Signature of extra-dimension(s): The enhanced open string pair production

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Based on JHEP0909(2009)093, JHEP12(2017)076, NPB934(2018)39 & arXiv:1808.04950, 1809.03806
(with San-San Xu, Qiang Jia, Zhihao Wu ...)

3rd East Asia Joint Workshop on Fields and Strings

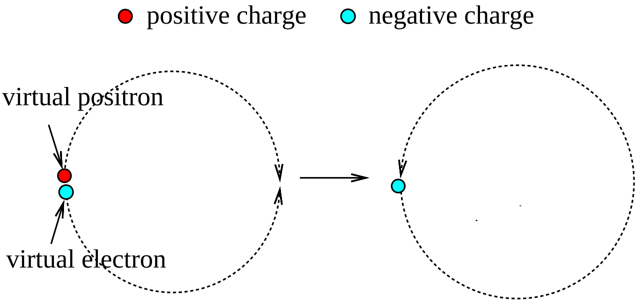
KIAS

November 5, 2018

QED Vacuum Fluctuations

VACUUM FLUCTUATION!

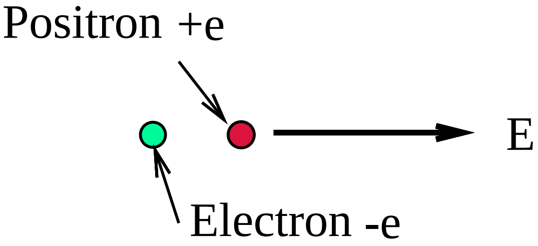
An anti-charge moving forward in time equivalent to a charge moving backward in time



$(\delta t \sim 10^{-21} s)$

QED Vacuum Fluctuations

Applying a constant electric field to QED vacuum, there is certain probability to create real electron and positron pairs from the vacuum fluctuations, called Schwinger pair production (1951).



$$(E \sim 10^{18}\, V/m)$$

The current lab E-field limit: $\sim 10^{10}\, V/m$

Stringy process

- Does there exist an analogous process in string theory?
- If so, can it be observable?
- If yes, what are the possible consequences and implications?

Stringy process

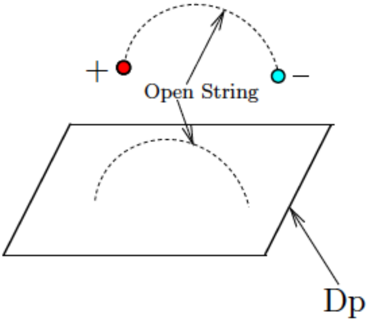
To address these three questions,

we will focus in this talk on the open string pair production due to systems of D-branes,

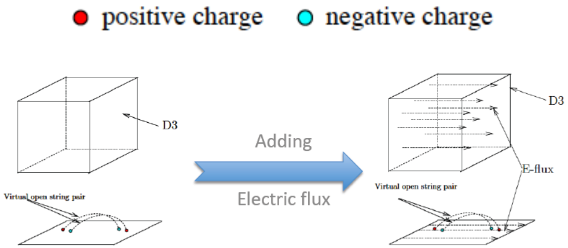
carrying constant electric and magnetic fluxes,

in Type II string theories and the resulting pair production can be significant under certain conditions.

D-branes



Take our 4-dim world as a D3 carrying an electric field



A single D3 brane carrying an electric field

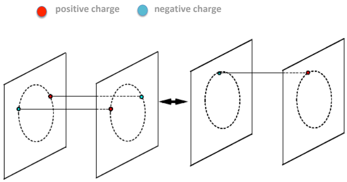
In a sharp contrast to the Schwinger pair production in QED, there is no pair production here.

This is due to each virtual open string being charge neutral (zero net charge) and their two ends experiencing the same electric field.

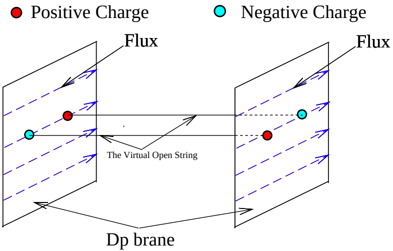
The open string pair production

In order to have the pair production, a possible choice is to let the two ends of the virtual charge-neutral open string experience different electric field.

A possible simplest setup for this is to consider two Dp branes in Type II string theory, placed parallel at a separation, with each carrying a different electric field.



