# ANOMALY CANCELLATION AND GLOBAL CONSISTENCY

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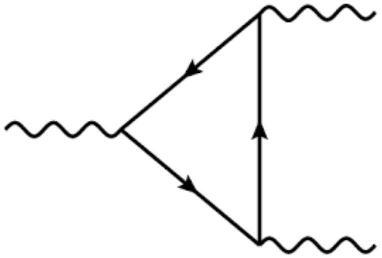
# Chiral anomaly

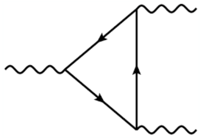
- Chiral anomaly [Adler] [Bell, Jackiw]



$$D^\mu j^{5,a}_\mu = -g^2 \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\rho\sigma} \mathrm{tr}(t_a\{t_b, t_c\})$$

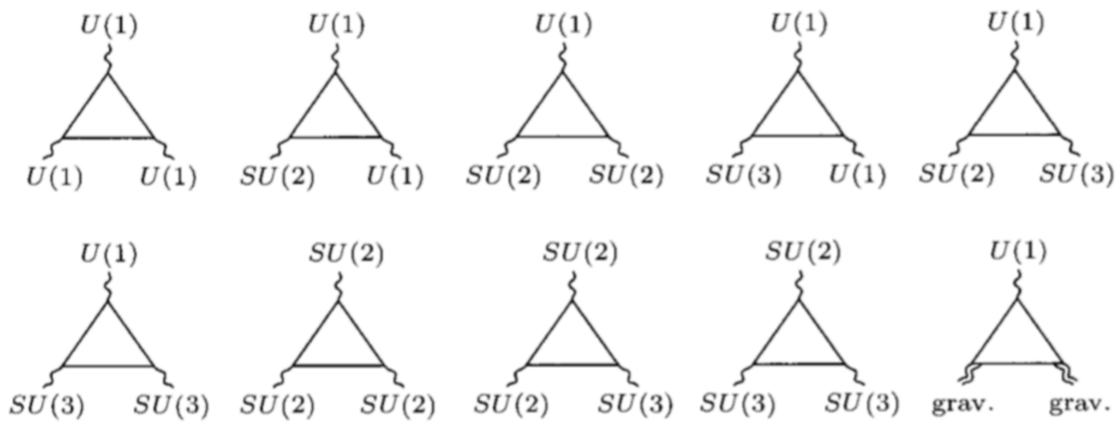
- Consistency: axial current conservation vs gauge symmetry
- They can be traded, but cannot be simultaneously removed.

- Cancellation: constrains low energy spectrum in terms of charge

$\Sigma$  = 0

# Guiding principle

- Good guiding principle of model building.
- Ex. $SU(3)\times SU(2)\times U(1)$



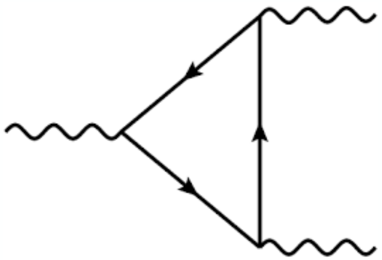

Constrains the spectrum uniquely

- $(3,2)_{1/6}$, $(3^*,1)_{2/3}$, $(3,1)_{1/3}$, $(1,2)_{1/2,}$, $(1,1)_{-1}$, $q$, $u^c$, $d^c$ ,$l$, $e^c$
- Singlets $(1,1)_0$ not constrained.

- New physics extensions
  - Grand Unification, SUSY breaking, ...

# Chiral anomaly

- Chiral anomaly [Adler] [Bell, Jackiw]



$$D^\mu j^{5,a}_\mu = -g^2 \epsilon^{\mu\nu\rho\sigma} F^b_{\mu\nu} F^c_{\rho\sigma} \mathrm{tr}(t_a \{t_b, t_c\})$$

$$F \wedge F \equiv \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- Consistency, low energy spectrum

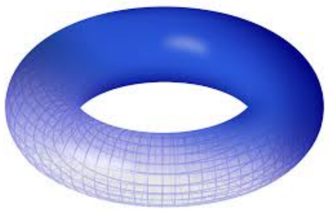
- Field theory is the low-energy limit of string theory.
- If string theory is UV finite (S-matrix, partition function), so is field theory.

- Adler-Bardeen theorem: anomaly is one-loop exact.

