Non-geometric states and Holevo information in AdS3/CFT2

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based on 1806.07595, 1808.02873 & 1810.01258 with Jiaju Zhang(Milano-Bicocca) & Wu-Zhong Guo(NCTS)

Motivations

- Recently, there are lots of discussions on quantum thermalization in the context of AdS/CFT duality, inspired by the holographic entanglement.
- One of the key issues is how we can characterize the local (dis-)similarity between a typical pure state and thermal state, and how general it should be?
- The general principle/criterion is formulated as the eigenstate thermalization hypothesis (Srednicki) or canonical typicality (Popescu et al).
- On the bulk side, the story becomes how we can characterize the microstates of a black hole, or can we tell them locally?

- In this talk, I will discuss two issues on quantum thermalization in the context of AdS3/CFT2.
- We will examine 1/c behaviors through short-interval expansion of entanglement entropy.
- In CFT side, 1/c effects will fine-grain the microstates and lead to expected failure of ETH for integrable models.
- In the bulk side, 1/c ~ G effects encode quantum gravity corrections.

1. Are all CFT states geometric?

- A folklore in 3D gravity, all the geometries are described by the Banados metrics: $$ds^2 = \frac{dy^2}{y^2} + \frac{L_\rho}{2}dz^2 + \frac{\bar{L}_\rho}{2}d\bar{z}^2 + \left(\frac{1}{y^2} + \frac{y^2}{4}L_\rho\bar{L}_\rho\right)dz\,d\bar{z},$$
- The holomorphic function $L_\rho(z)$ is related to the vev of boundary stress tensor: $$\langle T(z)\rangle_\rho = -\frac{c}{12}L_\rho(z), \quad \langle \bar{T}(\bar{z})\rangle_\rho = -\frac{c}{12}\bar{L}_\rho(\bar{z}).$$
- This seemingly suggests that every CFT state corresponds to a Banados metric.
- However, a simple example shows the opposite: the linear supposition of the CFT quantum states. This is because the bulk gravity is classical.
- *Can we formulate a general criterion for (non-)geometric states?*

Geometric state criterion

- By studying the (short-interval expansion of) entanglement entropy, we find some states have no bulk description a la Banados metric.

- This is based on following simple observation: If a CFT state of order c stress tensor can be described by the Banados metric (order one in 1/c expansion), then

$$S_{RT} \sim \frac{A}{4G} \sim O(c)$$

- The criterion for a CFT2 state to be bulk geometric, its entanglement/Renyi entropy calculated from CFT should be also order c at most in the large c limit.

2. Can we locally distinguish the microstates of a black hole?

- By ETH or canonical typicality, a typical state locally looks like thermal state.
- In dual gravity, this means that black hole microstate geometry (aAdS metric) locally looks like BTZ black hole.
- Can we distinguish the black hole's microstates?
- In our previous works 1610.01362, 1703.08724 & 1708.05090, we showed that the EE of heavy primary states are the same as the EE of thermal state at leading order of c.
- Can we have quantify this (in-)distinguishability?

Holevo information

- If we encode a random variable X (with outcome probability denoted by p_i) into a quantum state $\rho = \sum_i p_i \rho_i$

- By measuring the quantum state to obtain the outcome Y, then the maximum of the mutual information I(X;Y) is the Holevo information.

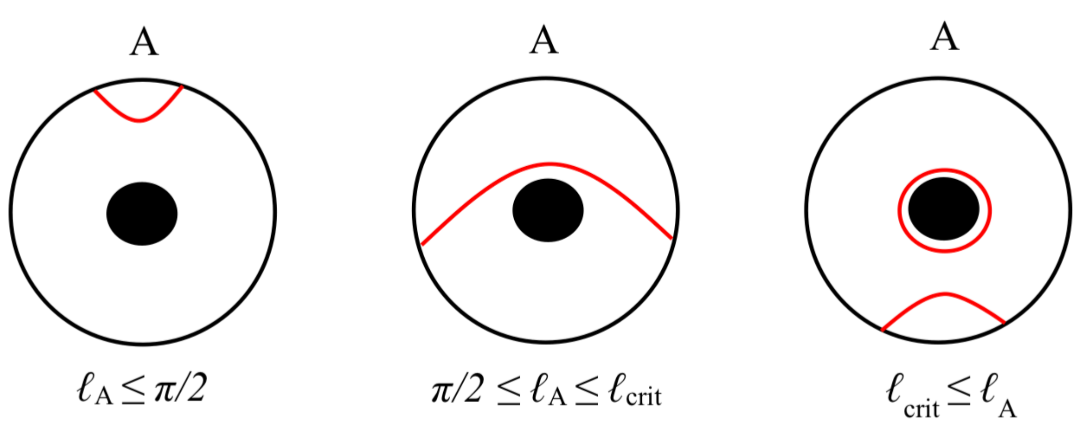
- Apply this to the local distinguishability of black hole microstates, the Holevo information is $\chi_A = S_A - \sum_i p_i S_{A,i}$

