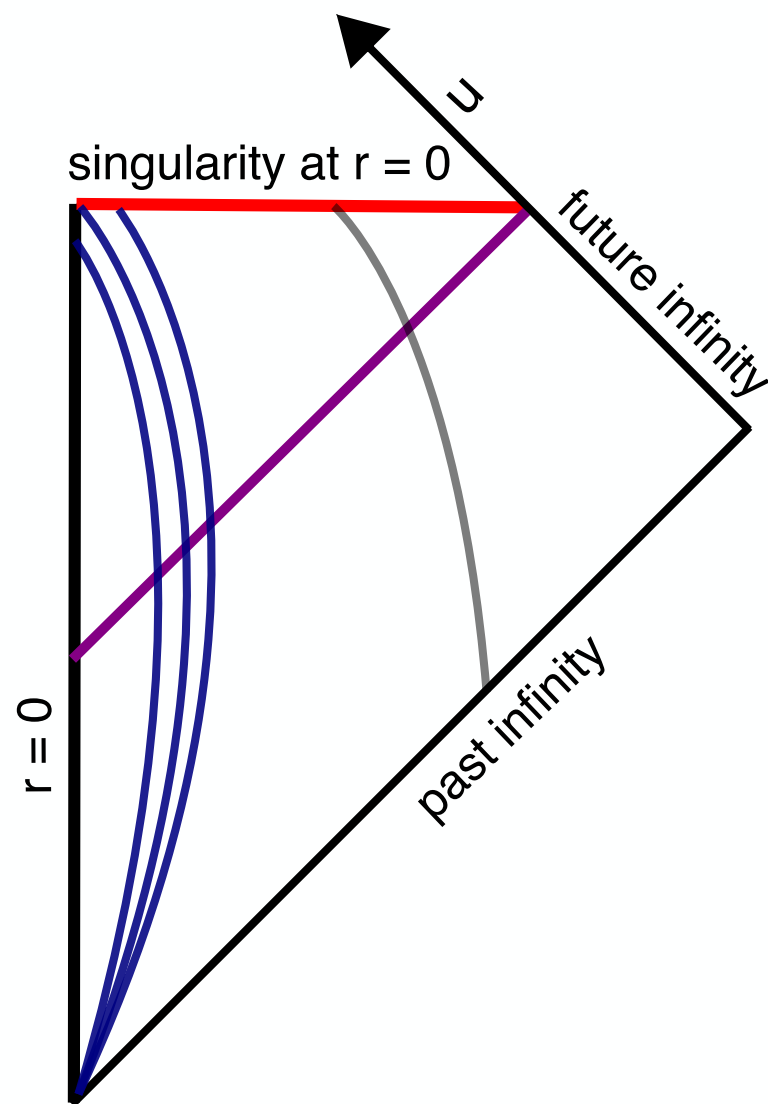


# **Vacuum Energy, Hawking Radiation and Black-Hole Geometry**

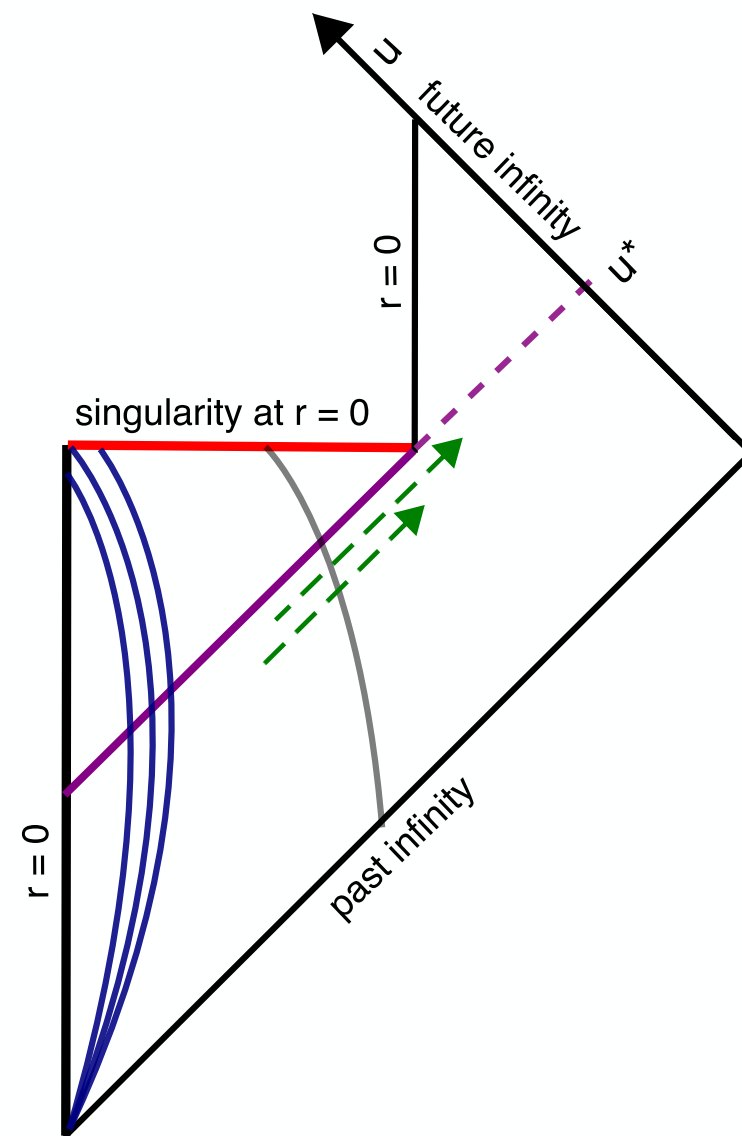
Pei-Ming Ho  
Physics Department  
National Taiwan University



