Vector Resonances and SIMPs

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The WIMP Miracle

Evidence for **Dark Matter** is robust: charge neutral, gravitational interacting, massive particle (not in SM).
Leading paradigm is the **W**eakly **I**nteracting **M**assive **P**article.

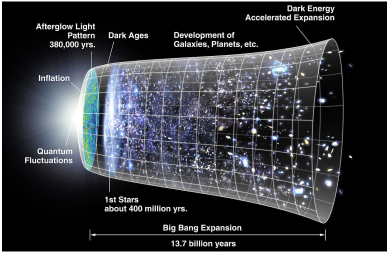


Image credit: Wikipedia

$\Gamma \gg H \rightarrow$ local equilibrium; $\Gamma \approx H \rightarrow$ decoupling
$\Gamma =$ interaction rate, $H =$ rate of expansion

$\Omega h^2 \sim 10^{-1} \rightarrow \sigma \sim G_F$

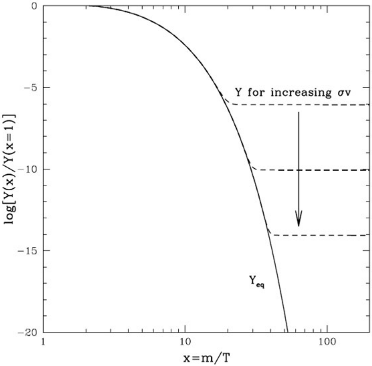
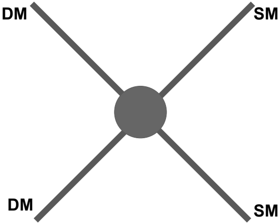


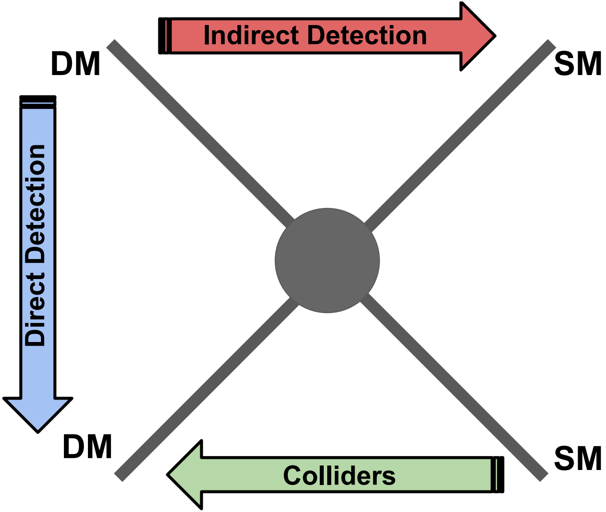
Image credit: PDG



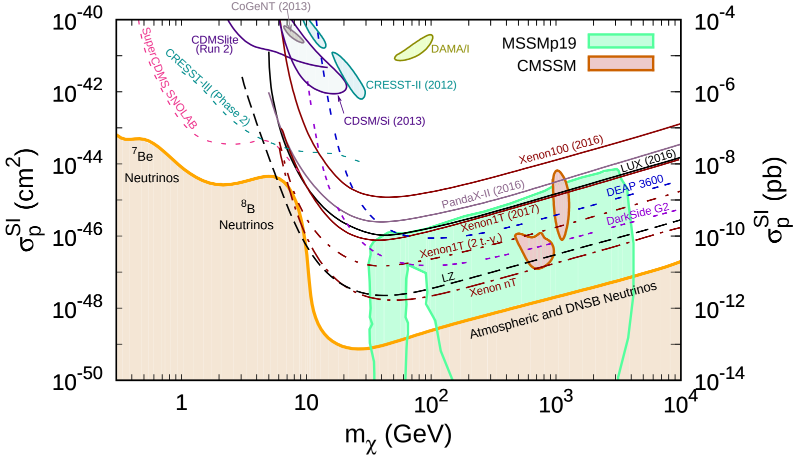
The WIMP Miracle:
simple thermal mech. predicts DM Relic with $m \sim 1$ TeV and 'weak' interactions.

$g^4/m^2 \sim$ 'weak scale'.

