

Microstates counting of magnetically charged AdS4 Black hole made of Wrapped M5-branes

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ArXiv : 1808.02797 with Nakwoo Kim (KyungHee U)
& WIP with Nakwoo Kim, Pando Zayas and James Liu (Michigan U)

A Magnetically charged AdS4 Black hole

Classically,

$$ds^2 = -\left(\rho - \frac{1}{2\rho}\right)^2 dt^2 + \left(\rho - \frac{1}{2\rho}\right)^{-2} d\rho^2 + \rho^2 ds^2(\Sigma_g)$$

$$F = \frac{dx_1 \wedge dx_2}{x_2^2} \quad (\text{Magnetic flux for U(1) gauge field along Riemann surface } \Sigma_{g>0})$$

BPS Solution for 4D $\mathcal{N}=2$ minimal gauged supergravity

$$I = \frac{1}{16\pi G_4} \int d^4x \sqrt{-g} \left(R + 6 - \frac{1}{4} F^2 \right) + (\text{fermions})$$

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In terms of AdS/CFT, the BH solution describes

RG : (3D $\mathcal{N}=2$ SCFT on $\mathbf{R}_t \times \Sigma_g$)  (1D SQM on \mathbf{R}_t)

With topological twisting : $(A^{(b.g)})_R = -\omega(\Sigma_g)$

$(\partial + (\omega + (A^{(b.g)})_R) \cdot) \epsilon = 0$ preserving 1/2 SUSY (2 supercharges)

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$$S_{\text{BH}} = \frac{A}{4G_4} = \frac{(g-1)\pi}{2G_4} + (\text{subleadings in } G_4)$$

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If the BH solution (AdS4 supergravity) can be embedded into an UV complete Quantum Gravity,

We may give a non-perturbative definition of d_{micro} (# of micorstates of BH), which should satisfy

- 1) $d_{\text{micro}}(g, G_4)$ is an non-negative integer (after including all corrections)

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2) $S_{\text{BH}} = \log d_{\text{micro}}(g, G_4) = \frac{(g-1)\pi}{2G_4} + (\text{subleadings in } G_4)$.

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$dx_1 \wedge dx_2$

In this talk,

- Embedding the BH into **M-theory** on AdS4x(...)
- $d_{micro}(g, G_4)$ using **AdS4/CFT3**
- Check of 1) integrality (at finite N)
2) Bekenstein-Hawking + sub-leading in large N

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A Magnetically charged AdS4 BH in M-theory

BH solution with asymptotically AdS4  Can be studied using AdS4/CFT3

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BH solution with asymptotically AdS4 \longrightarrow Can be studied using AdS4/CFT3

Two classes of well-established AdS4/CFT3 using M-theory

AdS4/CFT3 from M2-branes

$R^{1,2} \times \text{Cone}(Y_7)$ (Y_7 : Sasakian 7-manifold)
with N M2-branes on $R^{1,2}$

$\longrightarrow T_N[Y_7]$

3D $\mathcal{N}=2$ SCFT with global $U(1)_R \subset G = \text{ISO}(Y_7)$

M-theory on $\text{AdS}_4 \times Y_7$

\downarrow ($G_4 = \sqrt{\frac{27}{8N^3\pi^4} \text{Vol}(Y_7)}$)

4D $\mathcal{N}=2$ Gauged supergravity with $G = \text{ISO}(Y_7)$

$$S_{\text{BH}} = \frac{(g-1)\pi}{2G_4} = (g-1) \sqrt{\frac{2}{27\text{Vol}(Y_7)}} N^{3/2} \pi^3$$

Field theoretic description of $T_N[Y_7]$

[ABJM;08][HLLLP;09].....

e.g) $T_N[S^7/Z_k] = \text{ABJM model}$

AdS4/CFT3 from M5-branes

$R^{1,2} \times (T^*M) \times R^2$ (T^*M_3 : cotangent-bundle of 3-manifold M_3)
with N M5 branes on $R^{1,2} \times M_3$