## classical BH

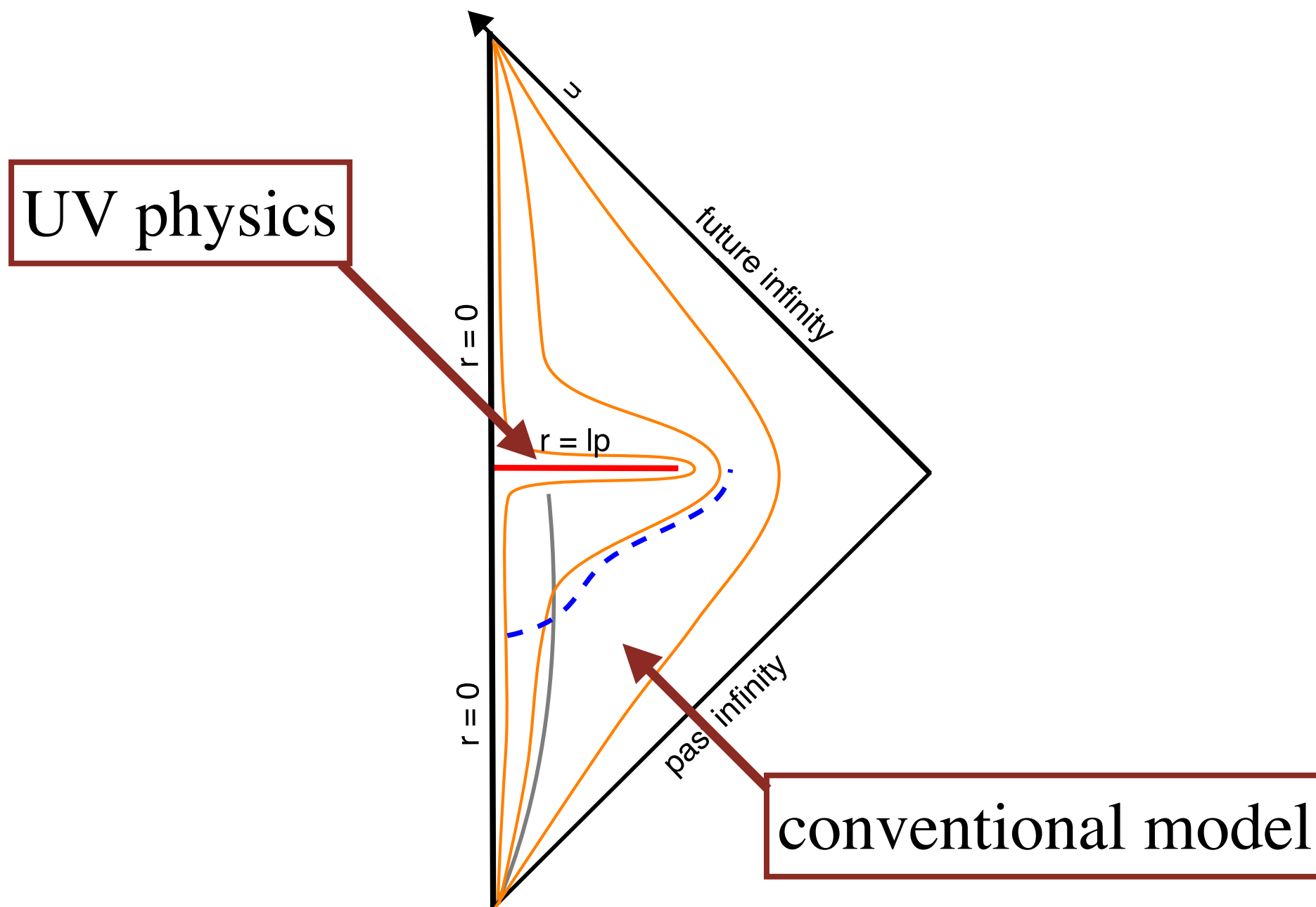


## conventional model