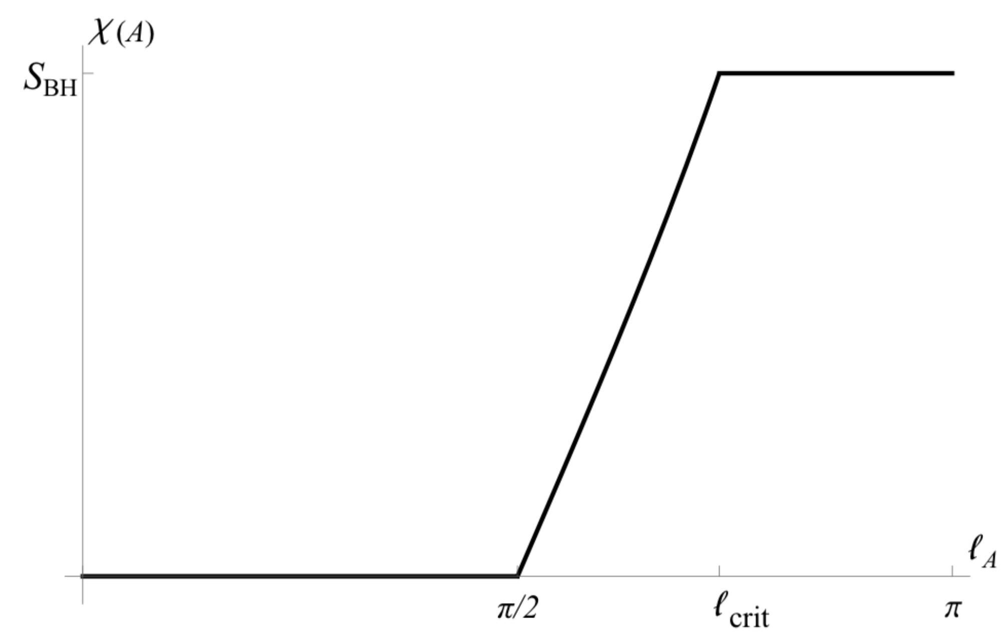
- χ_A is monotonically increasing with A.

- $$0 \leq \chi_A \leq S_{BH} = S_{thermal} := -\sum_i p_i \log p_i$$

- To calculate χ_A , the difficulty lies on how to take thermal average of EE for all CFT state.

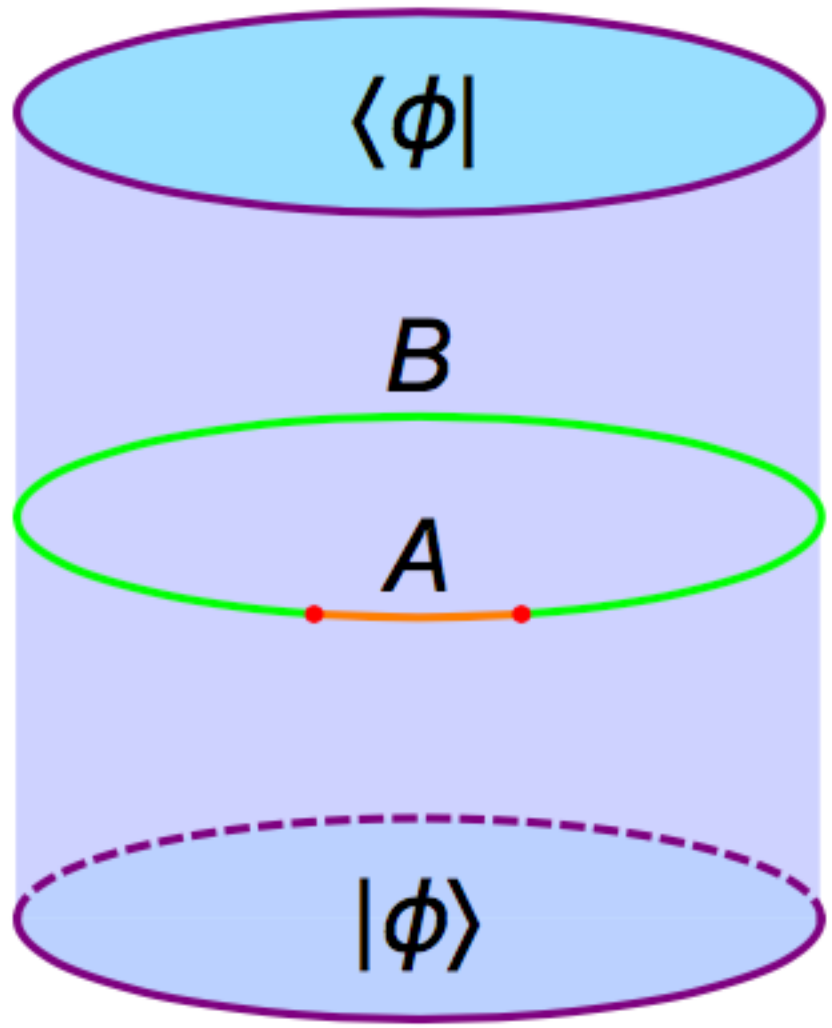
- To bypass this, Bao & Ooguri take the facts in the large c limit: (i) light sates are sparse, (ii) heavy states locally look thermal (not true for non-geometric states) (iii) verified by holographic EE





- One should expect the plateaux will be lifted by lifted by 1/c effect as I will show below.

Entanglement Entropy in 2D CFT



$\langle\phi|$
B
A
$|\phi\rangle$

Renyi entropy

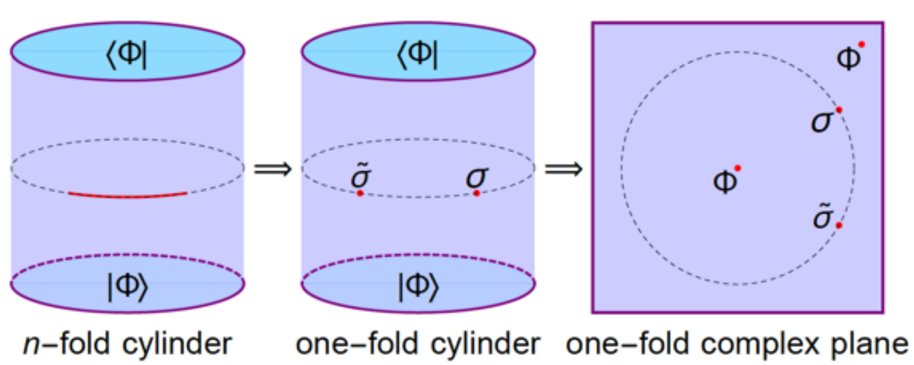
c.f. Calabrese & Cardy (2004)