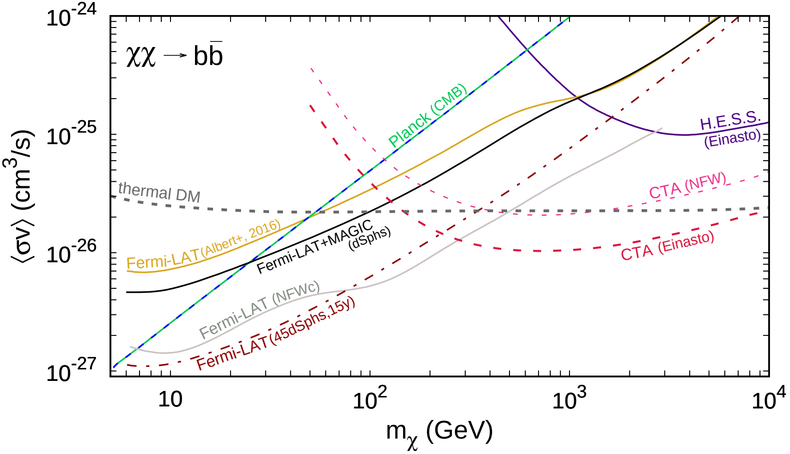
Annihilation leads to possible ways of detection:



Direct Detection



L. Roszkowski, E. M Sessolo, S. Trojanowski 2017



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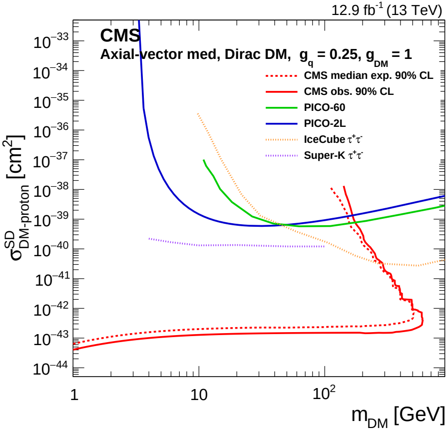
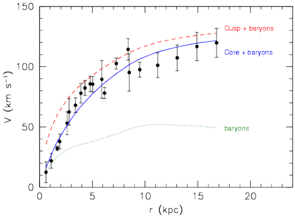


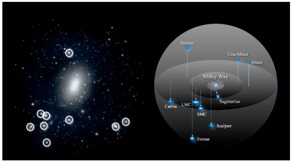
L. Roszkowski, E. M Sessolo, S. Trojanowski 2017

WIMPy Puzzles

Weak mass scale roughly fits correct relic density with mass $\sim TeV$, hence, the 'WIMP' miracle, however (D.H. Weinberg et al, 2013)...

- WIMP simulations too 'cuspy' compared to observations
- Missing halos/too-big-too-fail
- Diversity problem



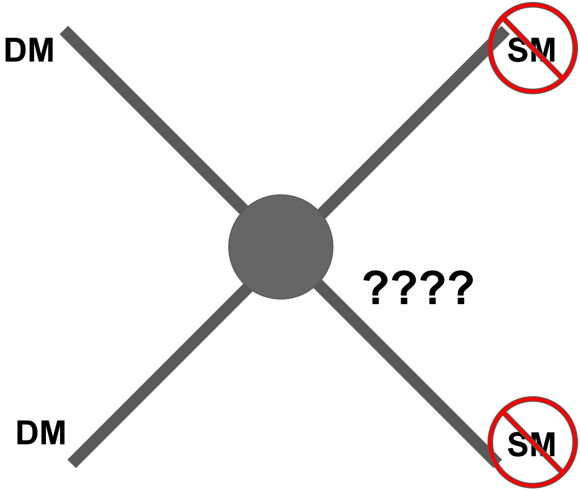


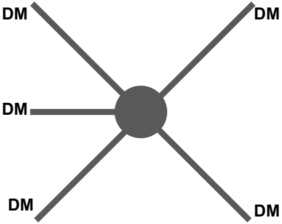
Introducing self interaction flattens DM density in Core*.
→ see S. Tulin,H.-B. Yu 2017 for a recent review ←

Constrained by Bullet Cluster observations $\sigma_{self} \lesssim 1\ cm^2/g$,

D.H. Weinberg et al, 2013

DM number density reduced by $\sigma_{ann.}$ for WIMPs, what if $\sigma_{ann.}$ to SM particles is effectively zero?





