Emergence of chiral zero modes in the type IIB matrix model

Asato Tsuchiya
Shizuoka Univ.

East Asia Joint Workshop on Fields and Strings
@KIAS, November 7th, 2018

Based on collaboration with
Kohta Hatakeyama (Shizuoka U.), Akira Matsumoto (Sokendai),
Jun Nishimura (Sokendai, KEK), Atis Yosprakob (Sokendai)

Introduction

Type IIB matrix model

IKKT ('96)

A proposal for nonperturbative formulation of superstring theory

$$S = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A^M, A^N][A_M, A_N] + \frac{1}{2}\bar{\psi}\Gamma^M[A_M, \Psi]\right)$$

$N \times N$ Hermitian matrices

A_M : 10D Lorentz vector $\quad (M = 0, 1, \ldots, 9)$

Ψ : 10D Majorana-Weyl spinor

$\Big\}$ SO(9,1) symmetry

Large-N limit is taken

Space-time does not exist a priori, but is generated dynamically from degrees of freedom of matrices

Euclidean model

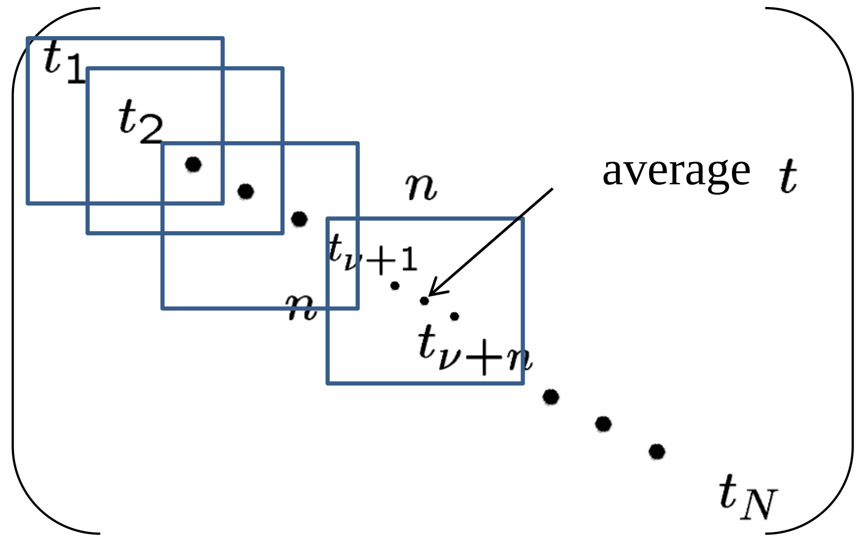
$A_0 = iA_{10}$

Aoki-Iso-Kawai-Kitazawa-Tada ('98) Krauth-Nicolai-Staudacher ('98) Austing-Wheater ('01), …………..

Here we study Lorentzian model

Emergence of time evolution

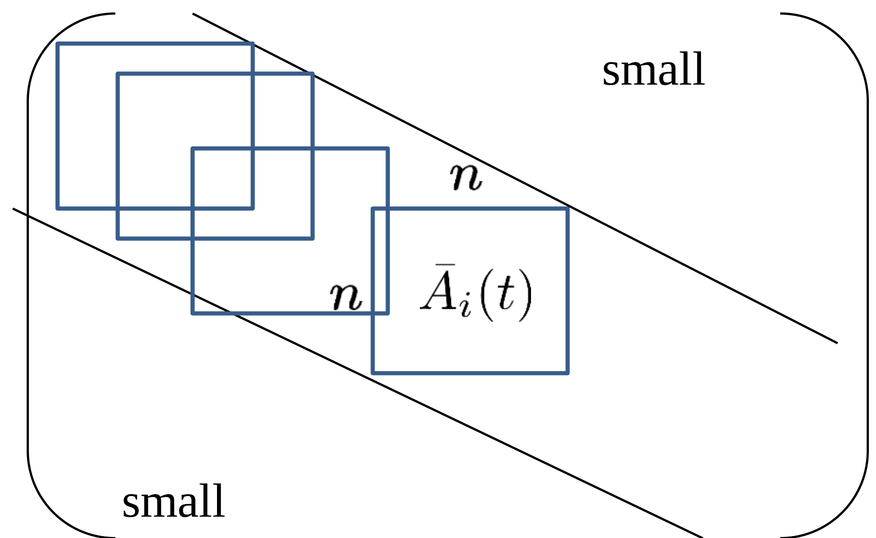
$A_0 =$



$t_1 < t_2 < \cdots < t_N$

These values are dynamically determined

$A_i =$



$(i = 1 \sim 9)$

Band-diagonal structure is observed, which is nontrivial

$\bar{A}_i(t)$ represents space structure at fixed time t

concept of "time evolution" emerges

SSB of SO(9) symmetry

Kim-Nishimura-A.T. ('11)

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\{\bar{A}_i(t)\bar{A}_j(t)\}$$

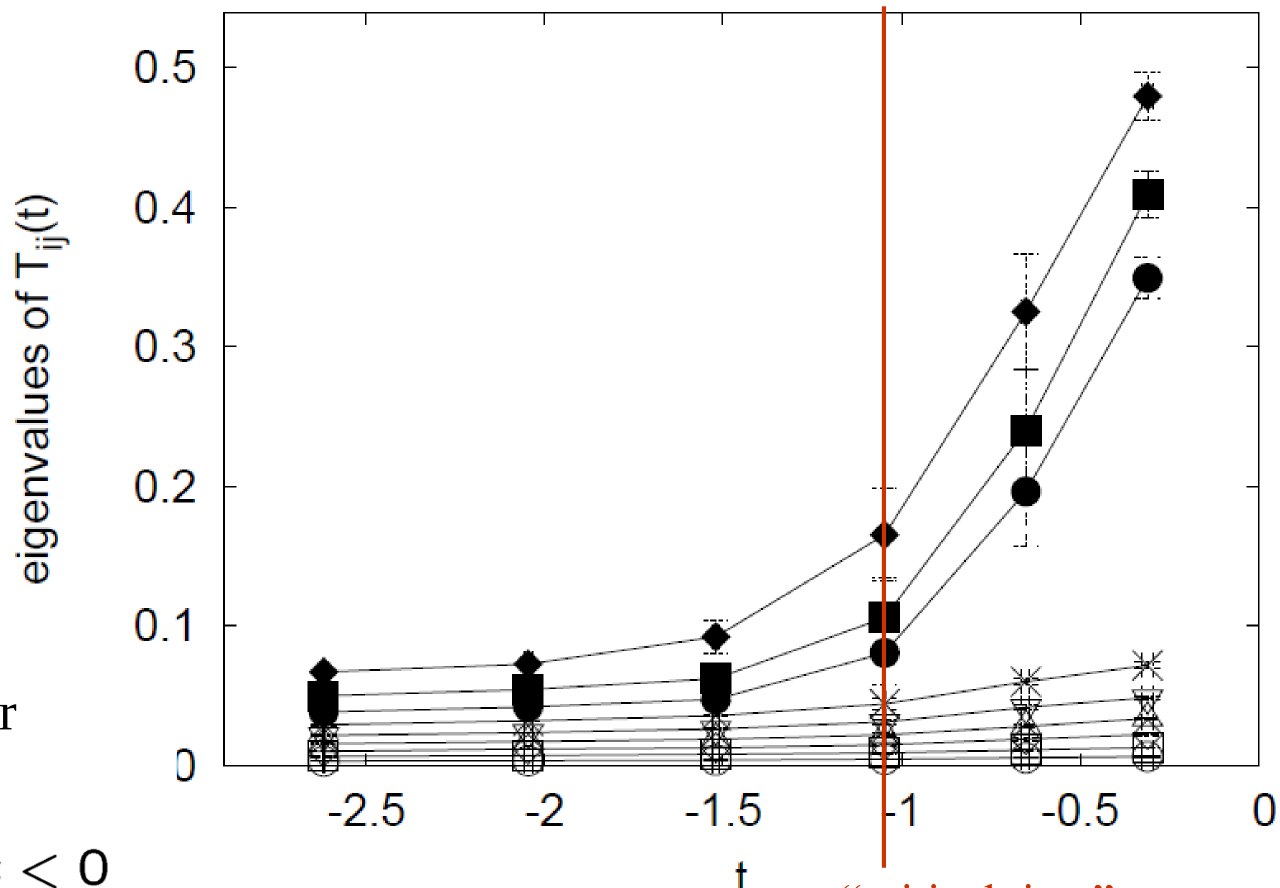
~moment of inertia tensor

$$SO(9) \xrightarrow{\text{SSB}} SO(3)$$

$i, j = 1 \sim 9$

$N = 16$

$\kappa = 4.0$



"critical time" t_c

symmetric under

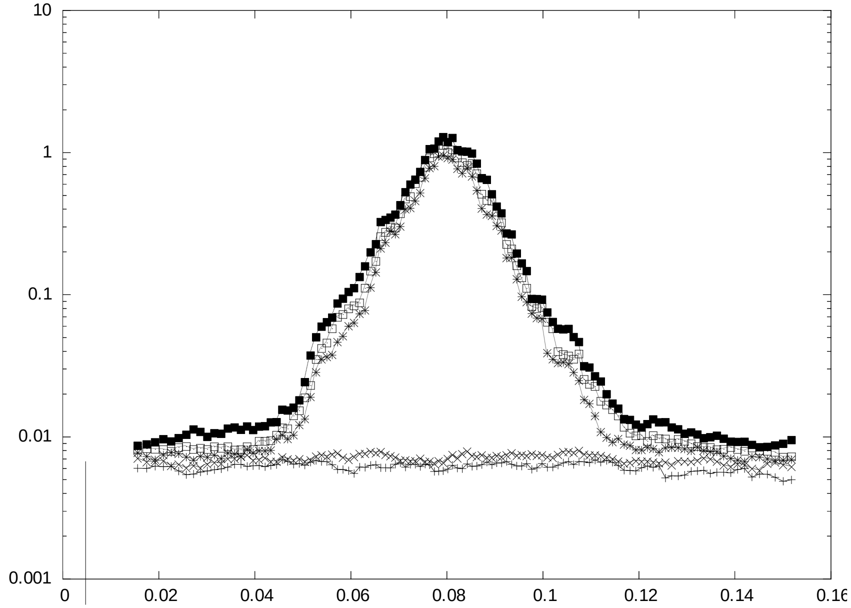
$t \to -t$

we only show $t < 0$

Emergence of expanding (3+1)d universe

Numerical simulation was updated recently by using the complex Langevin method

Nishimura-A.T. ('18)



Order of Planckian time
Exponential expansion

Questions

We are interested in dynamics at late times

- What is (3+1)d space-time structure like?
- Chiral fermions appear at low energy?