- Renyi entropy is defined as $S_n = -\frac{1}{n-1}\log \mathrm{tr}_A \rho_A^n,$
- As n —>1, we have entanglement entropy:

$$S = -\mathrm{tr}_A(\rho_A \log \rho_A).$$

- For 2D CFT, it can be obtained by 2-point function of twist operators of n-fold CFT on Riemann surface $\mathcal{R}$:

$$\mathrm{tr}_A \rho_A^n = \langle \mathcal{T}(\ell)\tilde{\mathcal{T}}(0)\rangle_{\mathcal{R}}, \quad h_{\mathcal{T}} = h_{\tilde{\mathcal{T}}} = \frac{c(n^2-1)}{24n}$$



1-fold CFT n-fold CFT n-fold CFT

OPE & Enumerating Quasi-primaries

$$\mathcal{T}(z)\tilde{\mathcal{T}}(w) = \frac{c_n}{(z-w)^{2h_{\mathcal{T}}}} \sum_K d_K \sum_{p=0}^{\infty} \frac{c_K^p}{p!} (z-w)^{h_K+p} \partial^p \Phi_K(w),$$

$$c_K^p := \frac{C^p_{h_K+p-1}}{C^p_{2h_K+p-1}}, \qquad d_K = \frac{1}{\alpha_K \ell^{h_K}} \lim_{z\to\infty} z^{2h_K} \langle \Phi_K(z) \rangle_{\mathcal{R}_{n,1}},$$

$\alpha_K =$ normalization of quasiprimary operator Φ_K

- Assuming translational symmetry, we have

$$\mathrm{tr}_A \rho_A^n = \frac{c_n}{\ell^{2h_{\mathcal{T}}}} \sum_{K \in CFT^n} d_K \ell^{h_K} \langle \Phi_K \rangle_{\mathcal{R}}$$

- Counting the quasiprimaries of n-fold CFT:

$$[x + (1-x)\mathrm{tr}\, x^{L_0}]^n = 1 + nx^2 + \frac{n(n+1)}{2} x^4 + \frac{n(n^2+3n+8)}{6} x^6 + \frac{n(n+1)(n^2+5n+30)}{24} x^8 + \cdots$$

$$\Phi_K^{j_1\cdots j_k} = \mathcal{X}_1^{j_1} \cdots \mathcal{X}_k^{j_k} \ \Rightarrow\ \langle \mathcal{X}_1^{j_1} \cdots \mathcal{X}_k^{j_k} \rangle_{\mathcal{R}} = \langle \mathcal{X}_1 \rangle_{\mathcal{R}} \cdots \langle \mathcal{X}_k \rangle_{\mathcal{R}} \quad \text{with}$$

$$b_{\mathcal{X}_1\cdots\mathcal{X}_k} := \sum_{j_1,\cdots,j_k} d^{j_1\cdots j_k}_{\mathcal{X}_1\cdots\mathcal{X}_k} \text{ with some constraints for } 0 \le j_1,\cdots,j_k \le n-1.$$

level	quasiprimary	?	#	#
0	1	-	1	1
2	T	✓	n	n
4	$\mathcal{A}$	×	n	$\frac{n(n+1)}{2}$
	TT	✓	$\frac{n_2}{2}$	
6	$\mathcal{B}$, $\mathcal{D}$	×	$2n$	$\frac{n(n^2+3n+8)}{6}$
	$T\mathcal{A}$	×	n_2	
	TTT	✓	$\frac{n_3}{6}$	
8	$\mathcal{E}$, $\mathcal{H}$, $\mathcal{I}$	×	$3n$	$\frac{n(n+1)(n^2+5n+30)}{24}$
	$T\mathcal{B}$, $T\mathcal{D}$	×	$2n_2$	
	$\mathcal{AA}$	✓	$\frac{n_2}{2}$	
	$TT\mathcal{A}$	✓	$\frac{n_3}{2}$	
	$TTTT$	✓	$\frac{n_4}{24}$	
9	$\mathcal{A}^{(9)}$	×	n	n
10	$\mathcal{A}^{(10,m)}$	×	$4n$	$\frac{n(n+1)(n+2)(n^2+7n+72)}{120}$
	$T\mathcal{E}$, $T\mathcal{H}$, $T\mathcal{I}$ $\mathcal{AB}$, $\mathcal{AD}$	×	$5n_2$	
	$TT\mathcal{B}$, $TT\mathcal{D}$	×	n_3	
	$T\mathcal{AA}$	✓	$\frac{n_3}{2}$	
	$TTT\mathcal{A}$	✓	$\frac{n_4}{6}$	
	$TTTTT$	✓	$\frac{n_5}{120}$	

level	quasiprimary	?	#	#
11	$\mathcal{A}^{(11,m)}$	×	$2n$	$n(n+1)$
	$T\mathcal{A}^{(9)}$	×	n_2	
12	$\mathcal{A}^{(12,m)}$	×	$7n$	$\frac{n(n+3)(n^4+12n^3+169n^2+438n+640)}{720}$
	$T\mathcal{A}^{(10,m)}$ $\mathcal{AE}$, $\mathcal{AH}$, $\mathcal{AI}$ $\mathcal{BD}$	×	$8n_2$	
	$\mathcal{BB}$, $\mathcal{DD}$	✓	n_2	
	$TT\mathcal{E}$, $TT\mathcal{H}$ $TT\mathcal{I}$	×	$\frac{3n_3}{2}$	
	$T\mathcal{AB}$, $T\mathcal{AD}$ $\mathcal{AAA}$	✓	$\frac{13n_3}{6}$	
	$TTT\mathcal{B}$, $TTT\mathcal{D}$ $TT\mathcal{AA}$	✓	$\frac{7n_4}{12}$	
	$TTTT\mathcal{A}$	✓	$\frac{n_5}{24}$	
	$TTTTTT$	✓	$\frac{n_6}{720}$	
13	$\mathcal{A}^{(13,m)}$	×	$3n$	$\frac{n(n+1)(n+2)}{2}$
	$T\mathcal{A}^{(11,m)}$ $\mathcal{AA}^{(9)}$	×	$3n_2$	
	$TT\mathcal{A}^{(9)}$	×	$\frac{n_3}{2}$	
$\cdots$				