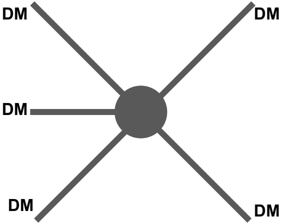
Freeze-out conditions become: $\Gamma \approx n_{DM}^2 \langle \sigma v^2 \rangle$, $\langle \sigma v^2 \rangle = \frac{\alpha^3}{m_{DM}^5}$

...then $m_{DM} \sim \alpha (T_{eq}^2 M_{pl})^{1/3} \sim 100$ MeV matches FO conditions and correct thermal relic with $\alpha \sim 4\pi$.

$\rightarrow$ **SIMP Miracle**

Stronlgy Interacting Massive Particle

Y. Hochberg, E. Kuflik, T. Volansky, J.G. Wacker, 2014



While the 3-to-2 sets the Relic, there should also be 2-to-2 interactions, $\sigma_{2to2} \approx \frac{\alpha^2}{m_{DM}^2} \approx 1\ cm^2/g$ is consistent with the SIMP Miracle.

The SIMP paradigm:

Strongly coupled, sub-GeV, model with 3-to-2 annihilation, and $\sigma_{scatter}/m_{DM} \approx 1\ cm^2/g$ for a rough upper-bound.

SIMPlest Model

Start with a strongly-coupled gauge theory ($SU(N_c)$):

$$\mathcal{L}_{SIMP} = -1/4 F^a_{\mu\nu} F^{\mu\nu a} + i\bar{Q}_i \not{D} Q_i$$

Has a global symmetry ($\mathcal{G}$) $SU(N_f)_L \times SU(N_f)_R$

Massive $Q's$ condense to composite states breaking $\mathcal{G}$ to $SU(N_f)_V$ (similar to SM QCD)

Note: The Gauge and global symmetries can be $SU(N_c)$, $Sp(N_c)$, etc.

Y. Hochberg, E. Kuflik, T. Volansky, J.G. Wacker, 2014

When the chiral symmetry is broken there are $N_f^2 - 1$ pNGB (**dark pions** are the DM in the SIMPLest model):

$$\Sigma = e^{i2\pi/f_\pi} \text{ and } \pi = T_a \pi_a$$

With interactions:

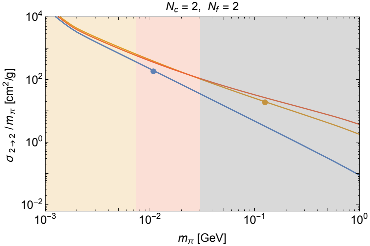
$$\mathcal{L}_\pi = \frac{f_\pi^2}{16} Tr \partial_\mu \Sigma \partial^\mu \Sigma^\dagger - 1/2 m_Q \mu^3 Tr \Sigma + c.c.,$$

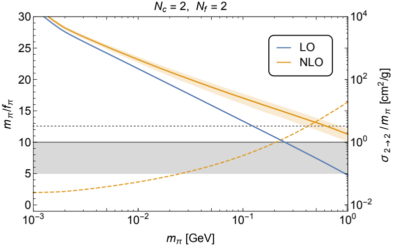
Expanding Σ leads to even-pion interaction terms (no 3-to-2 yet).

The 3-to-2 terms are generated from Wess-Zumino-Witten Term (Wess,Zumino 1971; Witten 1983):

$$\mathcal{L}_{WZW} = \frac{2N_c}{15\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} Tr \left[\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi \right]$$

Pion-only SIMP models generically require large m_π/f_π to be realistic ($m_\pi/f_\pi > 2\pi$ for $N_c, N_f = 3$); considerable NLO/NNLO corrections:





M. Hansen et al 2015

When LO breaks down in ChPT vector mesons should be included as well.

We expand the leading order ChPT with the vector meson $(N_f, N_c = 3)$:

$$\mathcal{L} = \mathcal{L} + m_V^2 Tr[V_\mu V^\mu] - i2g_{V\pi\pi} Tr[V_\mu[\partial^\mu \pi, \pi]] + \mathcal{L}_{Anom} + \ldots$$

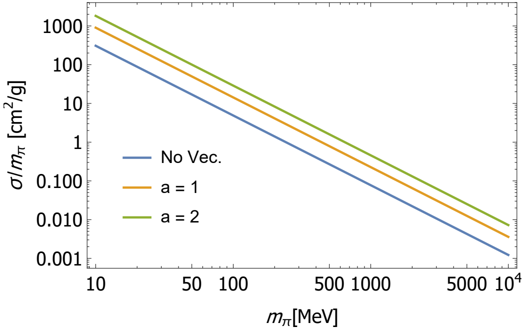
where $V_\mu = T^a V_\mu^a$, $m_V^2 = ag^2 f_\pi^2$, $g_{V\pi\pi} = 1/2ag$, and $\mathcal{L}_{Anom}$ are the anomalous terms containing $V\pi\pi\pi$ interactions.

