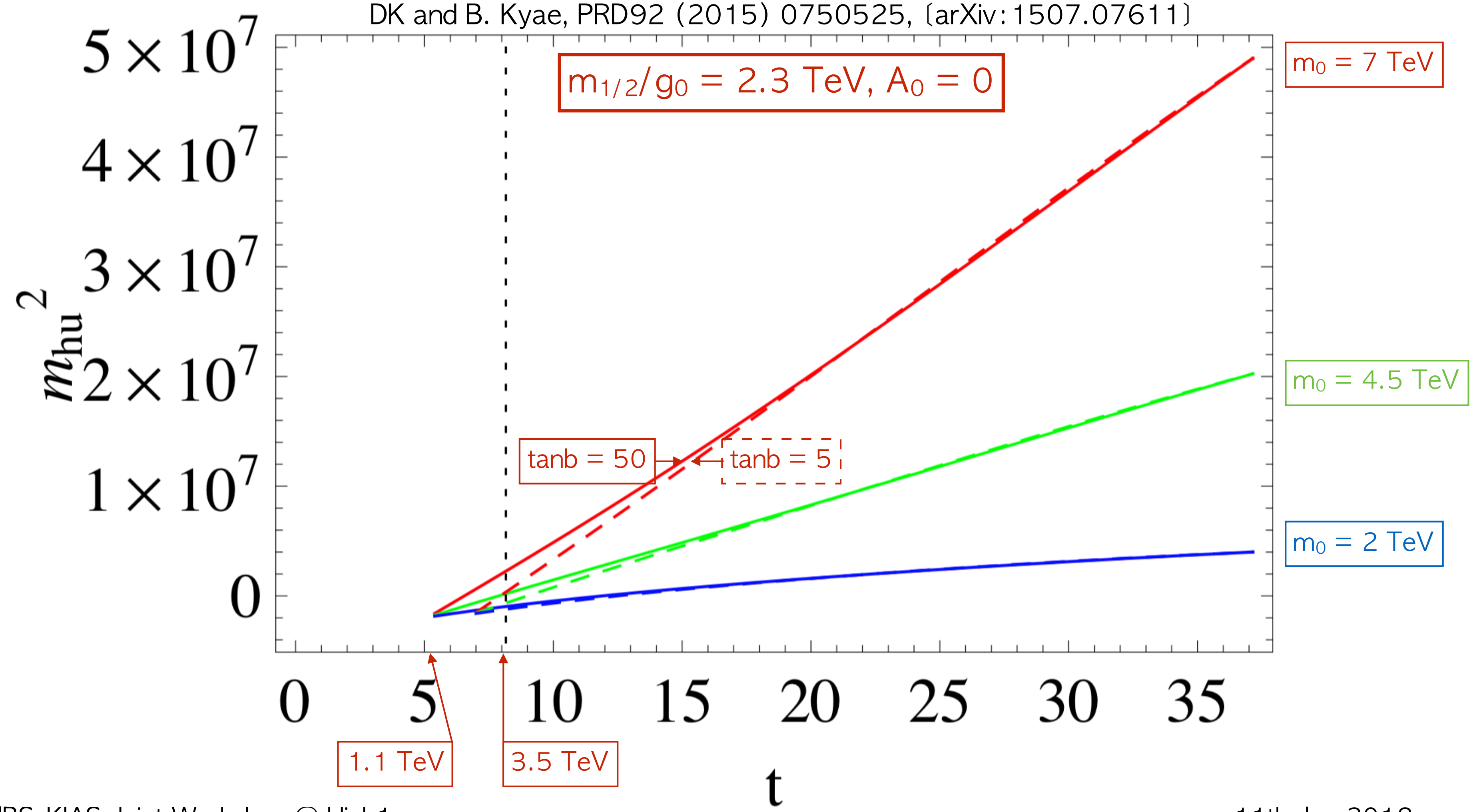
  Since μ term is natural, $\delta|\mu|^2$ is well-controlled.

- Fine-tuning in SUSY is how $\bar{m}_{H_u}^2$ is stabilized around the EW scale.

Focus Point Scenario

- Fortunately, we have a large set of such examples.

DK and B. Kyae, PRD92 (2015) 0750525, [arXiv:1507.07611]