EE in small interval expansion

$$S_A^{(n)} = \frac{c(n+1)}{12n}\log\frac{\ell}{\epsilon} - \frac{1}{n-1}\log\left(1+\sum_{k=1}^{n}\sum_{\{\mathcal{X}_1,\cdots,\mathcal{X}_k\}}\ell^{h_{\mathcal{X}_1}+\cdots+h_{\mathcal{X}_k}} b_{\mathcal{X}_1\cdots\mathcal{X}_k}\langle\mathcal{X}_1\rangle_{\mathcal{R}}\cdots\langle\mathcal{X}_k\rangle_{\mathcal{R}}\right)$$

$$S_A = \frac{c}{6}\log\frac{\ell}{\epsilon} + \sum_{k=1}^{\infty}\sum_{\{\mathcal{X}_1,\cdots,\mathcal{X}_k\}}\ell^{h_{\mathcal{X}_1}+\cdots+h_{\mathcal{X}_k}} a_{\mathcal{X}_1\cdots\mathcal{X}_k}\langle\mathcal{X}_1\rangle_{\mathcal{R}}\cdots\langle\mathcal{X}_k\rangle_{\mathcal{R}} \quad \text{with} \quad a_{\mathcal{X}_1\cdots\mathcal{X}_k} := -\lim_{n\to 1}\frac{b_{\mathcal{X}_1\cdots\mathcal{X}_k}}{n-1}.$$

Explicitly,

$$\begin{aligned}
S_A \;=\;& \frac{c}{6}\log\frac{\ell}{\epsilon} + \ell^2 a_T\langle T\rangle_\rho + \ell^4 a_{TT}\langle T\rangle_\rho^2 + \ell^6 a_{TTT}\langle T\rangle_\rho^3 \\
+\;& \ell^8\big(a_{\mathcal{AA}}\langle A\rangle_\rho^2 + a_{TT\mathcal{A}}\langle T\rangle_\rho^2\langle A\rangle_\rho + a_{TTTT}\langle T\rangle_\rho^4\big) \\
+\;& \ell^{10}\big(a_{T\mathcal{AA}}\langle T\rangle_\rho\langle A\rangle_\rho^2 + a_{TTT\mathcal{A}}\langle T\rangle_\rho^3\langle A\rangle_\rho + a_{TTTTT}\langle T\rangle_\rho^5\big) \\
+\;& \ell^{12}\big(a_{\mathcal{BB}}\langle B\rangle_\rho^2 + a_{\mathcal{DD}}\langle D\rangle_\rho^2 + a_{T\mathcal{AB}}\langle T\rangle_\rho\langle A\rangle_\rho\langle B\rangle_\rho + a_{T\mathcal{AD}}\langle T\rangle_\rho\langle A\rangle_\rho\langle D\rangle_\rho \\
+\;& a_{\mathcal{AAA}}\langle A\rangle_\rho^3 + a_{TTT\mathcal{B}}\langle T\rangle_\rho^3\langle B\rangle_\rho + a_{TTT\mathcal{D}}\langle T\rangle_\rho^3\langle D\rangle_\rho + a_{TT\mathcal{AA}}\langle T\rangle_\rho^2\langle A\rangle_\rho^2 \\
+\;& a_{TTTT\mathcal{A}}\langle T\rangle_\rho^4\langle A\rangle_\rho + a_{TTTTTT}\langle T\rangle_\rho^6\big) + O(\ell^{14}).
\end{aligned}$$

$$\begin{aligned}
& a_T = -\frac{1}{6},\quad a_{TT} = -\frac{1}{30c},\quad a_{TTT} = -\frac{4}{315c^2}, \\
& a_{\mathcal{AA}} = -\frac{1}{126c(5c+22)},\quad a_{TT\mathcal{A}} = \frac{1}{315c^2},\quad a_{TTTT} = -\frac{c+8}{630c^3}, \\
& a_{T\mathcal{AA}} = -\frac{16}{693c^2(5c+22)},\quad a_{TTT\mathcal{A}} = \frac{3}{385c^3},\quad a_{TTTTT} = -\frac{16(c+5)}{3465c^4}, \\
& a_{\mathcal{BB}} = -\frac{25}{123552c(70c+29)},\quad a_{\mathcal{DD}} = -\frac{70c+29}{18018c(2c-1)(5c+22)(7c+68)}, \\
& a_{T\mathcal{AB}} = -\frac{10}{1287c^2(70c+29)},\quad a_{T\mathcal{AD}} = \frac{5}{3003c^2(5c+22)},\quad a_{\mathcal{AAA}} = \frac{4(5c+64)}{3003c^2(5c+22)^2}, \\
& a_{TTT\mathcal{B}} = \frac{5(14c+43)}{9009c^3(70c+29)},\quad a_{TTT\mathcal{D}} = -\frac{2}{9009c^3},\quad a_{TT\mathcal{AA}} = -\frac{585c+10804}{90090c^3(5c+22)}, \\
& a_{TTTT\mathcal{A}} = \frac{2(33c+784)}{45045c^4},\quad a_{TTTTTT} = -\frac{2(11c^2+380c+1480)}{45045c^5}.
\end{aligned}$$