$\rightarrow$ Vector mesons as gauge fields of a local unbroken $SU(3)_V$.

Modified Self-Scattering

The $g_{V\pi\pi}$ terms modify the self-interaction ($m_V \approx 3m_\pi$):

$$\sigma_{2-to-2} = \frac{(81a^4g^4f_\pi^4 + 216a^2g^2f_\pi^2m_V^2 + 154m_V^4)m_\pi^2}{48\pi f_\pi^4 N_\pi^2 m_V^4}$$



Vector Chiral Anomaly

The inclusion of the vector mesons also generates a new chiral anomaly:

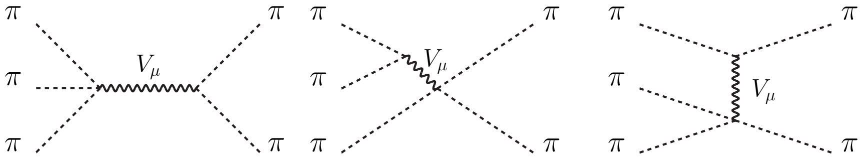
$$\mathcal{L}_1 = \frac{i15N_c c_1}{240\pi^2}\epsilon_{\mu\nu\rho\sigma}Tr[\alpha_L^{3\,\mu\nu\rho}\alpha_R^\sigma - \alpha_R^{3\,\mu\nu\rho}\alpha_L^\sigma]$$

$$\mathcal{L}_2 = \frac{i15N_c c_2}{240\pi^2}\epsilon_{\mu\nu\rho\sigma}Tr[\alpha_L^\mu\alpha_R^\nu\alpha_L^\rho\alpha_R^\sigma],$$

where $\alpha_L^\mu = (\alpha_R^\mu)^\dagger = (\partial_\mu e^{i\pi/f_\pi})e^{-i\pi/f_\pi} - igV_\mu$

c_1 and c_2 are 'free parameters', though also govern coupling of vector mesons to gauge bosons, for simplicity we consider the QCD-like case of $c_1 - c_2 = -1$

$\mathcal{L}_{1,2}$ with $\mathcal{L}_{g_{V\pi\pi}}$ generate new 3-to-2 interactions:



Potential resonances near $m_V \approx 3m_\pi$ $(m_V \approx 2m_\pi)$

Near these resonance poles, care must be taken with the thermal average.

Narrow Width

Due to the degeneracy assumptions, the $V \to 3\pi$ decays are suppressed near the resonances ($m_V \sim 3\pi$ has PS suppression, $m_V \sim 2\pi$ kinematic):

$$\Gamma / m_V \propto a^2 g^2$$

$$m_V \propto a g^2$$

So possible to consistently have $\Gamma / m_V \ll 1$ and m_V fixed if $a \ll 1$.

In more QCD-like cases ($a = 2, 1$) then the width is large.

Thermal Average for $m_V \approx 3m_\pi$

$g = \frac{2m_V}{\sqrt{a}f_\pi}$, and $m_V = 3m_\pi\sqrt{1+\epsilon_R}$, the Thermal Average yields:

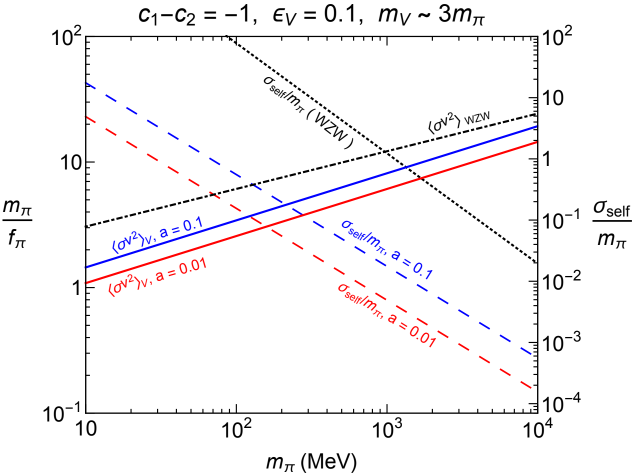
$$\langle \sigma v^2 \rangle \approx \frac{81\pi}{128} C_V \epsilon_R^4 x^3 e^{-3/2\epsilon_R x},$$

