Gravitational waves from phase transitions: an analytic approach

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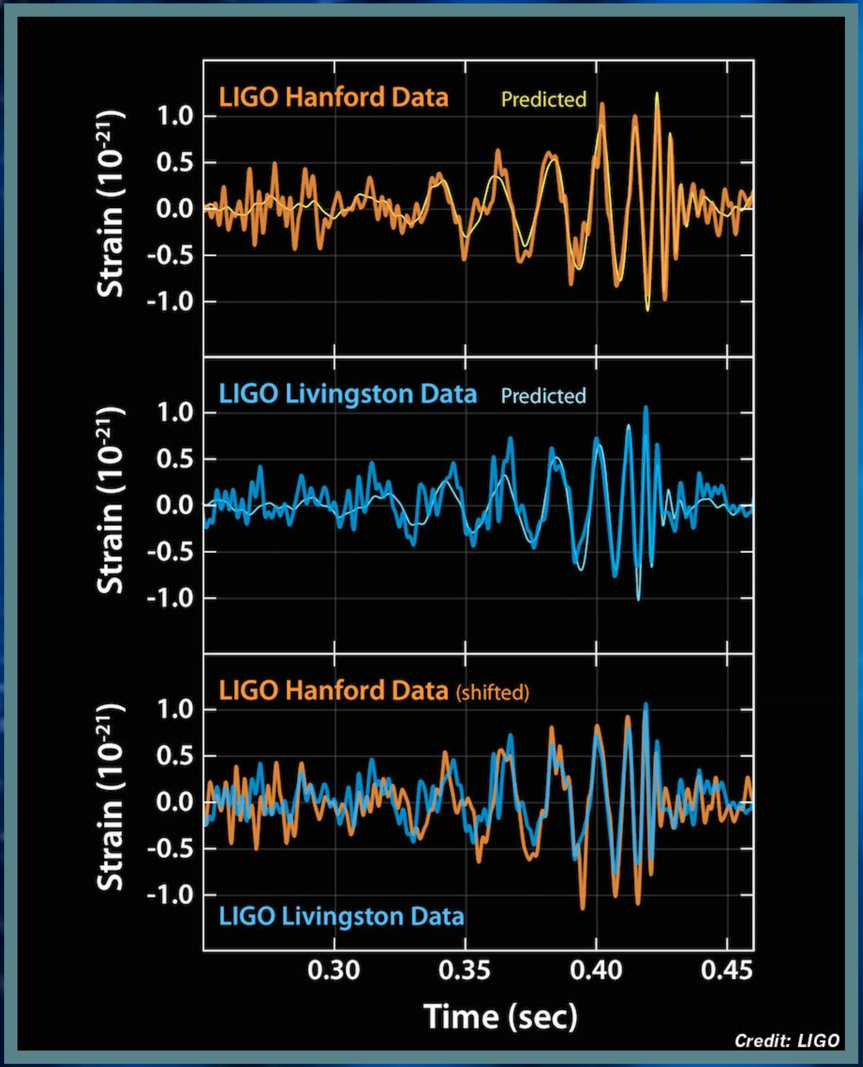
Based on arXiv:1605.01403 (PRD95, 024009) & 1707.03111
with Masahiro Takimoto (Weizmann Institute)
Jan. 8, 2018, High1 workshop

Introduction

ERA OF GRAVITATIONAL WAVES

- Detection of GWs from BH & NS binaries → **GW astronomy** has started



[LIGO]

- Black hole binary $36M_\odot + 29M_\odot \rightarrow 62M_\odot$

- Frequency ~ 35 to 250 Hz

- Significance > 5.1σ

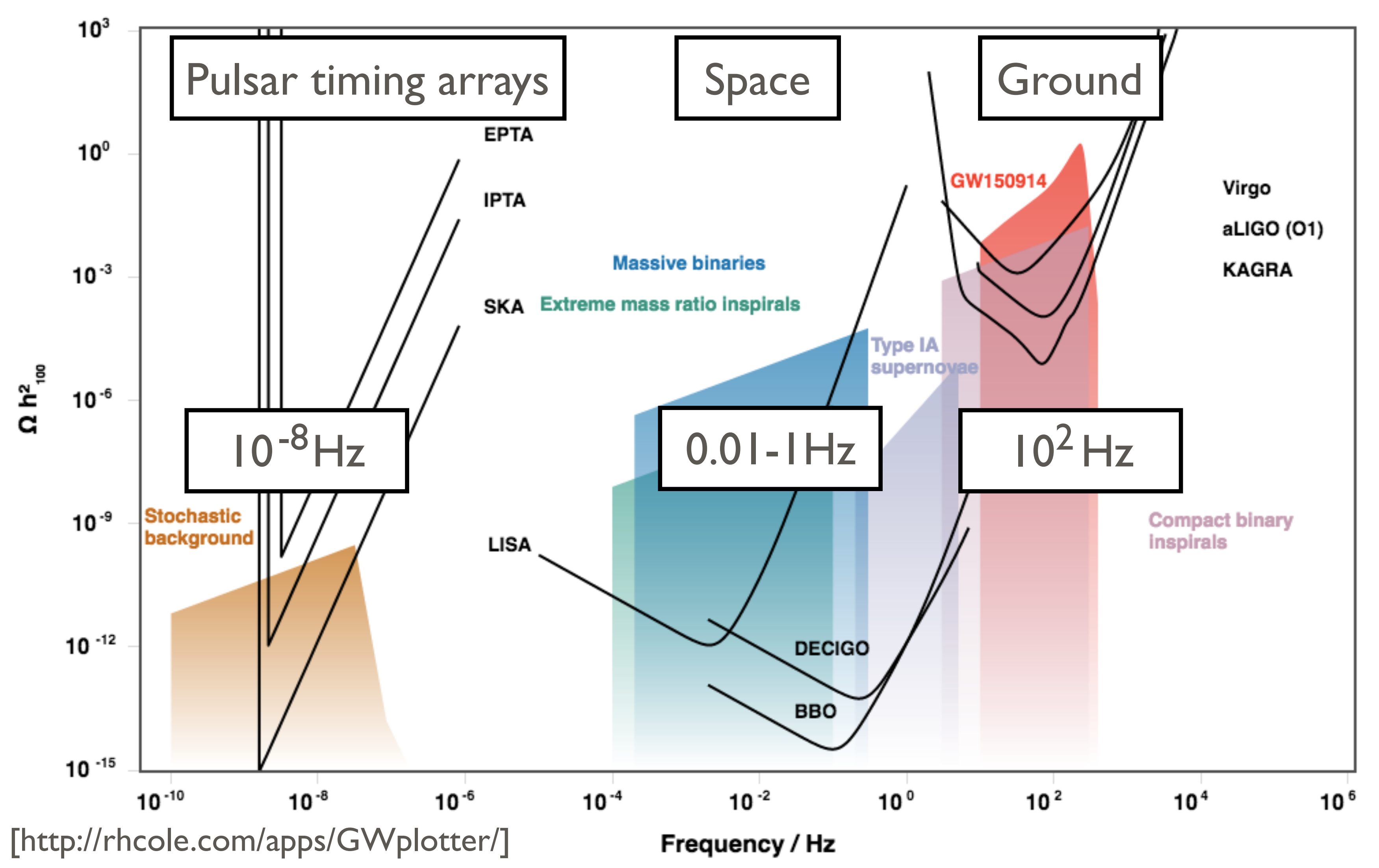
ERA OF GRAVITATIONAL WAVES

- Next will be **GW cosmology** with space interferometers

From Ando-san's talk @ JPS meeting 2014

ERA OF GRAVITATIONAL WAVES

- Sensitivity curves for current & future experiments

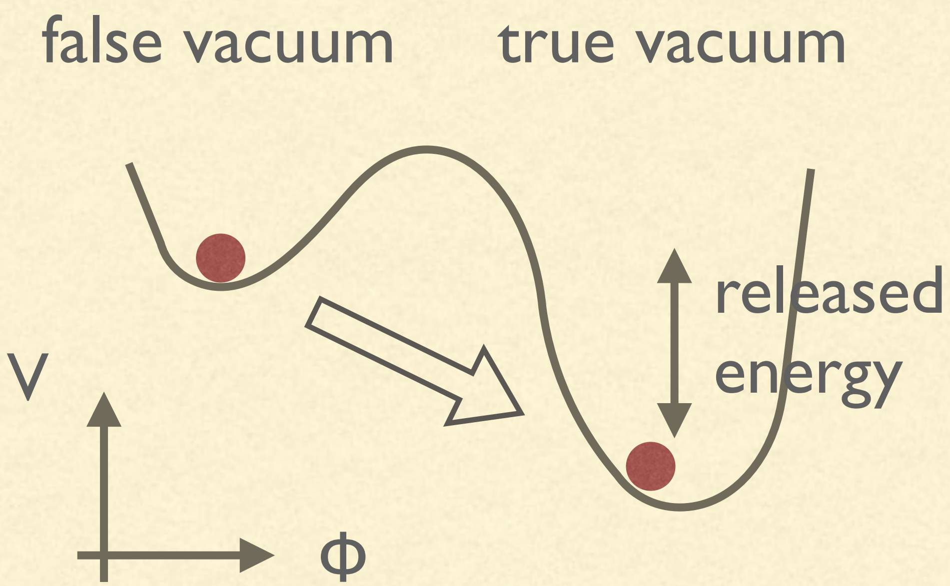


[http://rhcole.com/apps/GWplotter/]

ROUGH SKETCH OF PHASE TRANSITION & GW PRODUCTION

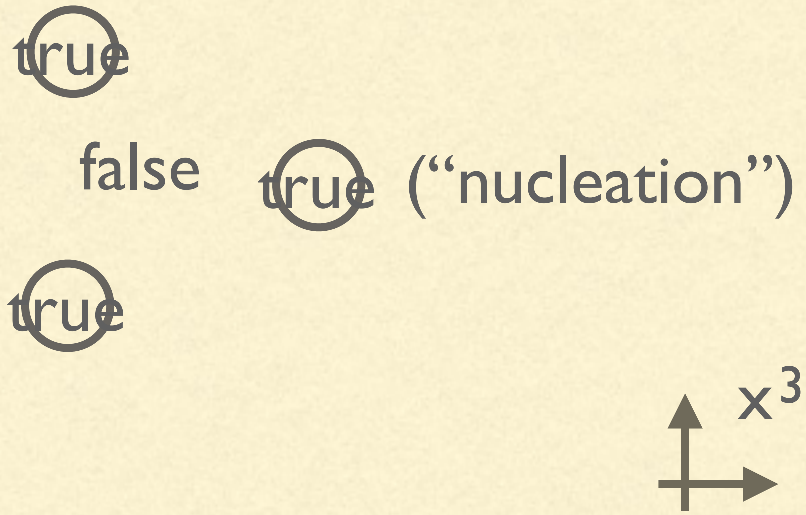
- How thermal first-order phase transition produces GWs

Field space



Quantum tunneling

Position space

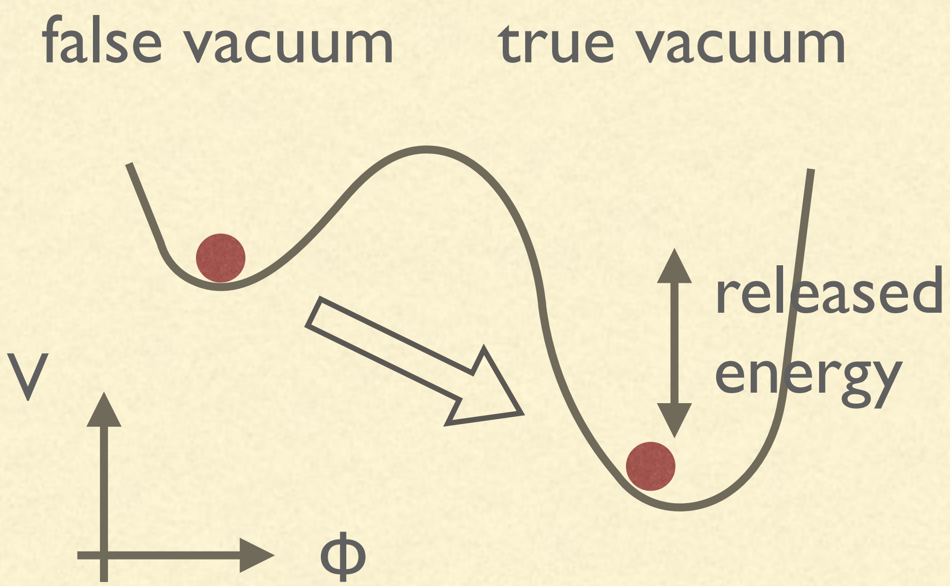


Bubble formation & GW production

ROUGH SKETCH OF PHASE TRANSITION & GW PRODUCTION

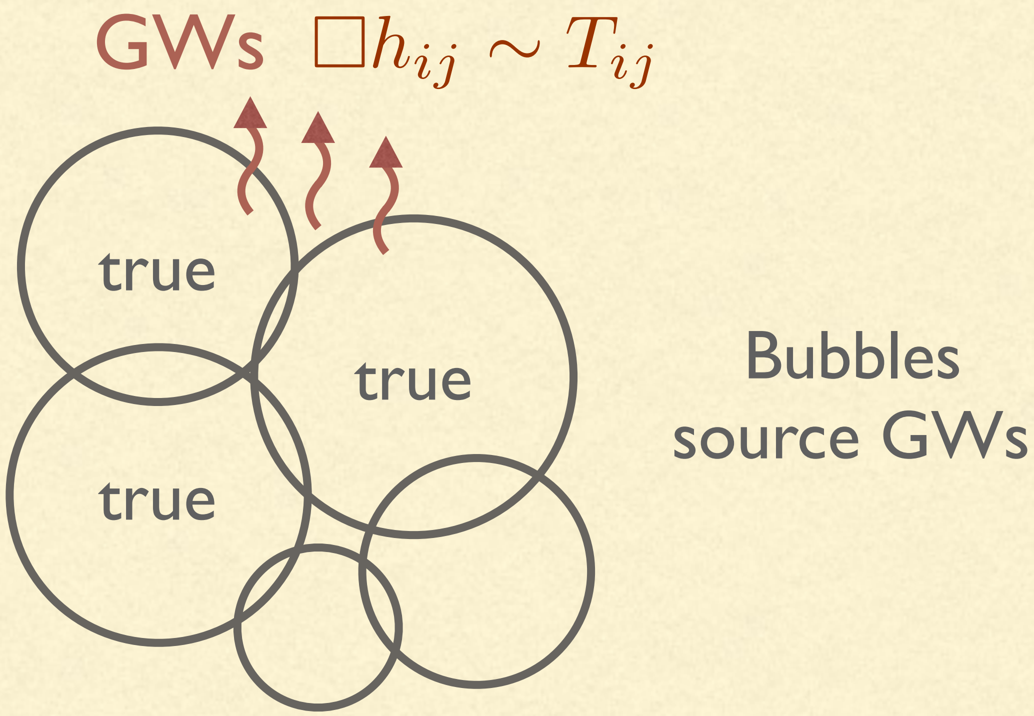
- How thermal first-order phase transition produces GWs

Field space



Quantum tunneling

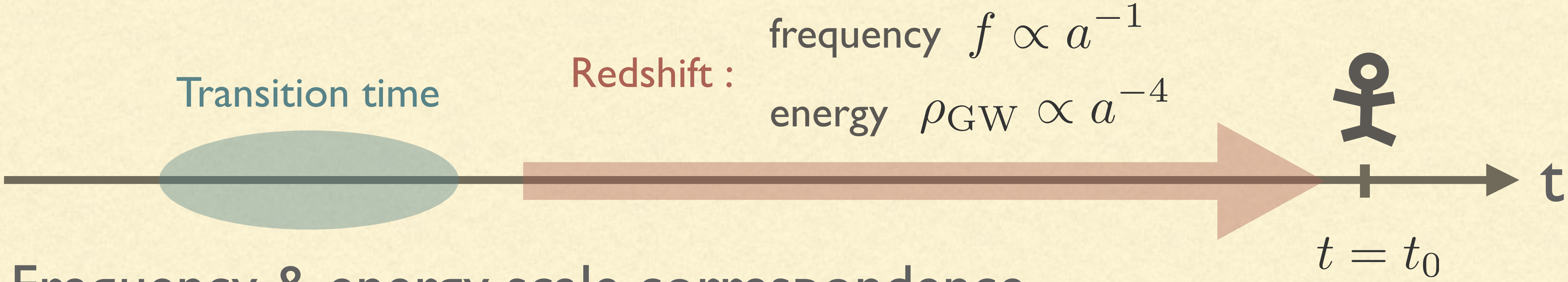
Position space



Bubble formation & GW production

ROUGH SKETCH OF PHASE TRANSITION & GW PRODUCTION

- GWs just redshift as non-interacting radiation after production



Transition time

Redshift :

frequency $f \propto a^{-1}$

energy $\rho_{\rm GW} \propto a^{-4}$

t

$t = t_0$

- Frequency & energy scale correspondence

$$\boxed{f_0 \sim 1\ \mathrm{Hz} \times \left(\frac{\beta}{H}\right)\left(\frac{T}{10^7\ \mathrm{GeV}}\right) \qquad \frac{\beta}{H} \sim \mathcal{O}(10^{1-4})}$$

f_0 : Present GW frequency

T : Temperature of the Universe @ transition time

0.1 - 1 Hz detectors sensitive to EW, TeV, PeV physics

TALK PLAN

✓ 0. Introduction

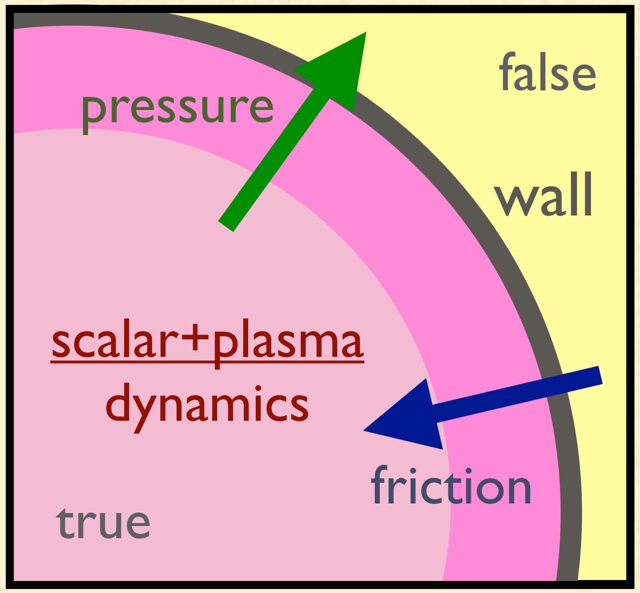
1. Bubble dynamics in first-order phase transitions

2. Analytic approach to GW production

3. Future prospects

BEHAVIOR OF BUBBLES

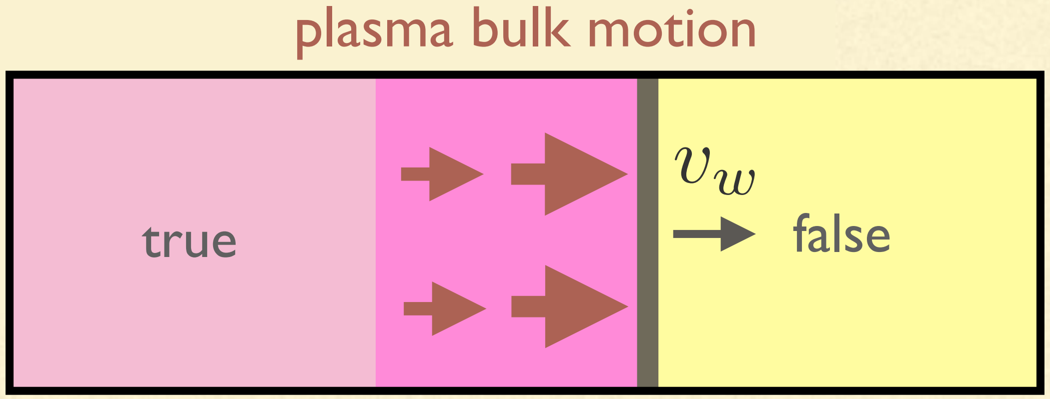
- Two main players : **scalar field & plasma** [e.g. Espinosa et al., JCAP06(2010)028]

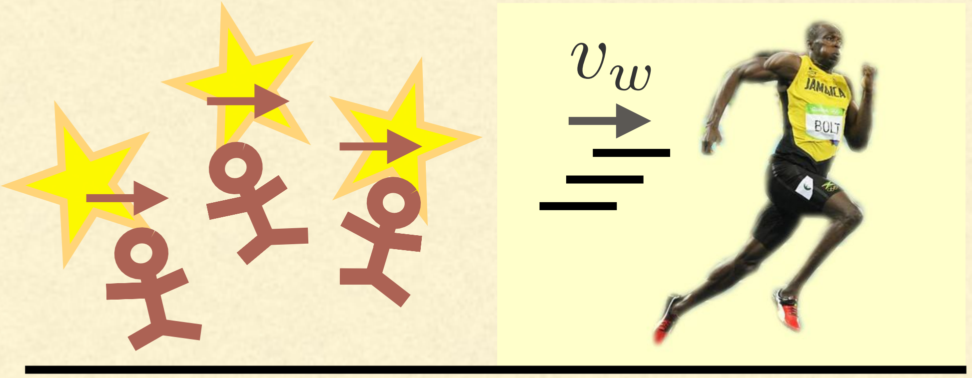


- Walls (where the scalar field value changes) want to expand ("pressure")

- Walls are pushed back by plasma ("friction")

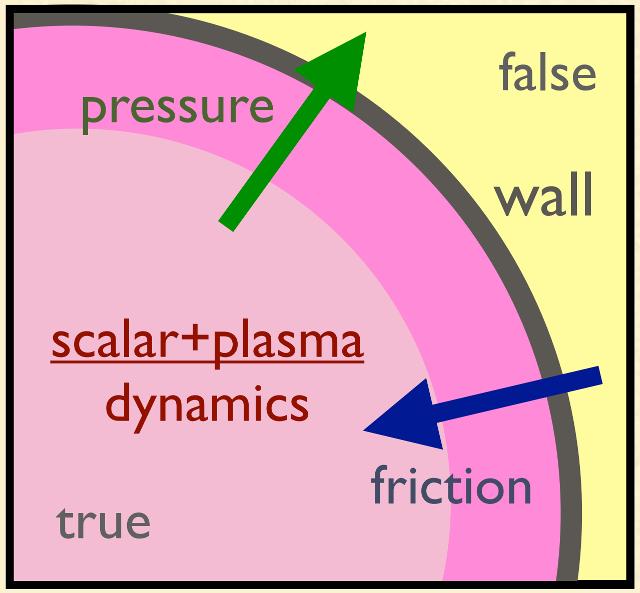
- Walls make thermal plasma motion : Case 1 $v_w \gtrsim 1/\sqrt{3}$ (Detonation)