$\Rightarrow$ Standard Model

Structure of extra dimensions

Classical dynamics dominates at late times

- The late–time behaviors are difficult to study by direct Monte Carlo methods, since larger matrix sizes are required.
- But the classical equations of motion are expected to become more and more valid at later times, since the value of the action increases with the space expansion.
- We develop a numerical algorithm for searching for classical solutions satisfying the most general ansatz with "quasi direct product structure"

  Stern, Steinacker, Yang,……

- Chiral zero modes are obtained as solutions of EOM.

  Cf.) Aoki-Iso-Suyama ('02)

  A. Chatzistavrakidis, H. Steinacker and G. Zoupanos ('11)

  Nishimura-A.T.('13)   Aoki-Nishimura-A.T.('14)

Plan of the present talk

1. Introduction
2. Analysis of classical EOM
3. Space-time structure and chiral zero modes from classical solutions
4. Conclusion and outlook

Analysis of classical EOM

Defining the Lorentzian model

- Lorentzian model

$$S_b \sim \boxed{2\mathrm{Tr}([A_0, A_i]^2)} \boxed{- \mathrm{Tr}([A_i, A_j]^2)}$$

opposite sign
not bounded below

Introduce IR cutoffs

$$\frac{1}{N}\mathrm{Tr}(A_0)^2 \leq \kappa$$
$$\frac{1}{N}\mathrm{Tr}(A_i)^2 \leq L^2$$

removed in $N \to \infty$

Equation of motion

$$S = -\frac{1}{4}\mathrm{Tr}([A^M, A^N][A_M, A_N]$$

$$\boxed{\begin{aligned}&[A^M,[A_M,A_0]]\\&[A^M,[A_M,A_i]]\end{aligned}} \begin{aligned}&+\alpha A_0 = 0\\&-\beta A_i = 0 \quad (i=1,\ldots,9)\end{aligned}$$

$\alpha,\ \beta$: Lagrange multiplier

constraints

$$\boxed{\begin{aligned}&\frac{1}{N}\mathrm{Tr}(A_0^2) = \kappa\\&\frac{1}{N}\mathrm{Tr}(A_i^2) = 1\end{aligned}}$$

corresponding to IR cutoffs

Configuration with "quasi direct product structure"

Nishimura-A.T.('13)

$$A_\mu = \boxed{X_\mu} \otimes \boxed{M} \quad (\mu = 0, \ldots, 3)$$

$$A_a = \boxed{1_{N_X}} \otimes \boxed{Y_a} \quad (a = 4, \ldots, 9)$$

$M = 1$: direct product space-time

$N_X \times N_X \qquad N_Y \times N_Y \qquad N = N_X \times N_Y$

Each point on (3+1)d space-time has the same structure in the extra dimensions

This ansatz is compatible with Lorentz symmetry to be expected at late time

$$O_{\mu\nu} X_\nu = g[O]\, X_\mu\, g[O]^\dagger$$

$$O \in \mathrm{SO}(3,1) \qquad g[O] \in SU(N_X)$$

Chiral fermions in type IIB matrix model

It is reasonable that one can analyze massless modes of fermions from Dirac equation in 10d

$$\Gamma^M [A_M , \Psi] = 0$$

Ψ is Majorana-Weyl in 10d

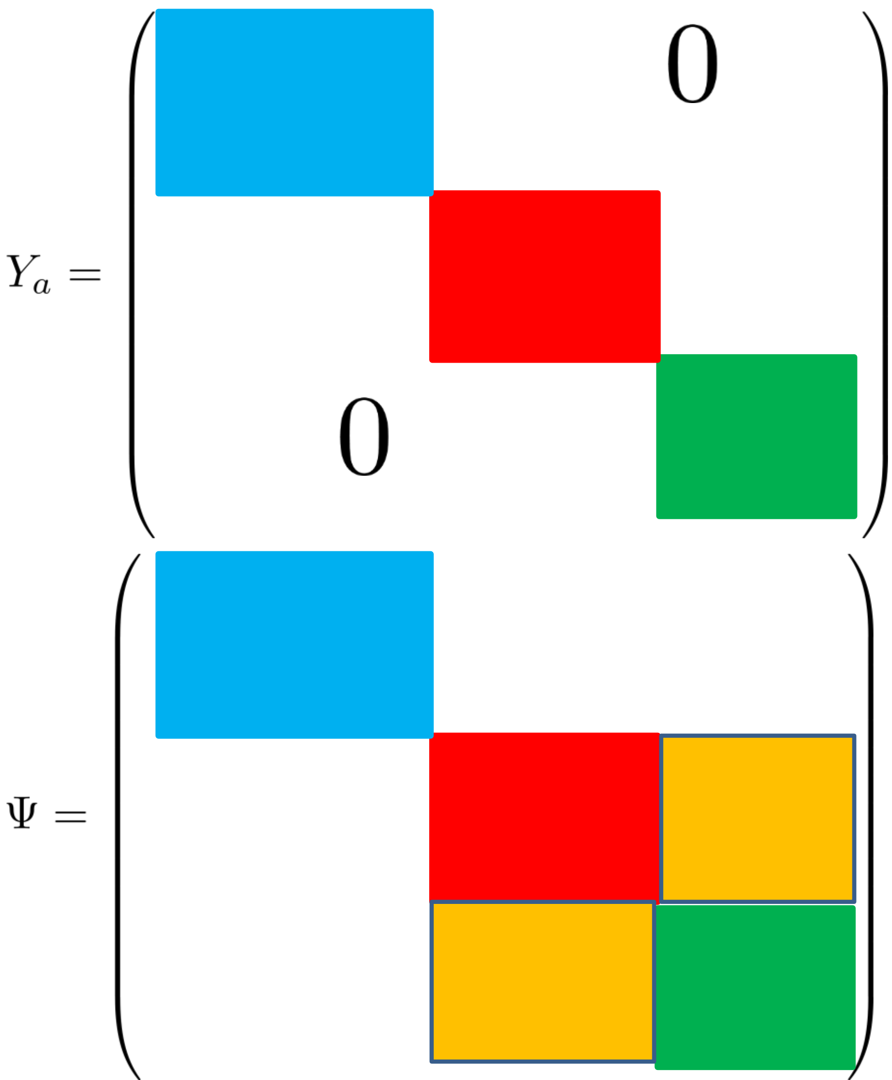
we demand Ψ to be chiral in 4d

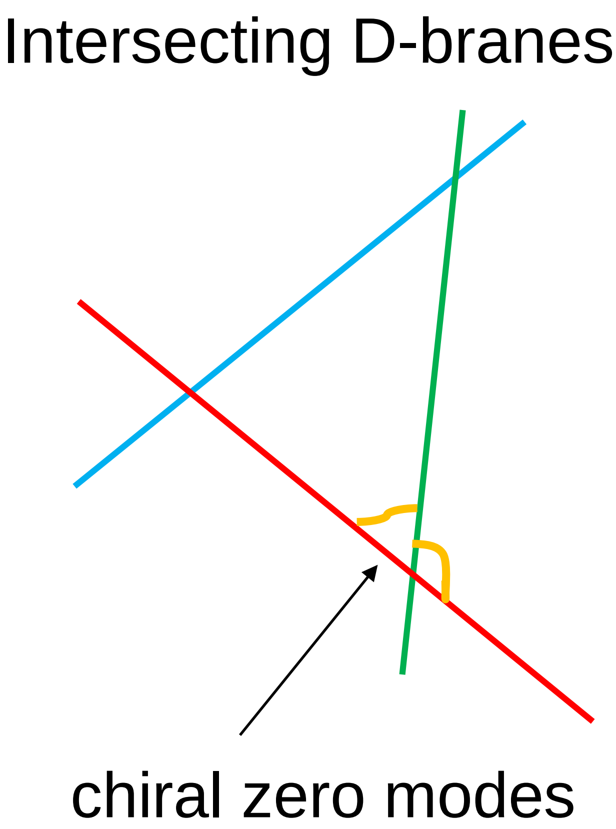
$$\Rightarrow \begin{cases} \Gamma^\mu [A_\mu , \Psi] = 0 \\ \Gamma^a [A_a , \Psi] = 0 \\ \Psi \text{ is chiral in 4d and 6d} \end{cases}$$

We examine spectrum of 6d Dirac operator $\Gamma^a [Y_a , *]$

zero eigenvectors → chiral zero modes in 4D

Structure of Ya and chiral zero modes





Algorithm for finding solutions

$$I = \mathrm{Tr}([A^M, [A_M, A_0]] + \alpha A_0)^2 + \mathrm{Tr}([A^M, [A_M, A_i]] - \beta A_i)^2$$

$$A_\mu = X_\mu \otimes M \quad (\mu = 0, \ldots, 3)$$

$$A_a = 1_{N_X} \otimes Y_a \quad (a = 4, \ldots, 9)$$

We search for configurations that gives $I = 0$

gradient descent algorithm

update configurations following

$$\delta X_\mu = -\epsilon \frac{\partial I}{\partial X_\mu^\dagger} \qquad \delta Y_a = -\epsilon \frac{\partial I}{\partial Y_a^\dagger} \qquad \delta M = -\epsilon \frac{\partial I}{\partial M^\dagger}$$