where the 3π resonance channel's C_V is:

$$C_V = \frac{225\sqrt{5}(c_1-c_2)^2 a^2 g^4 N_c^2 m_\pi^5}{256\pi^4 f_\pi^6 N_\pi^3 (9m_\pi^2 - m_V^2)^2}$$

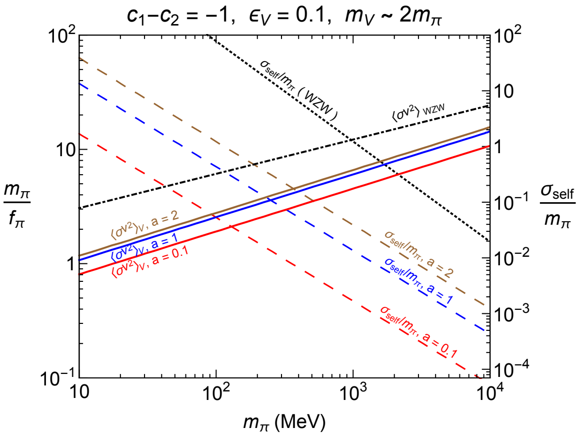
Similar equations exist for $m_V \approx 2m_\pi$.

In this case NWA is valid for vector mesons, $\sigma_{2-to-2,Vec.} \propto h(a^2 g^2)$, but $C_V \gg C_{WZW}$ as $m_V^2 \propto a g^2$ remains fixed (Vec. meson contribution dominates WZW to $\sigma_{3\pi\to 2\pi}$).


$c_1-c_2 = -1,\ \epsilon_V = 0.1,\ m_V \sim 3m_\pi$
10^2
10
1
10^{-1}
$\frac{m_\pi}{f_\pi}$
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\frac{\sigma_{self}}{m_\pi}$
10
10^2
10^3
10^4
m_π (MeV)
σ_{self}/m_π (WZW)
$\langle\sigma v^2\rangle$ WZW
$\langle\sigma v^2\rangle_V$, a = 0.1
$\langle\sigma v^2\rangle_V$, a = 0.01
σ_{self}/m_π, a = 0.1
σ_{self}/m_π, a = 0.01