BEHAVIOR OF BUBBLES

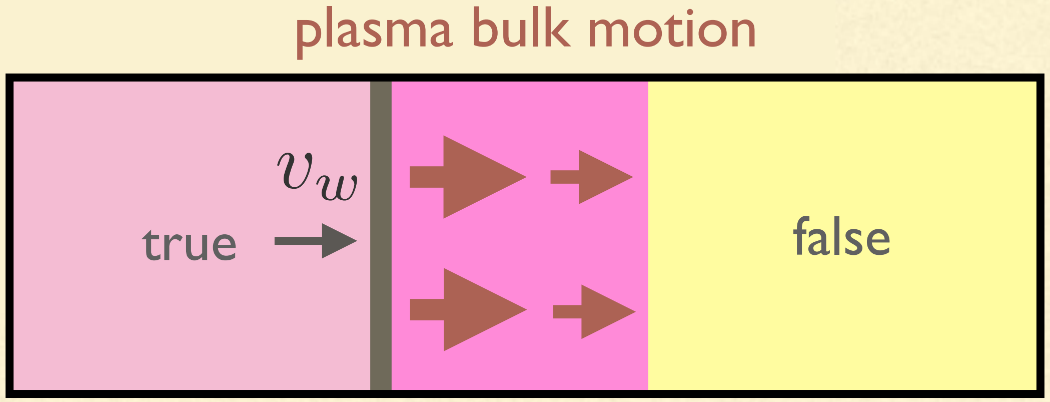
- Two main players : **scalar field & plasma** [e.g. Espinosa et al., JCAP06(2010)028]



- Walls (where the scalar field value changes) want to expand ("pressure")

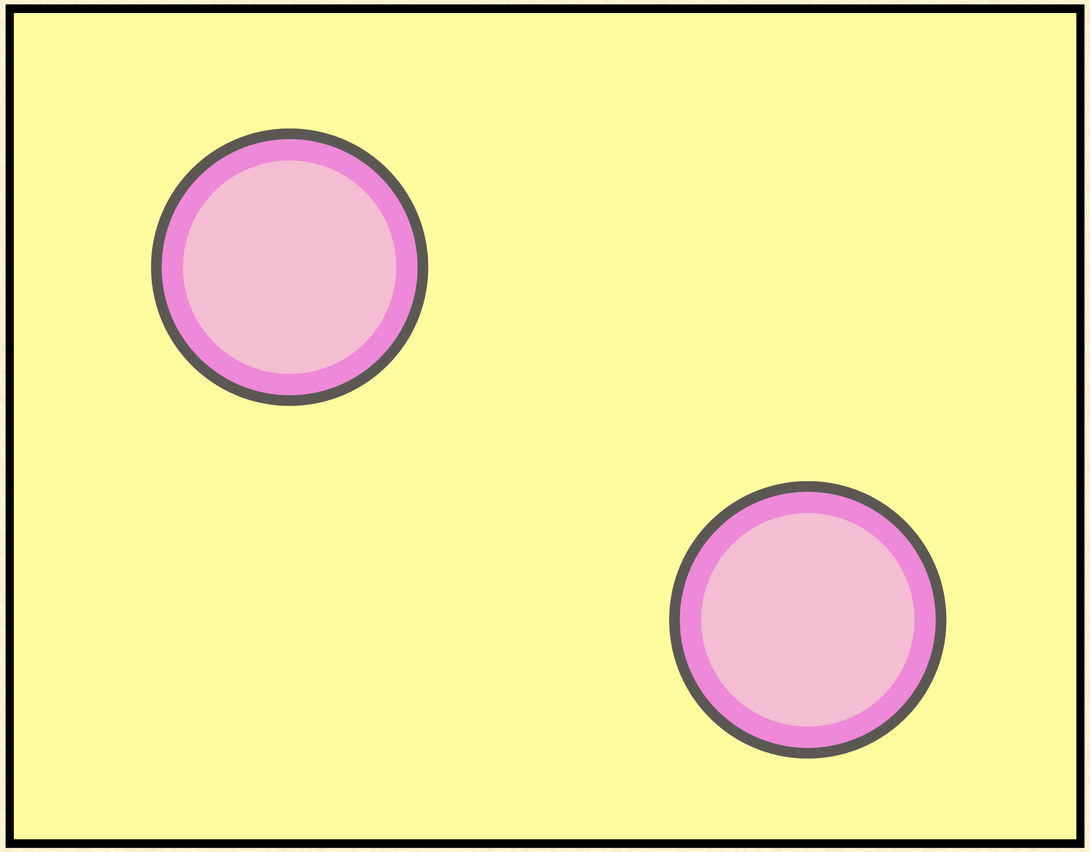
- Walls are pushed back by plasma ("friction")

- Walls make thermal plasma motion : Case 2 $v_w \lesssim 1/\sqrt{3}$ (Deflagration)



DYNAMICS AFTER COLLISION

Bubbles start to nucleate

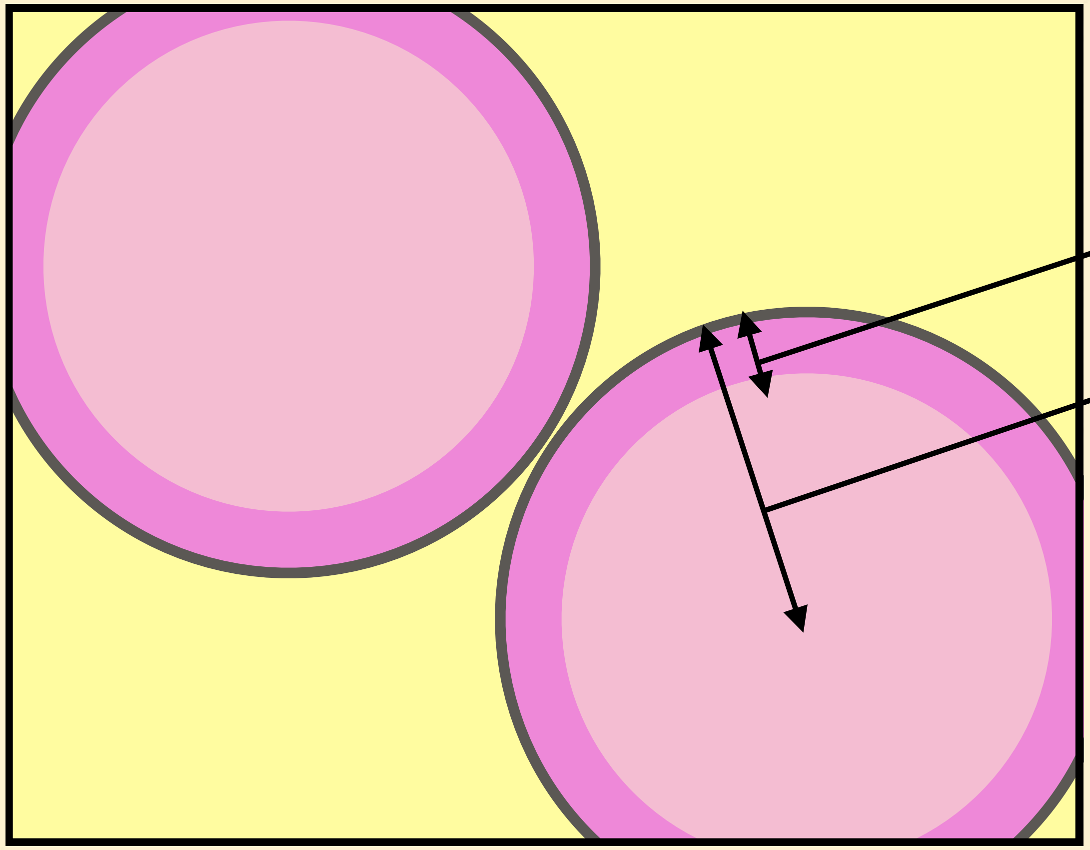


- Nucleation rate (per unit time & vol)

$$\Gamma(t) \propto e^{\beta t}$$

DYNAMICS AFTER COLLISION

Bubbles expand



- Bubbles expand, keeping

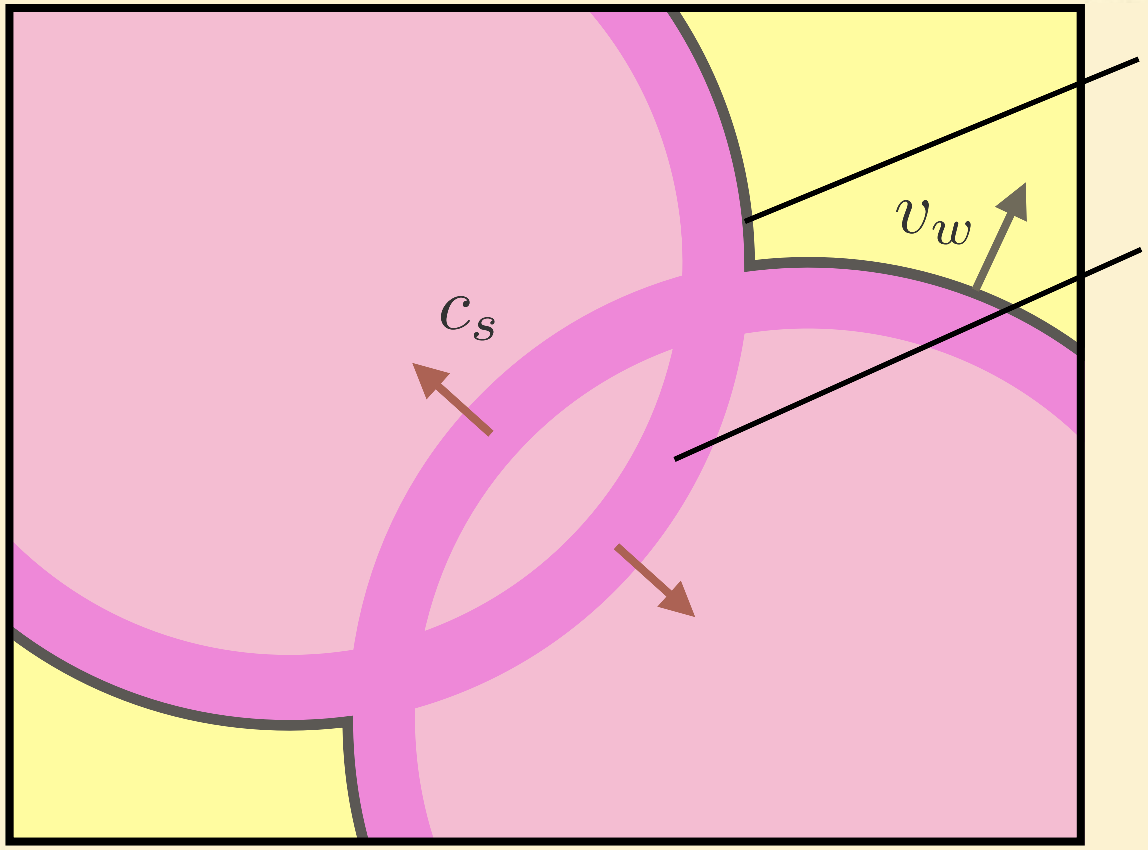
$$\frac{\text{(thickness of bulk motion)}}{\text{(bubble radius)}} = \text{const.} \left(\begin{matrix} \text{typically} \\ \sim 0.1 \end{matrix} \right)$$

- Bulk motion carries most of the released energy

- Typically collide $\Delta t \sim 1/\beta$ after nucleation

DYNAMICS AFTER COLLISION

Bubbles collide



- Scalar field damps soon after collision
- Plasma bulk motion continues to propagate

“sound waves”

$$\left(\partial_t^2 - c_s^2\nabla^2\right) u^i = 0$$

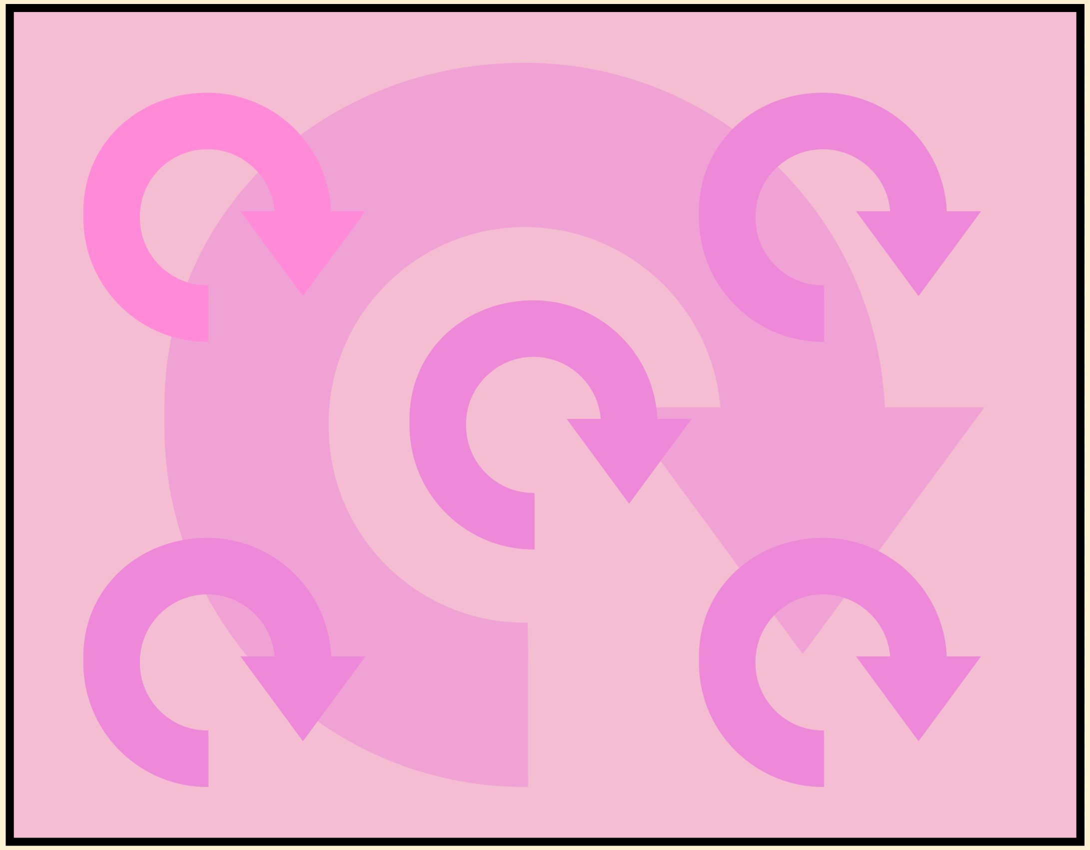
u^i : fluid velocity field

Note velocity changes from v_w to c_s

- Thickness of the bulk motion is fixed at the time of collision

DYNAMICS AFTER COLLISION

Turbulence develops



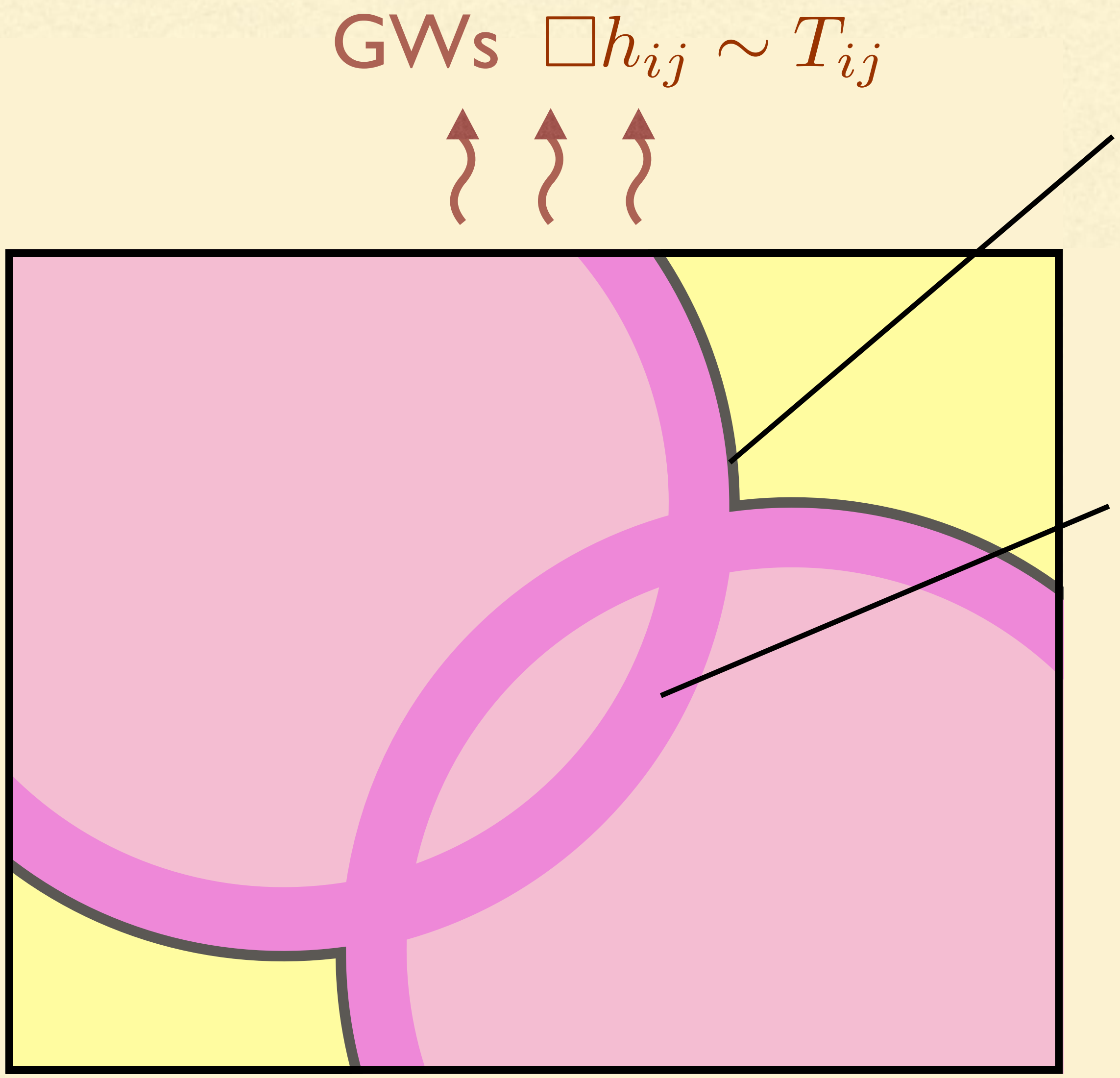
- Nonlinear effect appears at late times

“turbulence”

THREE SOURCES OF GWS

- Classification of GW sources

[e.g. Caprini et al., JCAP 1604(2016)]



1. Walls (energetically subdominant)

collide and damp soon

"bubble collision"

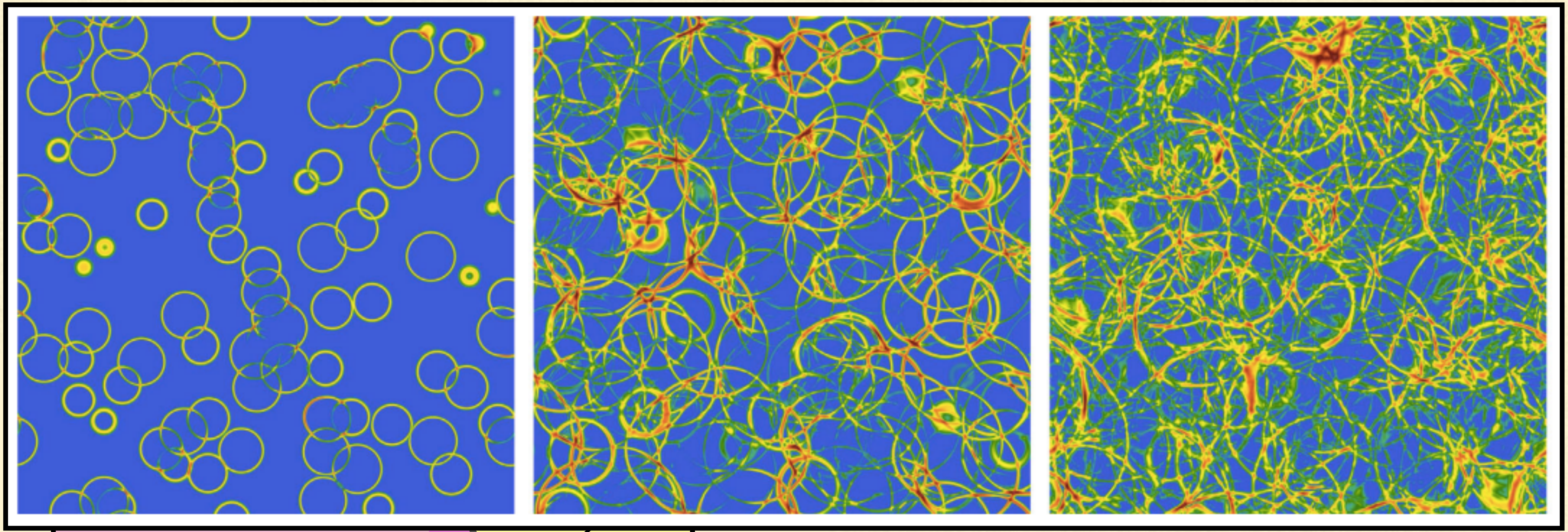
2. Plasma bulk motion continues to propagate

"sound waves"

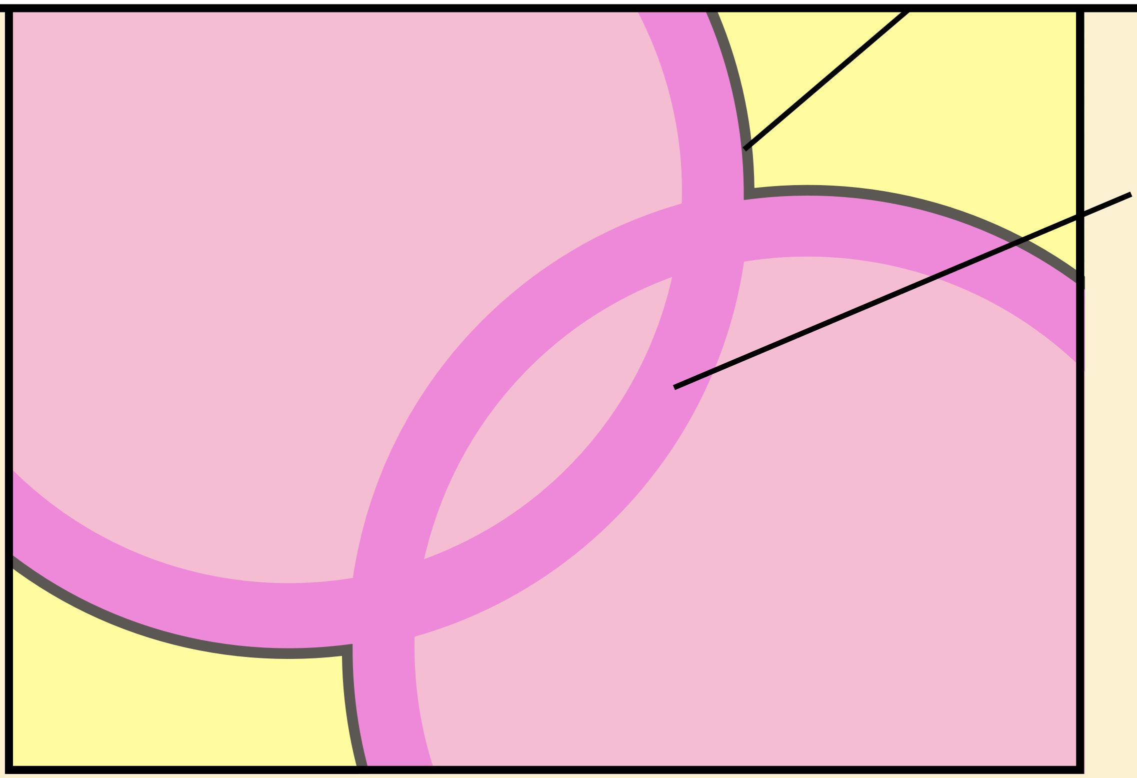
3. At late times,

sound waves develop into nonlinear regime

"turbulence"



“bubble collision” [Hindmarsh et al.‘15]



2. Plasma bulk motion continues to propagate

“sound waves”

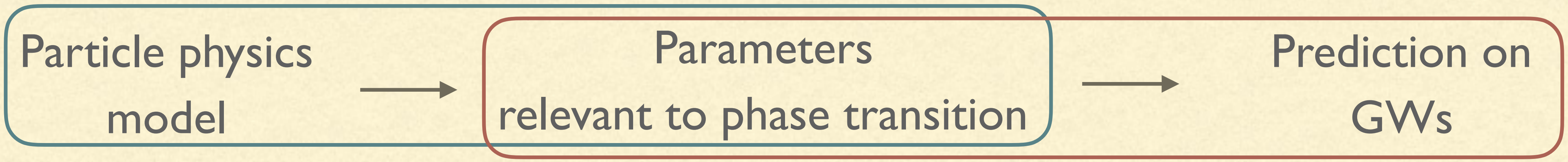
3. At late times,

sound waves develop into nonlinear regime

“turbulence”

NECESSITY OF ANALYTIC APPROACH

- What we do when we predict GWs in particle physics models



Particle physics model → Parameters relevant to phase transition → Prediction on GWs

e.g. - Transition temperature
- Nucleation rate ... and so on

- I want to make our understanding on ▭ "two-wheel":

Numerical understanding + Analytic understanding

- Why ? Imagine any successful field of physics e.g. CMB, lattice QCD, ...