The open string pair production



Stringy computations show indeed a non-vanishing pair production rate for this setup. However, this rate is usually vanishing small for any realistic electric fields and so has no any practical use.

This rate can be greatly enhanced if we add in addition a magnetic flux in a particular manner on each Dp.

The pair production rate

For this purpose, consider the electric/magnetic tensor $\hat{F}^1$ on one Dp brane and the $\hat{F}^2$ on the other Dp brane, respectively, as

$$\hat{F}^a = \begin{pmatrix} 0 & -f_a & 0 & 0 & 0 & \ldots \\ f_a & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & -g_a & 0 & \ldots \\ 0 & 0 & g_a & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}_{(p+1)\times(p+1)}, \qquad (2.1)$$

where f_a denotes the electric field ($|f_a| < 1$) while g_a the magnetic one ($|g_a| < \infty$) with $a = 1, 2$, and $6 \geq p \geq 3$. Note $\hat{F} = 2\pi\alpha' F$.

The pair production rate

For having the non-perturbative open string pair production rate, we first need to compute the open string one-loop annulus interaction amplitude between the two Dp branes. It is

$$\Gamma = \frac{2^2\,|f_1-f_2||g_1-g_2|}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty \frac{dt}{t^{\frac{p-1}{2}}}\,e^{-\frac{Y^2 t}{2\pi\alpha'}}\,\frac{(\cosh\pi\nu_0' t-\cos\pi\nu_0 t)^2}{\sin\pi\nu_0 t\,\sinh\pi\nu_0' t}$$
$$\times\prod_{n=1}^{\infty}\frac{[(1-2|z|^{2n}e^{-\pi\nu_0' t}\cos\pi\nu_0 t+|z|^{4n}e^{-2\pi\nu_0' t})(1-2|z|^{2n}e^{\pi\nu_0' t}\cos\pi\nu_0 t+|z|^{4n}e^{2\nu_0' t})]^2}{(1-|z|^{2n})^6(1-2|z|^{2n}\cos 2\pi\nu_0 t+|z|^{4n})(1-2|z|^{2n}\cosh 2\pi\nu_0' t+|z|^{4n})} \quad (2.2)$$

where Y is the brane separation, $|z| = e^{-\pi t} < 1$, and the electric parameter $\nu_0 \in [0,\infty)$ and the magnetic one $\nu_0' \in [0,1]$ are given by the electric fluxes and magnetic ones, respectively, as

$$\tanh\pi\nu_0 = \frac{|f_1-f_2|}{1-f_1 f_2}, \quad \tan\pi\nu_0' = \frac{|g_1-g_2|}{1+g_1 g_2}. \quad (2.3)$$

The open string pair production rate

- The integrand of the above amplitude has an infinite number of simple poles occurring on the positive real t-axis at $t_k = k/\nu_0$ with $k = 1, 2, \cdots$.
- Therefore this amplitude has an imaginary part, indicating the decay of the underlying system.
- The dynamical process responsible for this decay is the open string pair production. It occurs at each of these simple poles.
- The non-perturbative pair production rate is given as sum of the residues of the integrand at thess simple poles times π per unit worldvolume following Bachas-Porrati'92 as

The open string pair production rate

$$
\begin{aligned}
\mathcal{W} &= -2\,\mathrm{Im}\Gamma \\
&= \frac{8\,|f_1 - f_2||g_1 - g_2|}{(8\pi^2\alpha')^{\frac{p+1}{2}}} \sum_{k=1}^{\infty} (-)^{k-1} k \left(\frac{\nu_0}{k}\right)^{\frac{p-3}{2}} e^{-\frac{kY^2}{2\pi\nu_0\alpha'}} \\
&\times \frac{\left[\cosh\frac{k\pi\nu_0'}{\nu_0} - (-)^k\right]^2}{\sinh\frac{k\pi\nu_0'}{\nu_0}} Z_k(\nu_0, \nu_0'),
\end{aligned}
\tag{2.4}
$$

where

$$
Z_k(\nu_0, \nu_0') = \prod_{n=1}^{\infty} \frac{\left[1 - 2(-)^k e^{-\frac{2nk\pi}{\nu_0}} \cosh\frac{k\pi\nu_0'}{\nu_0} + e^{-\frac{4nk\pi}{\nu_0}}\right]^4}{\left[1 - e^{-\frac{2nk\pi}{\nu_0}}\right]^6 \left[1 - e^{-\frac{2k\pi}{\nu_0}(n-\nu_0')}\right] \left[1 - e^{-\frac{2k\pi}{\nu_0}(n+\nu_0')}\right]}.
\tag{2.5}
$$