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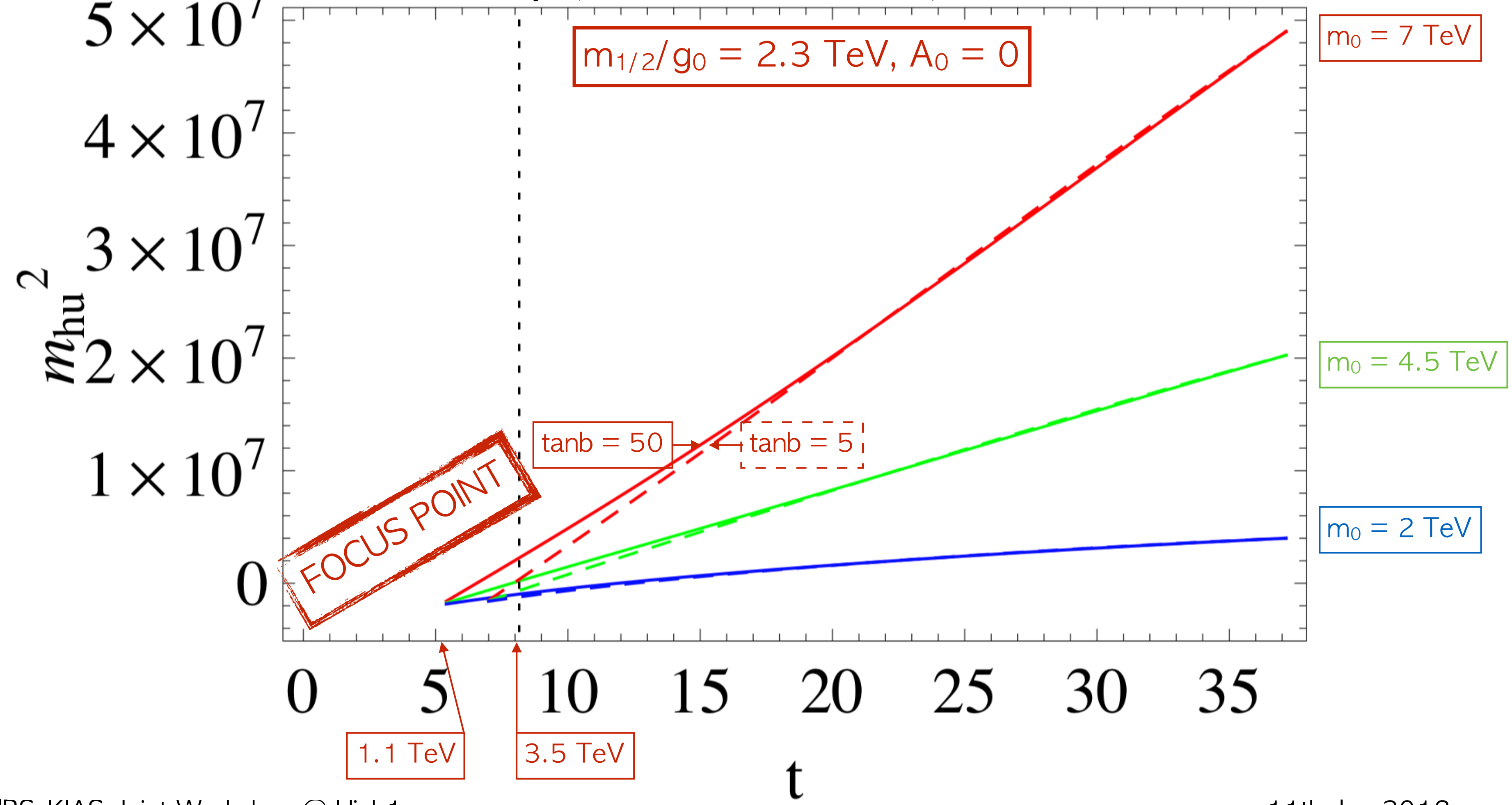
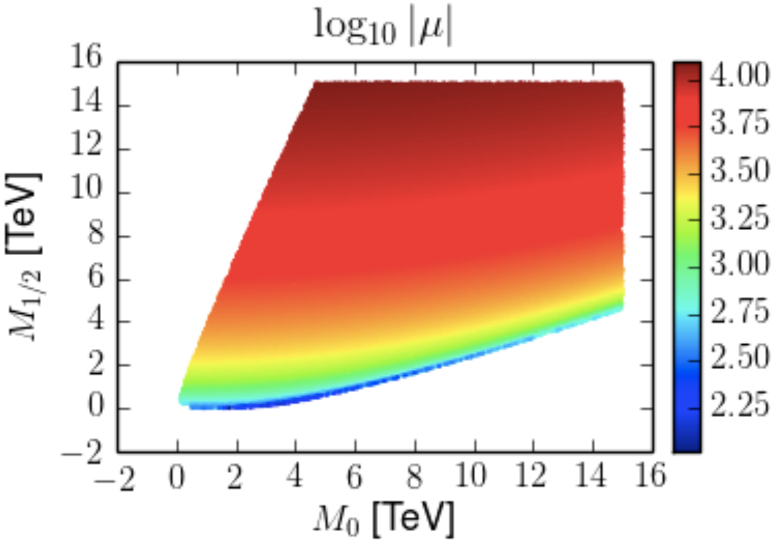
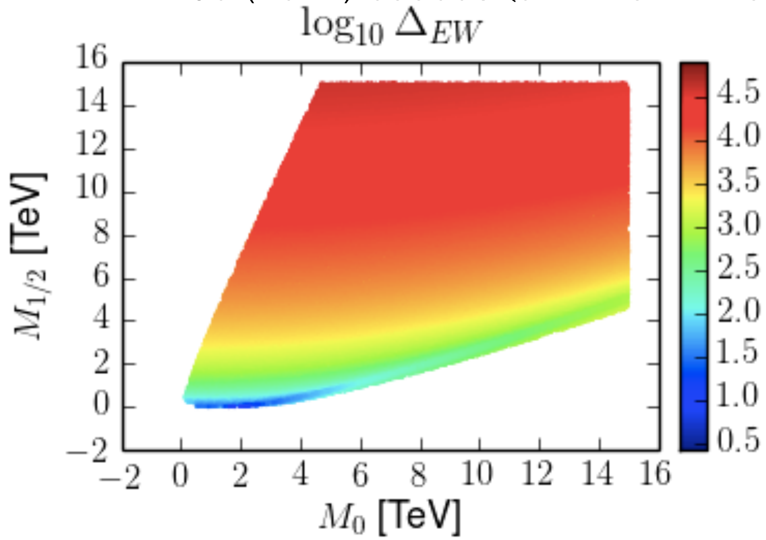
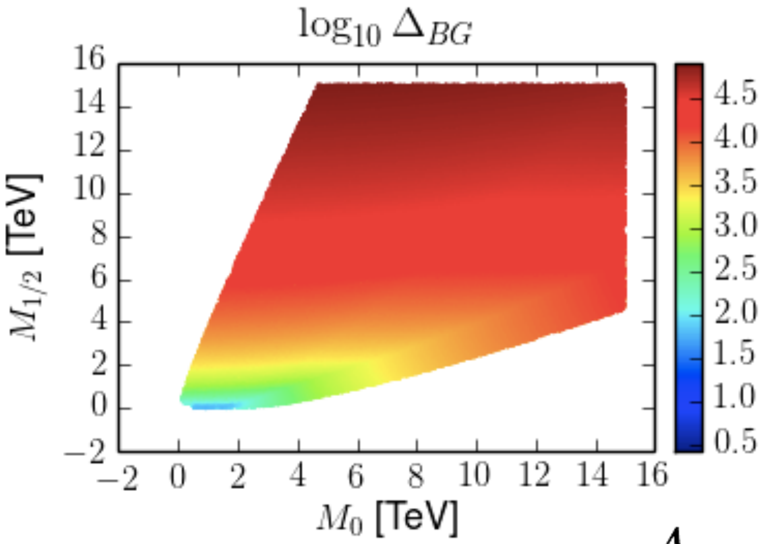