$\Rightarrow$ $\delta I \leq 0$

Space-time and chiral zero modes from classical solutions

Typical solutions

Our ansatz

$$\begin{cases} A_\mu = X_\mu \otimes M & (\mu = 0, \dots, 3) \\ A_a = 1_{N_X} \otimes Y_a & (a = 4, \dots, 9) \end{cases}$$

$$\begin{cases} M^3 = M \\ [M, Y_a] = 0 \\ [X^\nu, [X_\nu, X_0]] + \alpha X_0 = 0 \\ [X^\nu, [X_\nu, X_i]] - \beta X_i = 0 \quad (i = 1, 2, 3) \\ [Y^b, [Y_b, Y_a]] - \beta Y_a = 0 \end{cases}$$

$\Longrightarrow$ eigenvalues of M: -1, 0, 1

$$\Longrightarrow \begin{cases} [A^M, [A_M, A_0]] + \alpha A_0 = 0 \\ [A^M, [A_M, A_i]] - \beta A_i = 0 \quad (i = 1, \dots, 9) \end{cases}$$

Structure of M and Ya

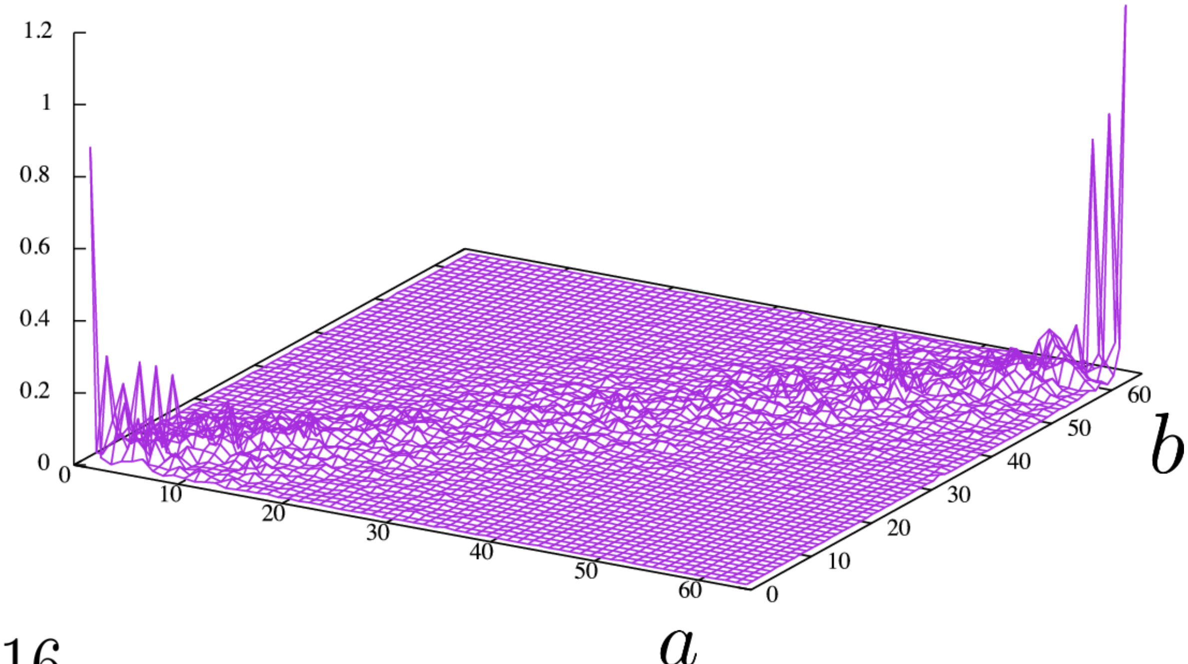
$$M = \begin{pmatrix} \begin{matrix} -1 & & \\ & \ddots & \\ & & -1 \end{matrix} & & \text{\Huge 0} \\ & \begin{matrix} 0 & & \\ & \ddots & \\ & & 0 \end{matrix} & \\ \text{\Huge 0} & & \begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix} \end{pmatrix}$$

$$Y_a = \begin{pmatrix} Y_a^{(-1)} & & \text{\Huge 0} \\ & Y_a^{(0)} & \\ \text{\Huge 0} & & Y_a^{(1)} \end{pmatrix}$$

Band diagonal structure of Xi

$$N_X = 64$$

$$\sum_{i=1}^{3} |(X_i)_{ab}|^2$$

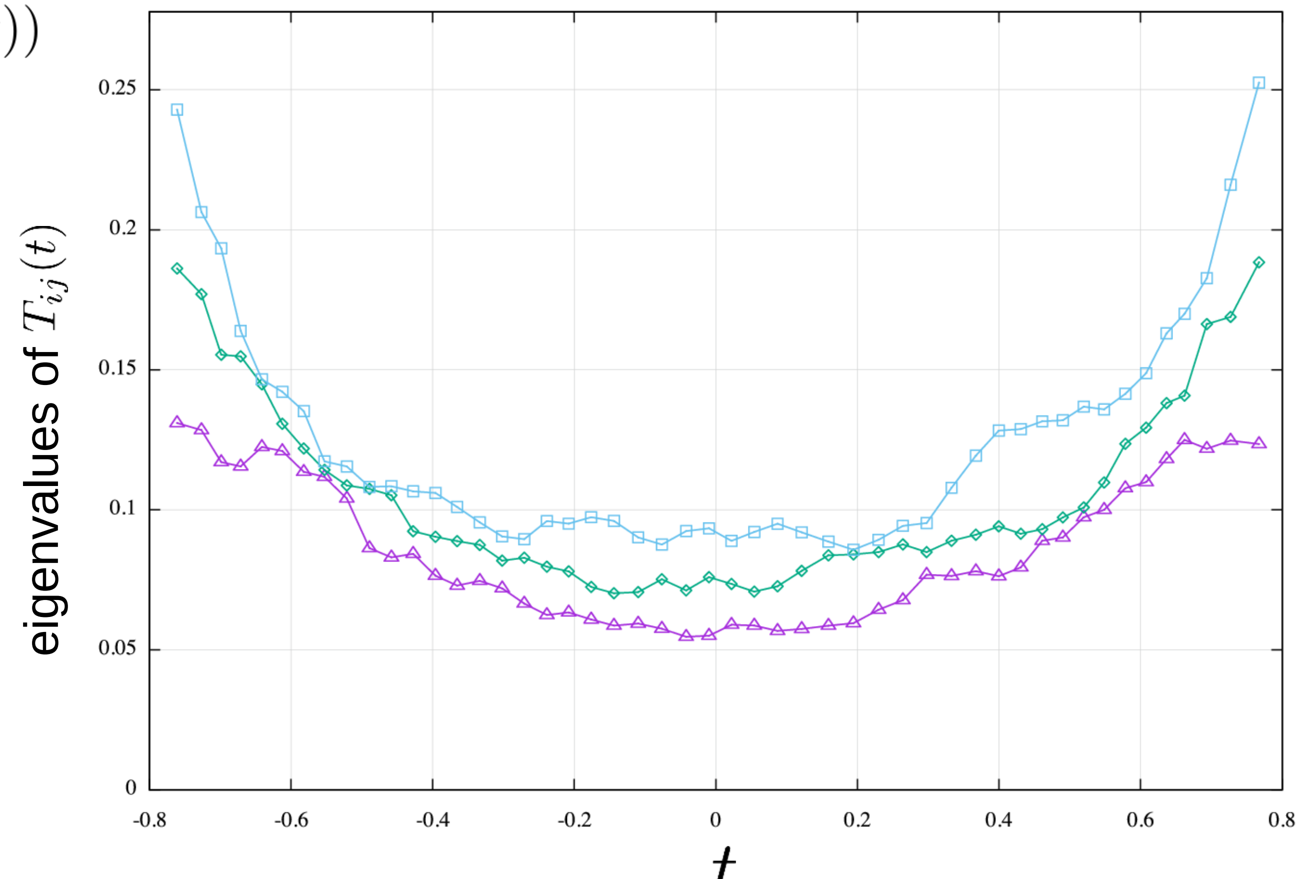


$$n = 16$$

Eigenvalues of Tij

$$T_{ij}(t) = \frac{1}{n}(\bar{X}_i(t)\bar{X}_j(t))$$

SO(3) symmetric



3d-3d ansatz

3d manifold and 3d manifold intersects at points

$$Y_a = \begin{array}{c} \\ \begin{matrix} N_Y^{(1)} \\ \\ N_Y^{(2)} \end{matrix} \end{array} \begin{array}{c} \begin{matrix} N_Y^{(1)} & \qquad\qquad \end{matrix} \\ \left(\begin{matrix} \text{3d} \; Y_a^{(1)} & \Psi \\ & \text{3d} \; Y_a^{(2)} \end{matrix} \right) \\ \begin{matrix} \qquad\qquad & N_Y^{(2)} \end{matrix} \end{array}$$

$$Y_1^{(1)} \neq 0, \quad Y_2^{(1)} \neq 0, \quad Y_3^{(1)} \neq 0$$
$$Y_4^{(1)} = Y_5^{(1)} = Y_6^{(1)} = 0$$