TALK PLAN

✓0. Introduction

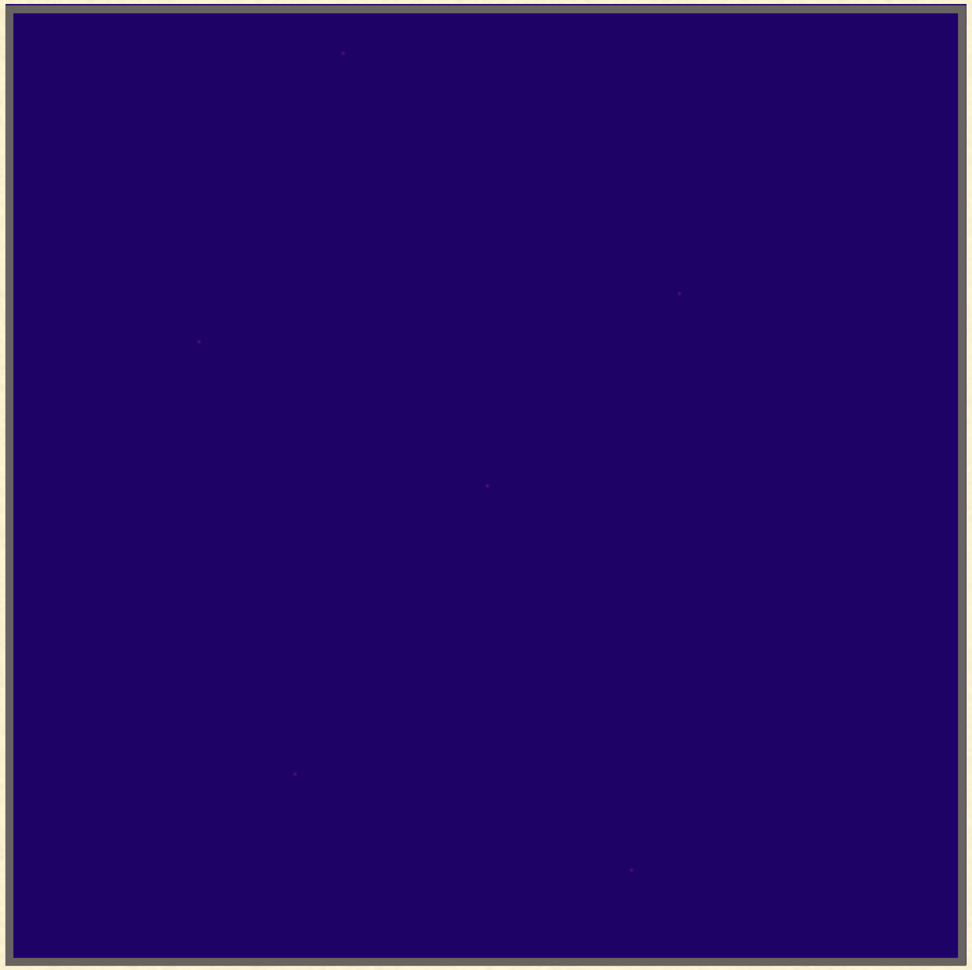
✓1. Bubble dynamics in first-order phase transitions

2. Analytic approach to GW production

3. Future prospects

THE SYSTEM WE WANT TO UNDERSTAND

- Let us solve the following system



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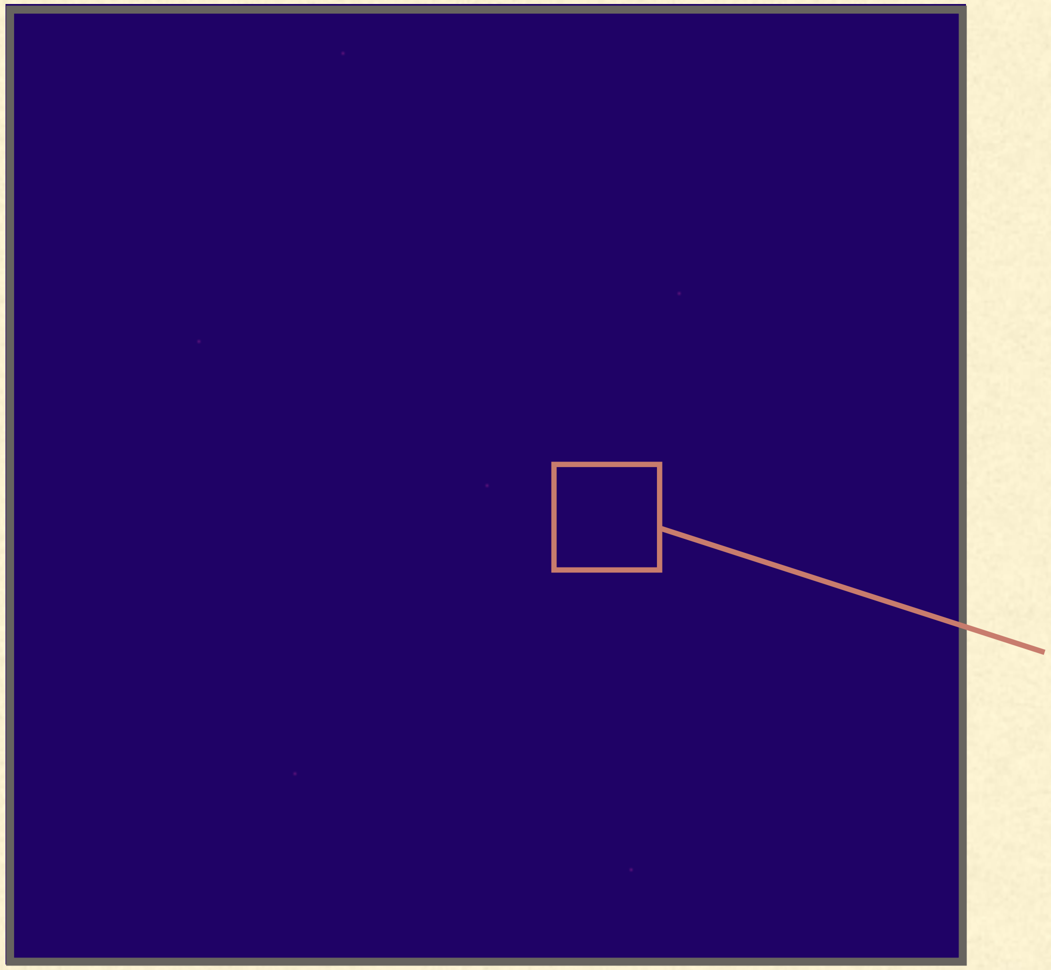


- Cosmic expansion neglected

- Bubbles nucleate with rate Γ

  (Typically $\Gamma \sim e^{\beta t}$ in thermal transitions)

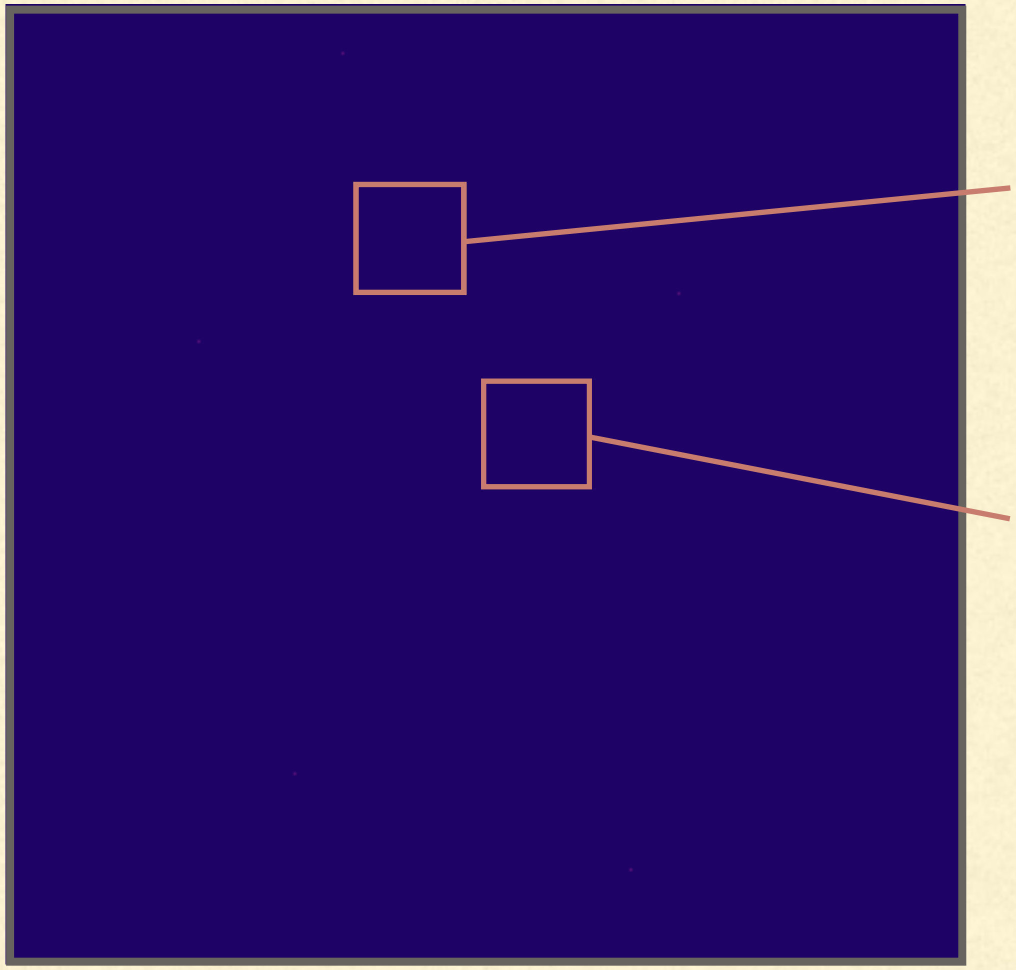
- Bubble shells

  (parametrizing both scalar & plasma bulk motion)

  are approximated to be thin

THE SYSTEM WE WANT TO UNDERSTAND

- Let us solve the following system



- Shells become more and more energetic

$$T_{ij} \propto \text{(bubble radius)}$$

- They lose energy & momentum after first collision

$$T_{ij} \propto T_{ij} \text{@ collision} \times \frac{\text{(bubble radius @ collision)}^2}{\text{(bubble radius)}^2}$$

× (arbitrary damping func. D)

THIS SYSTEM IS SOLVABLE

- We wrote down GW spectrum in this system analytically, essentially from causality

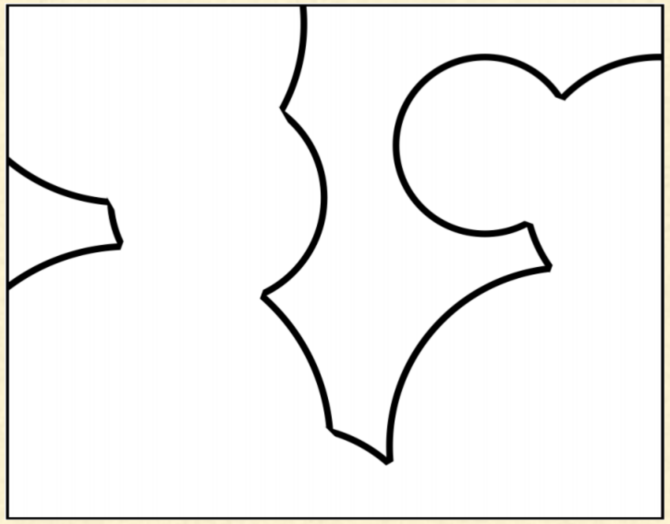
1707.03111:

After a short calculation

(1 year and half)

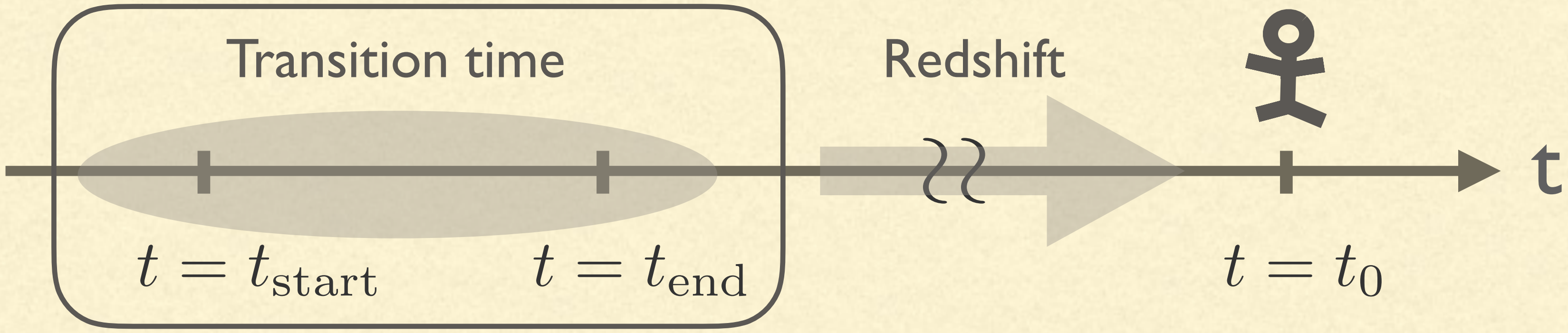
- Full derivation takes too long

  → we illustrate the derivation in a simplified setup : Envelope approximation



DEFINITION OF GW SPECTRUM

- Let's focus on transition time, since redshift after production is trivial



Transition time

Redshift

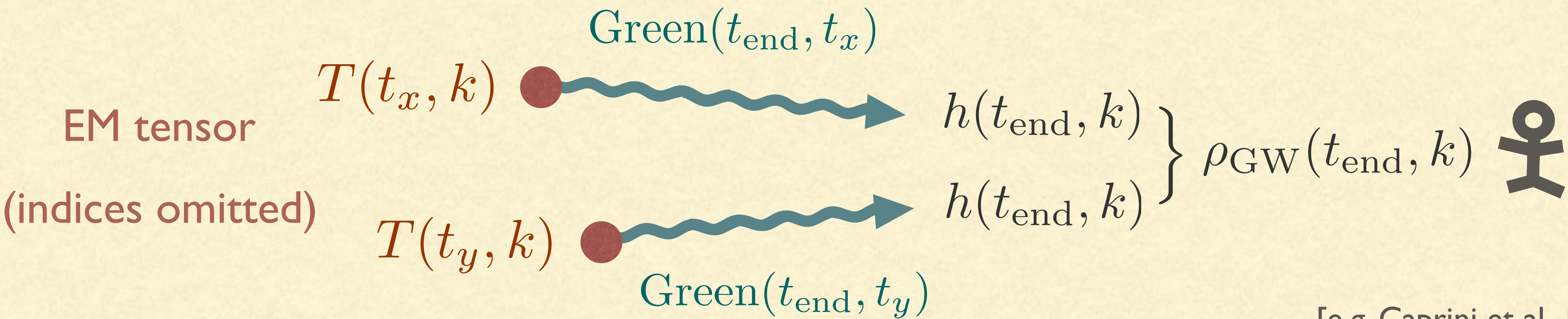
t

$t = t_{\rm start}$ $t = t_{\rm end}$

$t = t_0$

- What we want to know : $\rho_{\rm GW}(t_{\rm end}, k)$ = GW energy density per each wavenumber k

$$\rho_{\rm GW}(t_{\rm end}, k) \sim \langle h_{ij}(t_{\rm end}, k) h^*_{ij}(t_{\rm end}, k) \rangle \sim \int_{t_{\rm start}}^{t_{\rm end}} dt_x \int_{t_{\rm start}}^{t_{\rm end}} dt_y \, \cos(k(t_x - t_y)) \, {\rm F.T.} \left[\langle T_{ij}(t_x, \mathbf{x}) T_{ij}(t_y, \mathbf{y}) \rangle \right]$$



EM tensor

(indices omitted)

$T(t_x, k)$ $\xrightarrow{{\rm Green}(t_{\rm end}, t_x)}$ $h(t_{\rm end}, k)$

$T(t_y, k)$ $\xrightarrow{{\rm Green}(t_{\rm end}, t_y)}$ $h(t_{\rm end}, k)$

$\left.\begin{matrix} h(t_{\rm end}, k) \\ h(t_{\rm end}, k) \end{matrix}\right\} \rho_{\rm GW}(t_{\rm end}, k)$

[e.g. Caprini et al., PRD77 (2008)]

DEFINITION OF GW SPECTRUM

- Let’s focus on transition time, since redshift after production is trivial

- What we wa wavenumber k

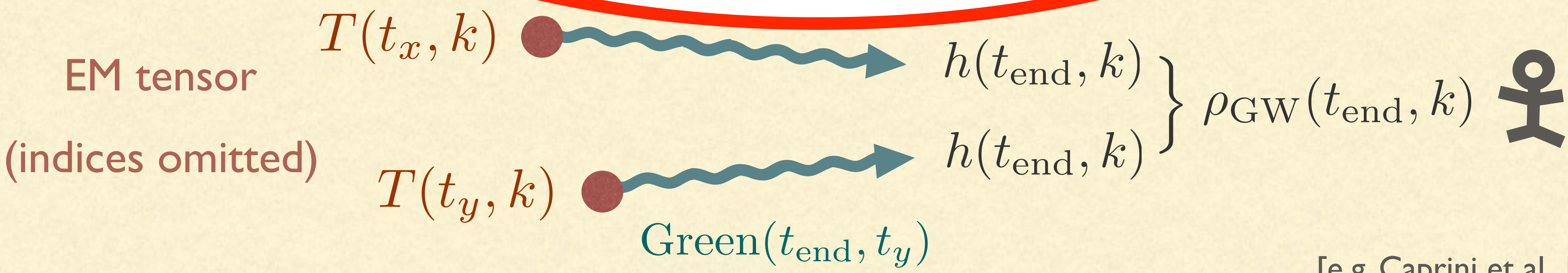
$\rho_{\rm GW}(t_{\rm end}, k) \sim \langle h_{ij}$ $_{ij}(t_x, \mathbf{x}) T_{ij}(t_y, \mathbf{y}) \rangle)]$

<u>Message</u>

GW spectrum is essentially

two-point ensemble average

< T T >



[e.g. Caprini et al., PRD77 (2008)]

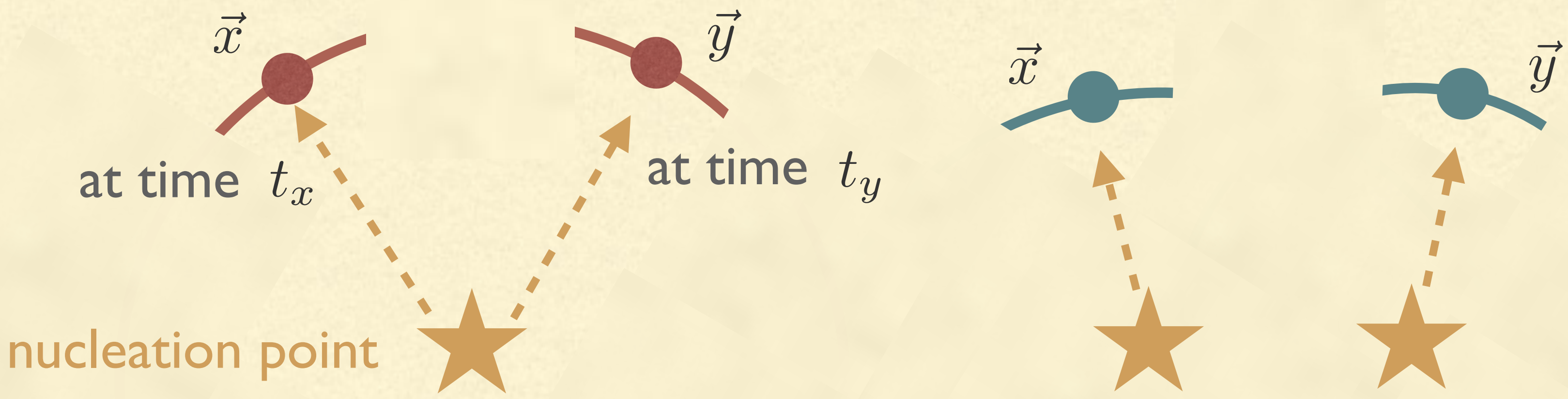
CALCULATION OF $\langle TT \rangle$

[Jinno & Takimoto '16 & '17]

- Calculating $\langle T(t_x, \vec{x}) T(t_y, \vec{y}) \rangle_{\rm ens}$ means ...