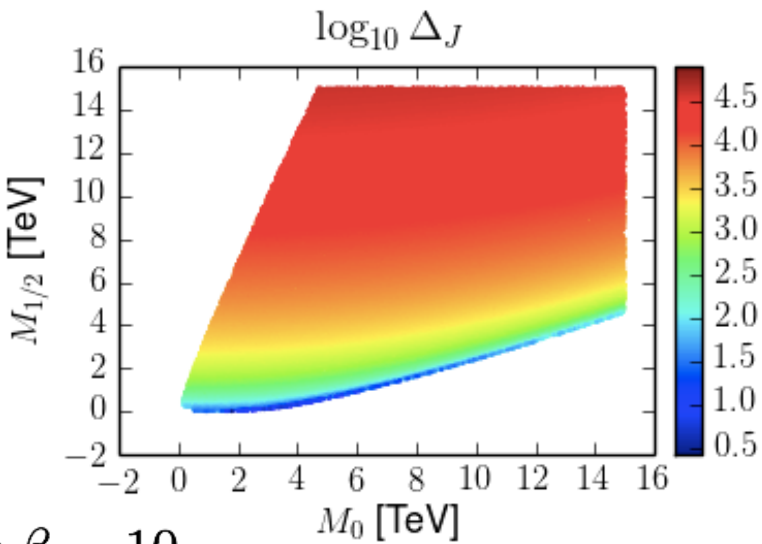
Focus Point Scenario in CMSSM

DK, P. Athron, C. Balazs, B. Farmer, E. Hutchison,
PRD90 (2014) 055008 [arXiv:1312.4150]





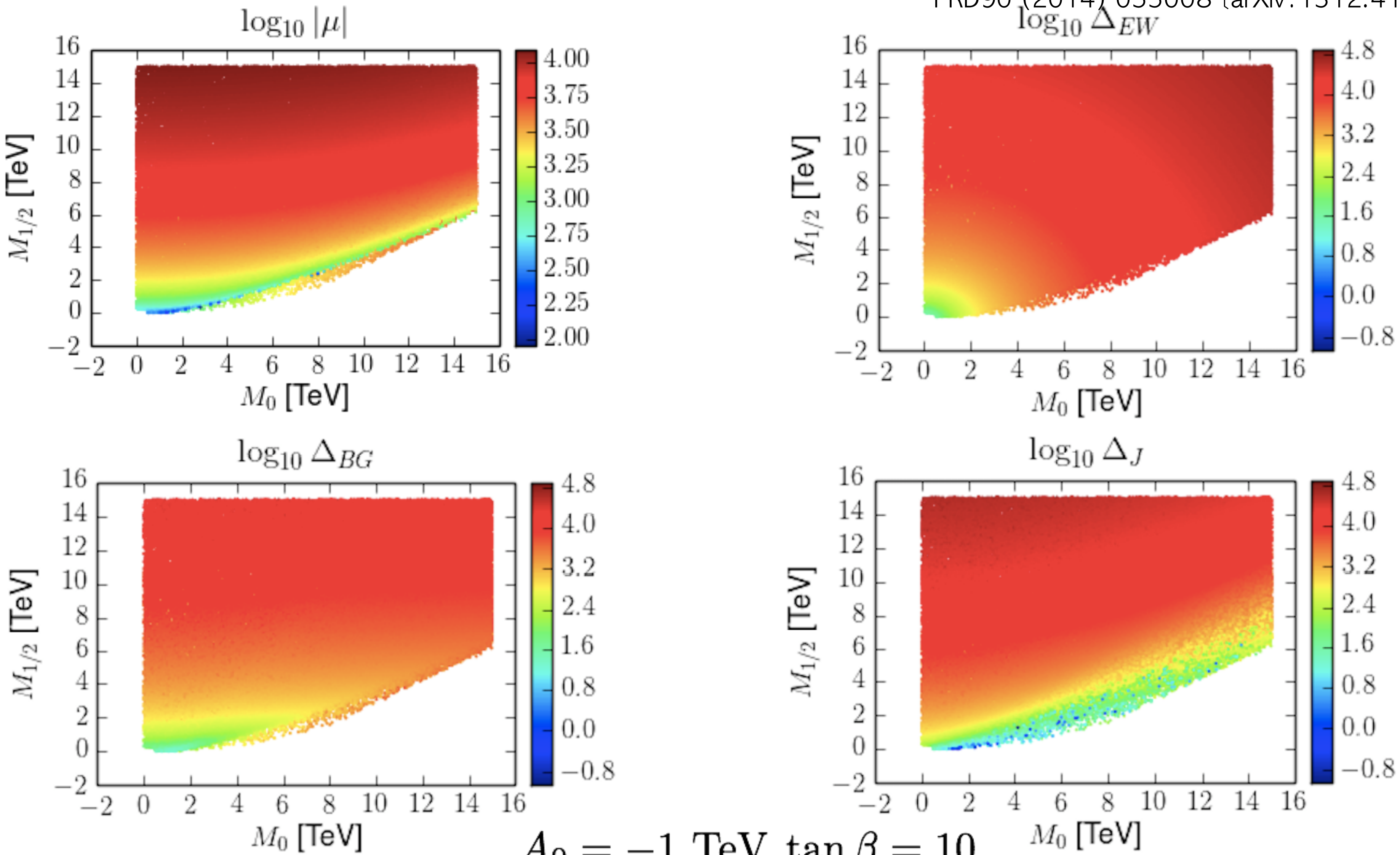




$$A_0 = -1\ \text{TeV}, \tan\beta = 10$$

Focus Point Scenario in CNMSSM

DK, P. Athron, C. Balazs, B. Farmer, E. Hutchison,
PRD90 (2014) 055008 [arXiv:1312.4150]



$$A_0 = -1\ \mathrm{TeV}, \tan\beta = 10$$

Focus Point Scenario

- In this scenario, $\delta \bar{m}_{H_u}^2$ is small and $\delta |\mu|^2$ dominates

$$-\frac{\delta M_Z^2}{2} \overset{t \gg 1}{\approx} \delta \bar{m}_{H_u}^2 + \delta \left|\mu\right|^2$$

- Thus the stability of EW scale can be measured by defining the fine-tuning as

$$\frac{\mu^2}{M_Z^2} \frac{\delta M_Z^2}{\delta \mu^2}$$

- This is another derivation of Barbieri-Giudice(-Ellis-Nanopoulos)'s fine-tuning measure, proposed in 1988 (1986):

$$\Delta_{BG} = \max_i \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i} \right|$$

- Note: Focus Point scenario was found 10 years later.
  Chan, Chattopadhyay, and Nath, PRD 58, 096004 (1998)
  Feng, Matchev and Moroi, PRD 61 (2000) 075005