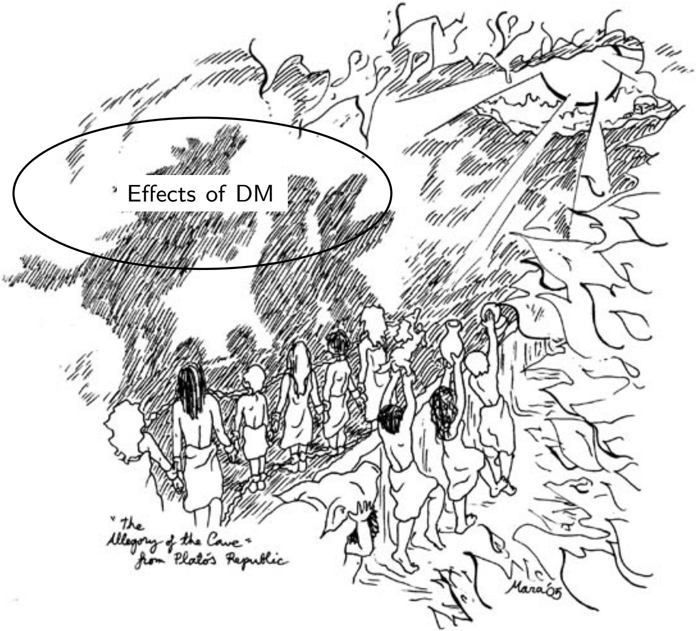
Results for $m_V \approx 2m_\pi$

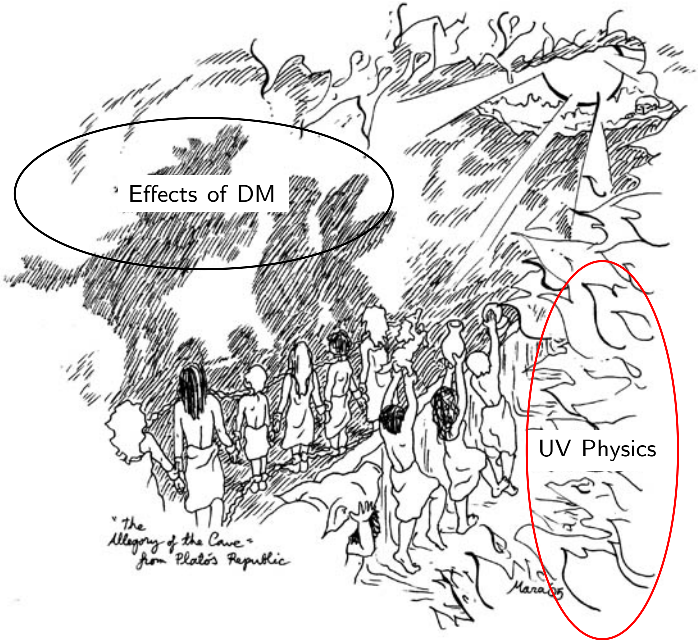
In the case of the $m_V \approx 2m_\pi$, NWA can be used even if a becomes larger:

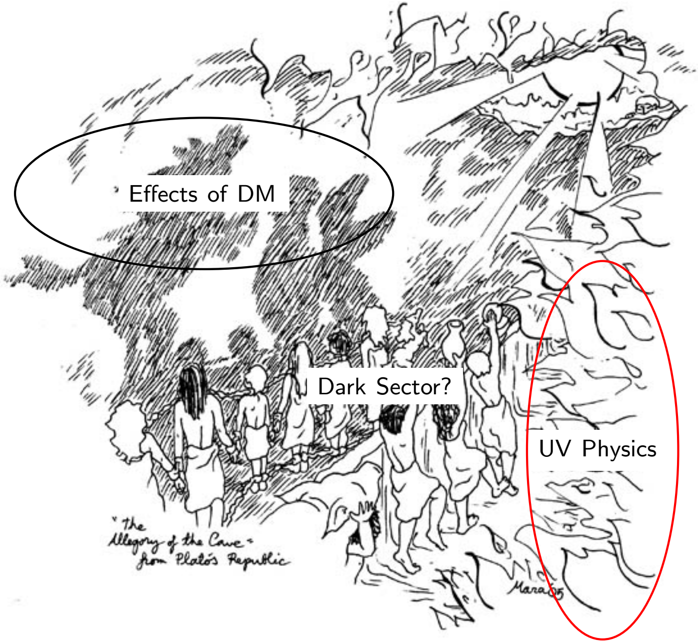



"The Allegory of the Cave" from Plato's Republic
Mara '05

Effects of DM




Effects of DM
UV Physics
"The Allegory of the Cave" from Plato's Republic
Mara 05


Effects of DM
Dark Sector?
UV Physics
"The Allegory of the Cave" from Plato's Republic
Mara 05

Summary & Conclusions

- Interesting alternative to WIMP paradigm with 3-to-2 freeze-out
- Addition of Vector Mesons in a composite model have important consequences
- Vector resonances in 3-to-2 alleviate tension ($\sigma_{scat.}$ vs. $m_\pi / f_\pi < 2\pi$) within minimal QCD-like SIMP model

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Some future questions:

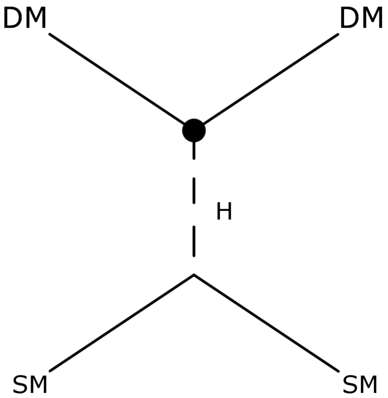
- Can the Kinetic Decoupling mechanism be self-contained?
- What are the details of Cosmological effects (SIDM)?

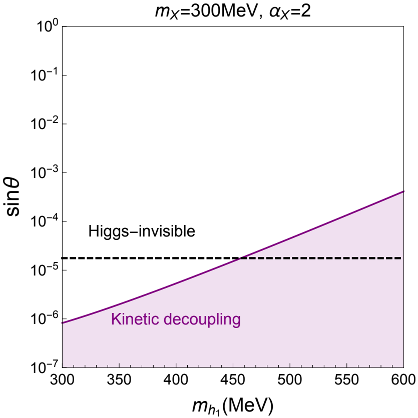
Thank you!