$\longrightarrow T_N[M_3]$

3D $\mathcal{N}=2$ SCFT, with global $U(1)_R$

M-Theory on Warped $\text{AdS}_4 \times M_3 \times S^4$ (for hyperbolic M_3)

\downarrow ($G_4 = \frac{3\pi^2}{2N^3 \text{vol}(M)}$)

4D $\mathcal{N}=2$ Gauged supergravity with $G = U(1)$

$$S_{\text{BH}} = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)\text{vol}(M)}{3\pi} N^3$$

Field theoretic description of $T_N[M_3]$

e.g) $T_{N=2}[\text{Thurston}] = U(1) + \text{Phi}$ with $k = -7/2$

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BH solution with asymptotically AdS4 \longrightarrow Can be studied using AdS4/CFT3

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AdS4/CFT3 from M2-branes	AdS4/CFT3 from M5-branes
<p>$R^{1,2} \times \text{Cone}(Y_7)$ (Y_7 : Sasakian 7-manifold) with N M2-branes on $R^{1,2}$</p> <p>$\longrightarrow T_N[Y_7]$</p> <p>3D $\mathcal{N}=2$ SCFT with global $U(1)_R \subset G = \text{ISO}(Y_7)$</p>	<p>$R^{1,2} \times (T^*M) \times R^2$ (T^*M_3: cotangent-bundle of 3-manifold M_3) with N M5 branes on $R^{1,2} \times M_3$</p> <p>$\longrightarrow T_N[M_3]$</p> <p>3D $\mathcal{N}=2$ SCFT, with global $U(1)_R$</p>
<p>M-theory on AdS4xY7</p> <p>\downarrow ($G_4 = \sqrt{\frac{27}{8N^3\pi^4} \text{Vol}(Y_7)}$)</p> <p>4D $\mathcal{N}=2$ Gauged supergravity with $G = \text{ISO}(Y_7)$</p> $S_{\text{BH}} = \frac{(g-1)\pi}{2G_4} = (g-1) \sqrt{\frac{2}{27\text{Vol}(Y_7)}} N^{3/2} \pi^3$	<p>M-Theory on Warped AdS4xM3xS4 (for hyperbolic M_3)</p> <p>[Pernici ;85] \downarrow ($G_4 = \frac{3\pi^2}{2N^3 \text{vol}(M)}$) [Gauntlet-Kim-Waldra;00]</p> <p>4D $\mathcal{N}=2$ Gauged supergravity with $G = U(1)$</p> $S_{\text{BH}} = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)\text{vol}(M)}{3\pi} N^3$
<p>Field theoretic description of $T_N[Y_7]$</p> <p>[ABJM;08][HLLLP;09].....</p> <p>e.g) $T_N[S^7/Z_k] = \text{ABJM model}$</p>	<p>Field theoretic description of $T_N[M_3]$</p> <p>[Dimoft-Gukov-Gaiotto;11][DG-Yonekura;18].....</p> <p>e.g) $T_{N=2}[\text{Figure}] = U(1) + \Phi$ with $k = -9/2$</p>



Non-perturbative definition of d_{micro} using AdS4/CFT3

Question : Which quantity in CFT3 corresponds to the d_{micro} of the BH ?

Hints:

BH : Asymptotic AdS₄ with $\partial(\text{AdS}_4) = \mathbf{R}_t \times \Sigma_g$  Near horizon AdS₂ × Σ_g ,

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Natural Answer : the number of ground states of 3d SCFT on Σ_g

d_{micro}

$$= \dim H^{E=0}(\text{3D } \mathcal{N} = 2 \text{ SCFT on } \Sigma_g)$$

$$= \# \text{ of supersymmetric ground states of (3D } \mathcal{N} = 2 \text{ SCFT on } \Sigma_g)$$

$$\text{cf) } \underline{d_{\text{micro}}^{\text{SUSY}} = \text{Tr}_{H^{E=0}}(\text{3D } \mathcal{N} = 2 \text{ SCFT on } \Sigma_g) (-1)^R = \text{Tr}_H(\text{3D } \mathcal{N} = 2 \text{ SCFT on } \Sigma_g) (-1)^R e^{-\beta E}}$$