The pair production rate enhancement

- The rate is highly suppressed by the brane separation Y and the integer k.
- For each given f_a and g_a, we can qualitatively understand this by noting that the mass for each produced open string is $kT_f Y$ with $T_f = 1/(2\pi\alpha')$. So the larger k or Y or both, the larger the mass is and therefore the more difficult the open string can be produced.
- The pure electric case can be obtained by setting $g_1 = g_2$ for which $\nu' = 0$ from (2.3). Now if we further set $f_1 = f_2$, i.e., the two ends of the open string experiences the same electric field, then $\nu_0 = 0$ from (2.3) and we have the rate $\mathcal{W} = 0$.
- This is consistent with that no Schwinger-type of pair production of a single D3 brane carrying a constant electric flux mentioned earlier.

The pair production rate enhancement

- For $f_1 \neq f_2$, the larger f_1 or f_2 is, the larger ν_0 and $|f_1 - f_2|$ are and the larger the rate $\mathcal{W}$ is.
- In general, the presence of magnetic fluxes enhances this rate. We here consider two special cases to show this explicitly.
- The first is the case of $g_1 = g_2 = g \neq 0$, we have from (2.4) and (2.3)

$$\frac{\mathcal{W}_{g_1=g_2=g}}{\mathcal{W}_{g_1=g_2=0}} = 1 + g^2. \tag{2.6}$$

The pair production rate enhancement

- Here

$$\mathcal{W}_{g_1=g_2=0} = \frac{32\nu_0|f_1-f_2|}{(8\pi^2\alpha')^{\frac{p+1}{2}}}\sum_{l=1}^{\infty}\frac{1}{(2l-1)^2}\left(\frac{\nu_0}{2l-1}\right)^{\frac{p-3}{2}} e^{-\frac{(2l-1)y^2}{2\pi\alpha'\nu_0}} \times \prod_{n=1}^{\infty}\left(\frac{1+e^{-\frac{2n(2l-1)\pi}{\nu_0}}}{1-e^{-\frac{2n(2l-1)\pi}{\nu_0}}}\right)^8 . \quad (2.7)$$

- The second is the case of $\nu_0'/\nu_0 \gg 1$. This says $\nu_0 \ll 1$ since $\nu_0' \in [0,1]$, implying $|f_1 - f_2| \ll 1$ from (2.3). For fixed $\nu_0' \in [0,1]$ and a very small ν_0, the rate (2.4) can be approximated by its leading $k=1$ term as

$$\mathcal{W} \approx \frac{4|f_1-f_2||g_1-g_2|}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\,\nu_0^{\frac{p-3}{2}}\, e^{-\frac{y^2}{2\pi\alpha'\nu_0}}\, e^{\frac{\pi\nu_0'}{\nu_0}} . \quad (2.8)$$

The pair production rate enhancement

- The zero-magnetic flux rate (2.7) for the same small ν_0 is

$$\mathcal{W}_{g_1=g_2=0} \approx \frac{32\nu_0|f_1 - f_2|}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \nu_0^{\frac{p-3}{2}} e^{-\frac{y^2}{2\pi\alpha'\nu_0}}. \tag{2.9}$$

- The enhancement is then

$$\frac{\mathcal{W}}{\mathcal{W}_{g_1=g_2=0}} = \frac{|g_1 - g_2|}{8\nu_0} e^{\frac{\pi\nu_0'}{\nu_0}} \tag{2.10}$$

which can be huge given $\nu_0'/\nu_0 \gg 1$ and $\nu_0 \ll 1$.

The rate enhancement

Consider now $\nu_0'/\nu_0 \gg 1$ for $p = 3$ as an illustration!

To have a sense of enhancement, let us make a numerical estimation for illustration!

Take $\nu_0 = 0.02$ and $\nu_0' = 0.5$. This can be achieved with a moderate choice of $g_1 = -g_2 = 1$ (noting $|g_a| < \infty$) and $f_1 = 0.2$ with $f_2 = f_1 - \epsilon$ and $|f_1 - f_2| = |\epsilon| \approx \pi\nu_0(1 - f_1^2) = 0.06 \ll 1$.