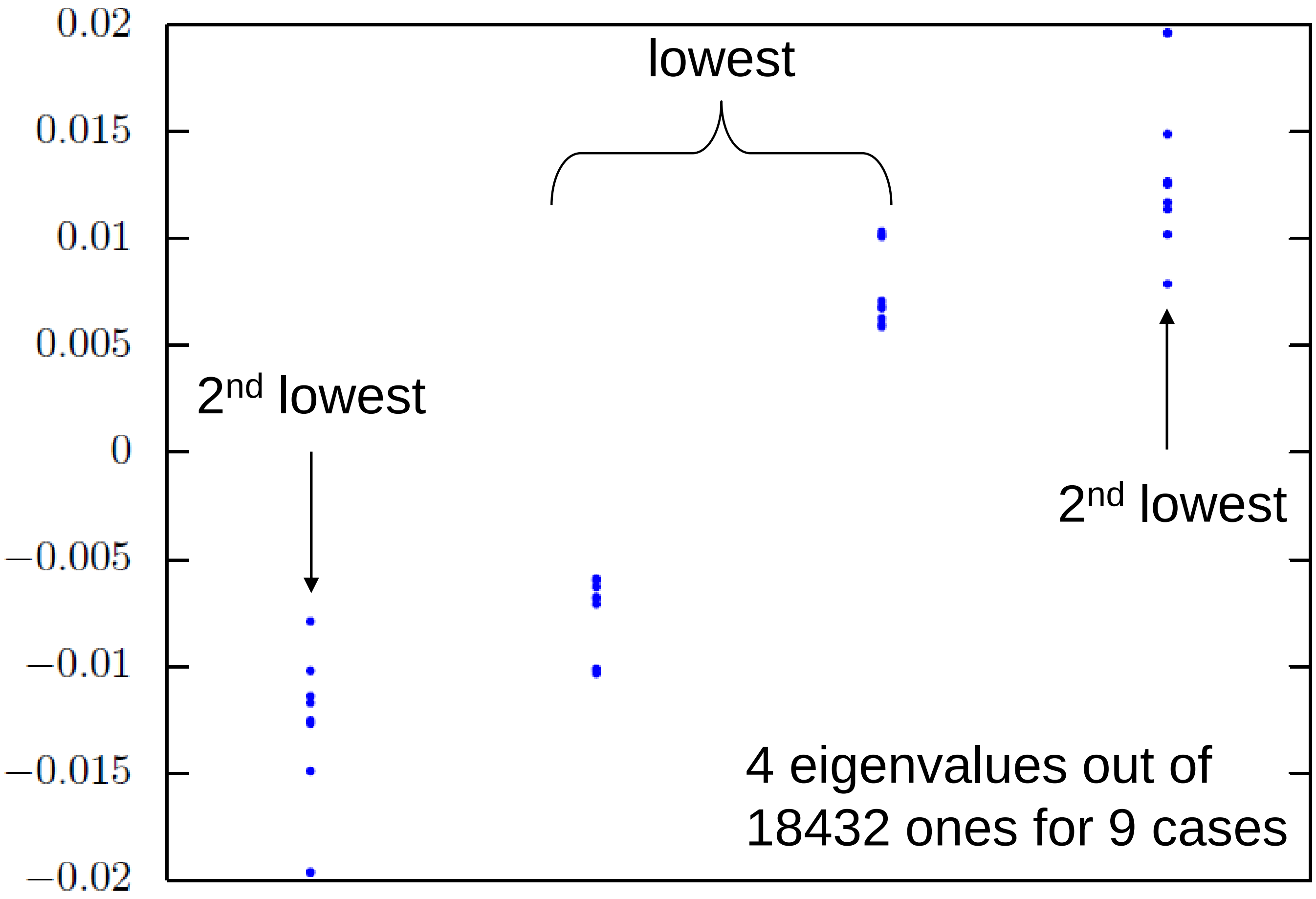
$$Y_1^{(2)} = Y_2^{(2)} = Y_3^{(2)} = 0$$
$$Y_4^{(2)} \neq 0, \quad Y_5^{(2)} \neq 0, \quad Y_6^{(2)} \neq 0$$

We solve $\Gamma^a (Y_a^{(1)} \Psi - \Psi Y_a^{(2)}) = \lambda \Psi$ $\quad \lambda \sim$ mass in 4d