EEs

- For the ground state of a CFT on a circle of size L:

$$\langle T\rangle_L=\frac{\pi^2 c}{6L^2},\quad \langle\mathcal{A}\rangle_L=\frac{\pi^4 c(5c+22)}{180L^4},\quad \langle\mathcal{B}\rangle_L=-\frac{62\pi^6 c}{525L^6},\quad \langle\mathcal{D}\rangle_L=\frac{\pi^6 c(2c-1)(5c+22)(7c+68)}{216(70c+29)L^6}$$

$$S_A=\frac{c}{6}\log\frac{\ell}{\epsilon}-\frac{\pi^2 c\ell^2}{36L^2}-\frac{\pi^4 c\ell^4}{1080L^4}-\frac{\pi^6 c\ell^6}{17010L^6}-\frac{\pi^8 c\ell^8}{226800L^8}-\frac{\pi^{10}c\ell^{10}}{2806650L^{10}}-\frac{691\pi^{12}c\ell^{12}}{22986463500L^{12}}+O(\ell^{14})$$

- For a low temperature state on a circle: $q:=e^{-2\pi\beta/L}<<1$

$$\langle T\rangle_{L,q}=\frac{\pi^2}{6L^2}[c-48q^2-72q^3-144q^4+O(q^5)],$$
$$\langle\mathcal{A}\rangle_{L,q}=\frac{\pi^4}{180L^4}[c(5c+22)+480(5c+22)q^2+2160(5c+22)q^3+30240(c+6)q^4+O(q^5)],$$
$$\langle\mathcal{B}\rangle_{L,q}=-\frac{2\pi^6}{525L^6}[31c-1008(120c+1)q^2-1512(720c+161)q^3-3024(1640c+841)q^4+O(q^5)],$$
$$\langle\mathcal{D}\rangle_{L,q}=\frac{\pi^6(2c-1)(7c+68)}{216(70c+29)L^6}[c(5c+22)+1584(5c+22)q^2+6696(5c+22)q^3+432(215c-638)q^4+O(q^5$$

$$\begin{aligned}S_A \;=\;& \frac{c}{6}\log\frac{\ell}{\epsilon}+\Big[-\frac{c}{36}+\frac{4q^2}{3}+2q^3+4q^4+O(q^5)\Big]\Big(\frac{\pi\ell}{L}\Big)^2+\Big[-\frac{c}{1080}+\frac{4q^2}{45}+\frac{2q^3}{15}\\
&+\frac{4(c-8)q^4}{15c}+O(q^5)\Big]\Big(\frac{\pi\ell}{L}\Big)^4+\Big[-\frac{c}{17010}+\frac{8q^2}{945}+\frac{4q^3}{315}+\frac{8(c-16)q^4}{315c}+O(q^5)\Big]\Big(\frac{\pi\ell}{L}\Big)^6\\
&+\Big[-\frac{c}{226800}+\frac{4q^2}{4725}+\frac{2q^3}{1575}-\frac{4(159c+728)q^4}{1575c}+O(q^5)\Big]\Big(\frac{\pi\ell}{L}\Big)^8\\
&+\Big[-\frac{c}{2806650}+\frac{8q^2}{93555}+\frac{4q^3}{31185}-\frac{104(295c+1312)q^4}{155925c}+O(q^5)\Big]\Big(\frac{\pi\ell}{L}\Big)^{10}\\
&+\Big[-\frac{691c}{22986463500}+\frac{5528q^2}{638512875}+\frac{2764q^3}{212837625}-\frac{8(21728429c+15283768)q^4}{212837625c}+O(q^5)\Big]\Big(\frac{\pi\ell}{L}\Big)^{12}\\
&+O(\ell^{14})\end{aligned}$$

Geometric vs Non-geometric

Conditions on expectation values of quasiprimaries

- As shown, the expectation values of the quasiprimaries dictates the entanglement/Renyi entropy.

- Requiring the Renyi entropy of A in a state ρ to be at most order c yield the following constraints:

$$\langle T\rangle_\rho = c\alpha(w)+\beta(w)+\frac{\gamma(w)}{c}+O\Big(\frac{1}{c^2}\Big),$$

$$\langle \mathcal{A}\rangle_\rho = c^2\alpha(w)^2+c\delta(w)+\epsilon(w)+O\Big(\frac{1}{c}\Big),$$

$$\langle \mathcal{B}\rangle_\rho = c^2\Big[\alpha'(w)^2-\frac{4}{5}\alpha(w)\alpha''(w)\Big]+c\zeta(w)+O(c^0),$$

$$\langle \mathcal{D}\rangle_\rho = c^3\alpha(w)^3+3c^2\alpha(w)[\delta(w)-\alpha(w)\beta(w)]+c\eta(w)+O(c^0),$$

$$\langle \mathcal{E}\rangle_\rho = c^2\Big\{\alpha''(w)^2+\frac{10}{63}[\alpha(w)\alpha^{(4)}(w)-7\alpha'(w)\alpha^{(3)}(w)]\Big\}+O(c),$$

$$\begin{aligned}\langle \mathcal{H}\rangle_\rho = {} & c^3\alpha(w)\Big[\alpha'(w)^2-\frac{4}{5}\alpha(w)\alpha''(w)\Big]+c^2\Big[-\alpha'(w)^2\beta(w)-2\alpha(w)\alpha'(w)\beta'(w)+\frac{4}{5}\alpha(w)^2\beta''(w)\\ & +\frac{8}{5}\alpha(w)\alpha''(w)\beta(w)+\alpha'(w)\delta'(w)-\frac{4}{5}\alpha''(w)\delta(w)-\frac{2}{9}\alpha(w)\delta''(w)+\frac{13}{9}\alpha(w)\zeta(w)\Big]+O(c),\end{aligned}$$

$$\begin{aligned}\langle \mathcal{I}\rangle_\rho = {} & c^4\alpha(w)^4+2c^3\alpha(w)^2[3\delta(w)-4\alpha(w)\beta(w)]+c^2[12\alpha(w)^2\beta(w)^2+4\alpha(w)^3\gamma(w)\\ & -12\alpha(w)\beta(w)\delta(w)+3\delta(w)^2-6\alpha(w)^2\epsilon(w)+4\alpha(w)\eta(w)]+O(c)\end{aligned}$$

c.f. n—>1 for EE:

$$\langle T\rangle_\rho = c\alpha(w)+O(c^0),$$

$$\langle \mathcal{A}\rangle_\rho = c^2\alpha(w)^2+O(c).$$

with $\alpha(w)$, $\beta(w)$, $\gamma(w)$, $\delta(w)$, $\epsilon(w)$, $\zeta(w)$, $\eta(w)$ being arbitrary order c^0 holomorphic functions.