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Recently people found that [Benini-Hristov-Zaffaroni ;'16].....

$$\text{Log}(d_{\text{micro}}^{\text{SUSY}}(T_N[Y_7], g)) \xrightarrow{N \rightarrow \infty} \frac{(g-1)\pi}{2G_4} = (g-1) \sqrt{\frac{2}{27\text{Vol}(Y_7)}} N^{3/2} \pi^3 + \text{sub-leading}$$

$$d_{\text{micro}}^{\text{SUSY}}(T_N[Y_7]) = d_{\text{micro}}(T_N[Y_7]) ??$$

Twisted index $d_{\text{micro}}^{\text{SUSY}}(g) = \text{Tr}_{H(3D \mathcal{N}=2 \text{ SCFT on } \Sigma_g)} (-1)^R e^{-\beta E}$

For $g = 1$ ($\Sigma_g = T^2$) case : It is just usual Witten index [Kim-Kim ;'10]
[Seiberg-Intrilligator ;'12]

For $g = 0$ (S^2) case [Benini-Zaffaroni ;'15]

For general g [Closset-Kim ;'16] [Benini-Zaffaroni ;'16]

For general $3d \mathcal{N} = 2$ theory with gauge G ,
the index can be written as finite sum over so called 'Bethe vacua'

$$d_{\text{micro}}^{\text{SUSY}}(g) = \sum_{\alpha: \text{Bethe-vacua}} (H^\alpha)^{g-1}, \quad [\text{Closset-Kim-Willet ;'17}]$$

Bethe vacua: solutions of eqn $\exp\left(2\pi i z_i \frac{\partial W}{\partial z_i}\right) = 1$, for $i = 1, \dots, \text{rank}(G)$

$W(z_1, \dots, z_{\text{rank}(G)})$: Twisted superpotential for $2d$ (2,2) theory
obtained by S^1 reduction keeping all infinity KK-modes

$$\text{Chiral field} : \delta W = \text{Li}_2\left(\prod z_i^{-Q_i}\right), \quad \text{CS term } \delta W = k_{ij} \text{Log}[z_i] \text{Log}[z_j]$$

$H^\alpha(z_1, \dots, z_{\text{rank}(G)})$: 'handle gluing operator',

$$\text{Log}[H] = -\log \text{Det}[\partial_{\log[z_i]} \partial_{\log[z_j]} \text{Log}[W]] + \sum_{\text{Chiral}} \text{Li}_1(z_i^{-Q_i})$$

Most recent studies on AdS4 BH microstates counting are about BH made of **M2-branes**

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(improperly quantized IR R-charge, additional Legendre transformation procedure)

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BH made of **M5-branes?**

$$\text{Log}(d_{\text{micro}}^{\text{SUSY}}(T_N[M_3],g)) \xrightarrow{N \rightarrow \infty} (g-1) \frac{N^3 \text{vol}(M_3)}{3\pi} ??$$

Bad : UV Gauge theory description is very ugly, no matrix model

Good : we can use the power of **3d-3d relation**

(Computation of perturbative sub-leading are doable)

$d_{micro}^{SUSY}(T_N[M_3], g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3] \text{ on } R^2 \times S^1) \sim (\text{SL}(N, C) \text{ Chern Simons theory on } M_3)$, **not duality but a relation**

M-theoretic derivation : $6dA_{N-1}(2,0)$ theory on $(R^2 \times S^1) \times M_3$

[Yamazaki-Terashima ;'11][Dimofte-Gukov-Gaiotto;11]
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$(T_N[M_3] \text{ on } R^2 \times S^1)$