We get 3 solutions for each of $Y_a^{(1)}$ and $Y_a^{(2)}$ ⇒ 3x3=9 cases

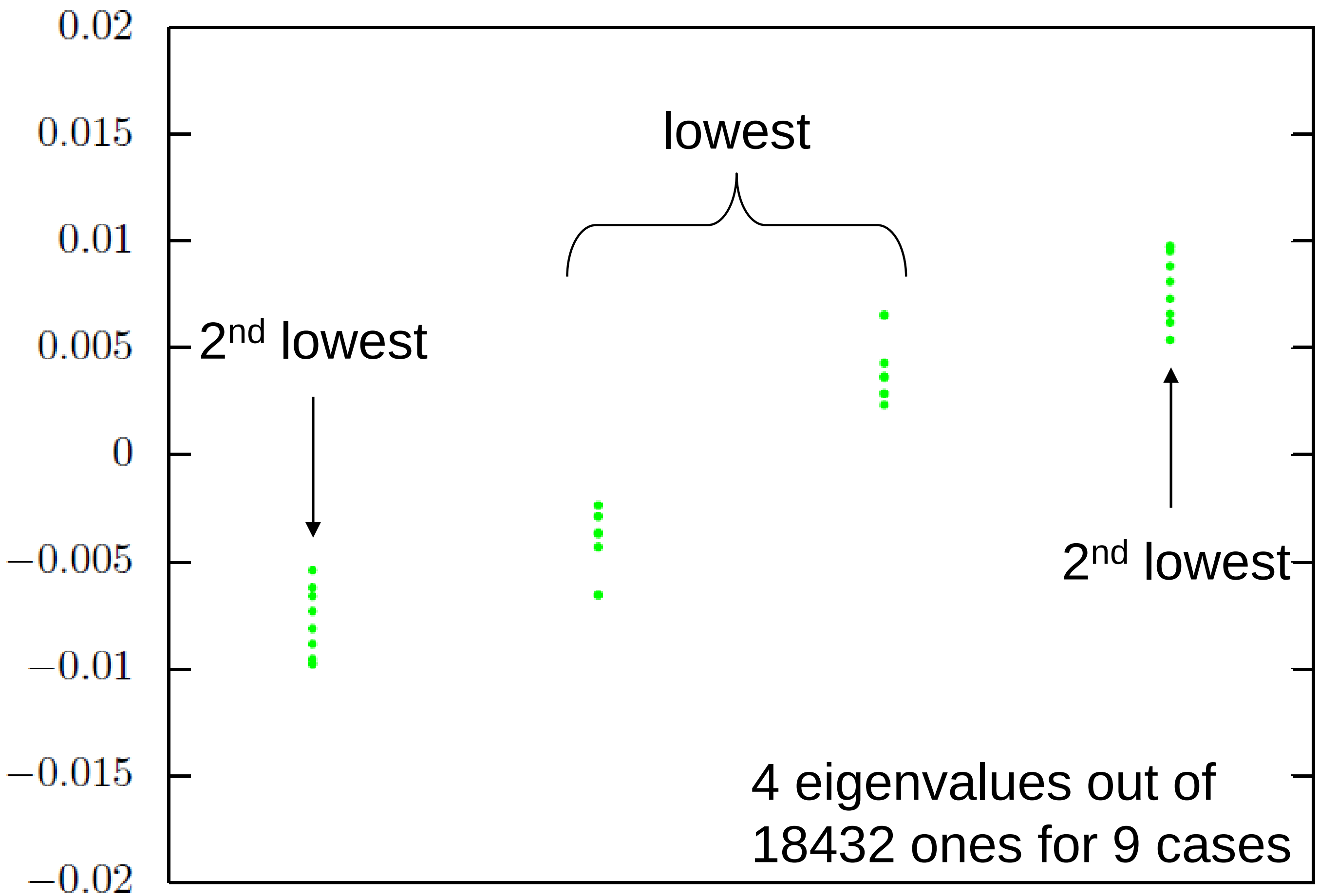
Spectrum of 6d Dirac operator

$$N_Y^{(1)} = N_Y^{(2)} = 32$$



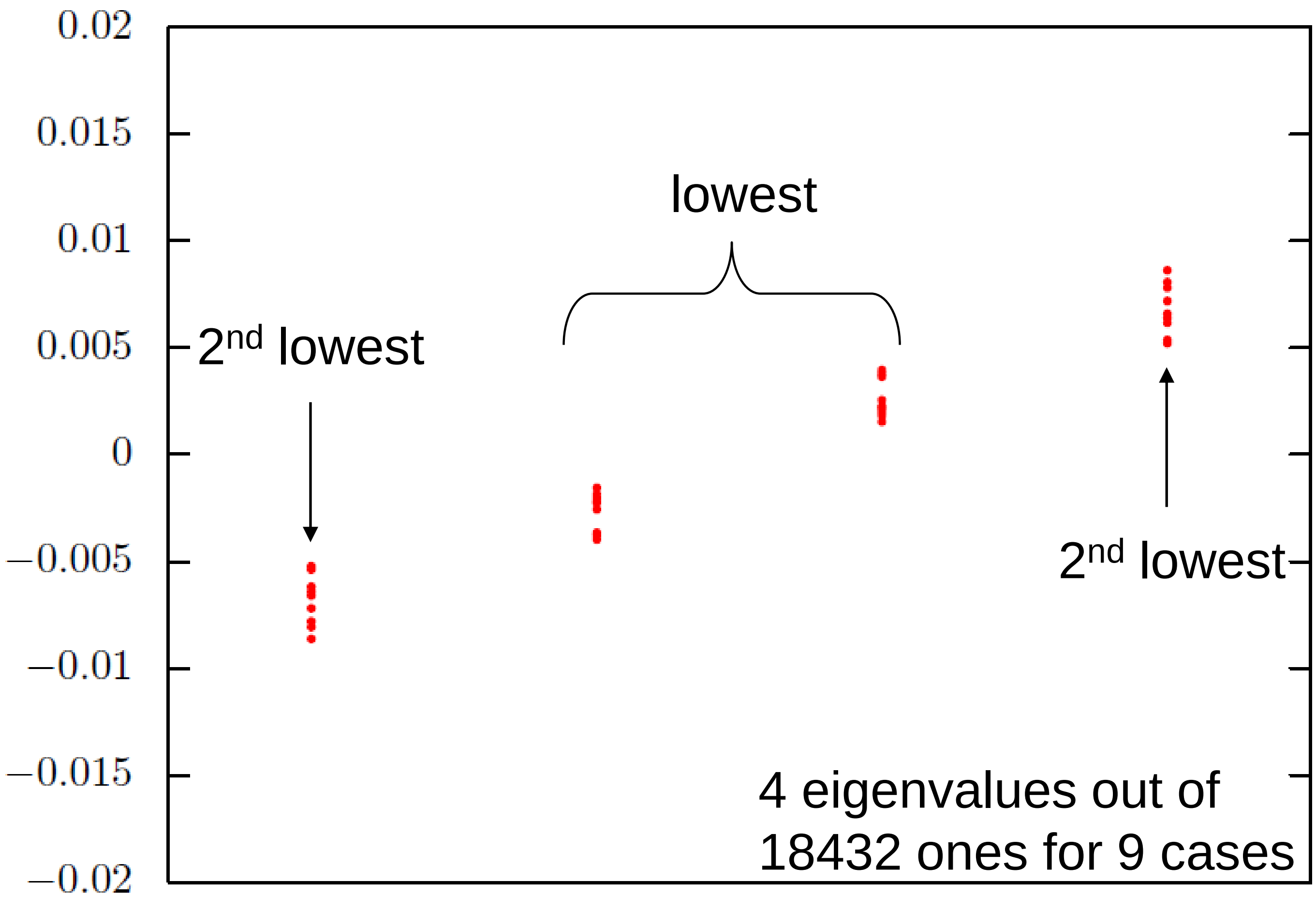
Spectrum of 6d Dirac operator

$$N_Y^{(1)} = N_Y^{(2)} = 48$$

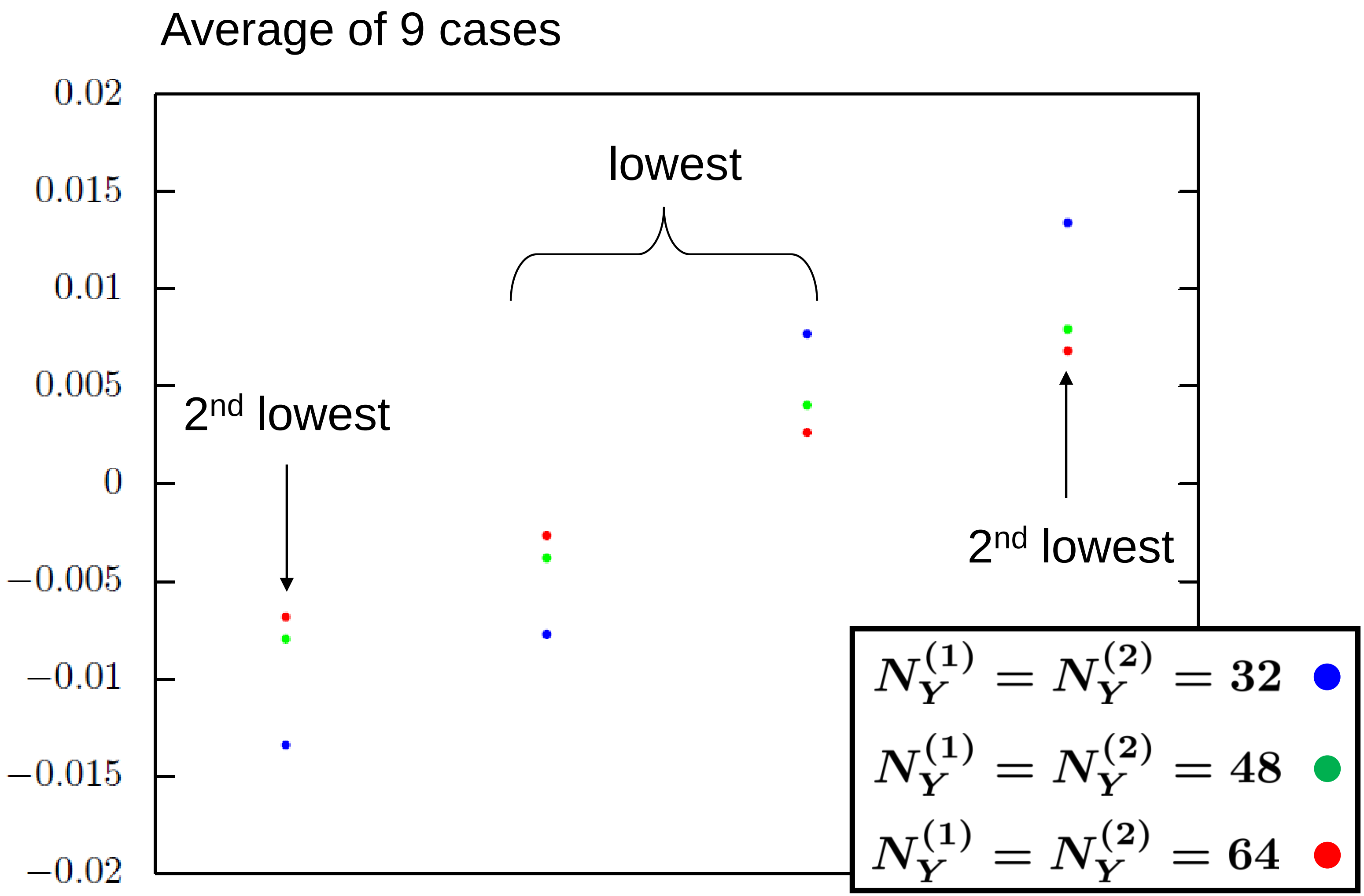


Spectrum of 6d Dirac operator

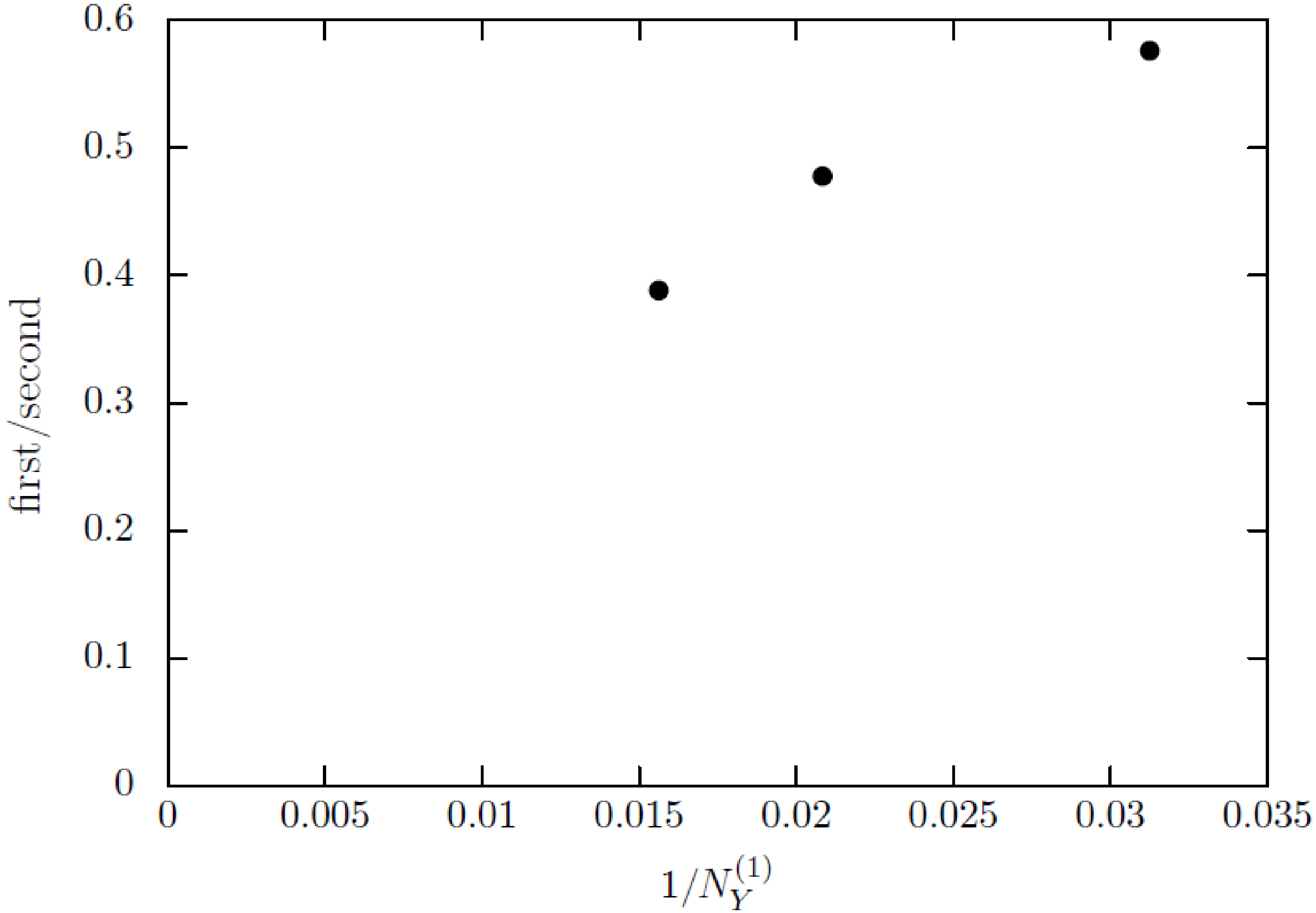
$$N_Y^{(1)} = N_Y^{(2)} = 64$$



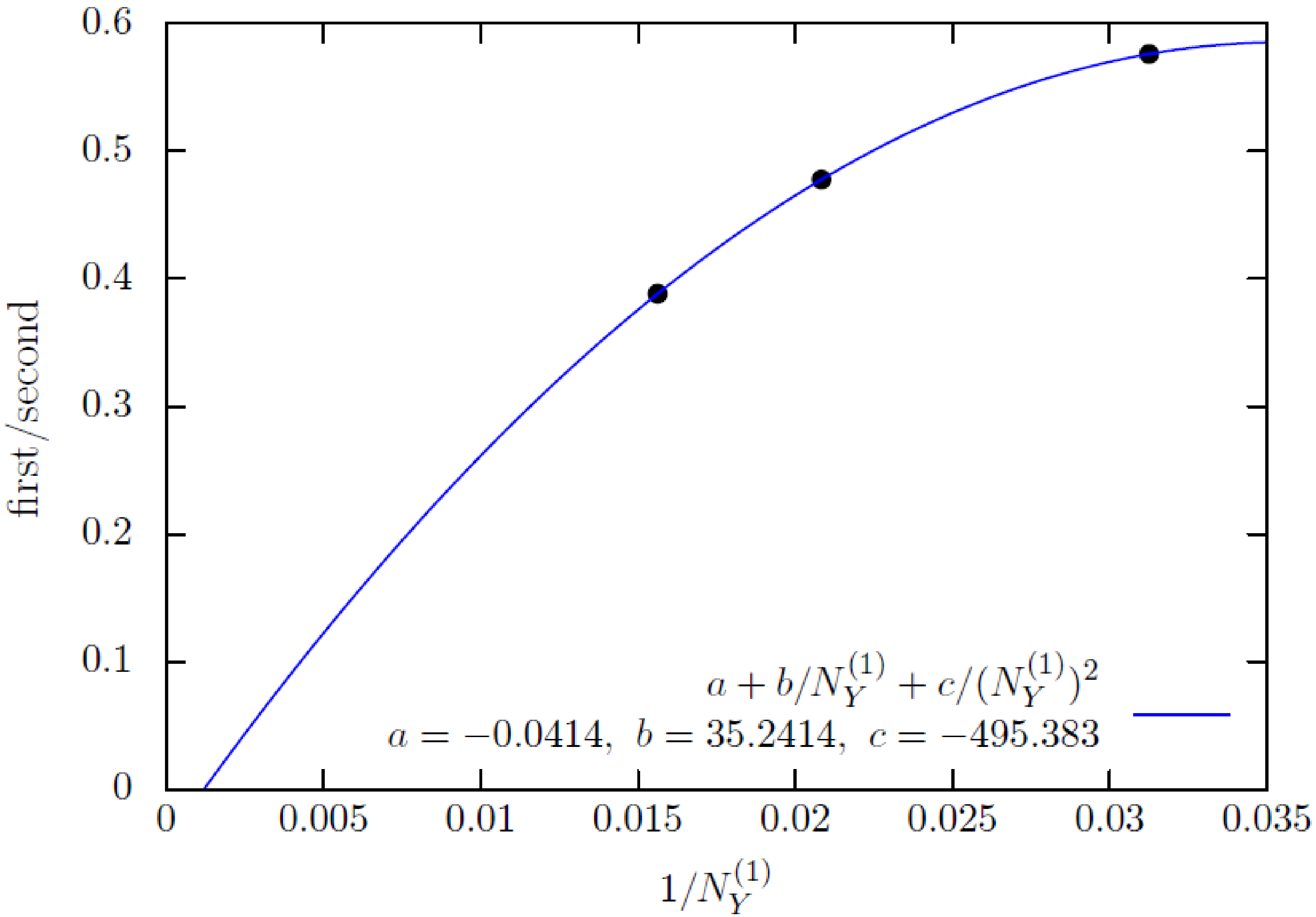
Spectrum of 6d Dirac operator



Lowest/2nd lowest

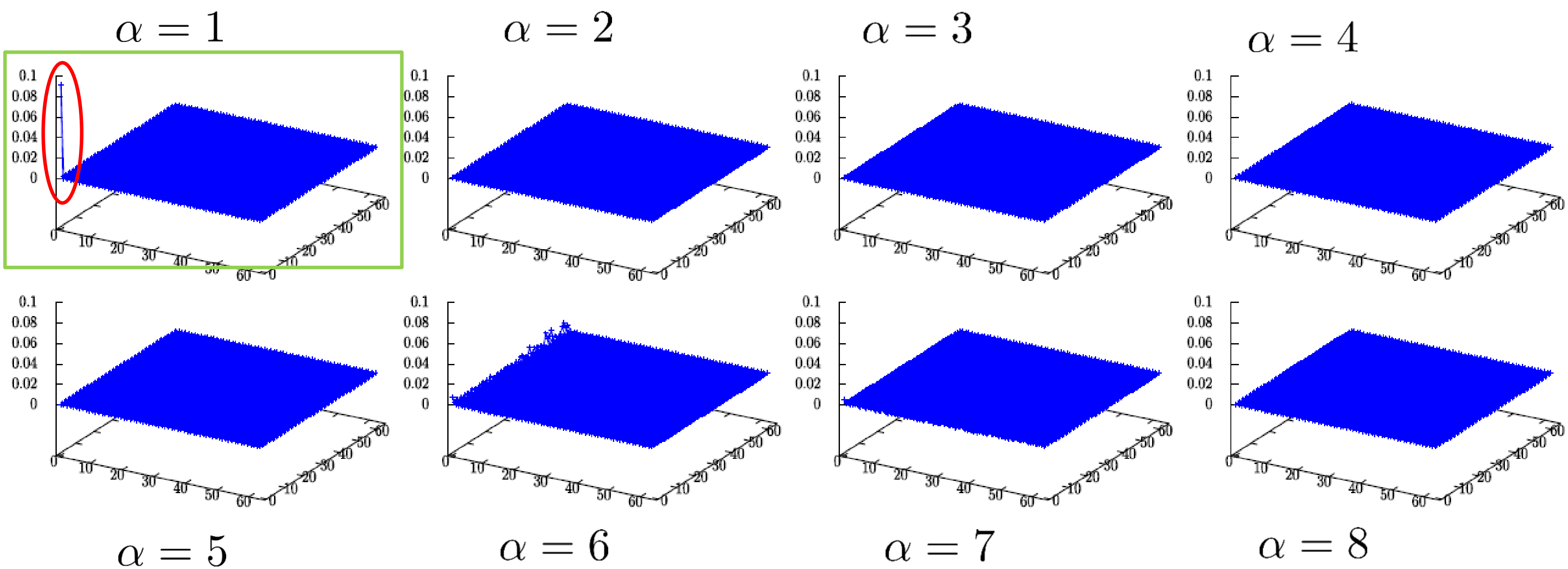