Examples

- Heavy primary states are geometrical as can be checked straightforwardly.

- Thermal states (dual to BTZ) satisfy the constraints as expected. The above twos are consistent with the ETH.

- A more nontrivial example is the ``coherent" CFT state, which is dual to a bulk moving particle:

$$\rho_{\phi(z_0)} = \frac{1}{\alpha_\phi}\Big(\frac{1-z_0\bar{z}_0}{\bar{z}_0}\Big)^{2h_\phi}\phi(z_0)|0\rangle\langle 0|\phi(1/\bar{z}_0).$$

- This coherent state example suggests existence of some kind of correspondence principle for the geometric states.

Correspondence Principle

- We look for the quantum observables (e.g.,KdV currents) whose one-point functions w.r.t. geometric states obey the classical counterpart of the equations of motion.

Quantum KdV equations: $\dot{T} = \frac{1-c}{6} T''' - 3(TT)' = -\frac{5c+22}{30} T''' - 3\mathcal{A}'.$

$$J^{\rho}_{2k}(w) \equiv \lim_{\substack{c \to \infty \\ L}} \frac{6^k}{c^k} \langle J_{2k}(w) \rangle_{\rho}$$

$$Q_{2k-1} = \int_0^L \frac{dw}{L} J_{2k}(w)$$

$$J_2 = T, \quad J_4 = \mathcal{A} + \frac{3}{10} T'', \quad J_6 = \mathcal{D} - \frac{25(2c+7)(7c+68)}{108(70c+29)} \mathcal{B} - \frac{2c-23}{108} \mathcal{A}'' - \frac{c-14}{280} T^{(4)},$$

$$J_8 = \mathcal{I} - \frac{5(3c+46)(350c^2+2315c-361)}{39(1050c^2+3305c-251)} \mathcal{H} + \frac{49(3c+46)(5c^2+46c+99)}{2860(105c+11)} \mathcal{E} + \frac{5c+94}{234} \mathcal{D}''$$
$$- \frac{490c^3+6713c^2+6887c+53032}{1404(70c+29)} \mathcal{B}'' + \frac{(c+7)(7c+89)}{11880} \mathcal{A}^{(4)} + \frac{7c^2+68c-315}{9072} T^{(6)}.$$

Classical KdV equations: $\dot{U} = U''' + 6UU'.$

$$J^{\rho}_2 = U, \quad J^{\rho}_4 = U^2, \quad J^{\rho}_6 = U^3 - \frac{1}{2} U'^2,$$

$$J^{\rho}_8 = U^4 + U^2 U'' + \frac{1}{5} U U^{(4)}.$$

non-examples

- As the classical gravity cannot reflect the quantum linear superposition principle, we expect the superpositions of the primary states are non-geometric. This is indeed the case.

- Besides, we find the following descendant states are non-geometric:

$$
\begin{aligned}
&|\phi^{(m)}\rangle \text{ with } h_\phi + m \sim O(c), \\
&|\tilde{\phi}\rangle \text{ with } h_\phi \sim O(c), \\
&|\tilde{\phi}^{(m)}\rangle \text{ with } h_\phi + m \sim O(c), \\
&|T^{(m)}\rangle \text{ with } m \sim O(c), \\
&|\mathcal{A}^{(m)}\rangle \text{ with } m \sim O(c).
\end{aligned}
$$

$$\tilde{\phi} := (T\phi) - \frac{3}{2(h_\phi + 1)}\phi''$$

Implications

- In perturbative quantum gravity, the metric is corrected by G~1/c correction, and thus we will not expect these effect will turn the non-non-geometric states into the geometric ones.

- By Cardy's formula, we find that the fraction of primary states is negligible in the thermodynamical limit. If most of the descendant states are non-geometric, they will be also non-thermal. This will then implies the canonical typicality of quantum thermalization fails in the large c 2D CFTs.

Holevo information of microstates

Thermal average of EEs

- To find the 1/c correction to the Holevo information, we need to find the thermal average of EEs over all states (inclu. primaries and descendants).

- In the small interval expansion of EE, this can be done by finding the thermal average of one-point functions (and its powers) of quasiprimaries. This relies on the following:

$$\sum_i p_i \langle \mathcal{X} \rangle_{\rho_i} = \langle \mathcal{X} \rangle_\beta, \quad \mathcal{X} = T, \mathcal{A},$$

$$\sum_i p_i \langle T \rangle_{\rho_i}^r X_i = \left(\frac{2\pi}{L}\right)^r \frac{\partial_\beta^r (\exp^{\frac{\pi c L}{12\beta}} \sum_i p_i X_i)}{\exp^{\frac{\pi c L}{12\beta}}}, \qquad \beta \ll L$$

$$\sum_i p_i \langle T \rangle_{\rho_i}^r = \left(\frac{2\pi \mathrm{i}}{L}\right)^{2r} \frac{(q\partial_q)^r Z(q)}{Z(q)}, \quad \text{with} \;\; Z(q) = q^{-\frac{c}{24}}[1 + q^2 + q^3 + 2q^4 + O(q^5)], \qquad q := e^{-\frac{2\pi\beta}{L}} \ll 1$$

$$\sum_i p_i \langle \mathcal{X} \rangle_{\rho_i} \langle \mathcal{Y} \rangle_{\rho_i} = \frac{1}{L} \int_{-L/2}^{L/2} dx \langle \mathcal{X}(x) \mathcal{Y}(0) \rangle_\beta \quad \Longleftarrow \quad [T, \mathcal{A}] = 0.$$