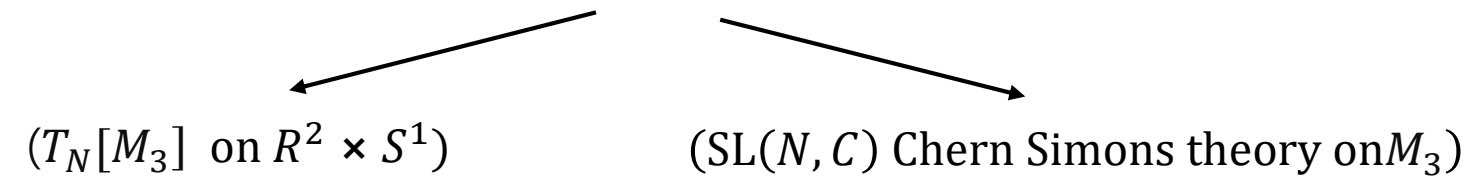
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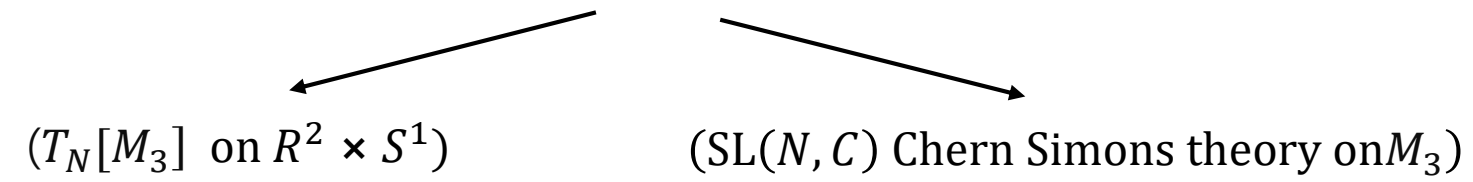


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Dictionary :

$T_N[M_3] \text{ on } R^2 \times S^1$	$\text{SL}(N, C) \text{ Chern Simons theory on } M_3$
Bethe vacuum α	$\text{SL}(N, C)$ irreducible flat connection A^α
Handle gluing operator H^α	$1/N \text{ Exp}[-2S^\alpha(1)]$

$$dA^\alpha + A^\alpha \wedge A^\alpha = 0$$

$$\frac{\delta \text{CS}[A]}{\delta A} = dA + A \wedge A$$

$$\text{CS}[A] = \int_M \text{tr} \left(A dA + \frac{2}{3} A^2 \right)$$

Recall that $d_{micro}^{SUSY}(g) = \sum_{\alpha: \text{Bethe-vacua}} (H^\alpha)^{g-1}$,

$S^\alpha(n)$: n loop perturbative expansion coefficient around a flat connection A^α

$$\log \int \frac{[d(\delta A)]}{(\text{gauge})} \text{Exp} \left[\frac{1}{2\hbar} \text{CS}[A^\alpha + \delta A] \right] \longrightarrow \frac{1}{\hbar} S^\alpha(0) + S^\alpha(0) + \dots \hbar^{n-1} S^\alpha(n) + \dots$$

$$S^\alpha(1) = \frac{1}{4} \text{Log} \left[\frac{(\det' \Delta_0^{(\alpha)})^3}{(\det' \Delta_1^{(\alpha)})} \right] \text{ (Ray-singer torsion)}$$

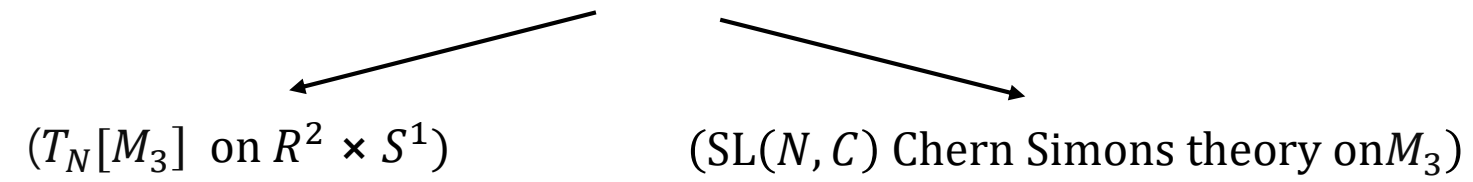
$$\Delta_n^{(\alpha)} = * d_A * d_A + d_A * d_A * , \quad d_A = d + A^\alpha \wedge$$