Lowest/2nd lowest



Profile of wave function for lowest ev

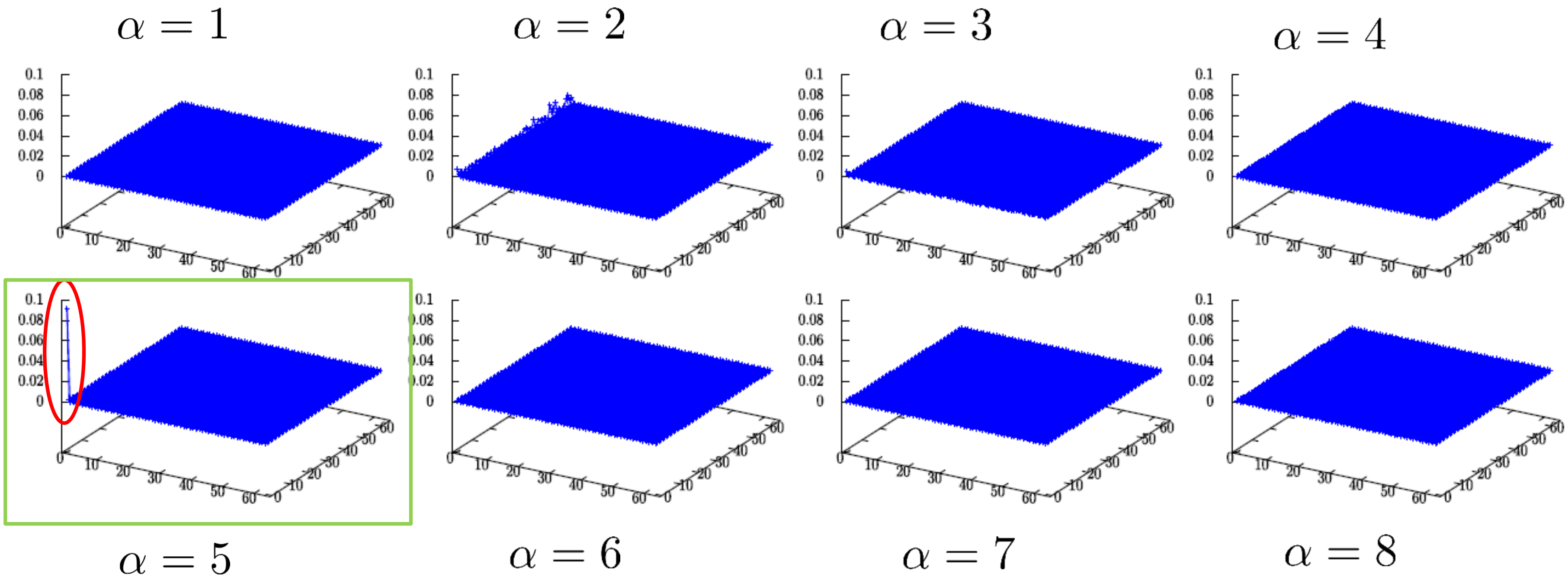
$$\Psi'_{R\alpha} = U_R \Psi_{R\alpha} V_R^{\dagger}$$ SVD for $\alpha = 1$



Localized ! $\Rightarrow$ Intersecting at a point

Profile of wave function for lowest ev

$$\Psi'_{L\alpha} = U_R \Psi_{L\alpha} V_R^\dagger$$



localized at different α → L-R asymmetry

Conclusion and outlook

Conclusion

- We developed a numerical method to search for classical solutions satisfying the most general ansatz with "quasi direct product structure". It works well.