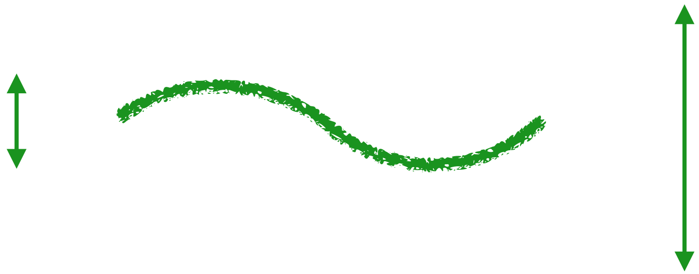
Fine-tuning Measure

Δ_{BG}

- Measures perturbative sensitivity of a low energy observable as the model parameters' fluctuation.

$$\Delta_{BG} = \sum_i \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i} \right| \sim \max_i \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i} \right| \sim \left| \frac{\mu^2}{M_Z^2} \frac{\partial M_Z^2}{\partial \mu^2} \right| = \frac{\mu^2}{M_Z^2}$$

M_Z

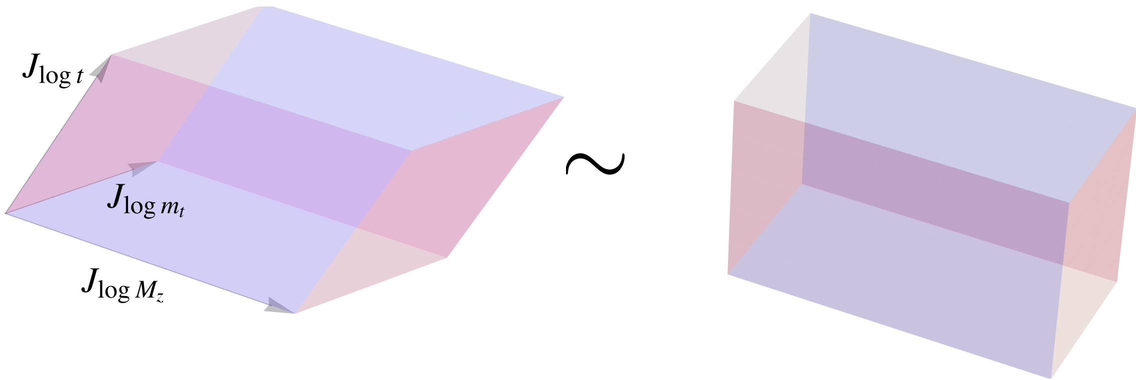


$p_i = \mu,\ B,\ m_0,\ m_{1/2}, A_0$

Δ_J

- Generalization: Encapsulates the correlation among the observables.

$$\Delta_J = \left|\left|\frac{\partial \ln \mathcal{O}_i}{\partial \ln p_j}\right|\right| \sim \frac{\delta V_{\mathcal{O}}}{\delta V_p}$$



- Note: Correlations among the high scale model parameters may reduce the fine-tuning at the EW scale. H. Baer, V. Barger, D. Mickelson (arXiv:1309.2984)

Δ_J

- Interestingly, this measure is systematically embedded in the program of Bayesian analysis for the new physics search, in form of the effective prior.

- DISCLAIMER:
  There is a controversy in model comparison due to the irreducible prior prob. dependency.
  Therefore, we focus on the param. estimation in a given model even though we present the evidence estimation for each model

Bayesian Analysis

Bayesian Naturalness [arXiv:1312.4150]

- In Bayesian Analysis, fine-tuning nature of the Jacobian factor penalizes unnatural parameter regions.

$$p(\mathcal{M}|\mathcal{D}) = \frac{p(\mathcal{D}|\mathcal{M})}{p(\mathcal{D})} p(\mathcal{M}) = \frac{1}{p(\mathcal{D})} \int p(\mathcal{D}|p_i) p(p_i) dp_i$$

- For example, in CMSSM

$$\int \mathcal{L} p(\mu, B, y) d\mu dB dy = \int \mathcal{L} \, |J_{\mathcal{T}_1}| \, p(M_Z, y, m_t) dM_Z dm_t dt$$

$$\mathcal{T}_1 : \{\mu, B, y\} \to \{M_z, t, m_t\}$$

M. E. Cabrera, J. A. Casas and R. Ruiz de Austri, JHEP 1005, 043 (2010) [arXiv:0911.4686]

Bayesian Analysis: Jacobian Effective Prior

- In Bayesian Analysis

$$\underbrace{p(\mathcal{M}|\mathcal{D})}_{\text{Posterior}} = \frac{p(\mathcal{D}|\mathcal{M})}{p(\mathcal{D})} p(\mathcal{M}) = \underbrace{\frac{1}{p(\mathcal{D})}}_{\text{Evidence}} \int \underbrace{p(\mathcal{D}|p_i)}_{\text{Likelihood}} \underbrace{p(p_i)}_{\text{Prior}} dp_i$$

- For CMSSM (a specific model)

$$\int \mathcal{L} p(\mu, B, y) d\mu dB dy = \int \mathcal{L} |J_{\mathcal{T}_1}| p(M_Z, y, m_t) dM_Z dm_t dt$$

$$\mathcal{T}_1 : \{\mu, B, y\} \rightarrow \{M_z, t, m_t\}$$

$$\Delta_J = \left| \frac{\partial \ln \mathcal{O}_j^2}{\partial \ln p_i^2} \right|$$

Fine-tuning sensitivity of physical data to the model parameters

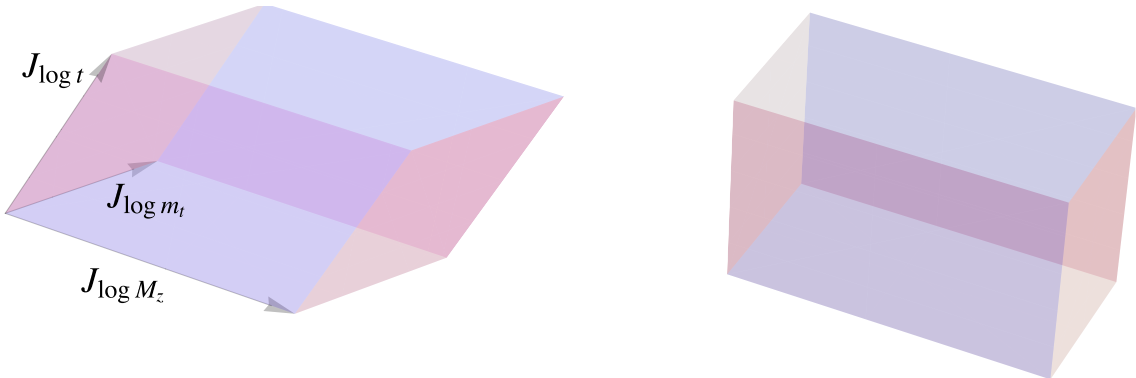
Bayesian Naturalness [arXiv:1312.4150]