(Laplacian acting on n -form twisted by A^α)

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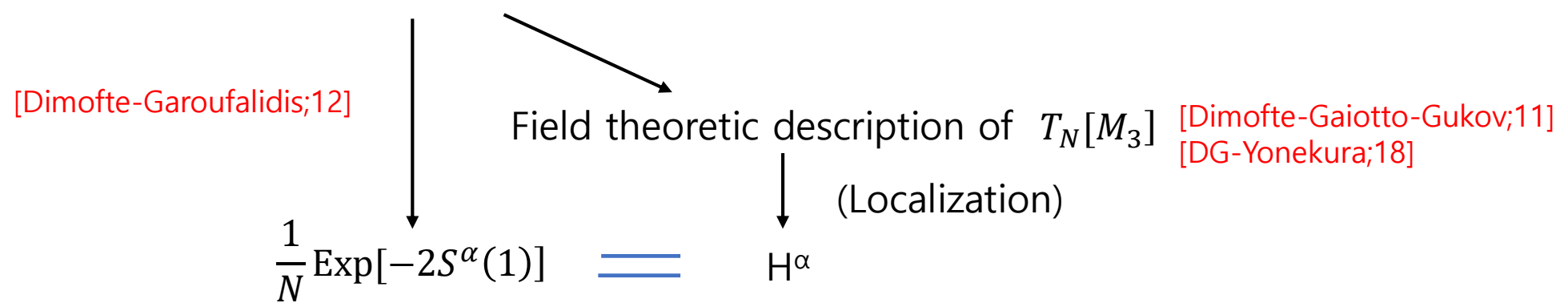
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Derivation : From $M = \left(\bigcup_{i=1}^k \Delta_i \bigcup_i^m S_i \right) / \sim$, Δ : (*ideal tetrahedron*), S : (*solid torus*)

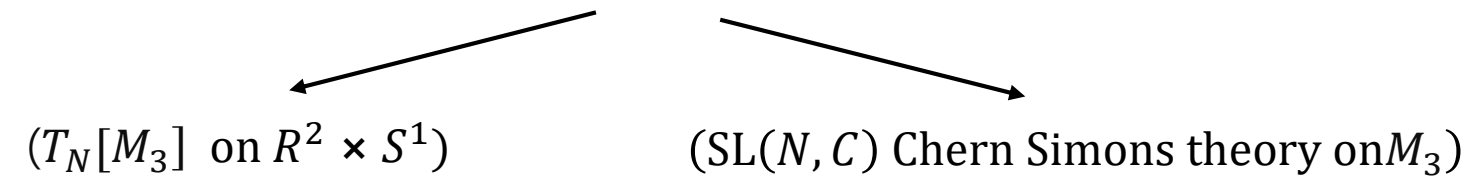


Observation, missing (?) M-theoretic derivation

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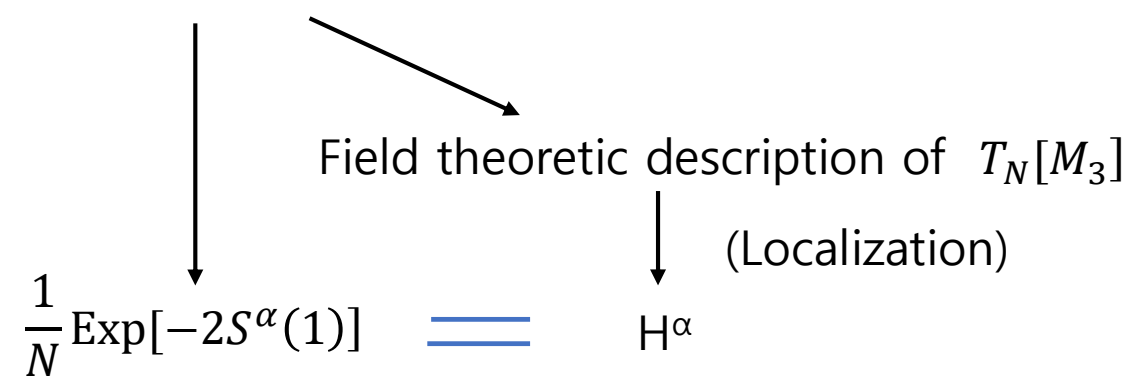
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The 1/N factor is crucial for