  - Fix spacetime points $x = (t_x, \vec{x})$ and $y = (t_y, \vec{y})$

  - Find bubble configurations s.t. EM tensor T is nonzero at x & y



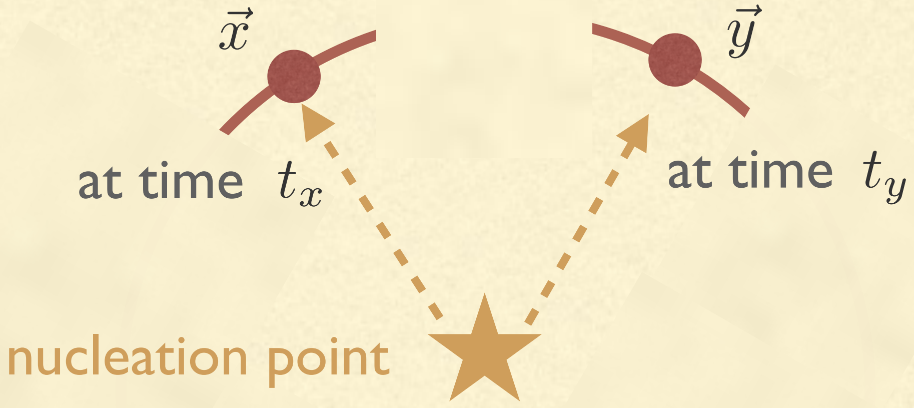
  - Calculate $\left\{ \begin{array}{c} \text{probability} \\ \text{value of } T(t_x, \vec{x}) T(t_y, \vec{y}) \end{array} \right\}$ for such configurations and sum up

CALCULATION OF $\langle TT \rangle$

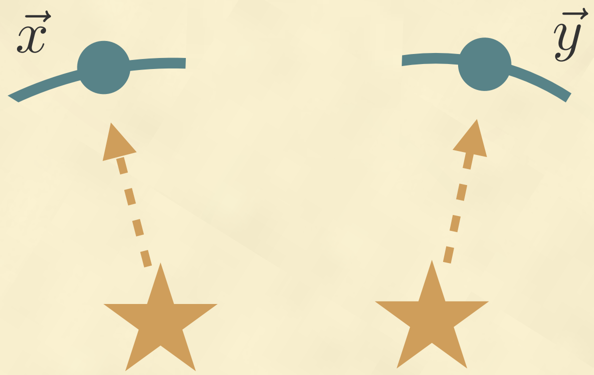
[Jinno & Takimoto '16 & '17]

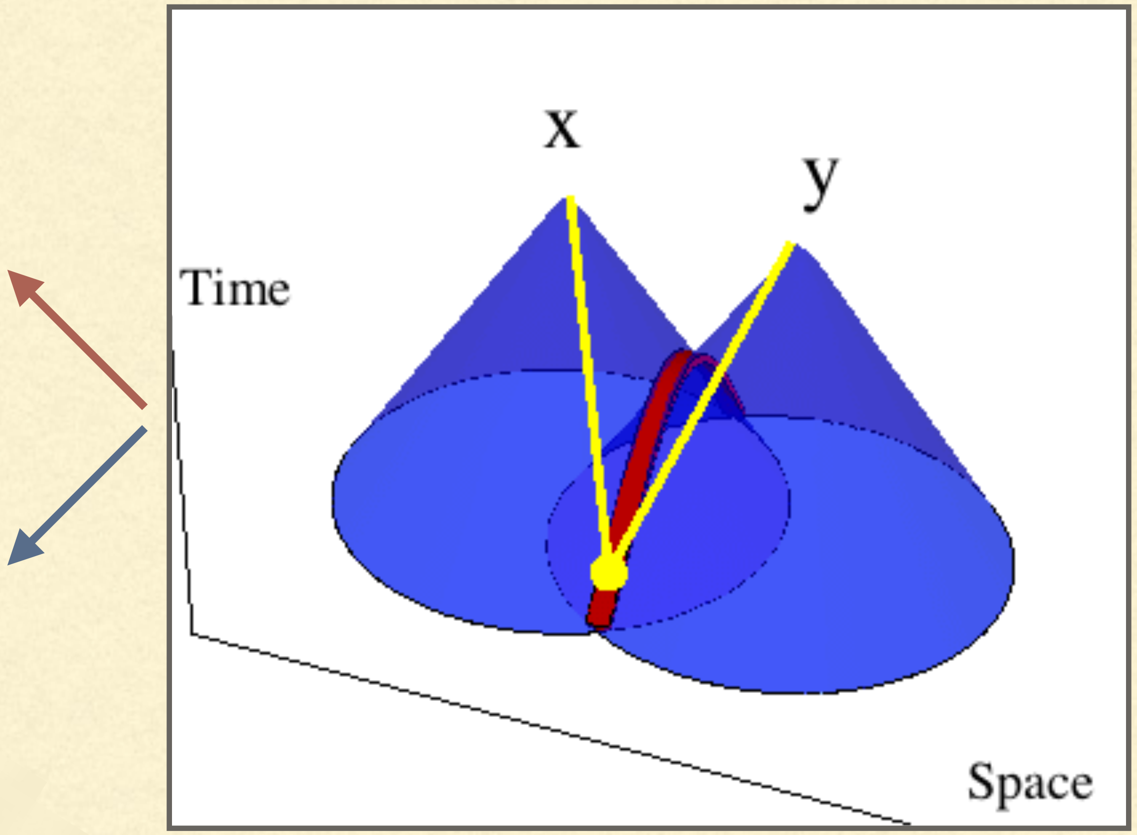
- Only two types of configurations exist :

  - Single-bubble



  - Double-bubble





FINAL RESULT FOR ENVELOPE CASE

- The spectrum becomes sum of two contributions

$$\boxed{\rho_{\rm GW}(t_{\rm end}, k) \propto \ \Delta^{(s)} + \Delta^{(d)}}$$

- Single-bubble spectrum

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_{x,y} \int_{|t_{x,y}|}^{\infty} dr_v \ v^3 k^3 \frac{e^{-\frac{r_v}{2}}}{\mathcal{I}(t_{x,y}, r_v)} \left[j_0(vkr_v)\mathcal{S}_0 + \frac{j_1(vkr_v)}{vkr_v}\mathcal{S}_1 + \frac{j_2(vkr_v)}{(vkr_v)^2}\mathcal{S}_2 \right] \cos(kt_{x,y})$$

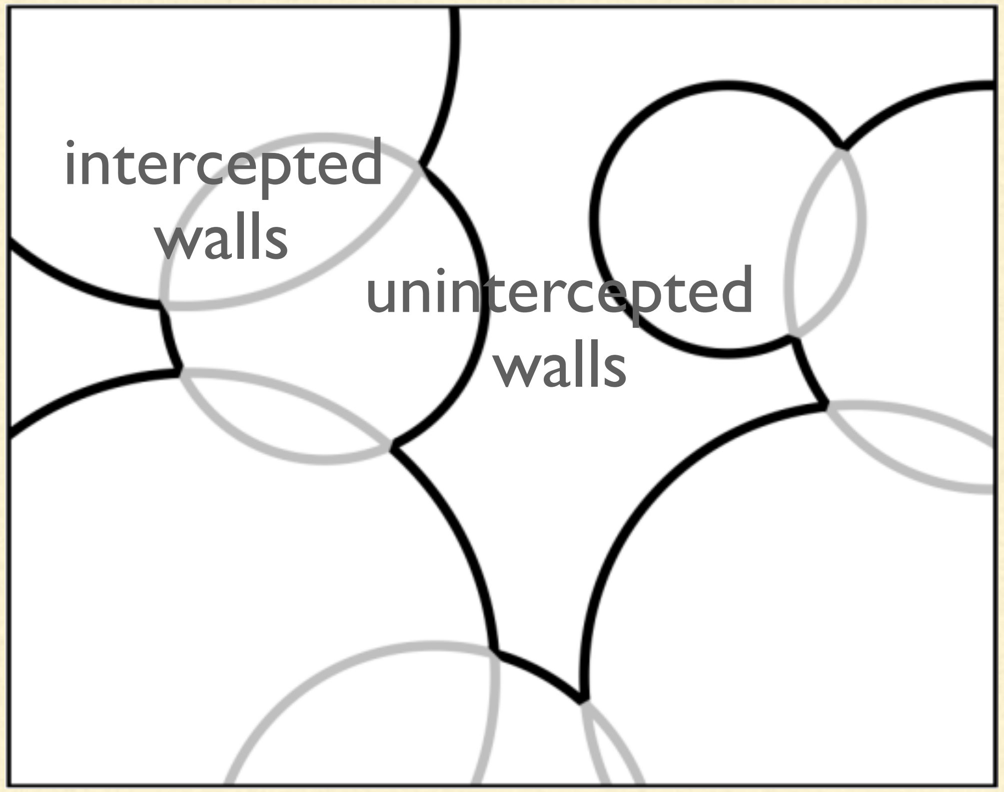
- Double-bubble spectrum

$$\Delta^{(d)} = \int_{-\infty}^{\infty} dt_{x,y} \int_{|t_{x,y}|}^{\infty} dr_v \ v^3 k^3 \frac{e^{-r_v}}{\mathcal{I}(t_{x,y}, r_v)^2} \frac{j_2(vkr_v)}{(vkr_v)^2} \ \mathcal{D}(t_{x,y}, r_v)\mathcal{D}(-t_{x,y}, r_v) \cos(kt_{x,y})$$

BEYOND THE ENVELOPE

- Interception (= collision) complicates the calculation

  once we consider "beyond the envelope"



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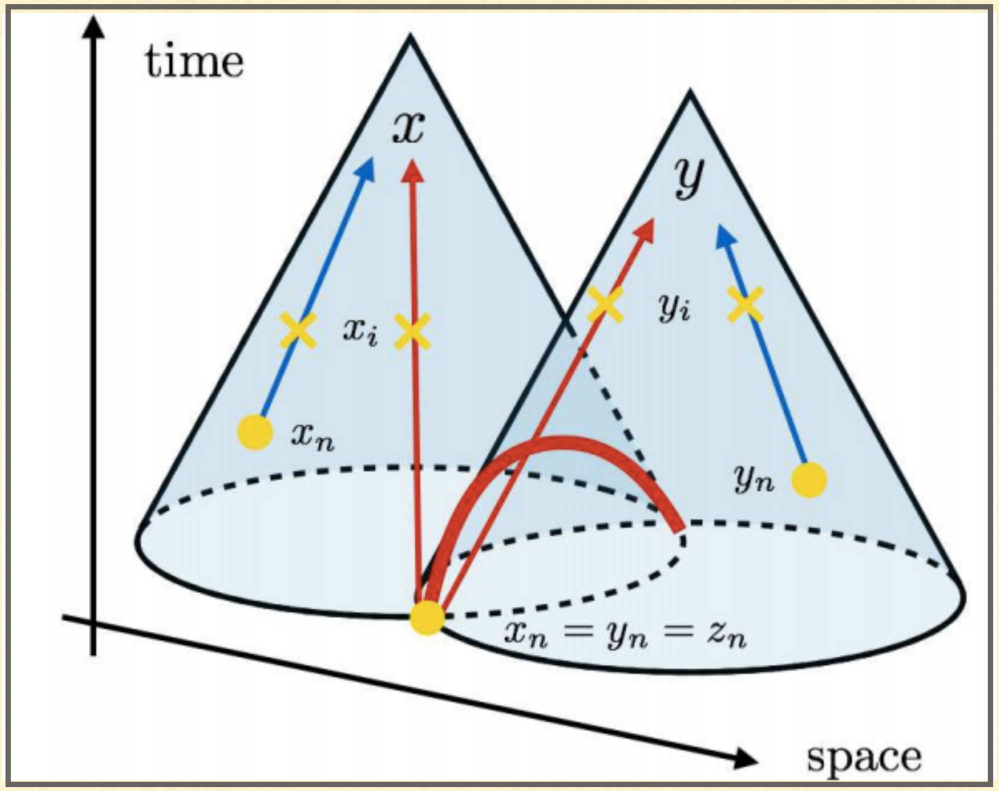


BEYOND THE ENVELOPE: FINAL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

1. single-bubble + 2. double-bubble

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi}$$

$$\frac{k^3}{3} \left[\begin{array}{l} e^{-I(x_i,y_i)}\, \Gamma(t_n)\, \dfrac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ \times \left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times}) \right] \\ \times\, \partial_{t_{xi}} \left[r_B(t_{xi}, t_n)^3 D(t_x, t_{xi}) \right] \partial_{t_{yi}} \left[r_B(t_{yi}, t_n)^3 D(t_y, t_{yi}) \right] \cos(k t_{x,y}) \end{array} \right]$$

BEYOND THE ENVELOPE: FINAL EXPRESSIONS

- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

① single-bubble

General nucleation rate

& wall velocity

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{\boxed{v|t_{x,y}|}}^{\infty} dr \int_{-\infty}^{t_{\max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi}$$

$$\frac{k^3}{3}\left[\begin{array}{l} e^{-I(x_i,y_i)}\boxed{\Gamma(t_n)}\dfrac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ \times\left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times})\right] \\ \times\, \partial_{t_{xi}}\left[r_B(t_{xi}, t_n)^3 \boxed{D(t_x, t_{xi})}\right] \partial_{t_{yi}}\left[r_B(t_{yi}, t_n)^3 D(t_y, t_{yi})\right] \cos(k t_{x,y}) \end{array}\right]$$

General damping function after collision

$T_{ij} \propto$ (bubble radius)$^{-2}$ × D

BEYOND THE ENVELOPE: FINAL EXPRESSIONS

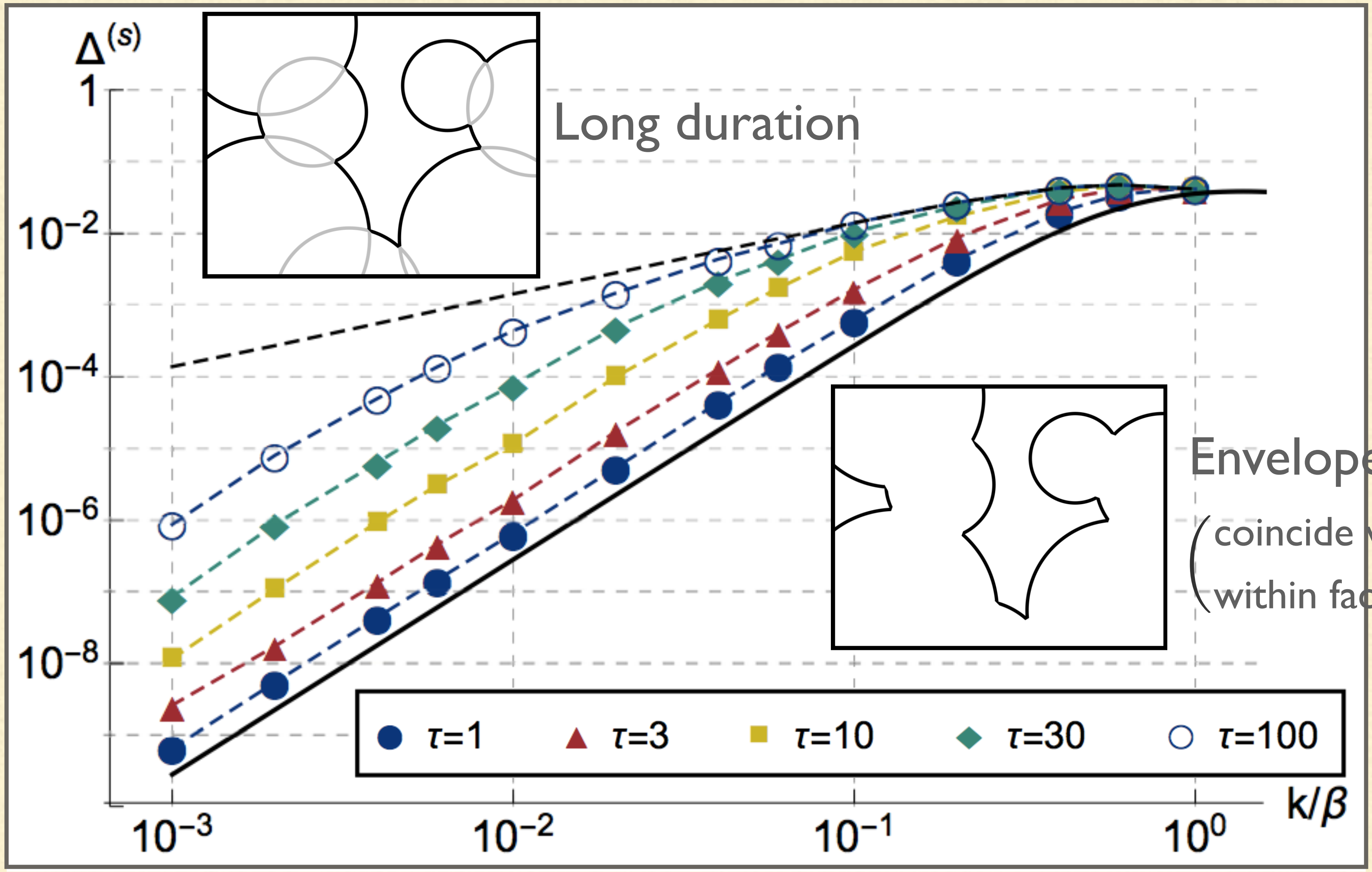
- Full expression reduces to only ~10-dim. integration [Jinno & Takimoto '17]

1. single-bubble + 2. double-bubble

$$
\begin{aligned}
\Delta^{(d)} = &\int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \\
&\int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_0^{2\pi} d\phi_{xn,yn} \\
&\frac{k^3}{3} \left[\begin{aligned} &\Theta_{\rm sp}(x_i, y_n)\Theta_{\rm sp}(x_n, y_i) e^{-I(x_i,y_i)} \Gamma(t_{xn})\Gamma(t_{yn}) \\ &\times r^2 \left[j_0(kr)\mathcal{K}_0(n_{xn}, n_{yn}) + \frac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn}, n_{yn}) + \frac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn}, n_{yn}) \right] \\ &\times \partial_{txi} \left[r_B(t_{xi}, t_{xn})^3 D(t_x, t_{xi}) \right] \partial_{tyi} \left[r_B(t_{yi}, t_{yn})^3 D(t_y, t_{yi}) \right] \cos(kt_{x,y}) \end{aligned} \right]
\end{aligned}
$$

NUMERICAL RESULT

- Single-bubble (Damping function $D = e^{-(t-t_i)/\tau}, \ t_i$: interception time)



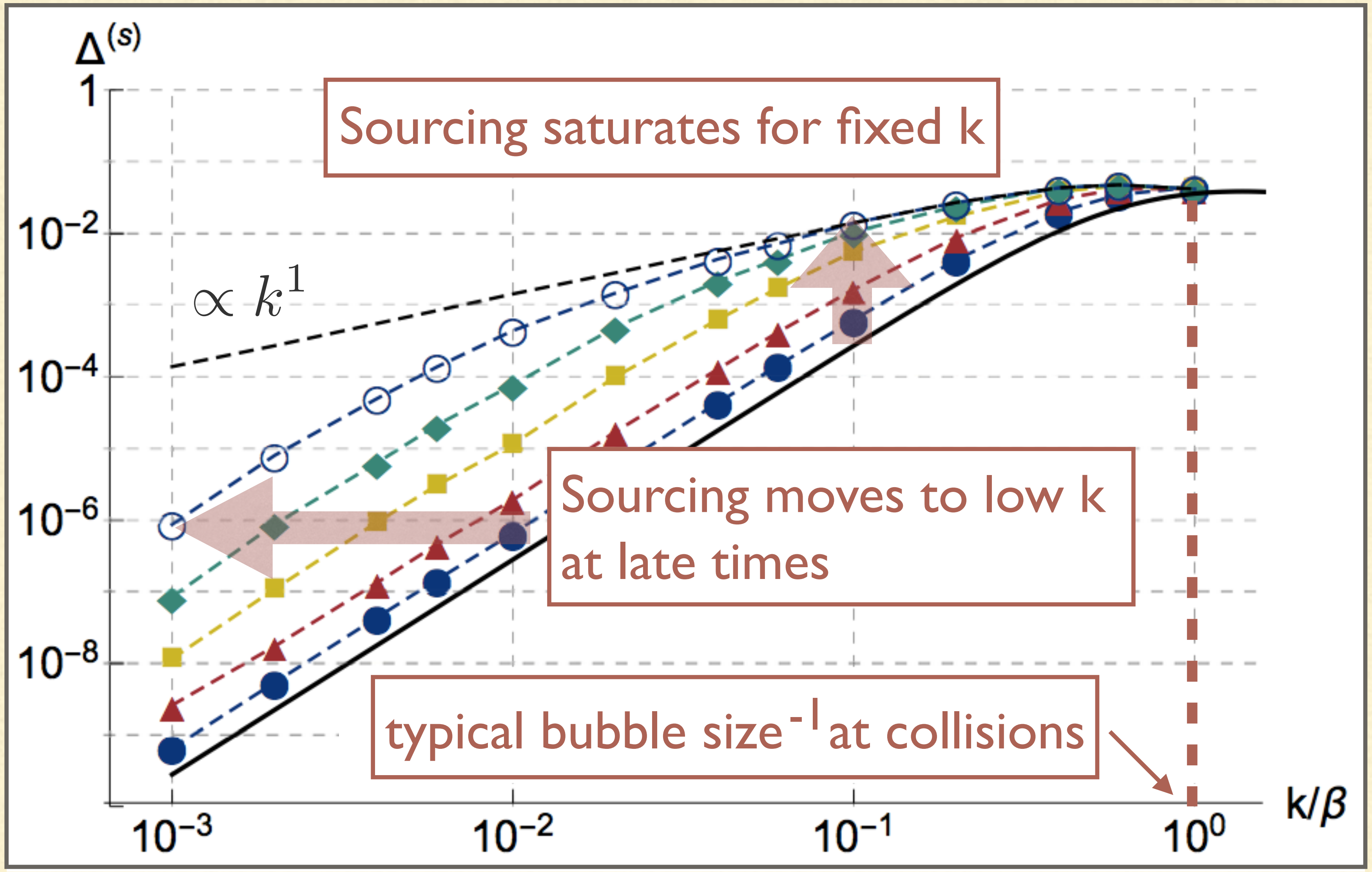
Long duration

Envelope

(coincide with [Huber&Konstandin '08] within factor 2)