The enhancement is then

$$\frac{|g_1 - g_2| e^{\frac{\pi\nu_0'}{\nu_0}}}{8\nu_0} = 1.6 \times 10^{35}!!! \qquad (2.11)$$

A huge number!

Possibility for detection

Let us explore under what conditions this rate itself can be large so a detection is possible.

Note $p \geq 3$. The $p = 3$ case gives the largest rate (JHEP12(2017)076,arXiv:1808.04950 &1809.03806) and the dimensionless rate, for example, for $p = 4$ is smaller by a factor of $(\nu_0/4\pi)^{1/2}$ and so on.

Let us give an estimate of this factor and see how large it is.

In practice, we can only control the brane on which we live, not the other brane. So we can set $f_2 = g_2 = 0$, for example, on the other brane.

Possibility for detection

Note $M_s = 1/\sqrt{\alpha'} \sim$ a few TeV upto $10^{16} \sim 10^{17}$ GeV (Berenstein, ARNPS64(2014)197),

Schwinger pair production $eE \sim m_e^2 = 2.5 \times 10^{-7}\mathrm{GeV}^2$, giving

$f_1 = 2\pi\alpha' eE = 2\pi\, m_e^2/M_s^2 \leq\sim 10^{-13} \ll 1$, so

$$\nu_0 = \frac{|f_1|}{\pi} = 2\frac{m_e^2}{M_s^2} \leq\sim 10^{-13} \to \left(\frac{\nu_0}{4\pi}\right)^{1/2} \sim 10^{-7} \ll 1 \qquad (2.12)$$

In other words, the dimensionless rate for any other $p > 3$ brane is at least smaller than that of the D3-brane by a factor of 10^{-7}!

So the pair production rate for D3 is the only hope for detection!

Possibility for detection

In terms of the lab. field E and B via

$$f_1 = 2\pi\alpha' eE, \qquad g_1 = 2\pi\alpha' eB, \tag{2.13}$$

the pair production rate for D3 brane is now (taking $k = 1$ in (2.4) with $Z_k(\nu_0, \nu_0') \approx 1$)

$$\mathcal{W} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh \frac{\pi B}{E} + 1\right]^2}{\sinh \frac{\pi B}{E}} e^{-\frac{\pi m^2}{eE}}, \tag{2.14}$$

where we have introduced a mass scale $m = T_f y = y/(2\pi\alpha')$.

Keep in mind, we need to have a nearby D3 brane for this rate!

Possibility for detection

Consider a nearby invisible (hidden or dark) D3

For this, we do not have a priori knowledge of the mass scale m,

$$\mathcal{W} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh \frac{\pi B}{E} + 1\right]^2}{\sinh \frac{\pi B}{E}} e^{-\frac{\pi m^2}{eE}}, \tag{2.15}$$

If $m \leq \mathcal{O}(m_e)$, we can falsify this rate once the applied fields are good for detecting the QED Schwinger rate.

For example, if $m = m_e$, a comparison with the QED Schwinger pair production rate

$$\mathcal{W}_{\rm QED} = \frac{(eE)(eB)}{(2\pi)^2} \coth\left(\frac{\pi B}{E}\right) e^{-\frac{\pi m_e^2}{eE}}, \tag{2.16}$$

can be made as

Possibility for detection

$$\frac{\mathcal{W}}{\mathcal{W}_{\rm QED}} = \frac{2\left[\cosh\frac{\pi B}{E}+1\right]^2}{\cosh\frac{\pi B}{E}} \geq 8,$$
$$\sim e^{\frac{\pi B}{E}} \to \infty, \quad \text{for } B/E \gg 1. \tag{2.17}$$

If on the other hand, $m \gg m_e$, a detection is hard.

Conclusion

- The pair production enhancement found is due to the interplay of non-perturbative Schwinger-type pair production and the presence of magnetic flux.
- The enhanced rate is the largest for $p = 3$ and for any $p > 3$, the corresponding rate can be at least seven orders of magnitude smaller. Adding more magnetic fluxes decrease the rate (Qiang Jia's talk).
- For observer living on the brane, he/she may use this enhanced rate to detect the current due to the particle/anti-particle pairs produced, which are the ends of the produced open string pairs.

Conclusion

If this detection can indeed be found and its dependence on the electric and magnetic fluxes are as predicted by stringy computations, we then

- find the existence of extra dimension(s),
- provide a new means to test the underlying string theories, and
- possibly single out our $(1+3)$-dim world.

THANK YOU!