- Solutions in general give expanding (and shrinking) (3+1)d space-times, which have smooth structure.

- Quasi direct product structure favors block-diagonal structure which can yield intersecting branes in extra dimensions. One can obtain chiral zero modes in 6d at intersecting points, which lead to the chiral fermions in (3+1) dimensions.

Conclusion

- The wave functions of chiral zero modes are localized. There is L-R asymmetry in the wave functions, which means the Yukawa coupling which is calculated from overlap of wave functions of fermions and Higgs field (fluctuation of Ya) is asymmetric w.r.t L and R.

  $\longrightarrow$ Chiral fermions in 4d at low energy?

Outlook

- We search for solutions by starting with various initial configurations to understand the variety of solutions.

- We expect that there exists a solution that realizes the Standard model or beyond the Standard model-like matter contents and that it is indeed selected in the sense that our Monte Carlo result is connected to such a solution.

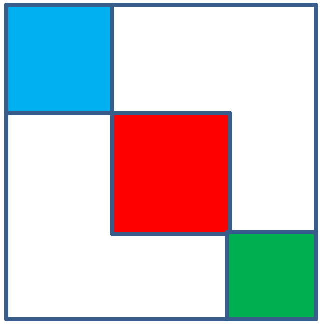
- Or we can calculate 1-loop effective actions around classical solutions we have found. We expect the effective action for the solution giving SM matter contents at low energy to be minimum.

Discussion

➢ Only 3 blocks?

To realize the Standard Model, more blocks seems to be needed.

(1) structure of blocks within a block is allowed for a classical solution, but seems non-generic.
Quantum effect might favor such a structure.



(2) We can generalize IR cutoffs as follows:

$$\frac{1}{N}\mathrm{Tr}((A_0^2)^p) = \kappa \qquad \frac{1}{N}\mathrm{Tr}((A_i^2)^p) = 1$$

We took p=1 in this talk for simplicity.
For p=2, arbitrary number of blocks are naturally obtained, because no constraints are obtained from $M^3 = M^3$
Indeed, p >1 seems to be required from universality

Azuma-Ito-Nishimura-A.T. ('17)

Discussion

- A different mechanism for getting chiral fermions

    more nontrivial solution having structure as $[M, Y_a] \neq 0$

    action of M on left and right modes are different

    Nishimura-A.T.('13)   Aoki-Nishimura-A.T.('14)

- Gauge groups?

    seem to come from a stack of multiple D-branes

    ~ identical blocks within a block

    ~ favored by quantum effect?

- Profile of D-branes and geometry of extra dimensions

Berenstein-Dzienkowski ('12), Ishiki ('15), Schneiderbauer-Steinaker ('16)

Evidences for nonperturbative formulation

(1) Manifest SO(9,1) symmetry and manifest 10D N=2 SUSY

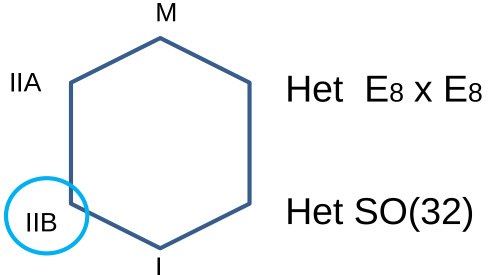
(2) Correspondence with Green-Schwarz action of Schild-type for type IIB superstring with κ symmetry fixed

(3) Long distance behavior of interaction between D-branes is reproduced

(4) Light-cone string field theory for type IIB superstring from SD equations for Wilson loops under some assumptions

Fukuma-Kawai-Kitazawa-A.T. ('97)

(5) Believing string duality, one can start from anywhere with nonperturbative formulation to tract strong coupling regime



Emergence of expanding (3+1)d universe

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\{\bar{A}_i(t)\bar{A}_j(t)\}$$

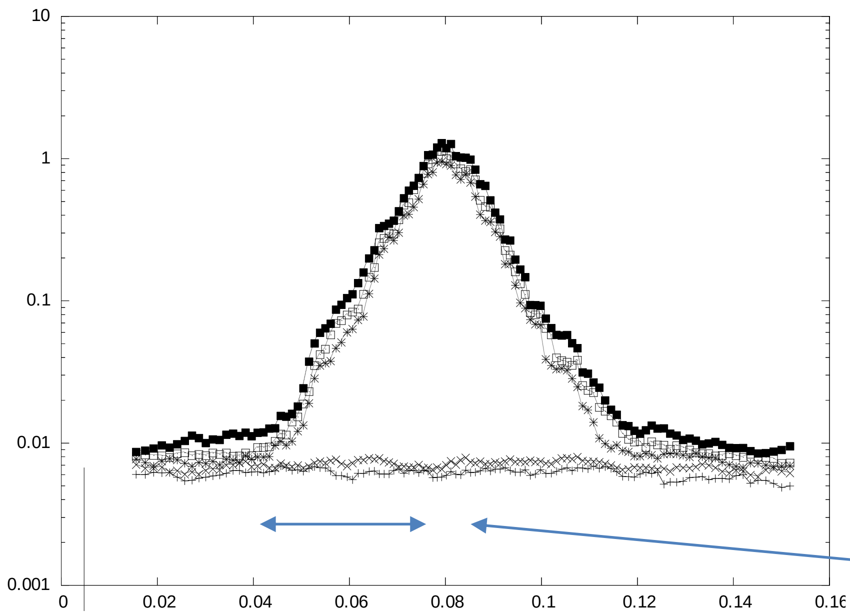
$$i,j = 1 \sim 9$$

~Moment of inertia tensor

Kim-Nishimura-A.T. ('11)

Nishimura-A.T. ('18)

Our numerical simulation suggests that expanding (3+1)-dimensional Universe emerges

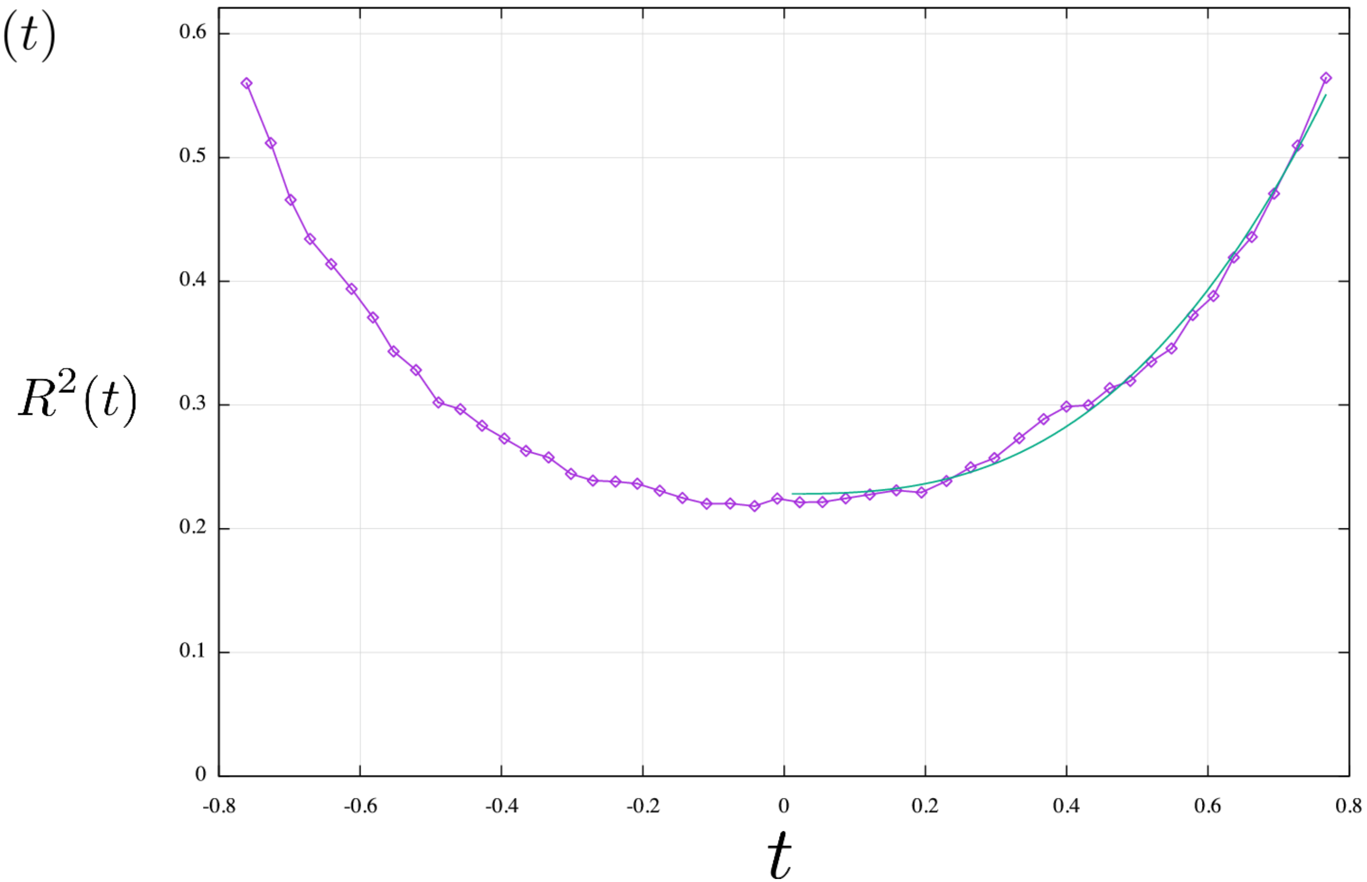


Order of Planckian time

Exponential expansion

R^2(t)