# Global consistency condition

In string theory,
anomaly cancellation is promoted to global consistency condition
from one-loop diagram.

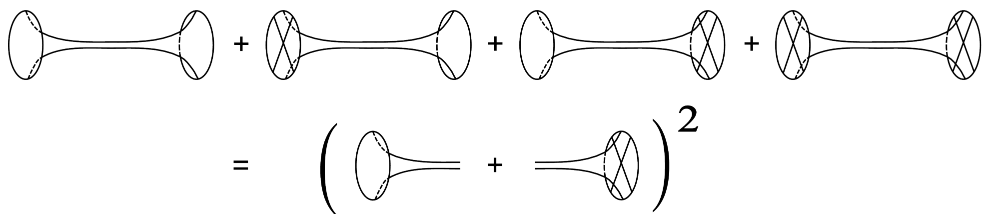
- Closed string:
  - Vacuum-to-vacuum (torus) diagram - modular invariance



  - Condition between gauge symmetry *F* and geometry *R*.
  - *F*, *R* in the low energy theory.

$$\mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F = 0$$

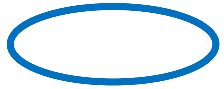
- Open string:
  - Cylinder and its twisted variants – RR tadpole cancellation.



  - Constrains the number of D-branes *n*.

# Modular invariance

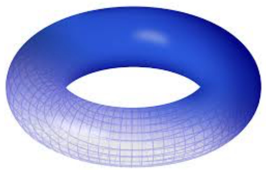
- Point particle on a circle of circumference *l* [Polchinski vol. 1]



$$Z_{S_1}(m^2) = V_d \int \frac{d^dk}{(2\pi)^d} \int_0^\infty \frac{dl}{2l} \exp[-(k^2+m^2)l/2]$$
$$= iV_d \int_0^\infty \frac{dl}{2l} (2\pi l)^{-d/2} \exp(-m^2 l/2) \,.$$

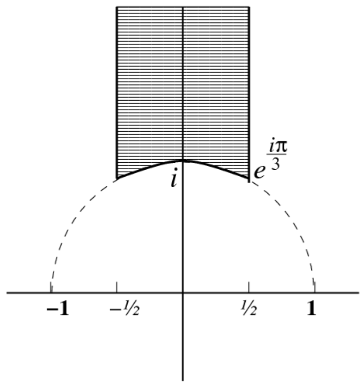
- UV divergent at $l \to 0$.

- String on a circle: no such region $\tau_2 \to 0$ in the moduli space



$$\sum_{i\in\mathscr{H}^\perp} Z_{S_1}(m_i^2) = iV_d \int_R \frac{d\tau d\bar\tau}{4\tau_2} (4\pi^2\alpha'\tau_2)^{-d/2} \sum_{i\in\mathscr{H}^\perp} q^{h_i-1}\bar q^{\tilde h_i-1}$$

- "Modular invariance cuts off the UV divergences without spoiling the space-time gauge invariance."



# Conclusion: partition function and elliptic genus

- Partition function $Z(\tau) = \mathrm{Tr}\, q^{\tilde{L}_0} \overline{q}^{L_0}$ $q = e^{2\pi i \tau}$

- Modular invariance (main topic today) UV finite condition.
- Anomaly free low energy field theory.
- Works well with perturbative heterotic string.

- Strongly coupled heterotic string
  + non-perturbative objects like N5 (M5 in M-theory)