High T case:

$$\sum_i p_i \langle T\rangle_{\rho_i} = -\frac{\pi^2 c}{6\beta^2}, \quad \sum_i p_i \langle T\rangle^2_{\rho_i} = \frac{\pi^3 c(\pi c L + 24\beta)}{36\beta^4 L}, \quad \sum_i p_i \langle T\rangle^3_{\rho_i} = -\frac{\pi^4 c(\pi^2 c^2 L^2 + 72\pi c\beta L + 864\beta^2)}{216\beta^6 L^2},$$

$$\sum_i p_i \langle T\rangle^4_{\rho_i} = \frac{\pi^5 c(\pi^3 c^3 L^3 + 144\pi^2 c^2 \beta L^2 + 5184\pi c\beta^2 L + 41472\beta^3)}{1296\beta^8 L^3},$$

$$\sum_i p_i \langle T\rangle^5_{\rho_i} = -\frac{\pi^6 c(\pi^4 c^4 L^4 + 240\pi^3 c^3 \beta L^3 + 17280\pi^2 c^2 \beta^2 L^2 + 414720\pi c\beta^3 L + 2488320\beta^4)}{7776\beta^{10} L^4},$$

$$\sum_i p_i \langle \mathcal{A}\rangle_{\rho_i} = \frac{\pi^4 c(5c+22)}{180\beta^4}, \quad \sum_i p_i \langle T\rangle_{\rho_i}\langle \mathcal{A}\rangle_{\rho_i} = -\frac{\pi^5 c(5c+22)(\pi c L + 48\beta)}{1080\beta^6 L},$$

$$\sum_i p_i \langle T\rangle^2_{\rho_i}\langle \mathcal{A}\rangle_{\rho_i} = \frac{\pi^6 c(5c+22)(\pi^2 c^2 L^2 + 120\pi c\beta L + 2880\beta^2)}{6480\beta^8 L^2},$$

$$\sum_i p_i \langle T\rangle^3_{\rho_i}\langle \mathcal{A}\rangle_{\rho_i} = -\frac{\pi^7 c(5c+22)(\pi^3 c^3 L^3 + 216\pi^2 c^2 \beta L^2 + 12960\pi c\beta^2 L + 207360\beta^3)}{38880\beta^{10} L^3},$$

$$\sum_i p_i \langle \mathcal{A}\rangle^2_{\rho_i} = \frac{\pi^7 c(5c+22)[7\pi c(5c+22)L + 480(7c+74)\beta]}{226800\beta^8 L},$$

$$\sum_i p_i \langle T\rangle_{\rho_i}\langle \mathcal{A}\rangle^2_{\rho_i} = -\frac{\pi^8 c(5c+22)[7\pi^2 c^2(5c+22)L^2 + 192\pi c(35c+262)\beta L + 40320(7c+74)\beta^2]}{1360800\beta^{10} L^2}.$$

$$\chi_A = \frac{2\pi^3\ell^4}{45\beta^3 L} - \frac{8\pi^4\ell^6(\pi c L + 12\beta)}{945 c\beta^5 L^2} + \cdots + O(\ell^{12}).$$

NB: order c^0 !!

To the order we considered, it vanishes in the thermodynamic limit, i.e., $L \longrightarrow \infty$ but with β, ℓ fixed

Low T case:

$$\sum_i p_i \langle T\rangle_{\rho_i} = \langle T\rangle_{\mathcal{T}} = \frac{\pi^2 c}{6L^2} - \frac{8\pi^2 q^2}{L^2} - \frac{12\pi^2 q^3}{L^2} - \frac{24\pi^2 q^4}{L^2} + O(q^5)$$

$$\sum_i p_i \langle T\rangle^2_{\rho_i} = \frac{\pi^4 c^2}{36L^4} - \frac{8\pi^4(c-24)q^2}{3L^4} - \frac{4\pi^4(c-36)q^3}{L^4} - \frac{8\pi^4(c-56)q^4}{L^4} + O(q^5)$$

$$\sum_i p_i \langle T\rangle^3_{\rho_i} = \frac{\pi^6 c^3}{216L^6} - \frac{2\pi^6(c^c-48c+768)q^2}{3L^6} - \frac{\pi^6(c^2-72c+1728)q^3}{L^6} - \frac{2\pi^6(c^2-112c+3840)q^4}{L^6} + O(q^5),$$

NB: order $1/c$!!

$$\begin{aligned}\chi_A \quad = \quad & \left[\frac{32q^2}{15c} + \frac{24q^3}{5c} + \frac{64q^4}{5c} + O(q^5)\right]\left(\frac{\pi\ell}{L}\right)^4 \\ & + \left[\frac{128(c-16)q^2}{315c^2} + \frac{32(c-24)q^3}{35c^2} + \frac{256(c-40)q^4}{105c^2} + O(q^5)\right]\left(\frac{\pi\ell}{L}\right)^6 \\ & + O(\ell^8)\end{aligned}$$

Long-interval cases

- For the long-interval case, we use the fact $S_{B,i} = S_{A,i}$, and the know result for thermal state EE:

$$S_B = \frac{c}{3}\log\left(\frac{\beta}{\pi\epsilon}\sinh\frac{\pi\ell}{\beta}\right) + \frac{\pi c L}{3\beta} - I(1-\exp^{-\frac{2\pi\ell}{\beta}}),$$

c.f. Chen & Wu, 1412.0761, 1506.03206

where $I(x)$ is the mutual inforamtion of two intervals on a complex plane with cross ration x.