NUMERICAL RESULT

- Single-bubble



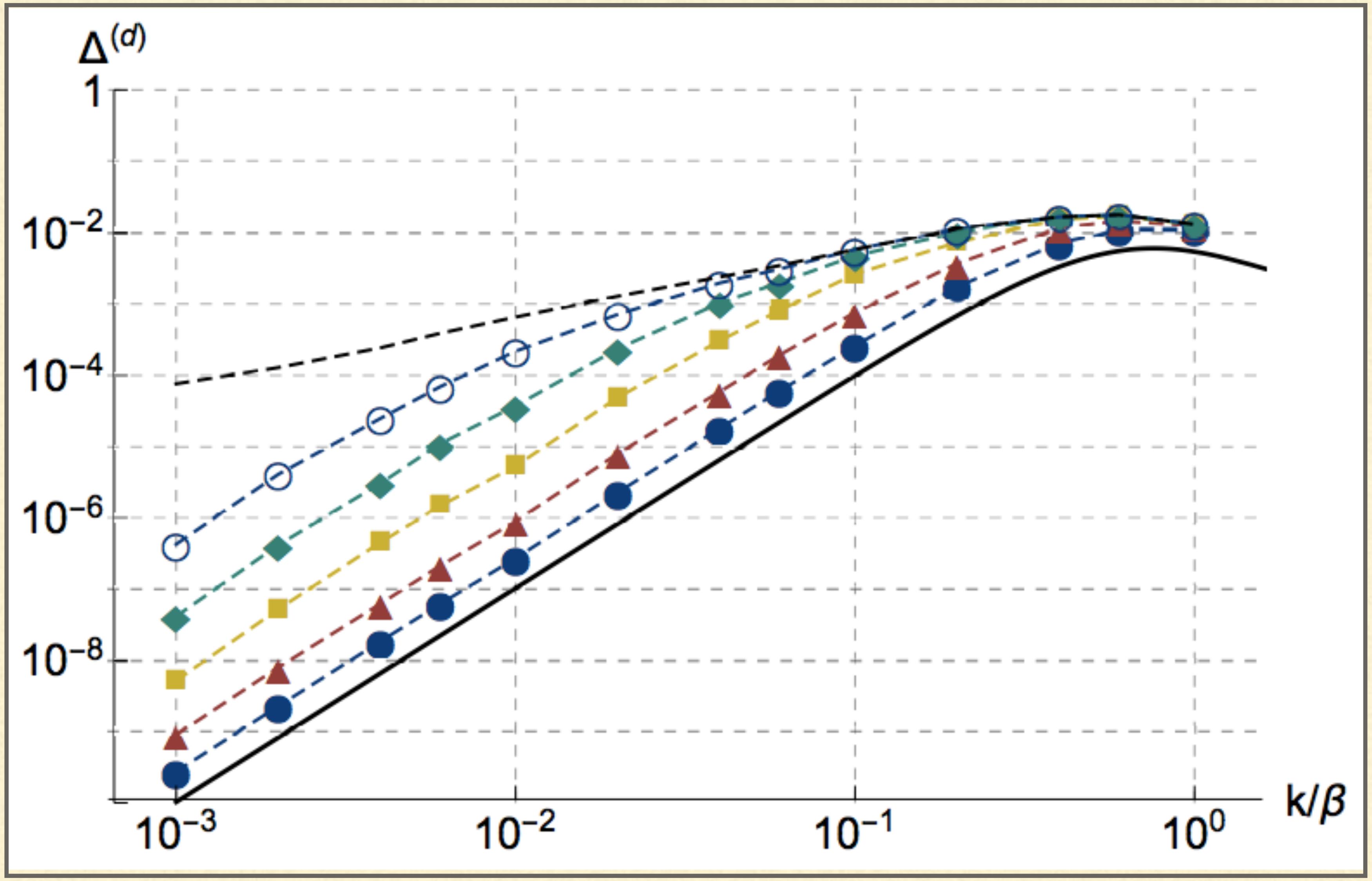
Sourcing saturates for fixed k

Sourcing moves to low k at late times

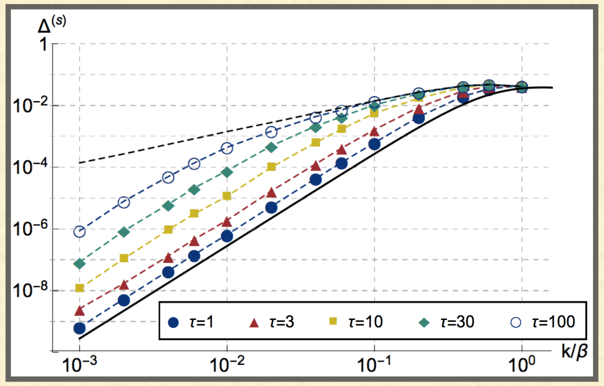
typical bubble size^{-1} at collisions

NUMERICAL RESULT

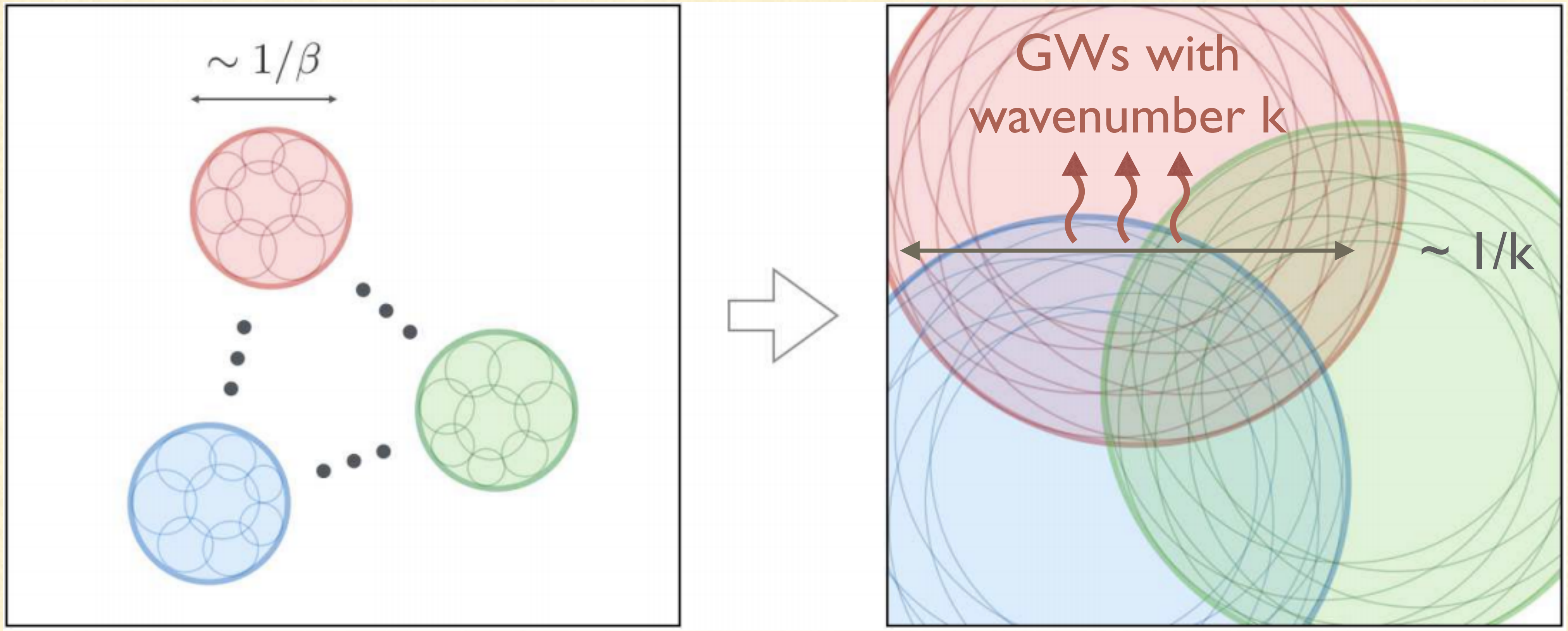
- Double-bubble



PHYSICAL INTERPRETATION

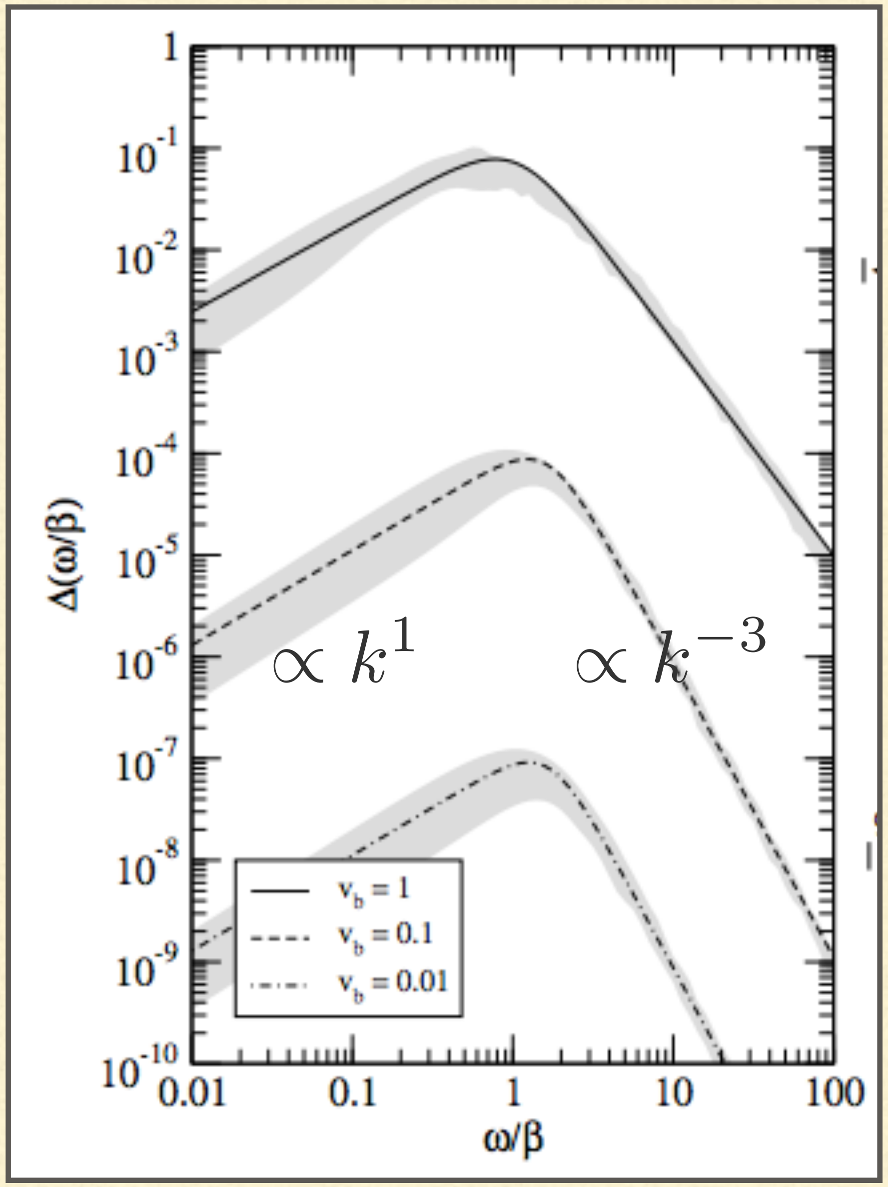


- Wavenumber k sourced when the typical bubble size grows to ~ 1/k



THIN-WALL NUMERICAL SIMULATION

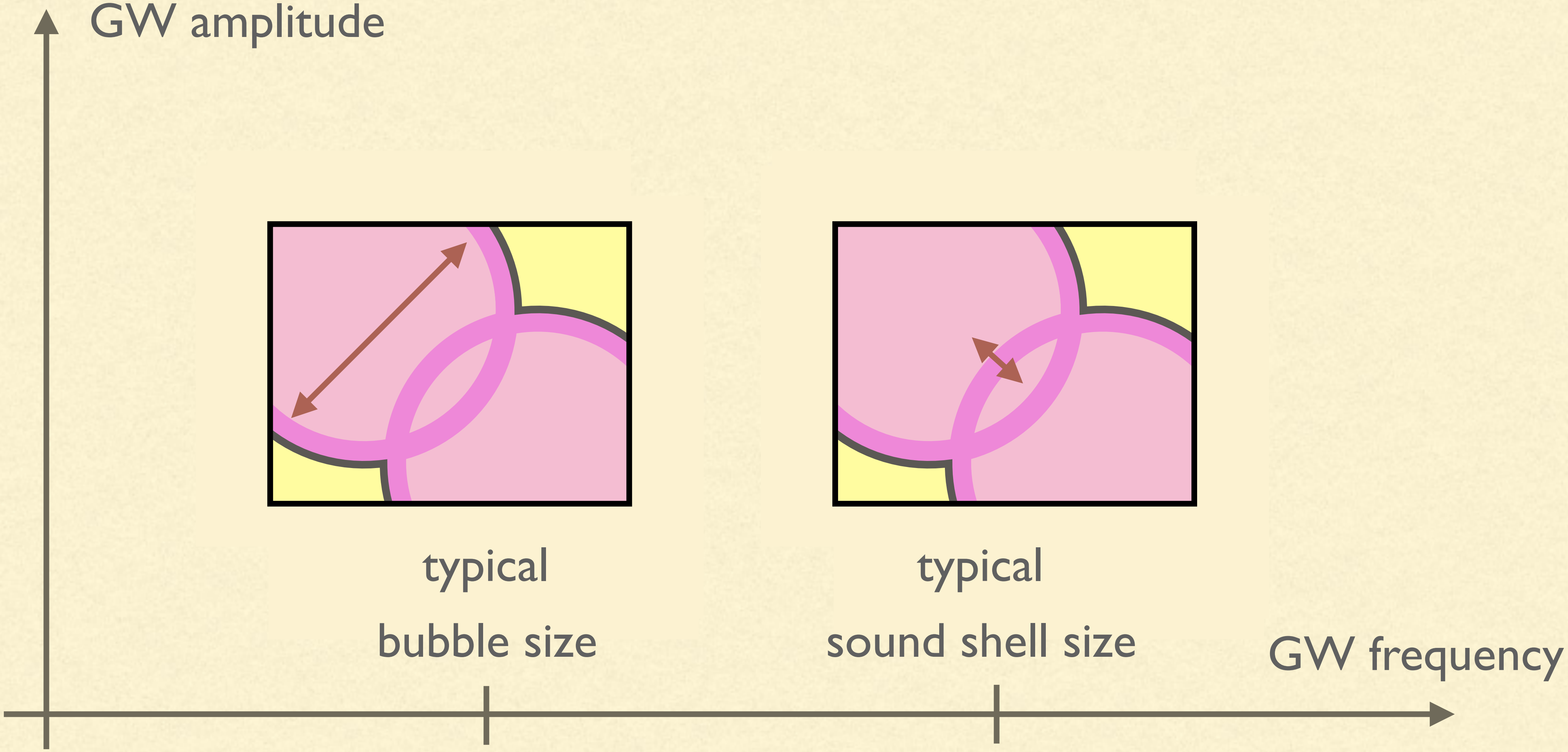
- Recently cross-checked by thin-wall numerical simulation by Prof. Konstandin



[Konstandin '17]

IMPLICATIONS: RICH & STRUCTUREFUL SPECTRUM

- What are the implications of our result to the REAL system?



GW amplitude

typical

bubble size

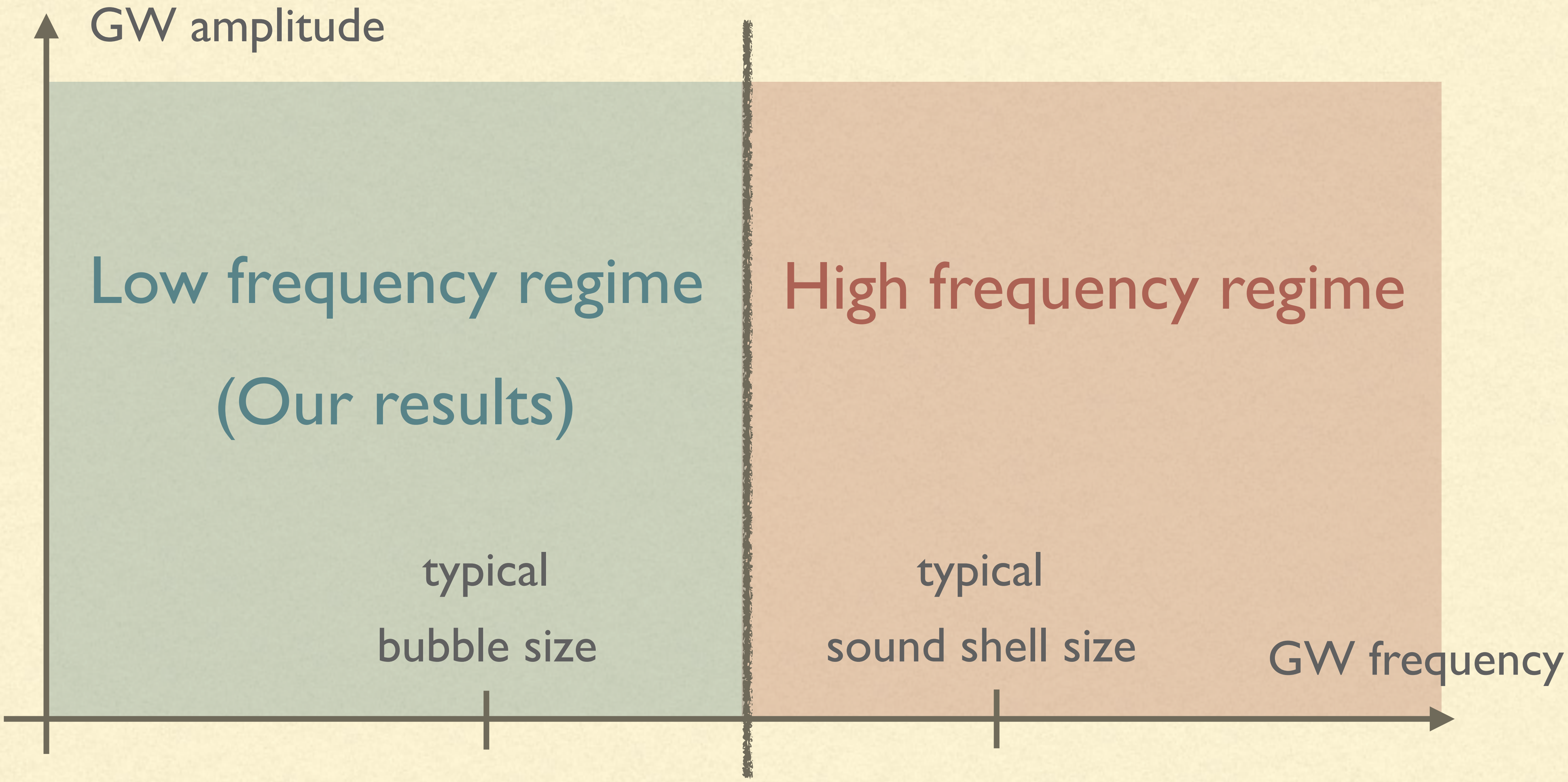
typical

sound shell size

GW frequency

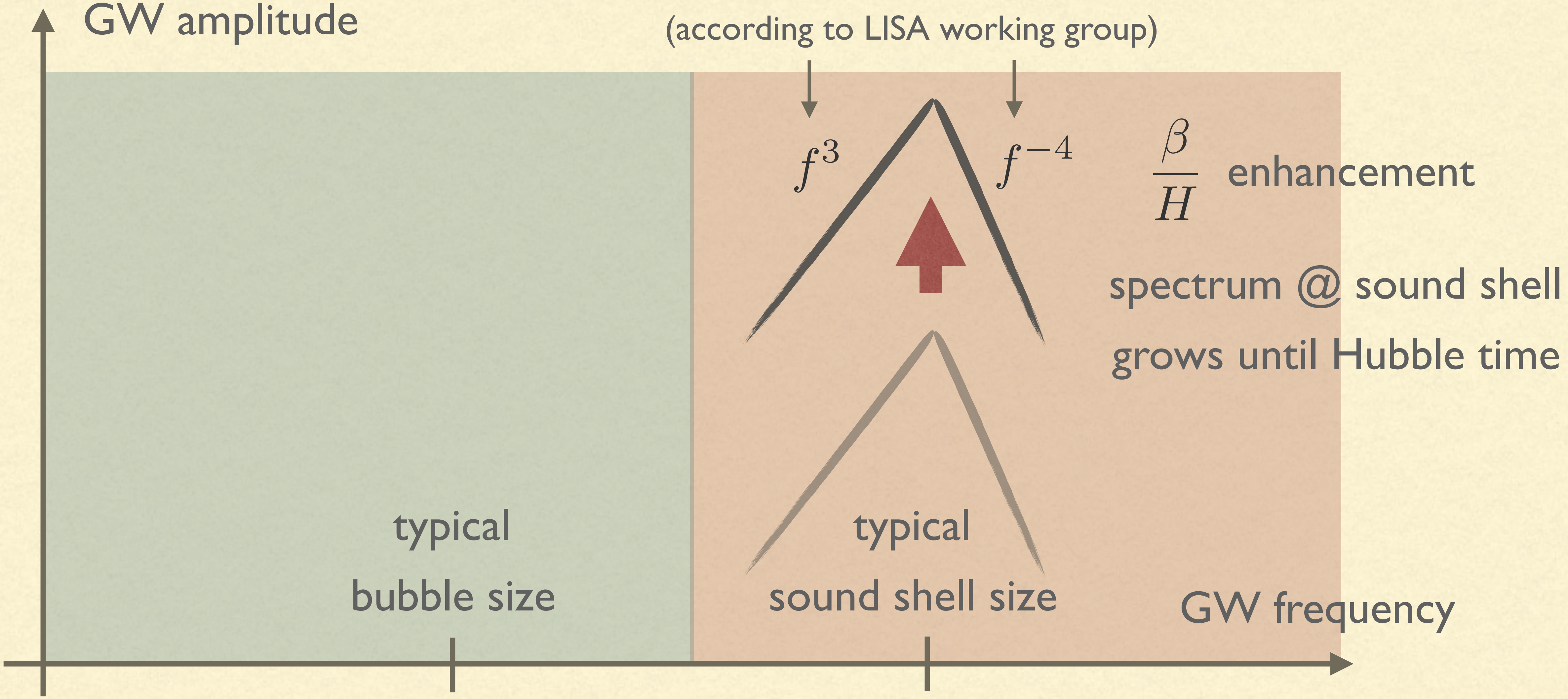
IMPLICATIONS: RICH & STRUCTUREFUL SPECTRUM

- What are the implications of our result to the REAL system?