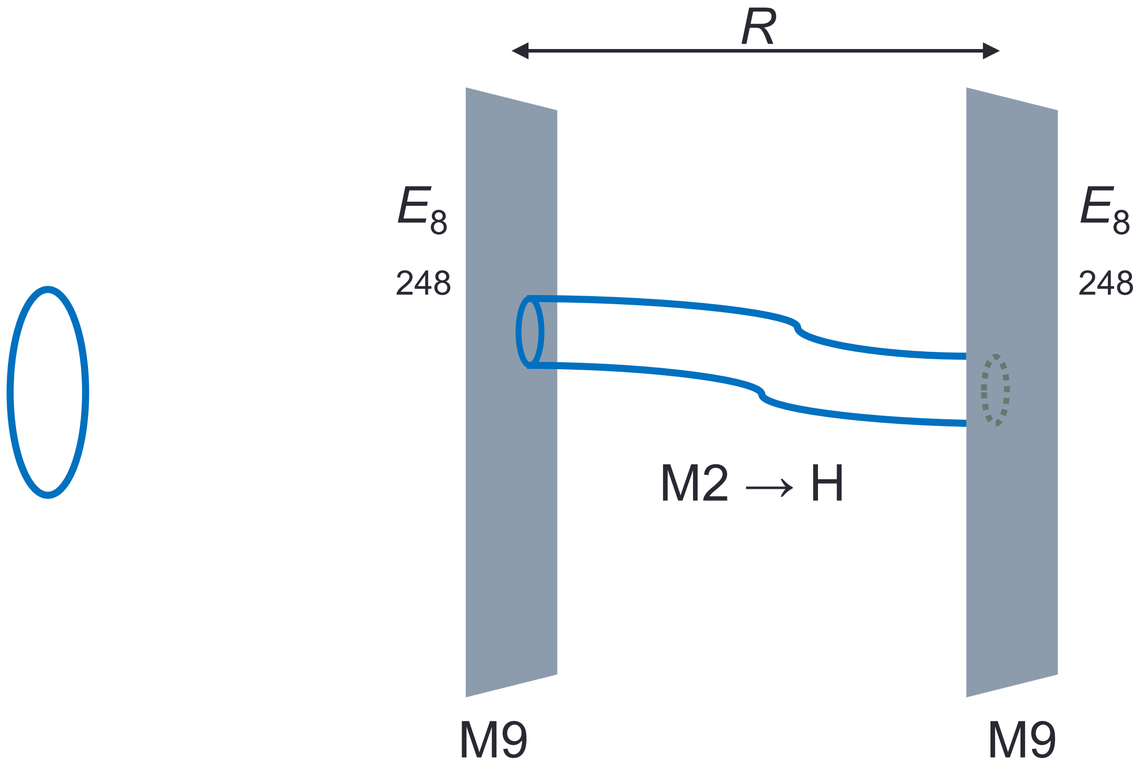
- Elliptic genus $Z(\tau) = \mathrm{Tr}\, q^{\tilde{L}_0} \overline{q}^{L_0} (-1)^F$
- Twisted version often gives us good enough information. Cf. ~~SUSY~~ [Witten]
- Can be calculated using topological string.

- Modular invariance of the elliptic genus gives global consistency condition.

[KSC, S.J.Rey]

# Strongly coupled heterotic string

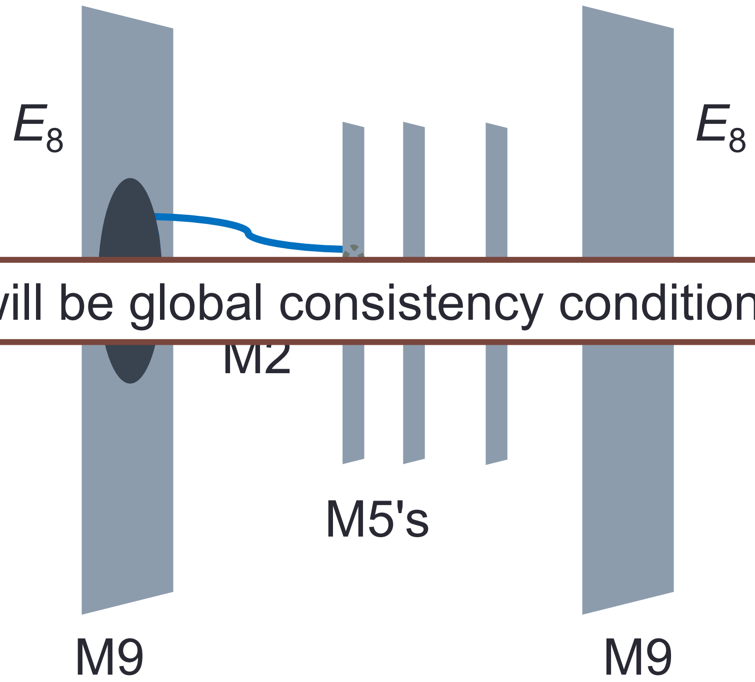
- M-theory on $S^1 / Z_2$ = heterotic string theory [Horava, Witten]
- At the ends of interval we have separated two $E_8$'s
- Heterotic string = M2-brane stretched between them.
- String tension & gauge coupling is proportional to $R$