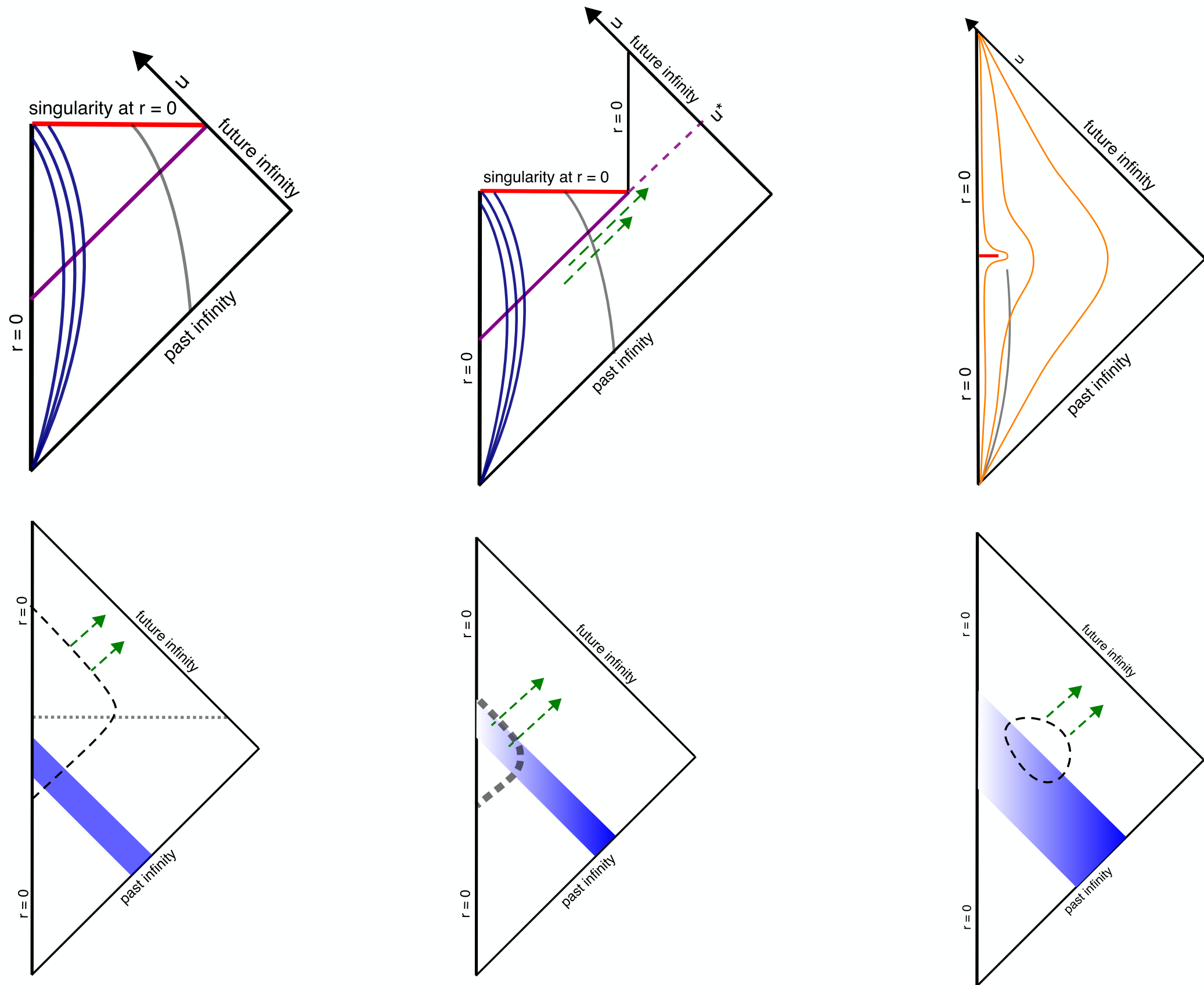
assuming no back reaction  
for Hawking radiation.  
[Hawking 1976]