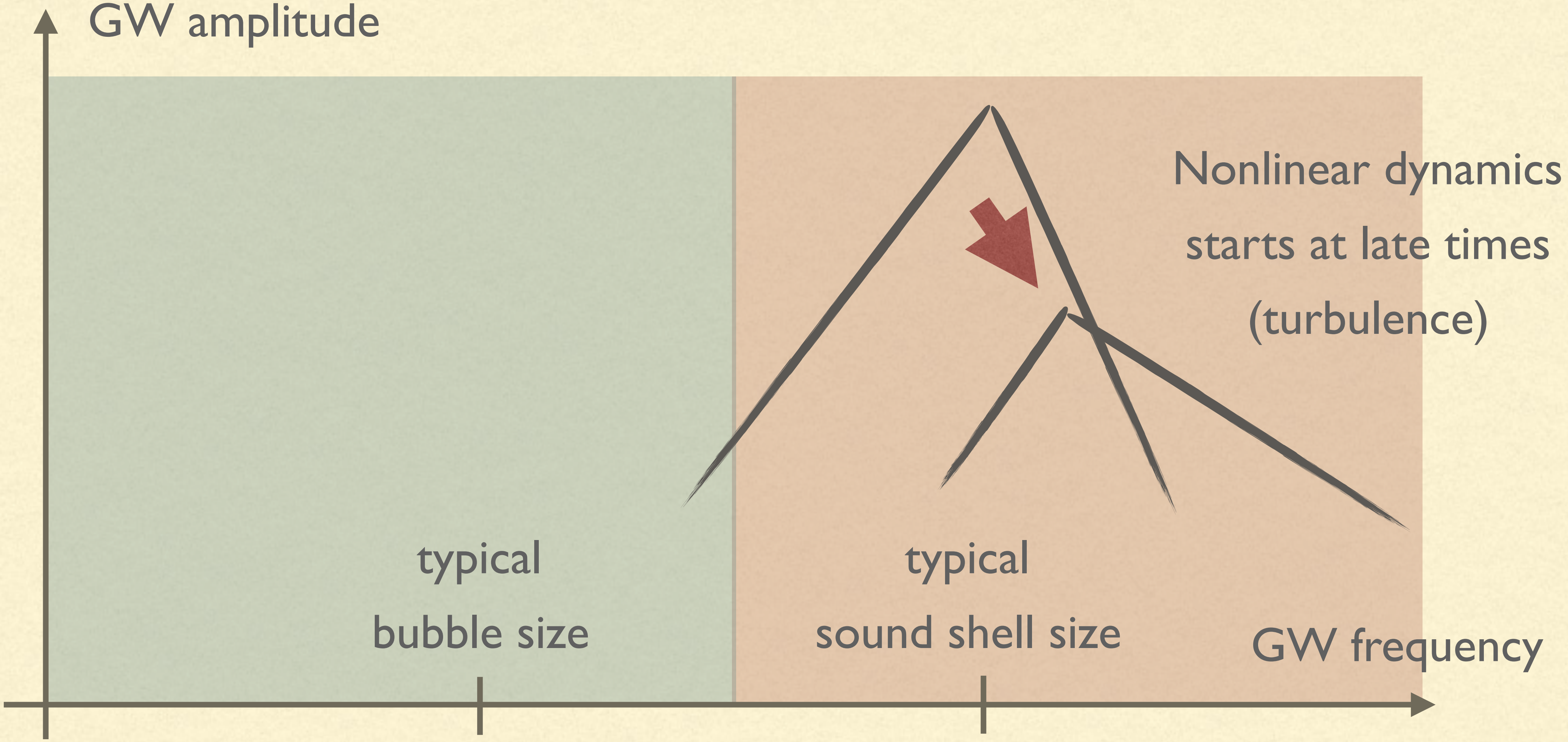
IMPLICATIONS: RICH & STRUCTUREFUL SPECTRUM

- GW spectrum discussed in the literature



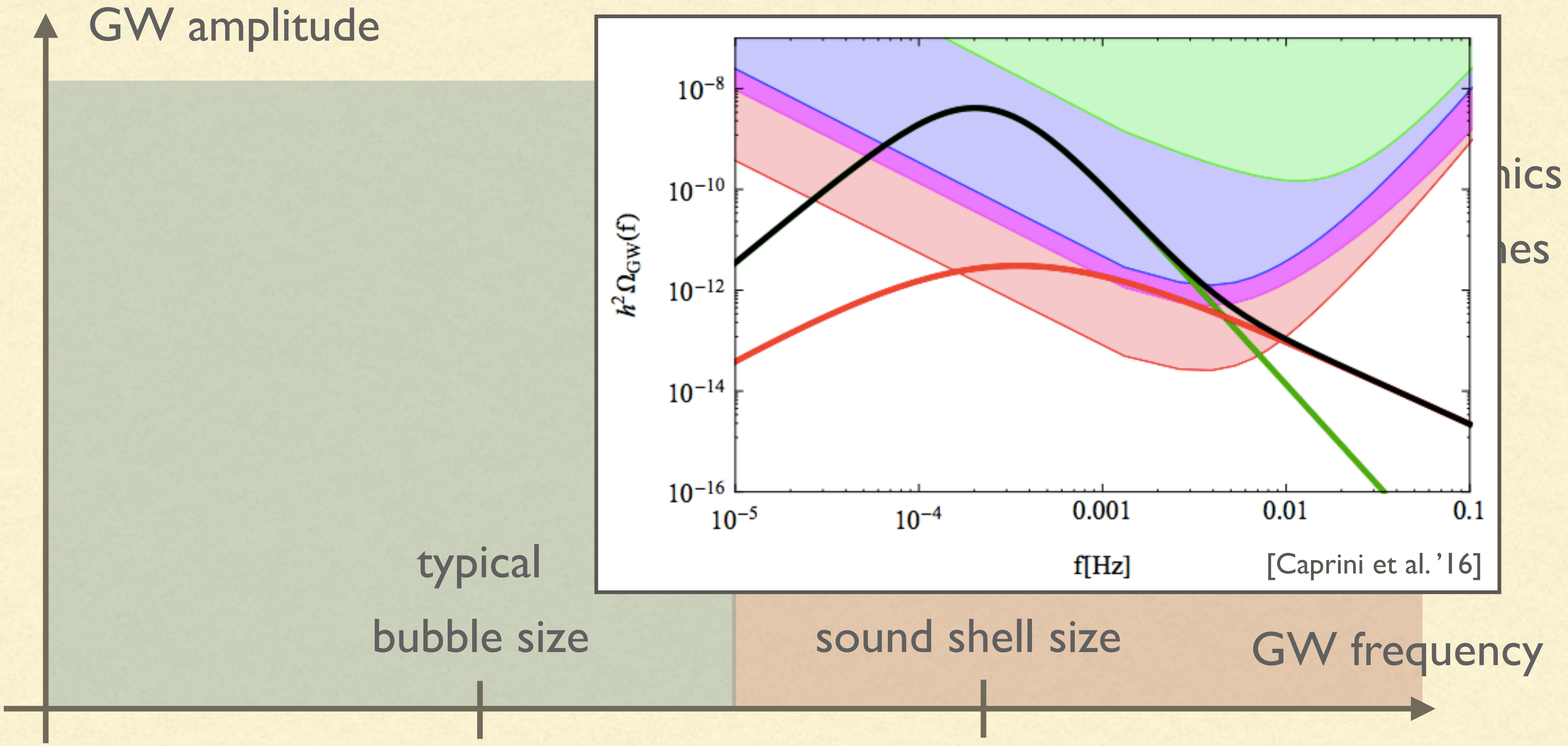
IMPLICATIONS:
RICH & STRUCTUREFUL SPECTRUM

- GW spectrum discussed in the literature



IMPLICATIONS: RICH & STRUCTUREFUL SPECTRUM

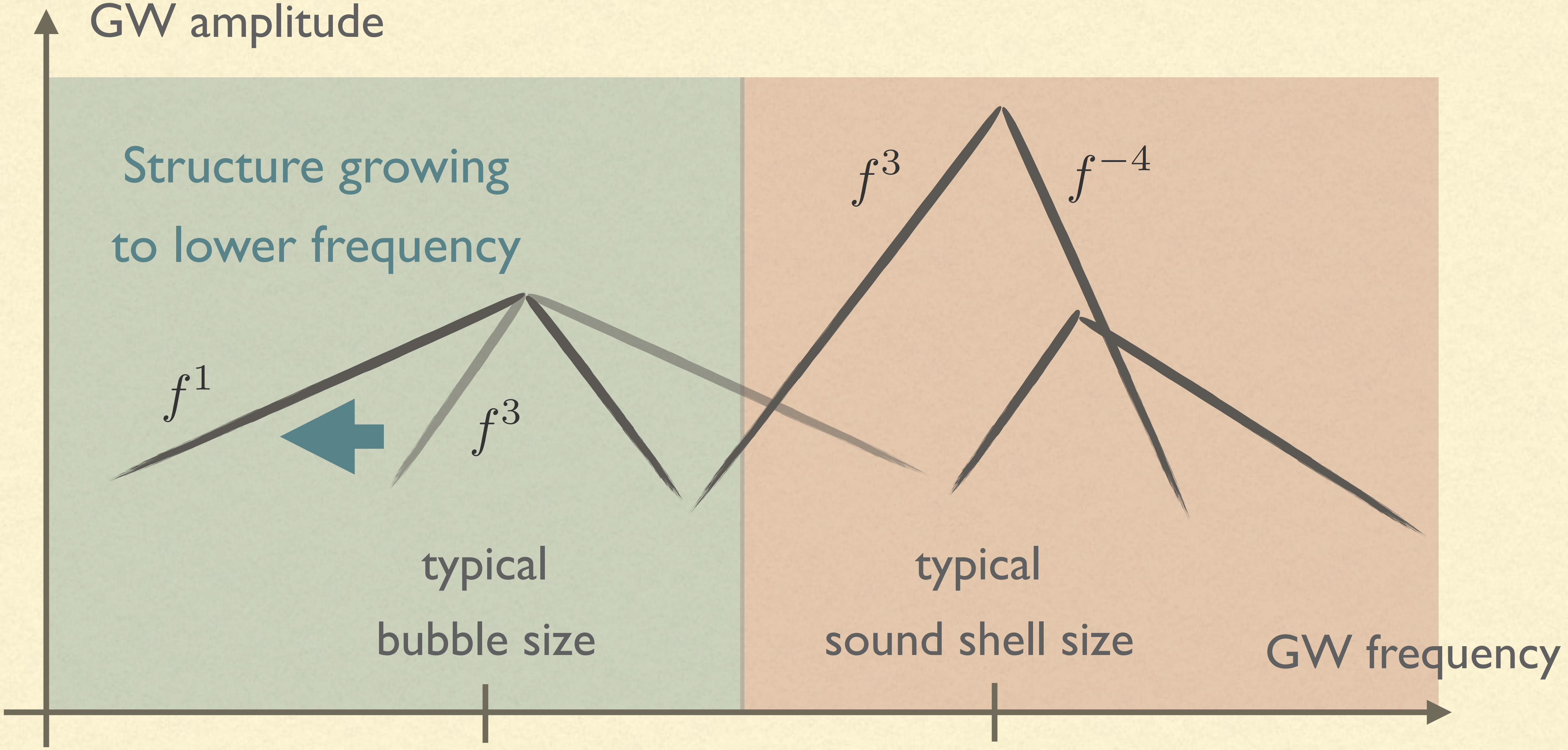
- GW spectrum discussed in the literature



[Caprini et al. '16]

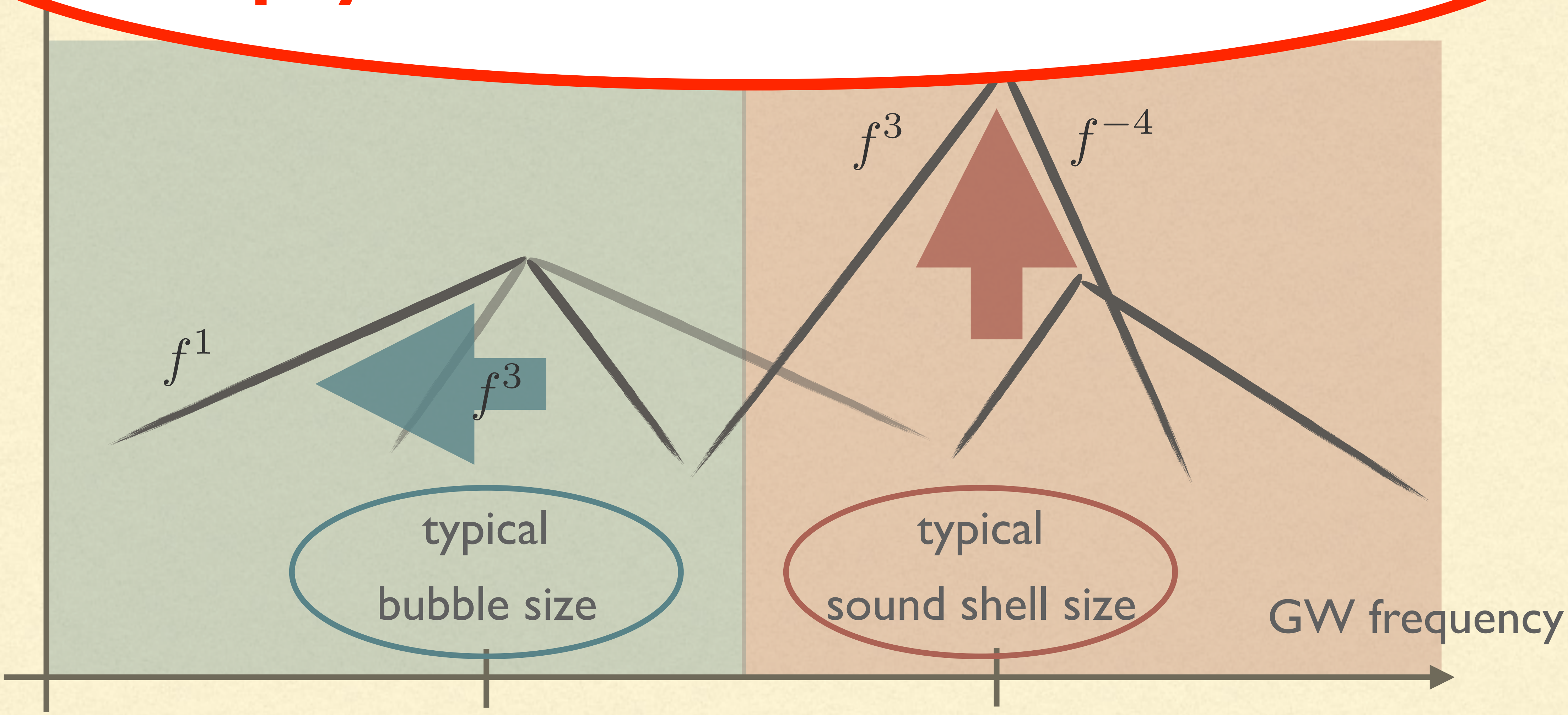
IMPLICATIONS: RICH & STRUCTUREFUL SPECTRUM

- GW spectrum may be more rich & structureful than previously thought



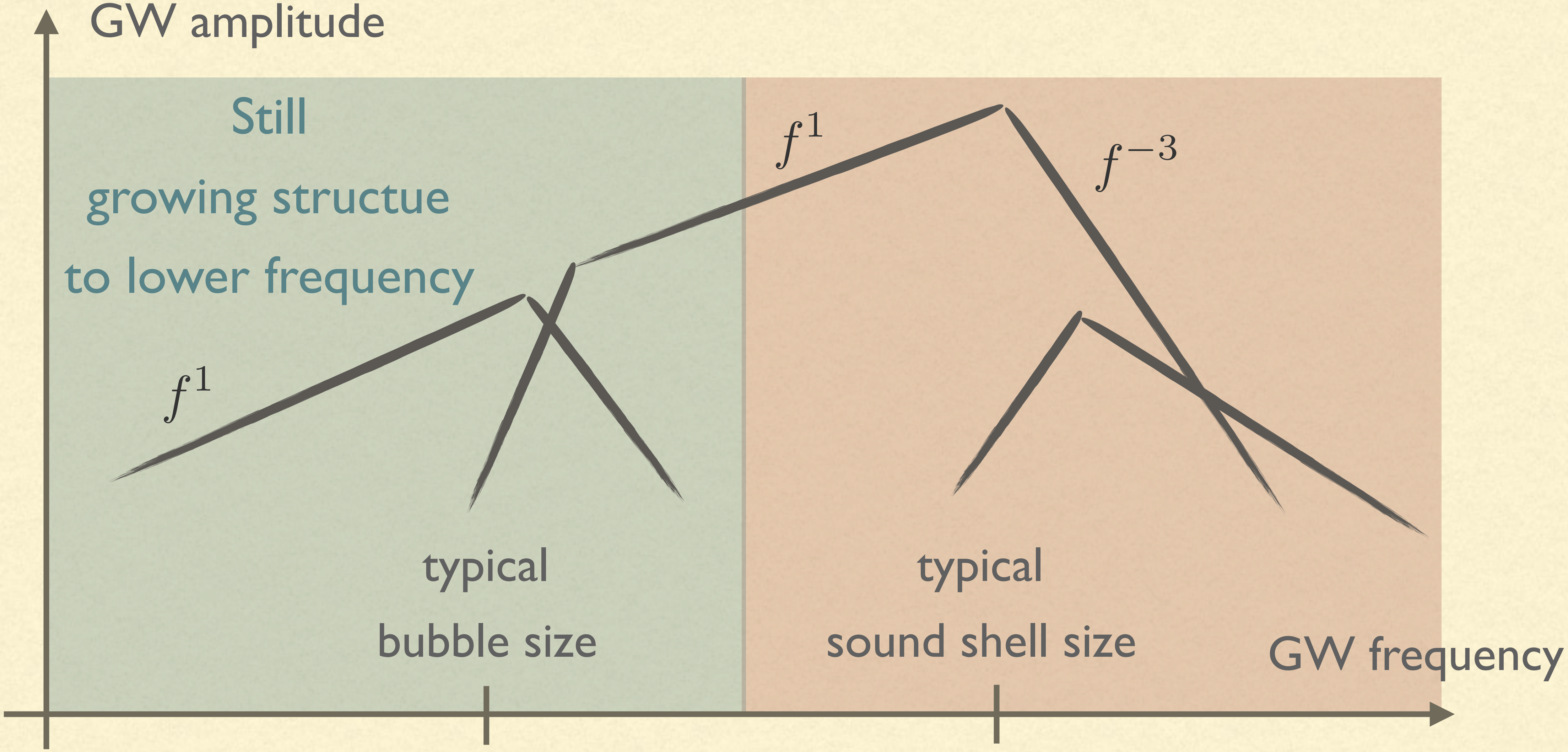
IMPLICATIONS:
RICH & STRUCTUREFUL SPECTRUM

All these structures tell us about the physics which drives the transition



IMPLICATIONS:
RICH & STRUCTUREFUL SPECTRUM

- Sound shell model for GW enhancement (for experts) [Hindmarsh '16]



TALK PLAN

✓ 0. Introduction

✓ 1. Bubble dynamics in first-order phase transitions

✓ 2. Analytic approach to GW production

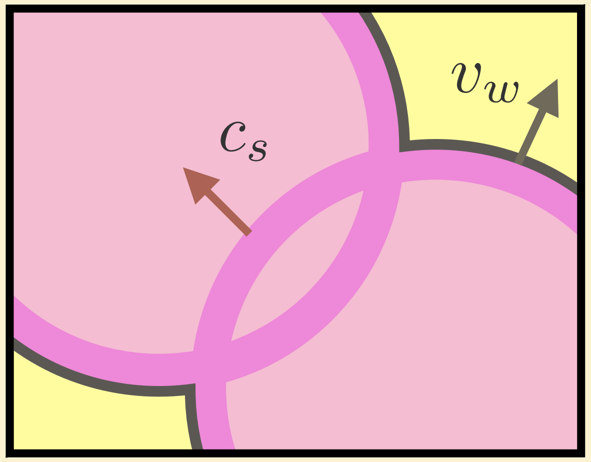
3. Future prospects

FUTURE PROSPECTS

- ***“Can we make GWs CMB?”***

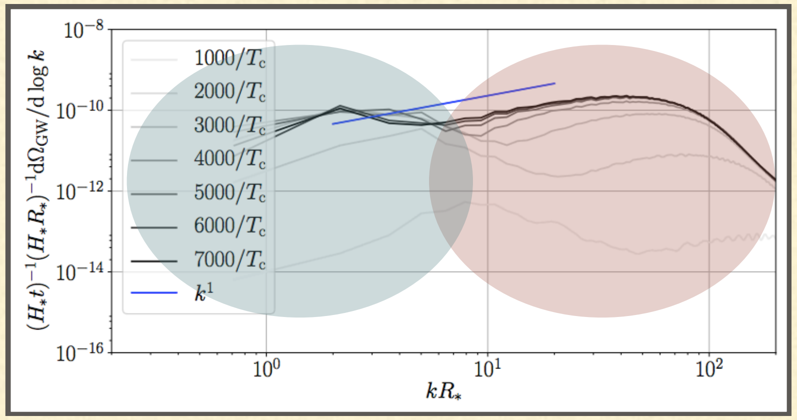
- Many things to do:

  - Make more realistic prediction
    by accommodating propagation velocity change



  - Cross-check with thin-wall numerical simulations → establish low-freq. regime

  - Subtracting above from full numerical simulation
    will tell us “really β/H enhancing part” of sound waves



[Hindmarsh et al. ’17]

  - Other questions e.g. how long sound waves last?
    We can divide problems into smaller pieces...