## Small instantons

- Solution to self-dual equation $F = {}^*F$ : $\mathrm{codim}_{\boldsymbol{R}} = 4$, $F \wedge F =$ integer
- Global consistency condition $dH = F_1 \wedge F_1 + F_2 \wedge F_2 - R \wedge R = 0$
- $E_8 \to G$ + commutant unbroken group

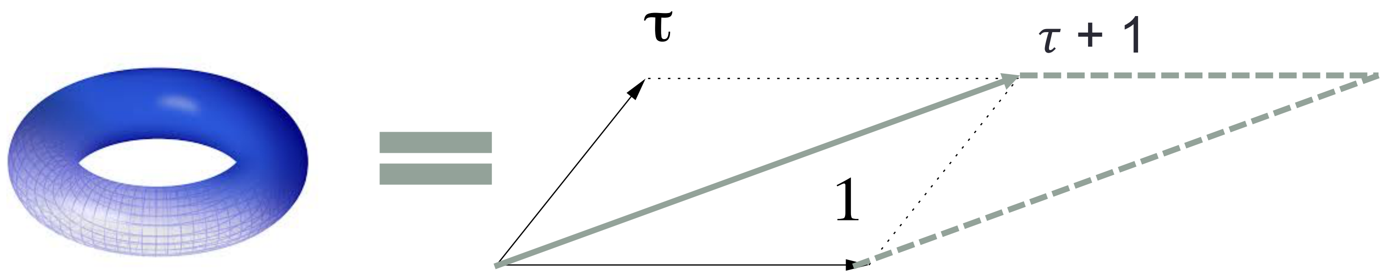
1. Shrinking to zero size = $E_8$ unbroken
2. Phase transition: NS5 detached into bulk. [Ganor, Hanany]




What will be global consistency condition in this case?

# Modular form

- Partition function is quasi-modular form.




- *SL*(2,***Z***) $\quad \tau \to \dfrac{a\tau + b}{c\tau + d} \qquad a, b, c, d \in \mathbf{Z}, \quad ad - bc = 1$

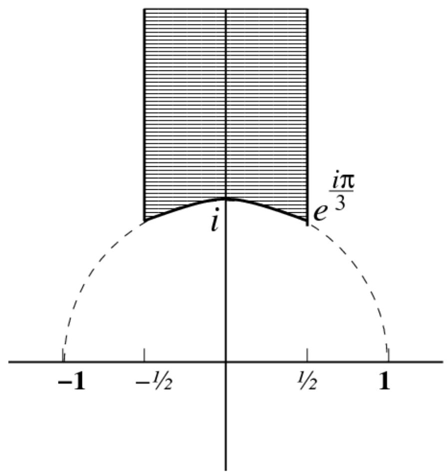
- Generated by

$$T{:}\,\tau \to \tau + 1,\ S{:}\,\tau \to -\frac{1}{\tau}$$

- Modular form

$$f\left(\frac{a\tau + b}{c\tau + d}\right) = (c\tau + d)^k f(\tau)$$

of weight *k*




- Partition function Z($\tau$) is quasi-modular form. (modular invariance + anomaly)

# Bianchi identity

- The partition function/elliptic genus is
- Invariant under *T*
- Non-invariant under *S* $$Z\left(-\frac{1}{\tau}\right) = Z(\tau)\exp\left(\frac{\pi i}{\tau}(\mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F)\right).$$
- The non-invariant piece can be expressed in term of $F \wedge F$, $R \wedge R$

Anomaly

- Non-invariant piece: Bianchi identity for Kalb-Ramond field

$$dH = \mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F$$

- Anomaly cancellation requires it to be zero $dH = 0$.

# Holomorpy vs modular invariance

- The building blocks of partition function/elliptic genus

$$\vartheta_1(z) = \eta(\tau)^3(2\pi i z)\exp\left(\sum_{k=1}^{\infty}\frac{B_{2k}}{(2k)(2k)!}E_{2k}(2\pi i z)^{2k}\right),$$

$E_{2k}$ :$2k^{\text{th}}$ Eisenstein series.