# Conventional model with singularity resolved



No event horizon if no singularity.

# Which is correct?





misconception

## “BH has event horizon”

- The notion of event horizon is global.  
No local implication.

- It needs a (to-be-resolved) singularity.

- Penrose diagram  $\neq$  local physics

$$10^{20} \text{ years} \neq \infty$$

$$a^3/\kappa \sim 10^{67} \text{ years for solar mass}$$



misconception

# **“Quantum correction too small to remove event horizon”**

- Hawking radiation makes a big difference.
- Accumulated tiny local changes can have large effect.
- $M$  vs  $M^3$  argument does not work.
- Quantum vacuum energy is only locally weak.  
Event horizon is not a local property.



misconception

# **“Apparent horizon must exist.”**

- Uniqueness demands spherical symmetry.
- Outer apparent horizon needs ingoing negative energy.  
(violation of weak energy condition)
- Astrophysical BH = huge red-shift factor



misconception

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The only thing about a black hole that does not need to be justified.



# Information Paradox

- String theory resolves singularity  
→ no event horizon → problem solved?
- Mathur:  
“Niceness conditions” must be violated. (e.g. firewall)  
 $O(1)$  correction needed at horizon → fuzzball
- How does low-energy effective theory break down?  
Why are the high energy events?



# Our Task

- What is really the geometry by time evolution according to semi-classical Einstein equations (with back reaction of vacuum energy)?

$$G_{\mu\nu} = \kappa T_{\mu\nu}^{class} + \kappa \langle \hat{T}_{\mu\nu} \rangle$$

- High energy events?



# Our Task

- What is really the geometry by time evolution according to semi-classical Einstein equations (with back reaction of vacuum energy)?

$$G_{\mu\nu} = \kappa T_{\mu\nu}^{class} + \kappa \langle \hat{T}_{\mu\nu} \rangle$$

- High energy events?

Horizon is not an assumption!



# Basic Assumptions

- Semi-classical Einstein equation:

$$G_{\mu\nu} = \kappa T_{\mu\nu}^{class} + \kappa \langle \hat{T}_{\mu\nu} \rangle$$

- Spherical symmetry.

$$ds^2 = - C(u, v) du dv + r^2(u, v) d\Omega^2$$

- Static black hole?

Black-hole formation/evaporation?



areal radius



# How can there be anything new?

- Nonlinear equations hard to solve exactly.
- conventional approximation:  
classical collapse  $\rightarrow$  horizon + evaporation  
ignore back reaction of vacuum energy  
(e.g. Schwarzschild metric outside matter)



# What's wrong with naive perturbation

- Schwarzschild metric is good approximation at a few Planck lengths away from the Schwarzschild radius (in vacuum).
- Perturbative expansion in  $\kappa$  is different when
$$r \sim a + O(\kappa/a)$$



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$$ds^2 = - (1 - a/r) dt^2 + \frac{dr^2}{1 - a/r} + d\Omega^2$$



# Conclusion Preview

- Back reaction of vacuum  $T_{\mu\nu}$  is important.
- 3 classes of static black-hole-like solutions with spherical symmetry
- 3 or more classes for gravitational collapses



# Static Black Holes

- The energy-momentum operator  $\langle T_{\mu\nu} \rangle$  in curved spacetime is different for different QFTs.
- Conformal matters are convenient because of trace anomaly.
- 2D massless field [Davies-Fulling-Unruh 1976][PMH-Matsuo 17 (1)]  
[PMH-Matsuo 17 (2)]
- 4D conformal matter [Christensen-Fulling 1977][PMH-Kawai-Matsuo-Yokokura 18]
- Literature [Solodukhin 04, 06; Fabbri-Farese-Navarro-Salas-Olmo-Sanchis-Alepuz 05 (1), 05 (2)]



# Energy-Mom. Tensor in 4D

[PMH-Kawai-Matsuo-Yokokura 18]

- Energy-momentum tensor in 4D is constrained by
  1. Conservation law
  2. Spherical symmetry
  3. Time independence
  4. Trace anomaly
- There is still one functional degree of freedom.
- Conclusion: [PMH-Kawai-Matsuo-Yokokura 18]  
**Event horizon needs fine-tuning!**



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- Conclusion: [PMH-Kawai-Matsuo-Yokokura 18]

**Event horizon needs fine-tuning!**

Not surprising?