- For CMSSM

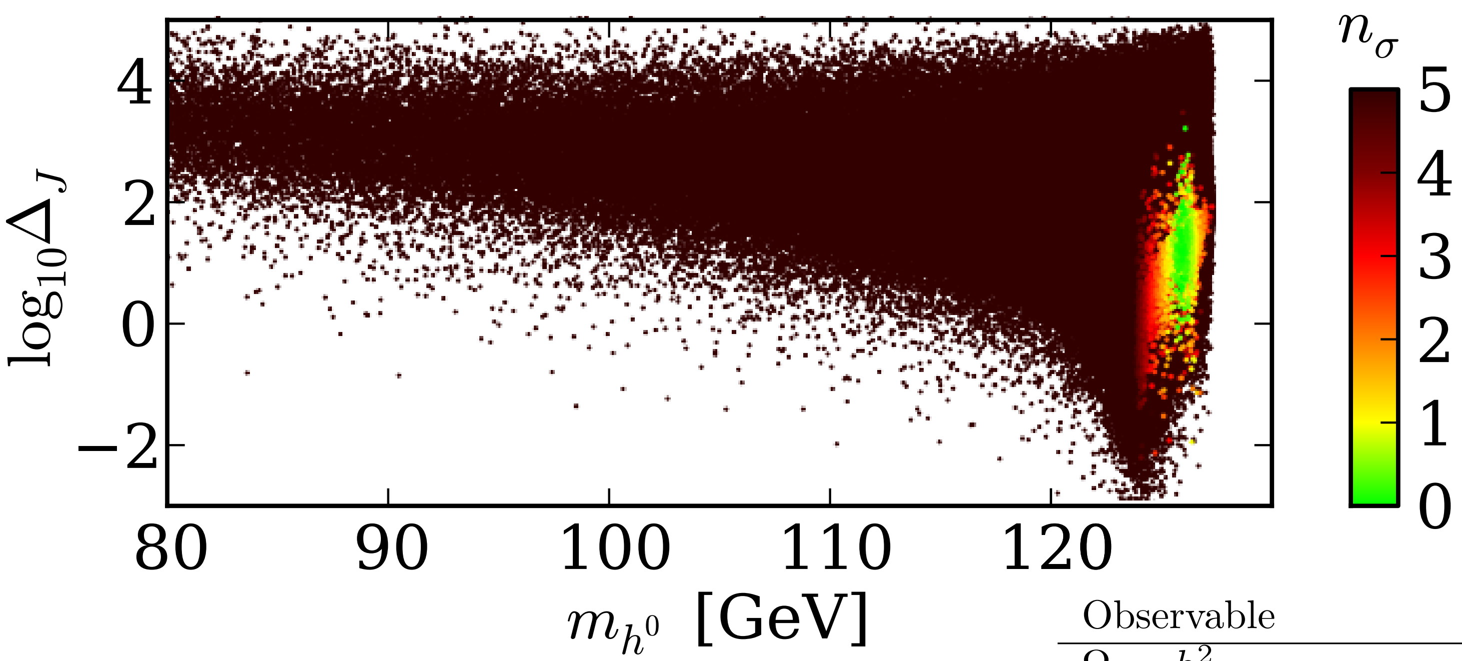
$$\Delta_J = \left| \frac{\partial \ln(M_Z^2, \tan^2\beta, m_t^2)}{\partial \ln(\mu^2, B^2, y_t^2)} \right|$$

- For CNMSSM

$$\Delta_J = \left| \frac{\partial \ln(M_Z^2, \tan^2\beta, s^2, m_t^2)}{\partial \ln(\lambda^2, \kappa^2, m_S^2, y_t^2)} \right|$$



Bayesian Analysis: Jacobian Effective Prior



- $A_0 = -2.5$ TeV, $\tan\beta = 10$
- A_λ, A_κ are set released

Observable	Experimental value
$\Omega_{DM}h^2$	0.1187 ± 0.0017
m_h	125.9 ± 0.4 GeV
$\mathrm{BR}\,(B_s \to \mu^+\mu^-)$	$(2.9 \pm 1.1) \times 10^{-9}$
$\mathrm{BR}\,(b \to s\gamma)$	$(343 \pm 21 \pm 7) \times 10^{-6}$
$\mathrm{BR}\,(B \to \tau\nu)$	$(114 \pm 22) \times 10^{-6}$
$m_{\tilde{\chi}_1^0}$	> 46 GeV
$m_{\tilde{\chi}_1^\pm}$	> 94 GeV if $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 3$ GeV
$m_{\tilde{q}}$	> 1.43 TeV
$m_{\tilde{g}}$	> 1.36 TeV