- In the holomorphic basis. All is modular form except $E_2$.

$$E_2\left(\frac{a\tau+b}{c\tau+d}\right) = (c\tau+d)^2E_2(\tau) - \frac{6ic}{\pi}(c\tau+d)$$

- The two can be traded.

$$\hat{E}_2(\tau,\bar{\tau}) = E_2(\tau) - \frac{6i}{\pi(\tau-\bar{\tau})}$$

$$\hat{E}_2\left(\frac{a\tau+b}{c\tau+d},\frac{a\bar{\tau}+b}{c\bar{\tau}+d}\right) = (c\tau+d)^2\hat{E}_2(\tau,\bar{\tau}).$$

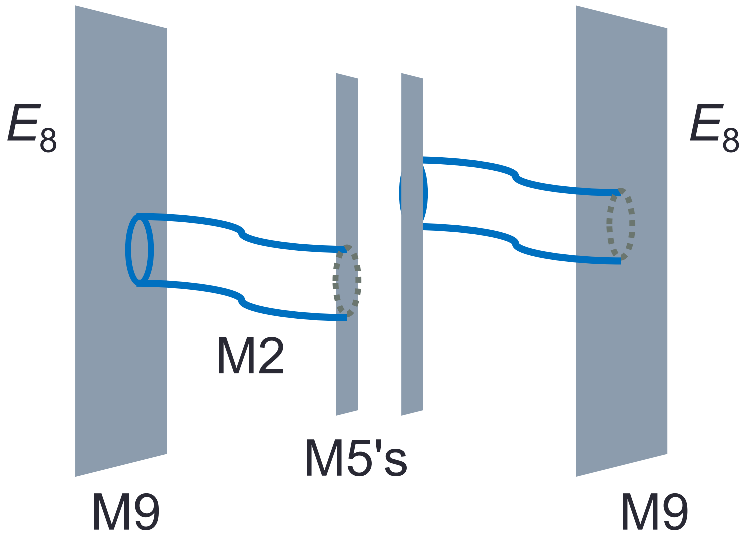
- But homomorphy lost. Cf. anomalous theory.
- Non-invariant phase: consistent relation among *F, R, n.*

# Elliptic genus, with M5-branes

- For strongly coupled heterotic string with M5-branes, we need elliptic genus

$$Z = \mathrm{Tr}\, q^{\tilde{L}_0} \overline{q}^{L_0} (-1)^F$$

- We can calculate it using defect operator via topological vertex [Minahan, Nemeschansky, Vafa, Warner] [Iqbal, Kozcaz, Vafa] [Haghighat et al.]




With probe M2-branes
M9-M2 vertex $D^{M9}_{L,\nu}, D^{M9}_{\nu^t,R}$
M2-M5-M2 vertex $D^{M5}_{\nu^t\mu}$

To regularize IR divergence we introduce Ω-deformation.

$$Z = D^{M9}_{L,\nu_1} D^{M5}_{\nu_1^t \nu_2} D^{M5}_{\nu_2^t \nu_3} \dots D^{M9}_{\nu_n^t,R}$$

# Application [KSC, S.J.Rey]

- The number of M5-branes *n*
- The probe M2-branes parameterized by Young tableaux $\nu_a$

$$Z = D^{M9}_{L,\nu_1} D^{M5}_{\nu_1^t \nu_2} D^{M5}_{\nu_2^t \nu_3} \dots D^{M9}_{\nu_n^t, R}$$

$$\prod_{a=1}^{n} D^{M5}_{\nu_a^t \nu_{a+1}} \left(-\frac{1}{\tau}\right) = \prod_{a=1}^{n} D^{M5}_{\nu_a^t \nu_{a+1}}(\tau) \exp\left[\frac{\pi i}{\tau}\left((|\nu_a| - |\nu_{a+1}|)^2 \epsilon_1 \epsilon_2 - (|\nu_a| + |\nu_{a+1}|)\epsilon_3 \epsilon_4\right)\right].$$

- With

$$Z\left(-\frac{1}{\tau}\right) = Z(\tau) \exp\left(\frac{\pi i}{\tau}(\mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F)\right).$$

Depends only on the size of the Young tableaux

- this is interpreted as

$$dH = \mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F = \sum_{i \in \mathrm{NS5}} \delta^4(z - z_i)$$

- cf. NS5 brane from small instanton.
  No information other than four-form. From *F*∧*F*

# Summary

- Anomaly cancellation is lifted to global consistency condition in string theory.
- 1-loop: embedded in partition function / elliptic genus
- We need also the effect of strongly coupled heterotic string and non-perturbative objects like NS5-branes.
- Requiring both holomorpy and modular invariance, we get global consistency condition.

$$dH = \operatorname{tr} R \wedge R - \operatorname{tr} F \wedge F = \sum_{i \in \mathrm{NS5}} \delta^4(z - z_i)$$

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