# 3 Classes Of Solutions

[PMH-Matsuo 17 (1)][PMH-Matsuo 17 (2)]

[PMH-Kawai-Matsuo-Yokokura 18]

1. event horizon: fine-tuning
2. “wormhole-like”: local minimum in areal radius  $r$  (no event horizon).
3. neither.

Buchdahl’s theorem ( $P \rightarrow \infty$  if  $R < 9a/8$ )

is in general violated (due to negative energy  
— weak energy condition violated).

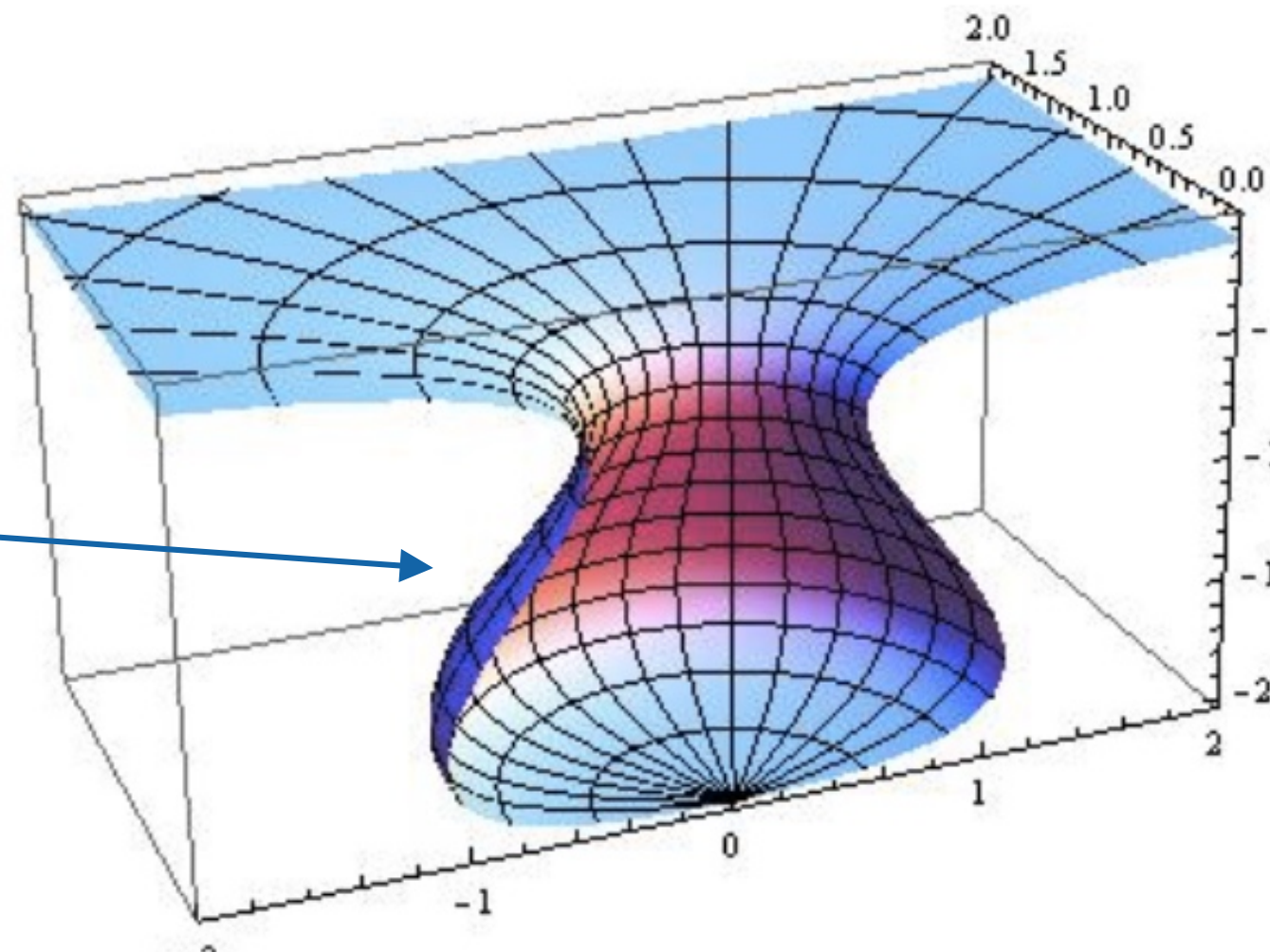


# Wormhole-Like Solution: local minimum in areal radius

- ▶ vacuum energy + incompressible fluid
- ▶ No horizon
- ▶ *Large pressure* when surface is not well outside the neck.

[PMH-Matsuo 17 (1), (2)]

Radial distance is short.



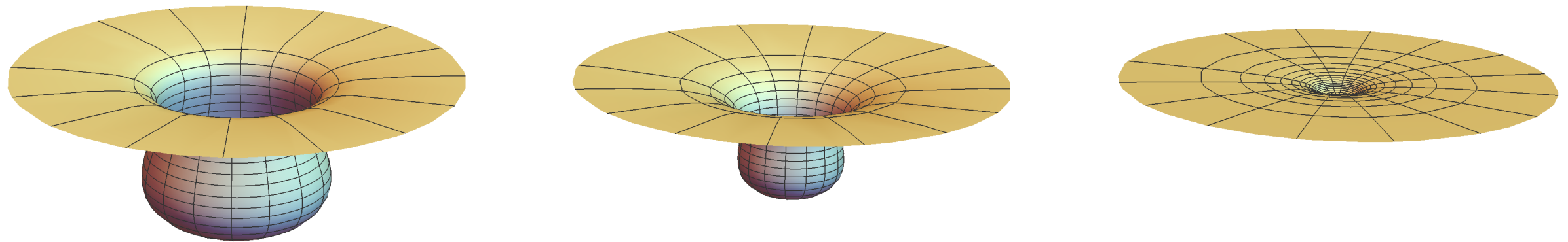


# Dynamical Cases: 3 Scenarios

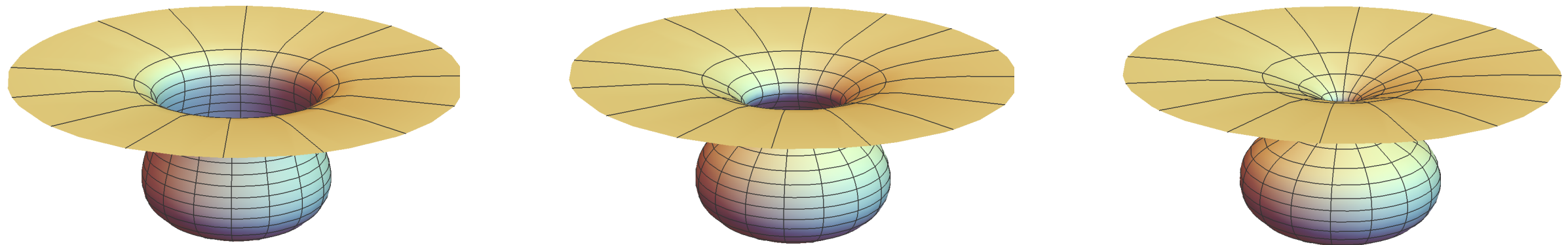
- apparent horizon: Collapsing wormhole evaporated  
[PMH-Matsuo 18]
- apparent horizon: Collapsing wormhole (nearly) decapitated [Parentani-Piran 1994]
- no apparent horizon: KMY Model  
[Kawai-Matsuo-Yokokura 13][Kawai-Yokokura 14, 15, 17][PMH 15, 15, 16]
- Other scenarios?



## **Wormhole evaporated** [PMH-Matsuo 18]



## **Wormhole decapitated** [Parentani-Piran 94]



information not lost

holography lost  $\leq$  macroscopic negative energy



# Conclusion

- Semi-classical Einstein equations solved ...
- Back reaction of vacuum energy
  - different classes of near-horizon geometries
- Near-horizon geometry sensitive to states.
- “High-energy events” for all examples.
- Generic statement about dynamical case?



**Thank you.**