Thank you for your time and attention!

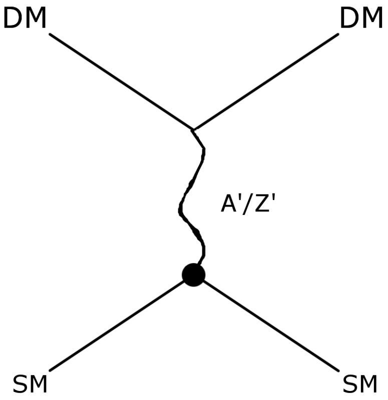
Backup Slides

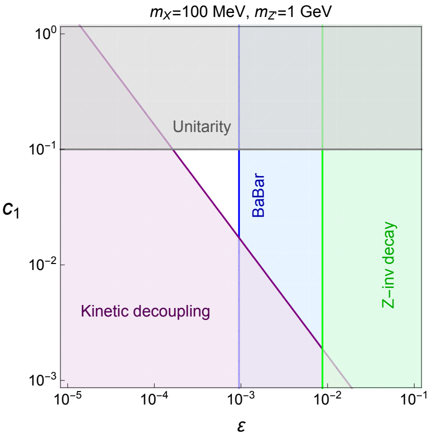
Higgs Portal





S.-M. Choi, Y. Hochberg, et al (2017)





S.-M. Choi, Y. Hochberg, et al (2017)

Boltzmann Equations for SU(3)

The Boltzmann Equations for the SIMP paradigm are:

$$\frac{dn_i}{dt} + 3n_i H = -\langle\sigma v^2\rangle_{ijk\to ml}(n_i n_j n_k - n_m n_l n_i^{eq}),$$

in the degenerate case $n_1 = n_2 = ... = n_{N_f^2-1} = n$ and $n_{DM} = \Sigma_i n_i = N_\pi n$. Summation of Boltzmann equations for each n_i:

$$\frac{n_{DM}}{dt} + 3n_{DM}H = -\langle\sigma_{3\pi\to 2\pi}v^2\rangle n_{DM}^2(n_{DM} - n_{DM}^{eq}),$$

Solving for m_π/f_π that yields $\Omega h^2 \approx 0.119$ yields a prediction for $\sigma_{scatt.}$, and valid parameter space should **satisfy two conditions**:

$m_\pi/f_\pi < 2\pi$ and $\sigma_{scatt.}/m_\pi < 1\ cm^2/g$

For $\phi_1\phi_2\phi_3 \to \phi_4\phi_5$:

$$\langle\sigma v^2\rangle = \frac{1}{n_1^{eq}n_2^{eq}n_3^{eq}}\frac{1}{s_i s_f}$$
$$\times \int d\Pi_1 d\Pi_2 d\Pi_3 d\Pi_4 d\Pi_5 f_1^{eq} f_2^{eq} f_3^{eq} (2\pi)^4 \delta^4(p) \mid \mathcal{M}_{3\to 2} \mid^2$$

v^2 is **not** the analog of two body case; we *explicitly expand* $\sigma_{3\to 2}$ in terms of $v_i's$ $(i = 1, 2, 3)$, with B-W velocity dependence, and integrate.

The results for the 3π **resonance** in NWA are $(x = m_\pi/T)$:

$$\frac{81\pi}{128} C_V \left(\frac{m_V^2 - 9m_\pi^2}{9m_\pi^2}\right)^2 x^3 e^{-3/2x\frac{(m_V^2-9m_\pi^2)}{9m_\pi^2}}$$

We found an error in arXiv:1702.07860 which has been corrected here.

And for 2π **resonance** in NWA:

$$6\sqrt{3}\pi C_V \left(\frac{m_V^2 - 4m_\pi^2}{4m_\pi^2}\right)^2 x e^{-2x\frac{(m_V^2-4m_\pi^2)}{4m_\pi^2}} I_2\left(x\frac{(m_V^2-4m_\pi^2)}{4m_\pi^2}\right),$$

where $I_2(y)$ is the modified Bessel function of the first kind.

Unitarity Constraint