High T case:

NB: order c^0 correction!!

$$\chi_B = \frac{\pi c L}{3\beta} - \frac{2\pi^3(4\pi L - 7\beta)\ell^4}{315\beta^4 L} + \frac{32\pi^5\ell^5}{3465\beta^5} + \frac{8\pi^4(32\pi^2L^2 - 143\pi\beta L)\ell^6}{135135\beta^6 L^2} + \cdots + O(\ell^{11}, 1/c).$$

Low T case:

$$\chi_B = S(L) - \left[\frac{32\pi\beta(\beta^2+L^2)(4\beta^2+L^2)}{15L^5}q^2 + O(q^3)\right]\left(\frac{\pi\ell}{L}\right)^4 + O(\ell^5).$$

Micro-canonical ensemble (MCE)

- We can also consider the Holevo information in the MCE with fixed high energy E, i.e., $p_i = \frac{\delta(E - E_i)}{\Omega(E)}, \text{ with } \Omega(E) = \sqrt{\frac{\pi c L}{6E}} I_1\Big(\sqrt{\frac{2\pi c L E}{3}}\Big),$
- We can obtain the multi-point functions/EEs in MCE by inverse Laplace transform of the ones in Canonical ensemble (CE), similar to the trick deriving the Cardy's formula for $\Omega(E)$.

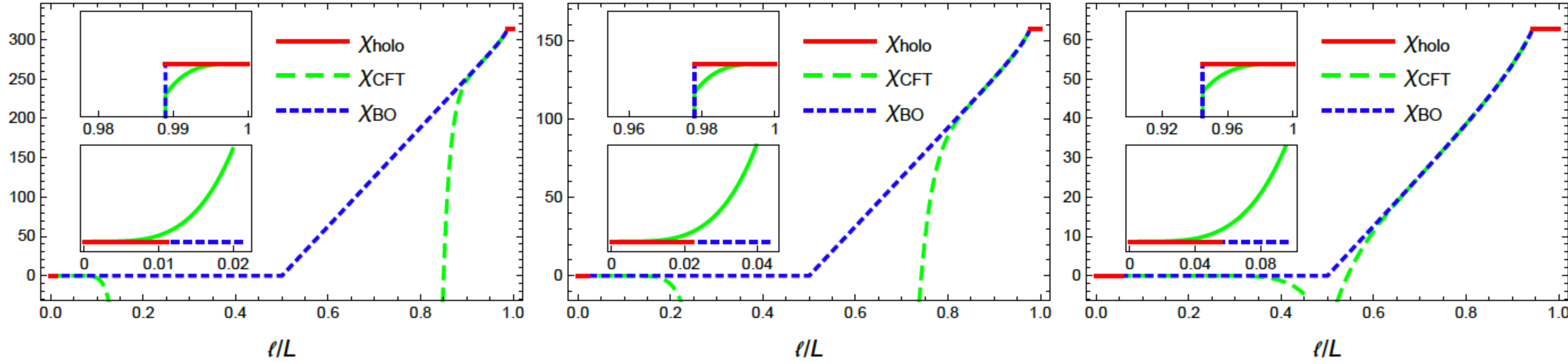
Short-interval: $$\chi_A = \frac{\pi^3 \ell^4 [\pi c L (I_3 - I_1) + 24\lambda I_2]}{540\lambda^4 L I_1} + \cdots + O(\ell^{12})$$

NB: order c !!

Long-interval: $$\chi_B = S(L) + O(\ell^{12})$$

There are also non-universal contribution from non-identity families.

c=30 here!



(a) $\beta/L = 0.1$ (b) $\beta/L = 0.2$ (c) $\beta/L = 0.5$

$$\chi_{\mathrm{holo}}(\ell) = \begin{cases} 0 & \ell < \frac{\beta}{2\pi}\log 2 \\ \frac{\pi c L}{3\beta} & \ell > L - \frac{\beta}{2\pi}\log 2 \end{cases}.$$

$$\chi_A = \frac{2\pi^3\ell^4}{45\beta^3 L} - \frac{8\pi^4\ell^6(\pi c L + 12\beta)}{945 c\beta^5 L^2} + \cdots + O(\ell^{12}).$$

$$\chi_B = \frac{\pi c L}{3\beta} - \frac{2\pi^3(4\pi L - 7\beta)\ell^4}{315\beta^4 L} + \frac{32\pi^5\ell^5}{3465\beta^5} + \frac{8\pi^4(32\pi^2 L^2 - 143\pi\beta L)\ell^6}{135135\beta^6 L^2} + \cdots + O(\ell^{11}, 1/c).$$

$$\chi_{\mathrm{BO}}(\ell) = \begin{cases} 0 & \ell < L/2 \\ \frac{c}{3}\log\frac{\sinh\frac{\pi\ell}{\beta}}{\sinh\frac{\pi(L-\ell)}{\beta}} & L/2 < \ell < L - \frac{\beta}{2\pi}\log 2, \\ \frac{\pi c L}{3\beta} & \ell > L - \frac{\beta}{2\pi}\log 2 \end{cases}$$

Implications & Conclusions

- We see that the Holevo information got 1/c corrections but is consistently vanishing as interval size goes to zero.
- Recall that Holevo information=average of relative entropy over all states, including descendants.
- Since most of states are descendants (at least in the thermodynamic limit), many of which we shown are non-geometric and should contribute with higher order of c. $$\frac{\Omega_p(E)}{\Omega(E)} \sim \mathrm{e}^{-L\left[\sqrt{\frac{2\pi c\varepsilon}{3}}-\sqrt{\frac{2\pi(c-1)}{3}\left(\varepsilon+\frac{\pi}{6L^2}\right)}\right]} \to 0 \text{ as } L \to \infty.$$
- Thus, our results implies there is a subtle cancellation of non-geometric state's contributions in the average of relative entropy to yield order c Holevo information.
- This implies that even the non-perturbative quantum gravity effect looks wild, it is miraculously to yield averaging classical geometry of BTZ.