CONCLUSION

- We developed an analytic approach to GW production in phase transitions,

  which will describe the low-frequency regime of GW spectrum

- Will help to interpret numerical simulation results

  and to gain insight on the physics encoded in the spectrum

- Still many things to do: Let's prepare for LISA

Backup

SOURCES FOR COSMOLOGICAL GWS

- Inflationary quantum fluctuations ("primordial GWs")
- Preheating (particle production just after inflation)
- Cosmic strings, Domain walls
- First-order phase transition can occur in many physics models

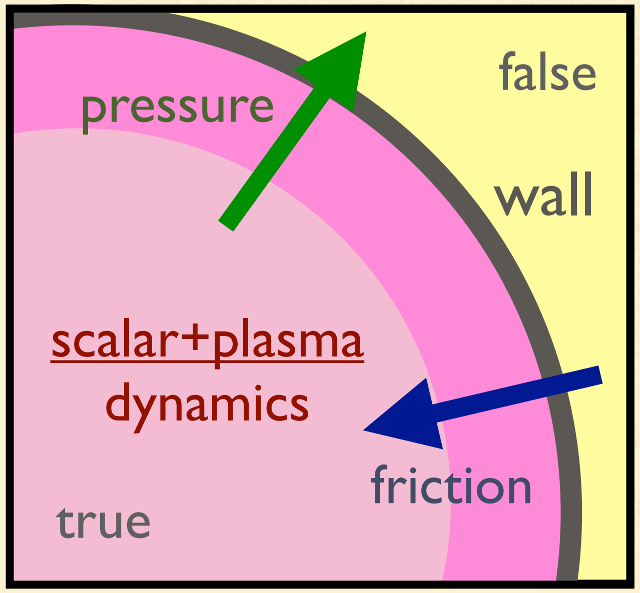
- Electroweak sym. breaking (w/ extension)
- B-L breaking
- PQ sym. breaking
- Breaking of GUT group
- Strong dynamics

... and so on

BEHAVIOR OF BUBBLES

- Two main players : **scalar field & plasma**

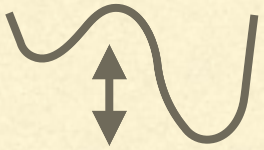
[e.g. Espinosa et al., JCAP06(2010)028]



- Walls (where the scalar field value changes) want to expand ("pressure")

- Walls are pushed back by plasma ("friction")

- Pressure & friction are determined by

Pressure ~ $\alpha \equiv \frac{\rho_{\rm released}}{\rho_{\rm rad}}$ ~  / $\rho_{\rm rad}$

Friction ~ η (coupling btw. scalar field & plasma) × v_w (wall velocity)

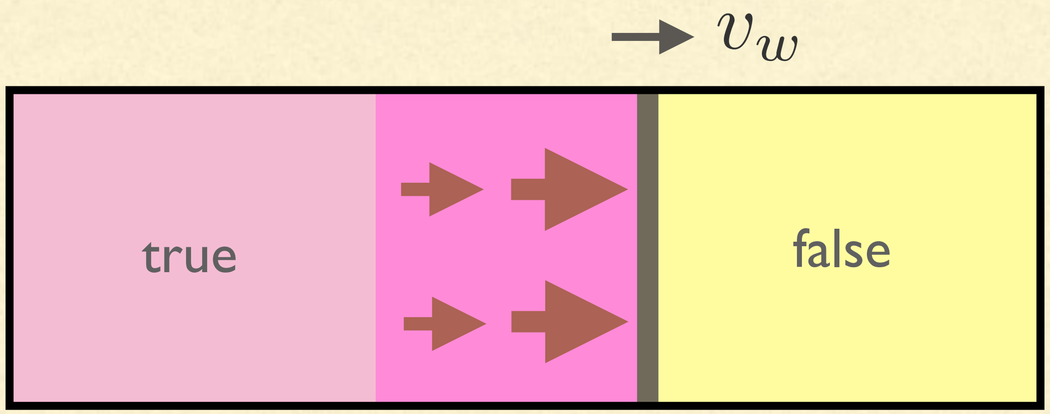
BEHAVIOR OF BUBBLES

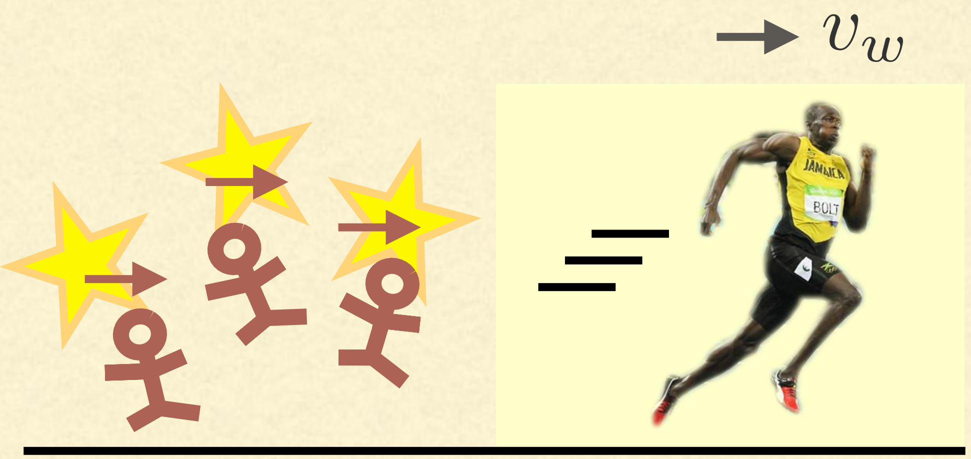
- “Pressure vs. friction” gives terminal velocity of walls

$$\begin{array}{l}\text{Pressure} \sim \alpha \\ \text{Friction} \sim \eta \times v_w\end{array} \quad \to \quad v_w \begin{cases}\text{increasing in } \alpha \\ \text{decreasing in } \eta\end{cases}$$

- Walls drag fluid as they propagate

Large v_w ($\gtrsim 1/\sqrt{3}$) : Detonation





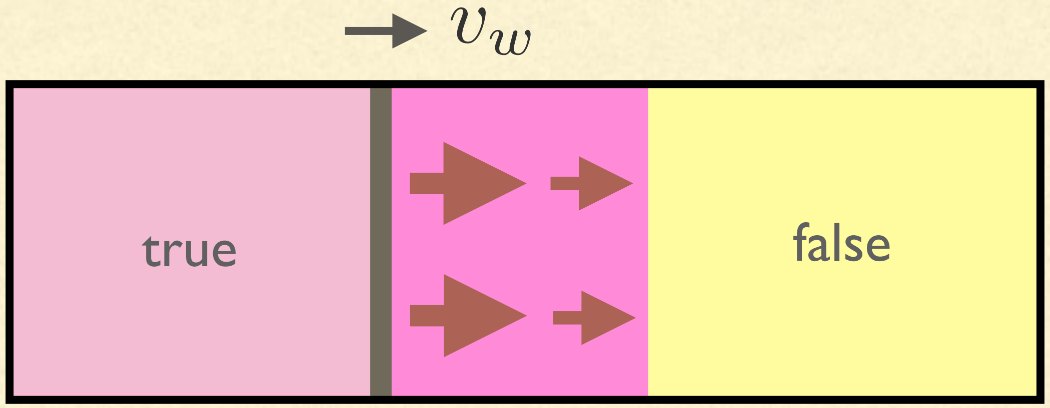
BEHAVIOR OF BUBBLES

- “Pressure vs. friction” gives terminal velocity of walls

$$\begin{array}{lll}\text{Pressure} & \sim & \alpha \\ \text{Friction} & \sim & \eta \times v_w\end{array} \quad \rightarrow \quad v_w \begin{cases}\text{increasing in } \alpha \\ \text{decreasing in } \eta\end{cases}$$

- Walls drag fluid as they propagate

Small v_w ($\lesssim 1/\sqrt{3}$) : Deflagration



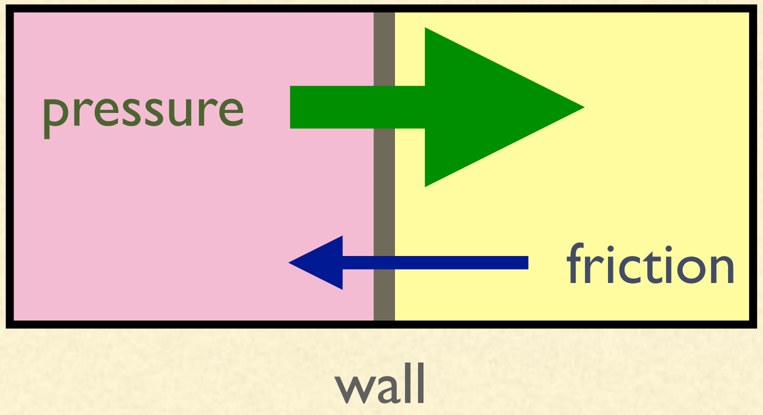
plasma bulk motion

BEHAVIOR OF BUBBLES

- Understanding until ~ 2016

[e.g. Bodeker & Moore, JCAP 0905 (2009) 009
Espinosa et al., JCAP 1006 (2010) 028]

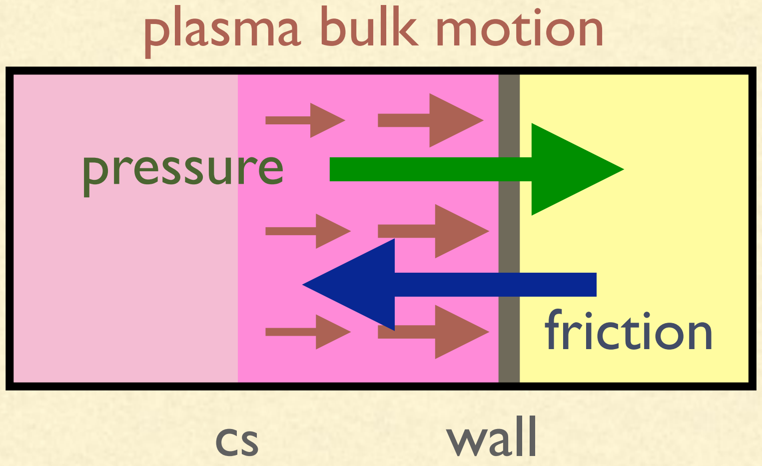
$\alpha \gtrsim \mathcal{O}(0.1)$: Large energy release → Runaway



- Plasma friction cannot balance with pressure
- Walls approach the speed of light
- Energy accumulates in <u>walls</u>

$\alpha \lesssim \mathcal{O}(0.1)$: Small energy release → Terminal velocity

(to experts :
this is detonation case)



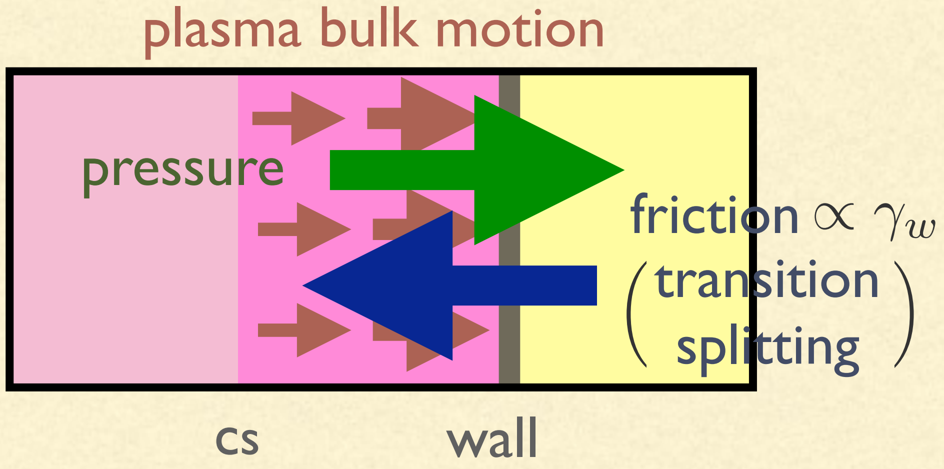
- Plasma friction gets balanced with pressure
- Walls approach terminal velocity
- Energy accumulates in <u>plasma bulk motion</u>

BEHAVIOR OF BUBBLES

- Understanding from 2017 ~

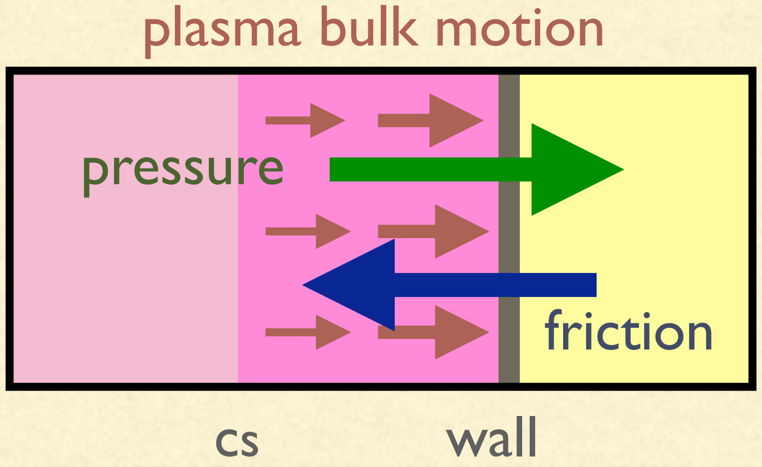
[Bodeker & Moore ’17]

$\alpha \gtrsim \mathcal{O}(0.1)$: Large energy release → ~~Runaway~~ High-terminal velocity



- Plasma friction does balance with pressure
- Walls approach high terminal velocity
- Energy accumulates in plasma bulk motion

$\alpha \lesssim \mathcal{O}(0.1)$: Small energy release → Low-terminal velocity



- Plasma friction gets balanced with pressure
- Walls approach terminal velocity
- Energy accumulates in plasma bulk motion

GRAVITATIONAL WAVES ?

- Transverse-traceless part (tensor part) of the metric (2dof)

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + \boxed{2h_{ij}})dx^i dx^j \qquad h_{ii} = \partial_i h_{ij} = 0$$

- Action is similar to massless scalar

$$S_{\rm grav} = \int d^4x\sqrt{-g}M_P^2\left[\frac{1}{2}\dot{h}_{ij}^2 - \frac{1}{2a^2}(\nabla h_{ij})^2\right] \qquad M_P h_{ij}$$: canonical

- Coupled to the energy-momentum tensor of the system

$$\Box h_{ij} = 8\pi G K_{ij,kl} T_{kl}$$

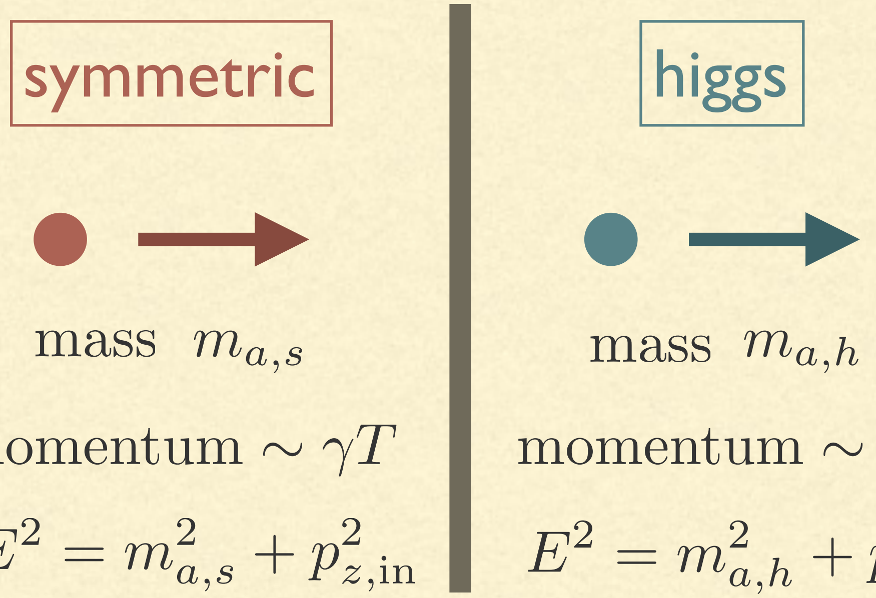
projection to
transverse-traceless modes

energy-momentum tensor
(coming from bubbles)

FRICTION ON THE WALL

- 1 → 1 process

wall rest frame



mass $m_{a,s}$

momentum $\sim \gamma T$

$E^2 = m_{a,s}^2 + p_{z,\text{in}}^2$

mass $m_{a,h}$

momentum $\sim \gamma T$

$E^2 = m_{a,h}^2 + p_{z,\text{out}}^2$

- number density γ-enhanced
- contribution from each particle γ-suppressed

$$\boxed{\Delta p_{1\to1} = p_{z,\text{in}} - p_{z,\text{out}} \simeq \frac{m_{a,\text{h}}^2 - m_{a,\text{s}}^2}{2E}}$$

$\sim \gamma T^3$

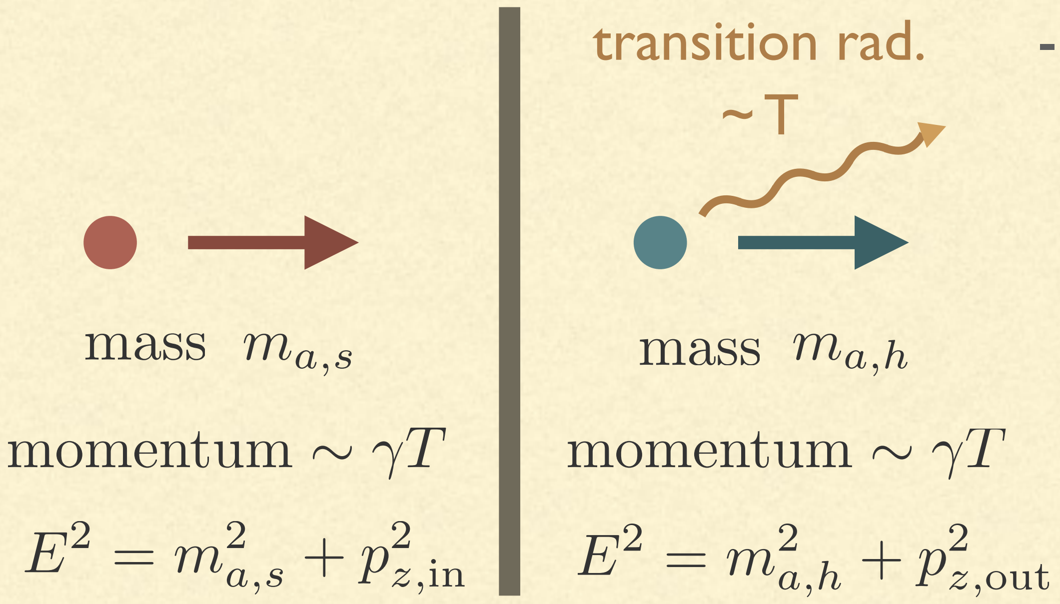
$\sim 1/\gamma T$

$$\boxed{\mathcal{P}_{1\to1} = \sum_a \nu_a \int \frac{d^3p}{(2\pi)^3} f_a(p) \times \frac{m_{a,\text{h}}^2 - m_{a,\text{s}}^2}{2E} = \sum_a \nu_a \int \frac{d^3p}{(2\pi)^3 2E} f_a(p) \left(m_{a,\text{h}}^2 - m_{a,\text{s}}^2\right)}$$

FRICTION ON THE WALL

- 1 → 2 process

wall rest frame



transition rad.

~ T

mass $m_{a,s}$

momentum $\sim \gamma T$

$E^2 = m_{a,s}^2 + p_{z,\mathrm{in}}^2$

mass $m_{a,h}$

momentum $\sim \gamma T$

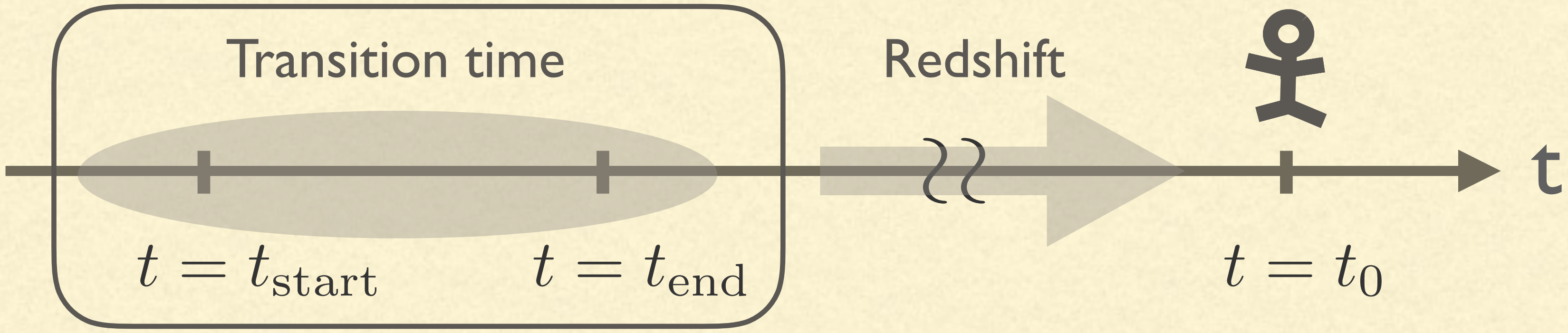
$E^2 = m_{a,h}^2 + p_{z,\mathrm{out}}^2$

- number density γ-enhanced
- contribution from each particle almost constant in γ

$$\boxed{\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}\omega} &= \int_0^{2\pi}\int_0^{\pi/2} \frac{\mathrm{d}W}{\mathrm{d}\Omega \mathrm{d}\omega} \sin\theta\ \mathrm{d}\theta\ \mathrm{d}\phi \\ &= \frac{e^2}{2\pi c}\left[\frac{(1+\beta^2)}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right]\end{aligned}}$$

DEFINITION OF GW SPECTRUM

- Let's focus on transition time, since redshift after production is trivial



Transition time

Redshift

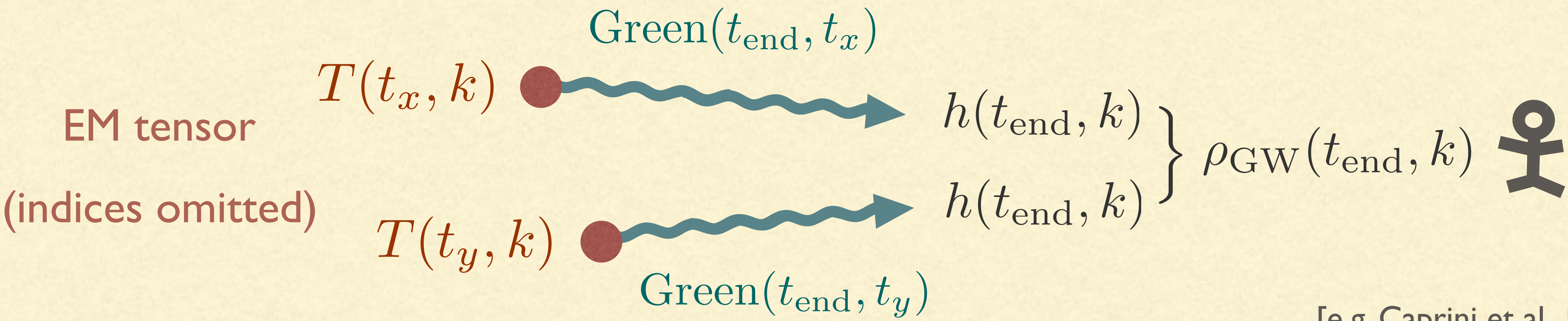
t

$t = t_{\rm start}$ $t = t_{\rm end}$

$t = t_0$

- What we want to know : $\rho_{\rm GW}(t_{\rm end}, k)$ = GW energy density per each wavenumber k

$$\rho_{\rm GW}(t_{\rm end}, k) \sim \langle h_{ij}(t_{\rm end}, k) h^*_{ij}(t_{\rm end}, k) \rangle \sim \int_{t_{\rm start}}^{t_{\rm end}} dt_x \int_{t_{\rm start}}^{t_{\rm end}} dt_y \, \cos(k(t_x - t_y)) \, {\rm F.T.} \left[\langle T_{ij}(t_x, \mathbf{x}) T_{ij}(t_y, \mathbf{y}) \rangle \right]$$



$\text{Green}(t_{\rm end}, t_x)$

$T(t_x, k)$

EM tensor

(indices omitted)

$$\left. \begin{array}{l} h(t_{\rm end}, k) \\ h(t_{\rm end}, k) \end{array} \right\} \rho_{\rm GW}(t_{\rm end}, k)$$

$T(t_y, k)$

$\text{Green}(t_{\rm end}, t_y)$

[e.g. Caprini et al., PRD77 (2008)]

CALCULATION OF $\langle TT \rangle$

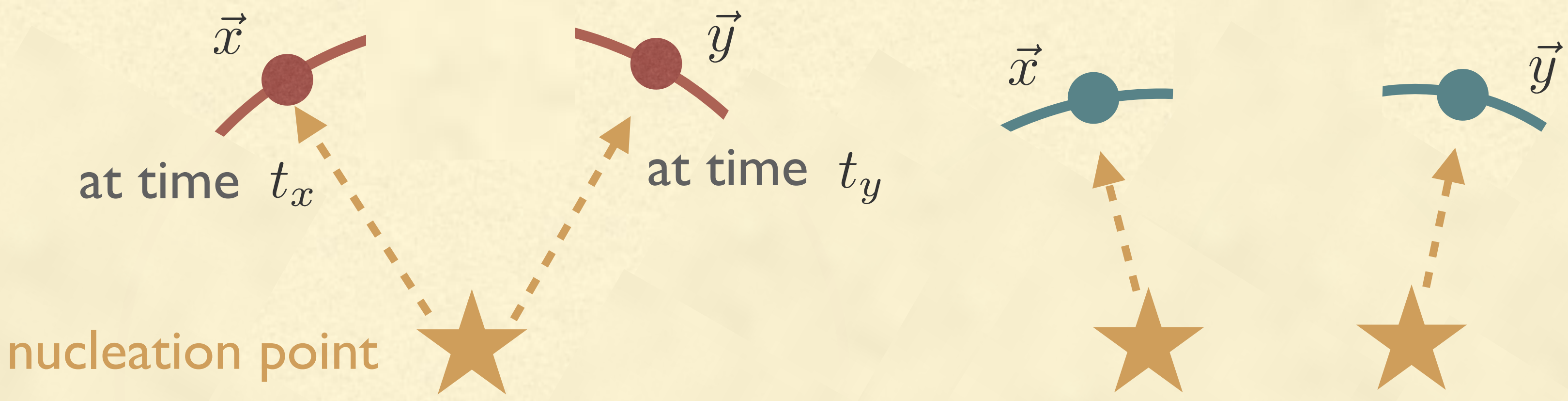
[Jinno & Takimoto '16 & '17]

- Calculating $\langle T(t_x,\vec{x})T(t_y,\vec{y})\rangle_{\rm ens}$ means ...