$$R^2(t) = \frac{1}{n}\mathrm{Tr}\bar{X}_i^2(t) = T_{ii}(t)$$

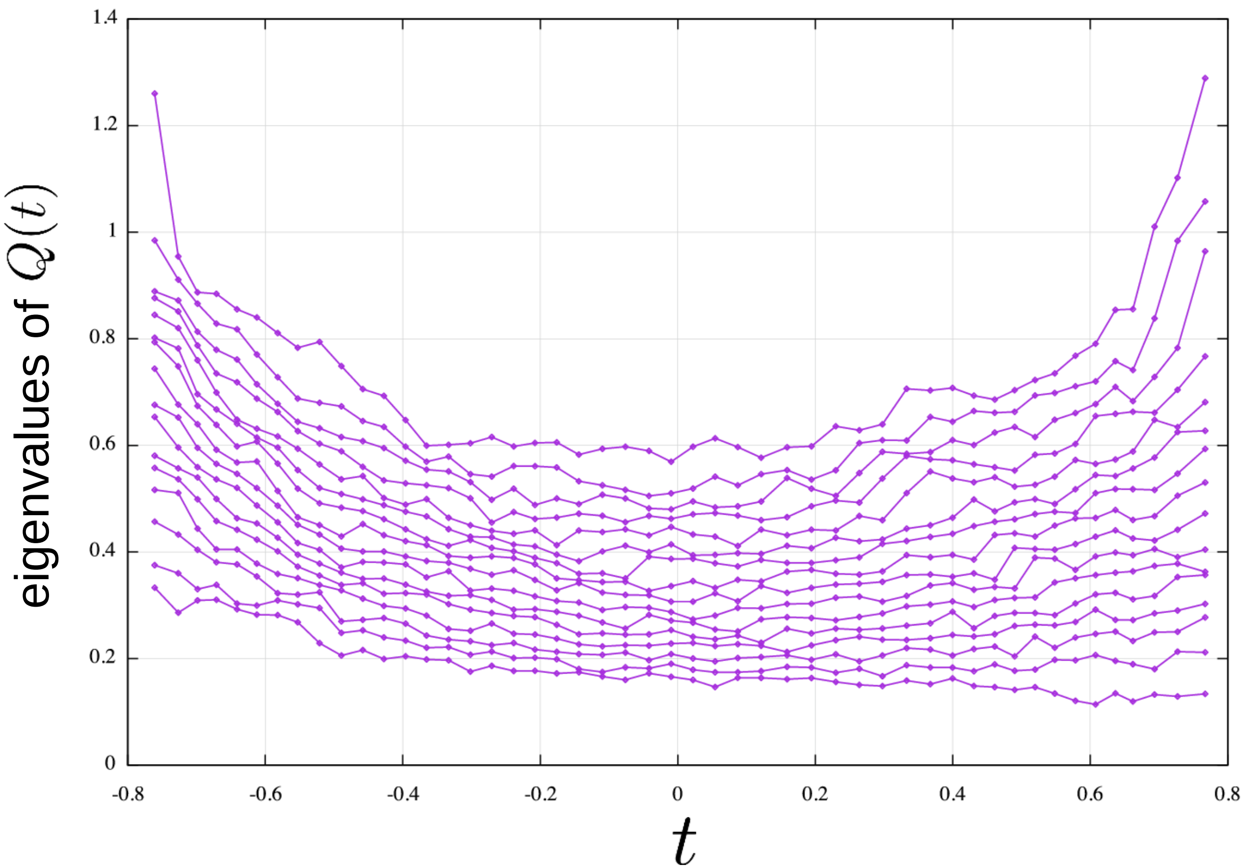


Space-time structure

$$Q(t) = \sum_{i=1}^{3} \bar{X}_i(t)^2$$

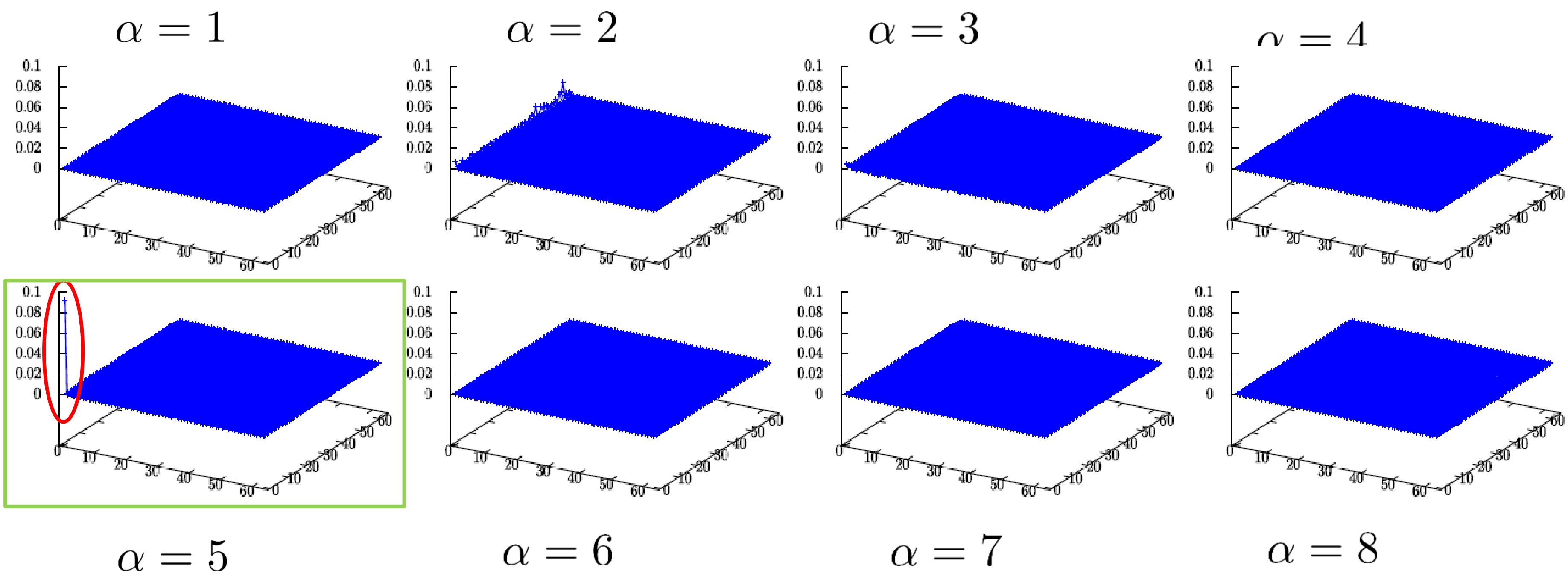
dense distribution

⇨ smooth manifold



Profile of wave function for lowest ev

$$\Psi'_{L\alpha} = U_L \Psi_{L\alpha} V_L^{\dagger}$$ SVD for $\alpha = 5$

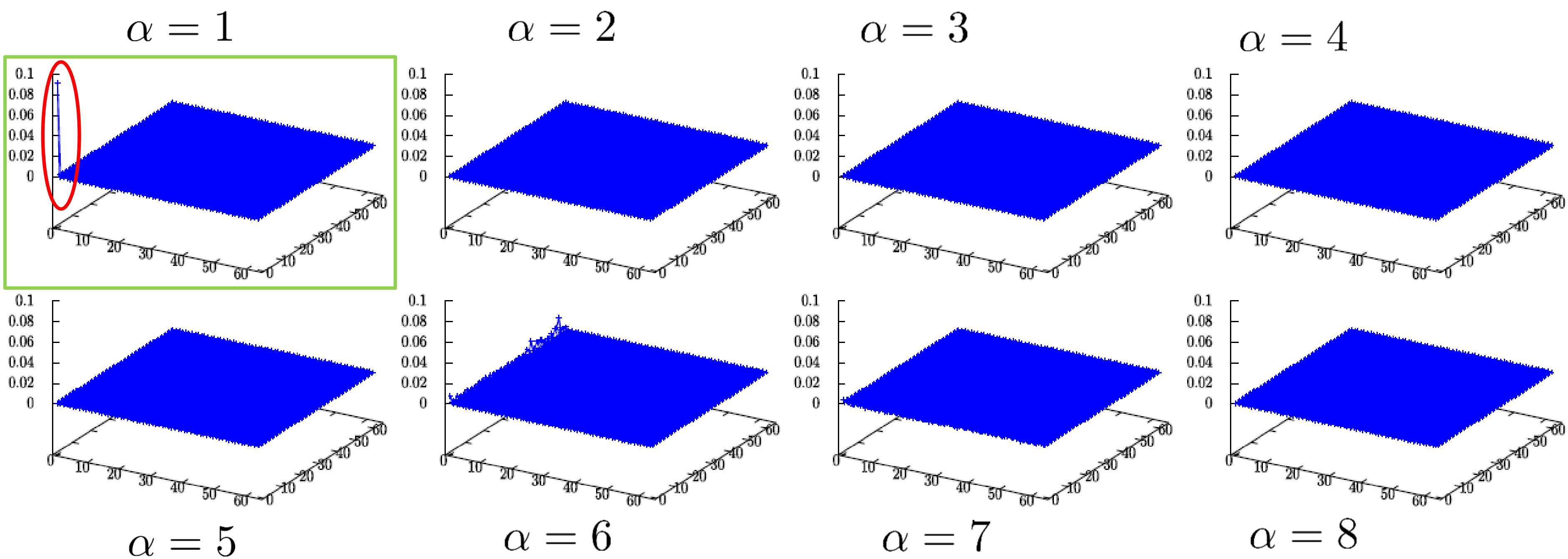


Localized !

Intersecting at a point

Profile of wave function for lowest ev

$$\Psi'_{R\alpha} = U_L \Psi_{R\alpha} V_L^\dagger$$



localized at different α → L-R asymmetry