Result

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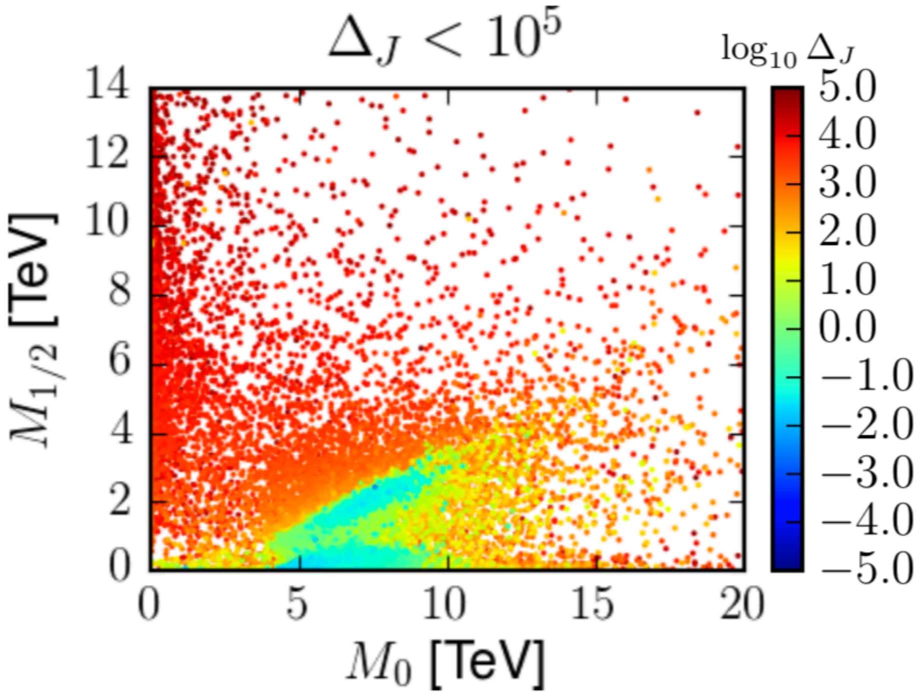
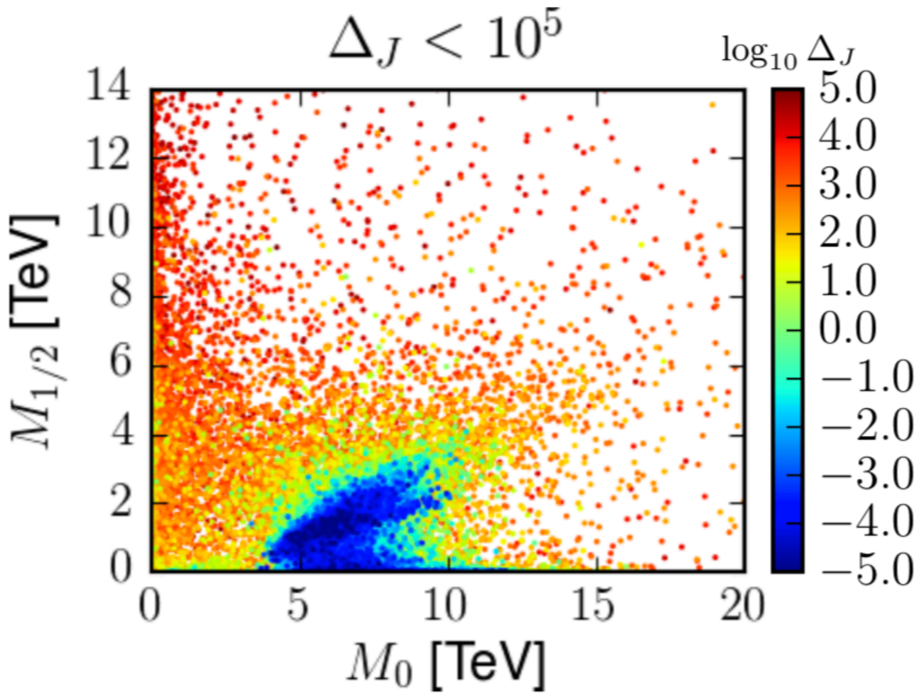
- Reexamined the Focus Point features