- Fix spacetime points $x=(t_x,\vec{x})$ and $y=(t_y,\vec{y})$

- Find bubble configurations s.t. EM tensor T is nonzero at x & y ,

i.e. bubble shells are on x & y



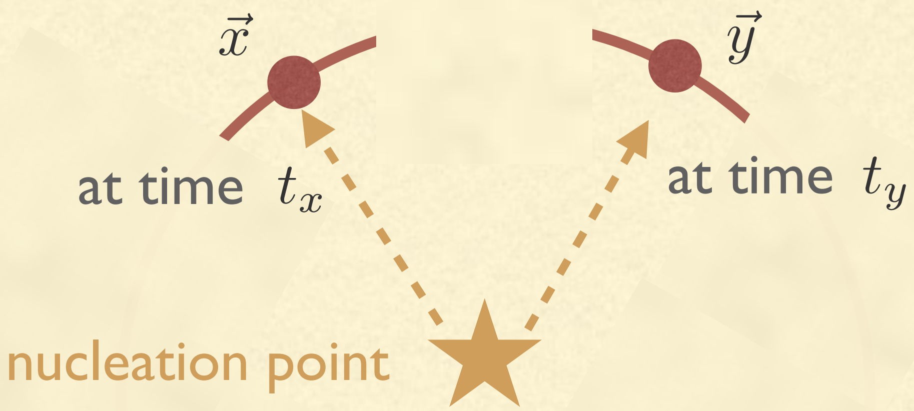
- Calculate $\left\{ \begin{array}{l} \text{probability for such a configuration to occur} \\ \text{value of } T(t_x,\vec{x})T(t_y,\vec{y}) \text{ for such a configuration} \end{array} \right\}$ and sum up

CALCULATION OF $\langle TT \rangle$

[Jinno & Takimoto '16 & '17]

- Only two types of configurations exist :

  - Single-bubble

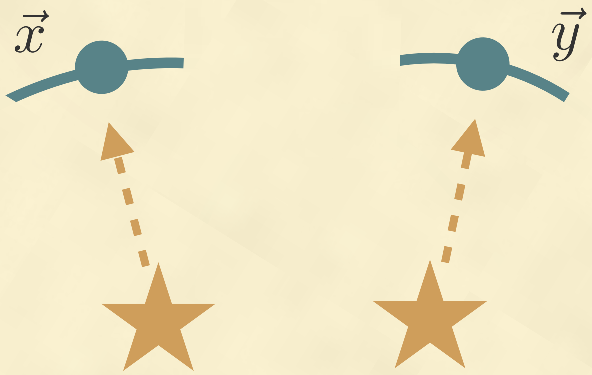


$\vec{x}$ $\vec{y}$

at time t_x at time t_y

nucleation point

  - Double-bubble



$\vec{x}$ $\vec{y}$

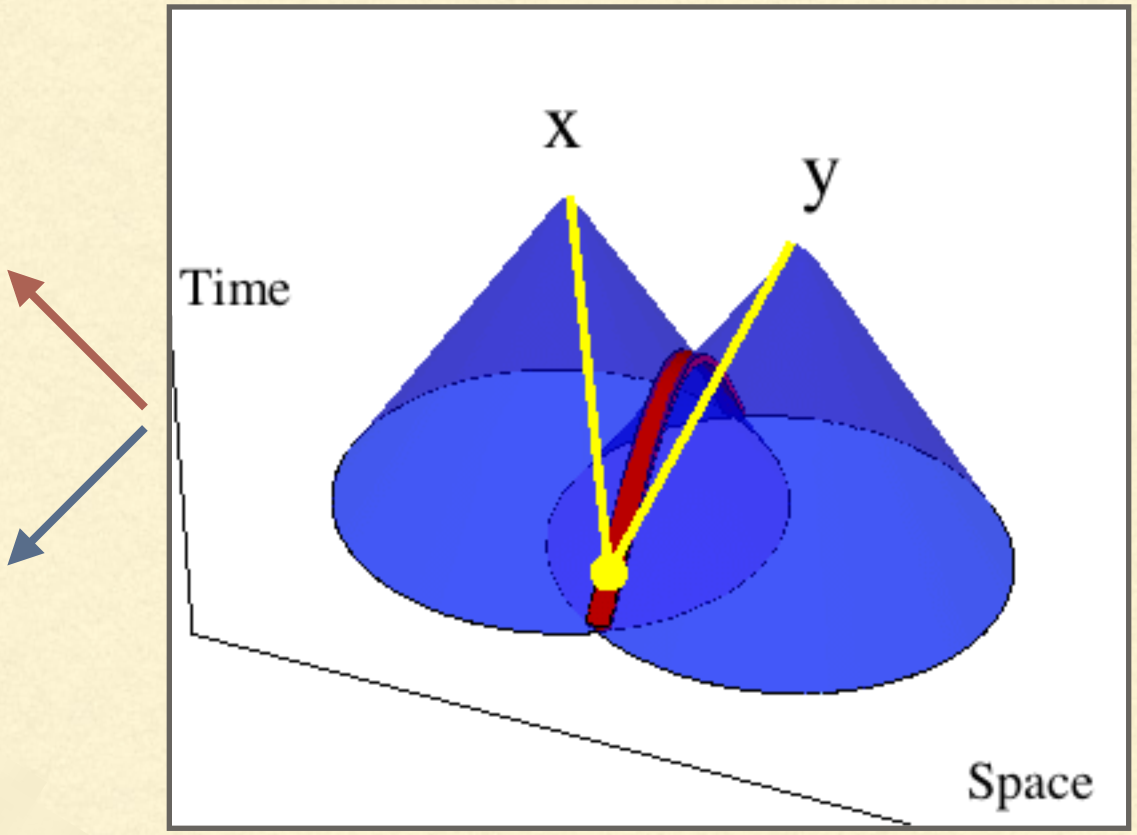


ILLUSTRATION: SINGLE-BUBBLE SPECTRUM

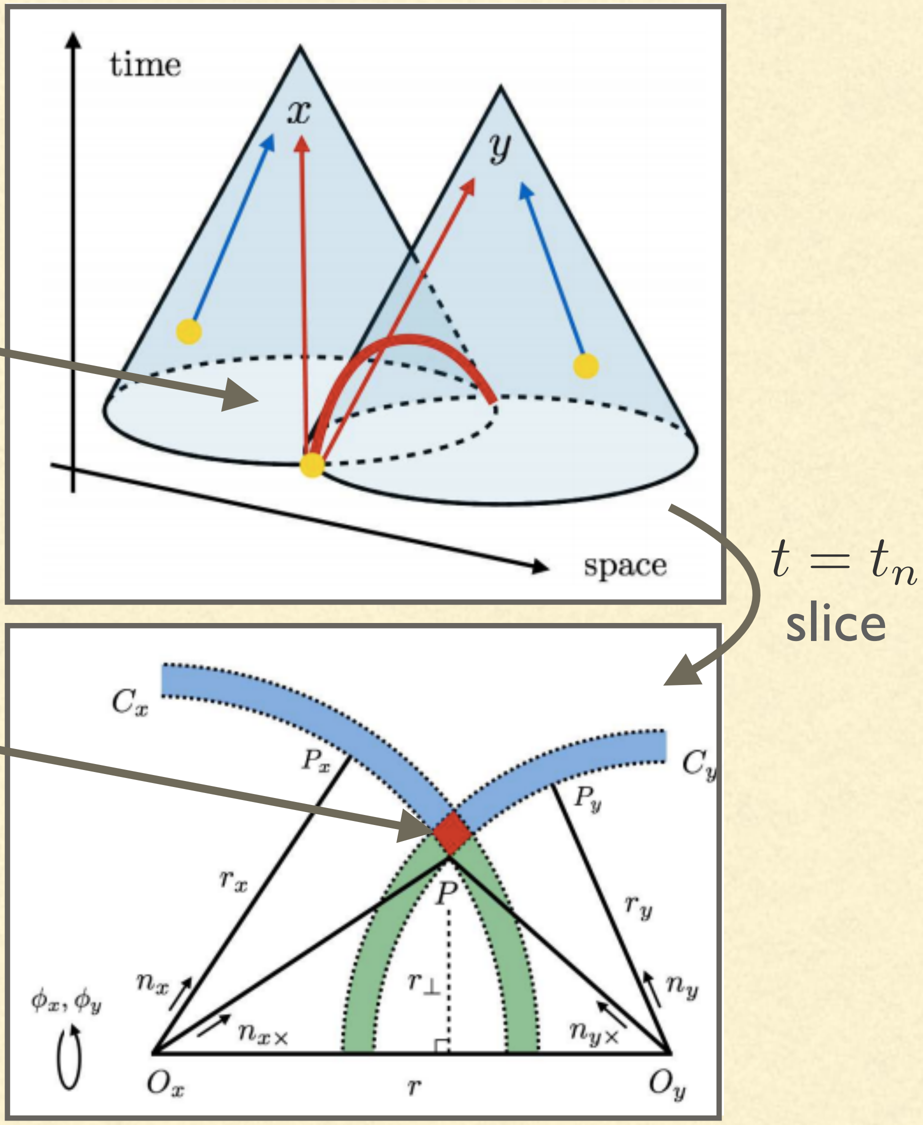
- Necessary and sufficient conditions

  - No bubbles nucleate inside past cones

    (probability denoted by $P(x,y)$)

  - One bubble nucleates inside the red diamond

    within infinitesimal time interval $t_n \sim t_n + dt_n$



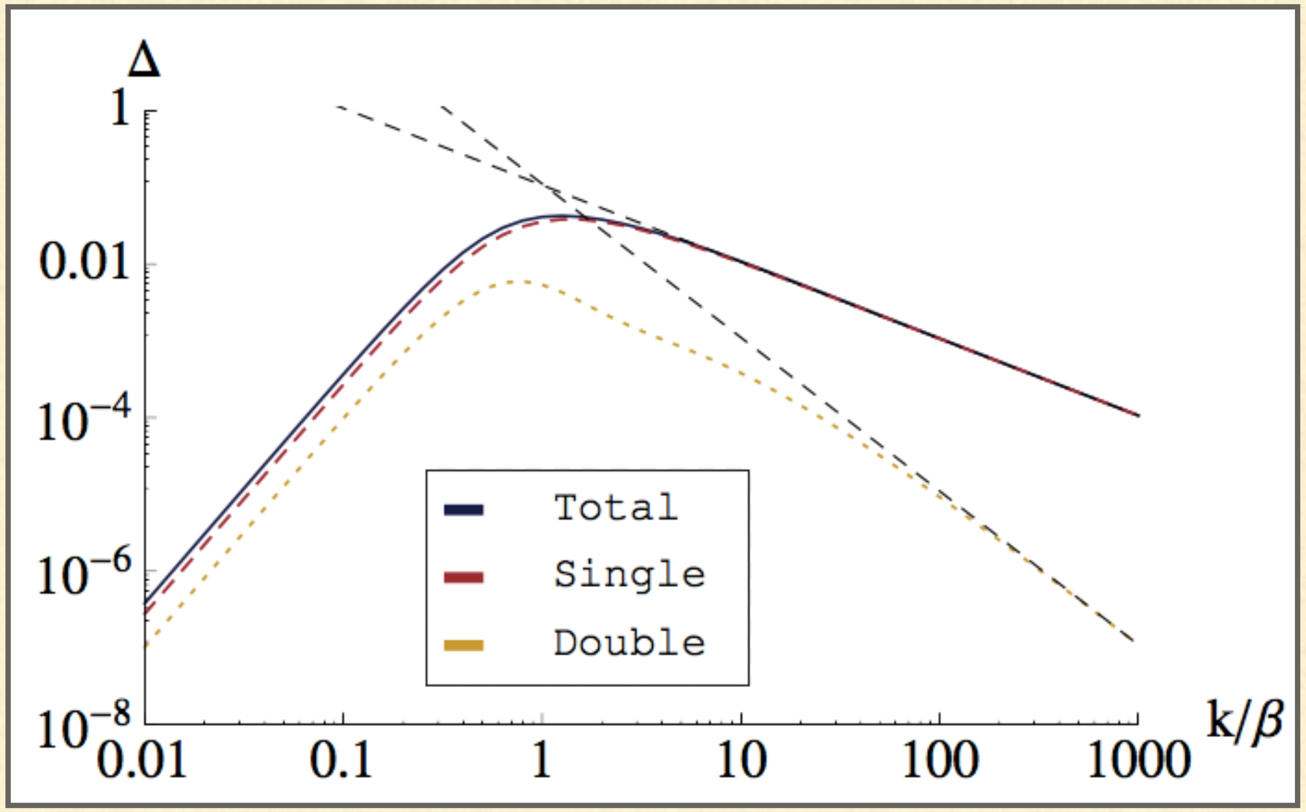
- Resulting expression

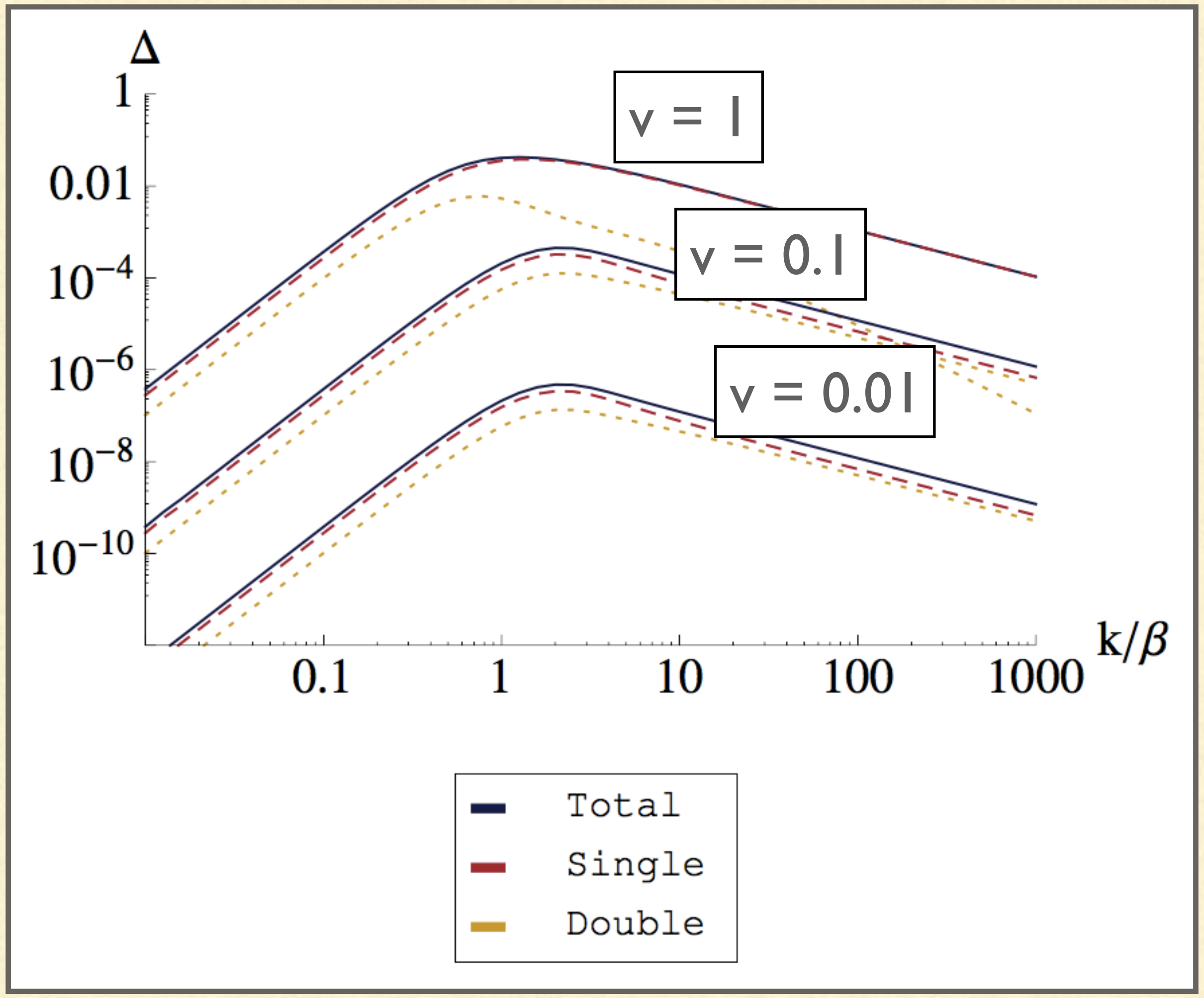
$$\langle T(x)T(y)\rangle^{(s)}_{\rm ens}$$

$$= P(x,y)\int dt_n \left(\begin{array}{l}\text{prob. for one bubble}\\ \text{to nucleate}\\ \text{in the red diamond}\end{array}\right)\left(\begin{array}{l}\text{value of } T(x)T(y)\\ \text{realized in each case}\end{array}\right)$$

NUMERICAL RESULT FOR ENVELOPE CASE

- Consistent with numerical simulation within factor ~2

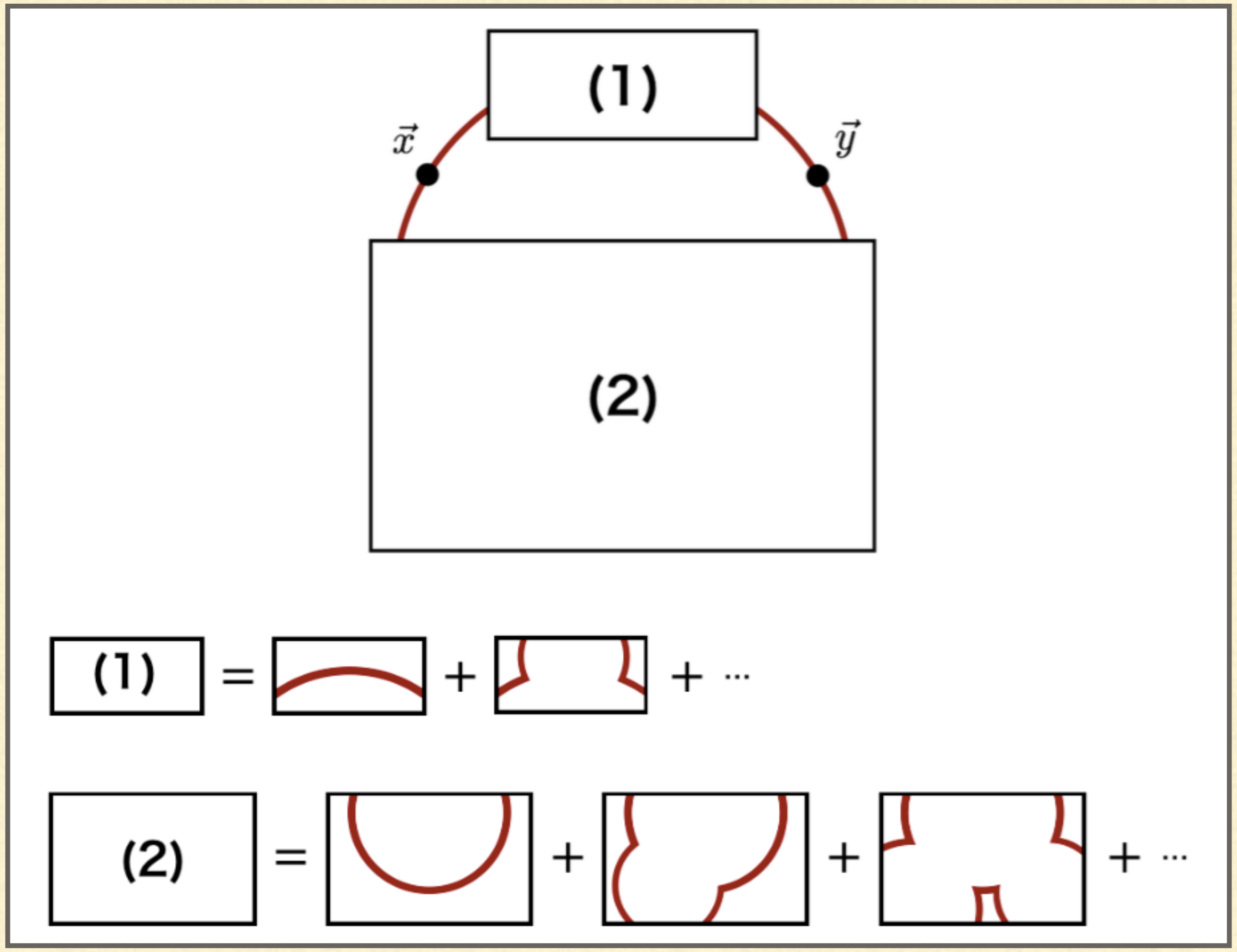




[Jinno & Takimoto '16]

WHY SINGLE-BUBBLE MATTERS

- Illustration with envelope



[Jinno & Takimoto, PRD95 (2017)]

- Two bubble-wall fragments must remain uncollided until they reach x and y

- Other parts of the bubble might have collided already

- In this sense, breaking of spherical sym. is automatically taken into account

SELF INTRODUCTION

- Name : Ryusuke Jinno / 神野 隆介 / 진노 류스께
- Career : 2016/3 : Ph.D @ Univ. of Tokyo (supervised by Takeo Moroi)

  2016/4-8 : JSPS fellow @ KEK, Tsukuba, Japan

  2016/9- : Research Fellow @ IBS-CTPU, Korea
- Research interest & recent works (~ 1 year)

  - Inflation : Hillclimbing inflation (unexplored branch of inflatoinary attractor)

    Hillclimbing Higgs inflation (new realization of Higgs inflation)
  - (P)reheating : Preheating in Higgs inflation (discovery of main preheating channel)
  - Gravitational waves : Analytic approach to GW poduction