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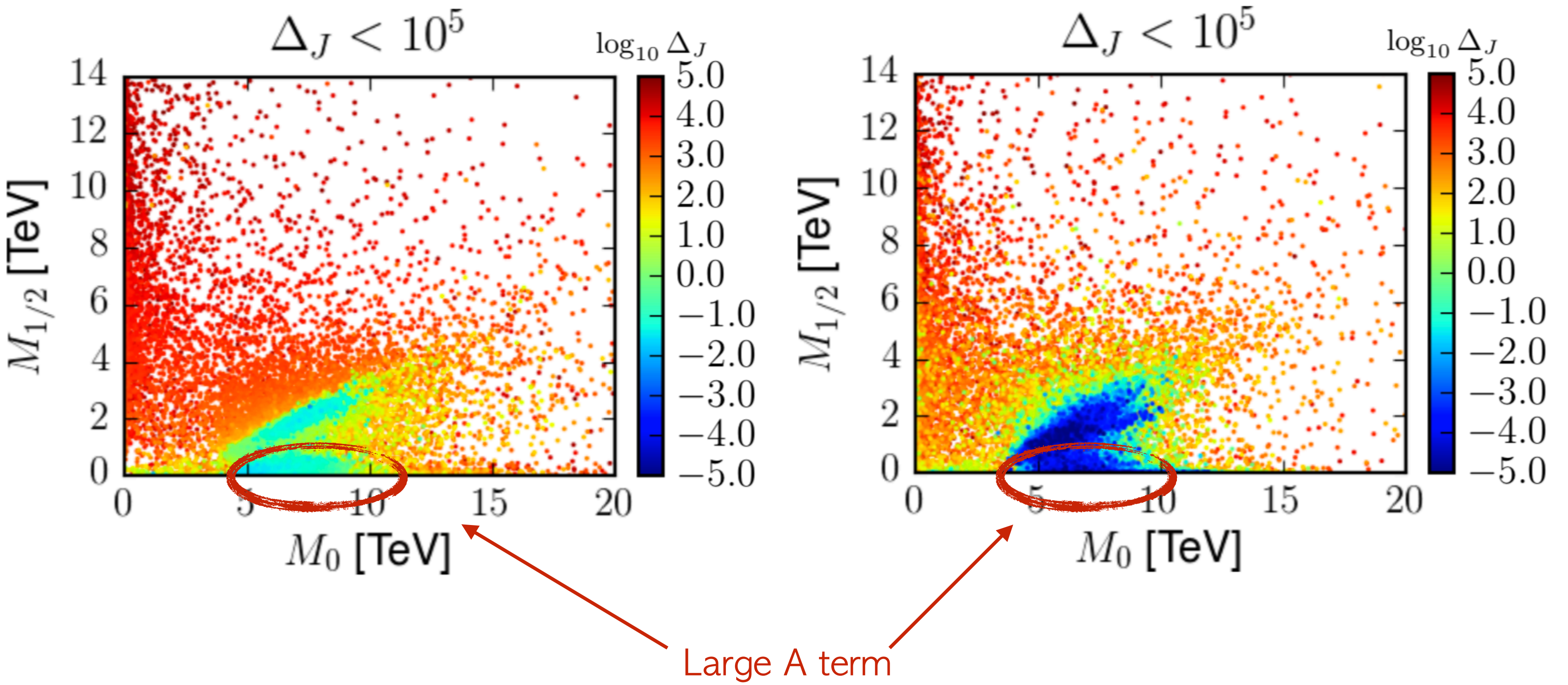
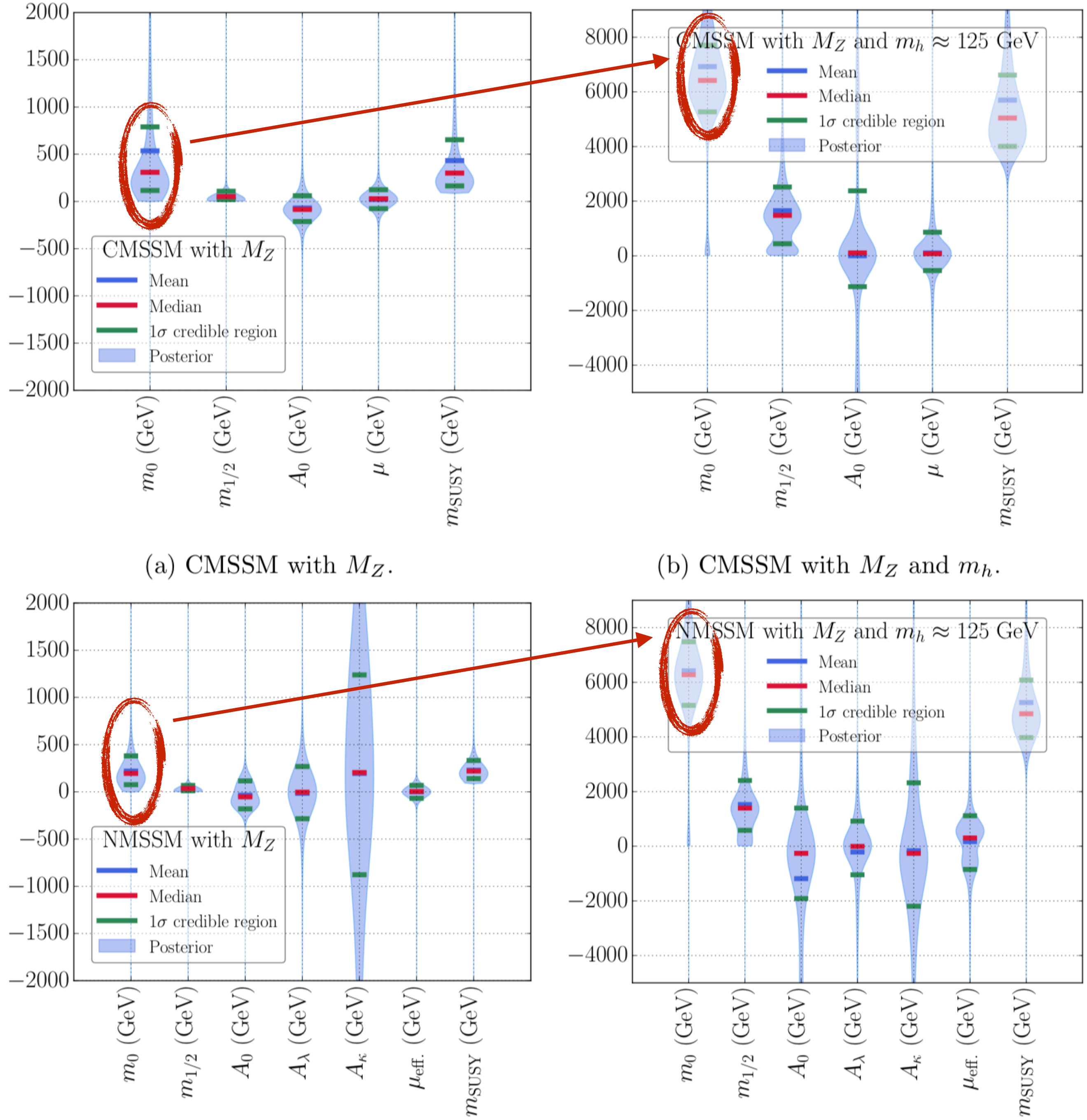
- Reexamined the Focus Point features



Result

- Minimum fine-tuning measures found in the scans

	M_Z		M_Z and $m_h \approx 125\,\mathrm{GeV}$	
	CMSSM	NMSSM	CMSSM	NMSSM
$\Delta_{\mathcal{J}}\big\vert_{M_{\mathrm{GUT}}}$	3×10^{-9}	2×10^{-10}	0.004	8×10^{-7}
$\Delta_{\mathcal{J}}\big\vert_{m_{\mathrm{SUSY}}}$	6×10^{-7}	2×10^{-10}	0.005	8×10^{-7}
Δ_{EW}	0.3	0.3	48.7	47.4
Δ_{BG}	0.1	0.2	451.9	133.2



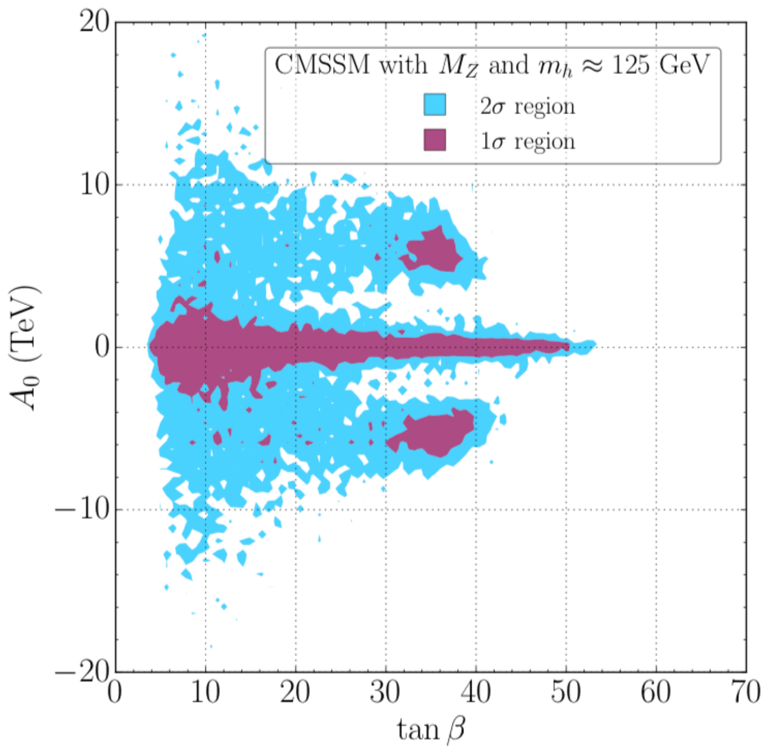
(a) CMSSM with M_Z.

(b) CMSSM with M_Z and m_h.

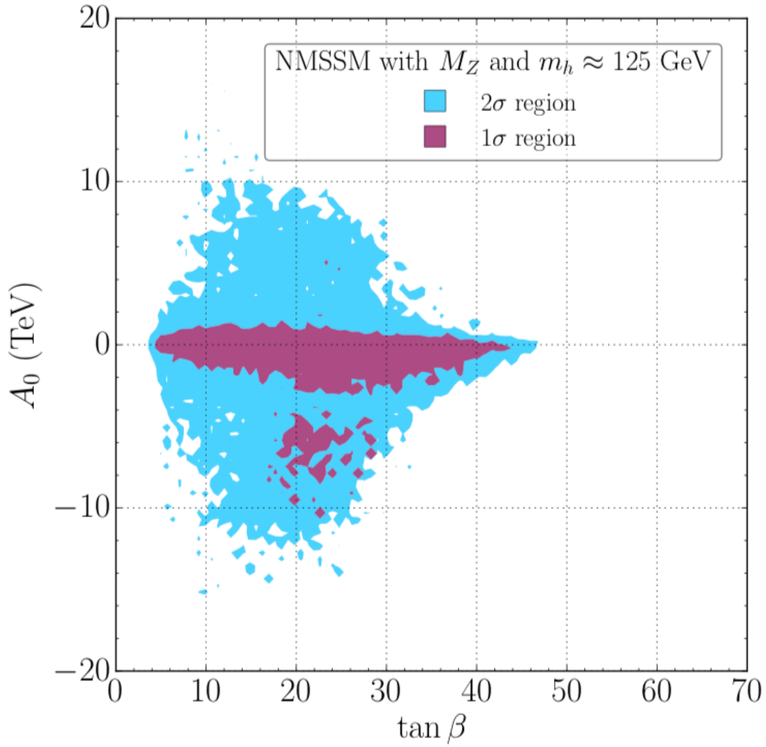
(c) NMSSM with M_Z.

(d) NMSSM with M_Z and m_h.

Result



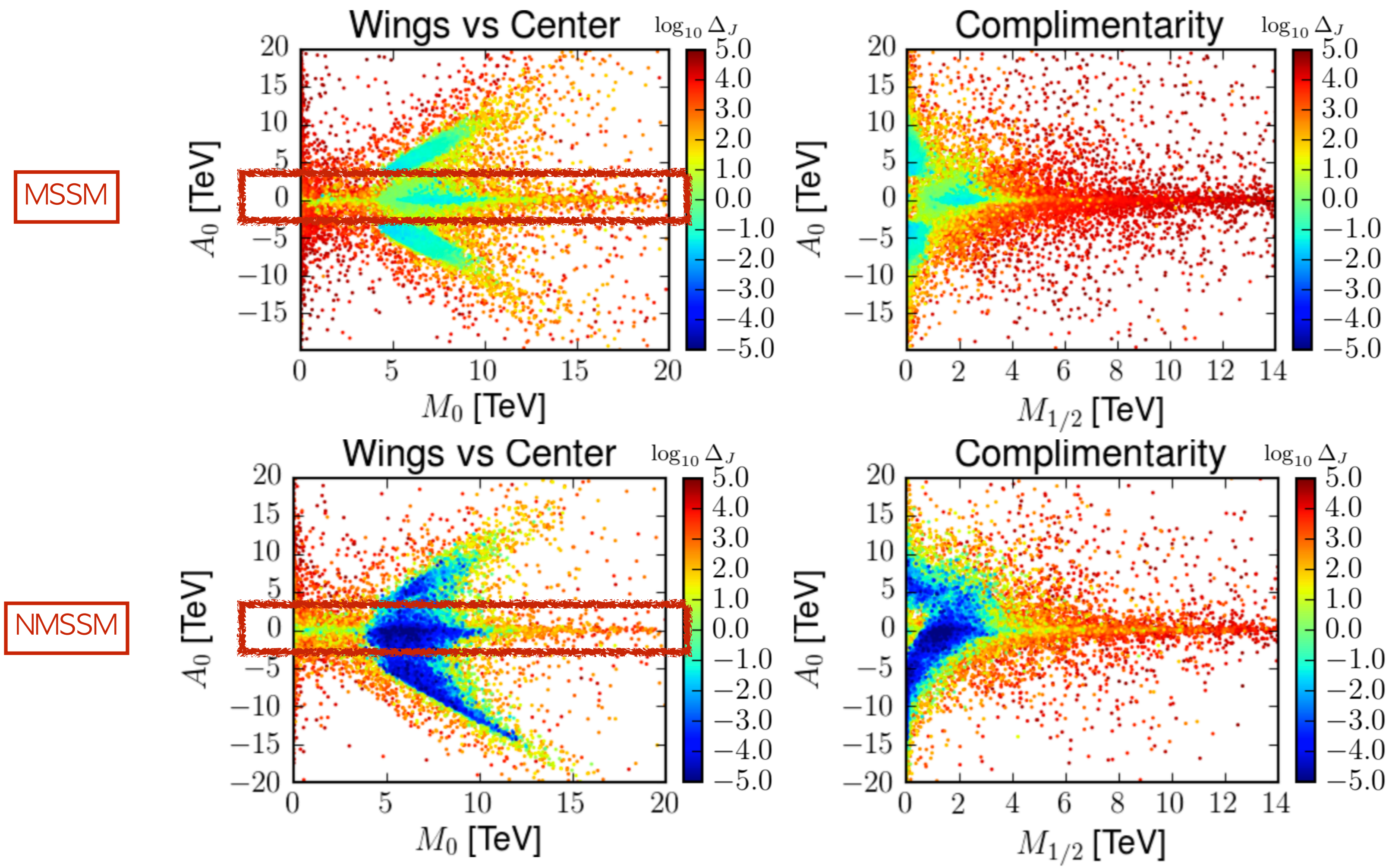
(a) CMSSM credible regions.



(c) NMSSM credible regions.

Future prospect

- Strong Correlations are found among model parameters.



Summary

- Physical consideration to find natural model parameters such as the Focus Point scenario, can be understood in terms of the Fine-tuning measure which is developed independently.

- Generalized fine-tuning measure is well accommodated with Bayesian approach through the effective prior probability.

- Two categories of benchmark points are well separated physically, and are interesting to study further.