- 1) Integrality $d_{micro}(T_N[M_3], g)$
- 2) Correct subleading # $\log N$ of S_{BH}

Observation, missing (?) M-theoretic derivation

$d_{micro}^{SUSY}(T_N[M_3], g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3] \text{ on } R^2 \times S^1) \sim (\text{SL}(N, C) \text{ Chern Simons theory on } M_3)$, not duality but a relation

Dictionary :

$T_N[M_3] \text{ on } R^2 \times S^1$	$\text{SL}(N, C) \text{ Chern Simons theory on } M_3$
Bethe vacuum α	$\text{SL}(N, C) \text{ irreducible flat connection } A^\alpha$
Handle gluing operator H^α	$1/N \text{ Exp}[-2S^\alpha(1)]$

$$S^\alpha(1) = \frac{1}{4} \text{Log} \left[\frac{(\det' \Delta_0^{(\alpha)})^3}{(\det' \Delta_1^{(\alpha)})} \right] \text{ (Ray-singer torsion)}$$

$$d_{micro}^{SUSY}(T_N[M_3], g) = N^{1-g} \sum_{\alpha} \exp(-2S^\alpha(1)[M_3 ; N])^{g-1}$$

We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to study BH entropy !!

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Using the expression, Let us check followings

1) $d_{micro}^{SUSY}(T_N[M_3], g)$ is an integer (after including all corrections)

$$2) S_{\text{BH}} = \log d_{micro}^{SUSY}(T_N[M_3], g) = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)\text{vol}(M)}{3\pi} N^3 + (\text{subleadings in } 1/N).$$

Integrality of $d_{micro}^{SUSY}(T_N[M_3], g)$

Irreducible flat connection : $dA^\alpha + A^\alpha \wedge A^\alpha = 0$

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Conjecture (?)

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e.g) $M_3 = \text{B}_5$, $N=2$

$$\{\exp(-2S^\alpha(1)[M_3; N])\}_{\alpha=1,2,3,4}$$

$$=\{-7.62152 - 2.27598 i, -7.62152 + 2.27598 i, 6.95966, 10.2834\}$$

$$\longrightarrow \{d_{micro}^{SUSY}(T_N[M_3], g)\}_{g=0,1,2,\dots} = \{0, 4, 1, 65, 97, 1045, \dots\}$$

Large N of $d_{micro}^{SUSY}(T_N[M_3],g)$

$$d_{micro}^{SUSY}(T_N[M_3],g) = N^{1-g} \sum_{\alpha} \exp(-2S^{\alpha}(1)[M_3 ; N])^{g-1}$$

Two canonical irreducible flat connections A^{hyp} and $A^{\overline{hyp}}$

$$A^{hyp} = \rho_N[\omega + ie] , \quad A^{\overline{hyp}} = \rho_N[\omega - ie]$$

$\rho_N: \mathfrak{su}(2) \rightarrow \mathfrak{su}(N), N - \text{dimensional irred representation}$

ω : spin connection
 e : vielbein

for unique hyperbolic metric on M satisfying $R_{\mu\nu} = -2g_{\mu\nu}$

Both of them can be locally considered as $\mathfrak{so}(3)$ valued 1 forms

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Using following mathematical facts [Muller;14]

$$\text{Re}[S^{hyp}(1)[M_3; N]] \xrightarrow{N \rightarrow \infty} -\frac{N^3 \text{vol}(M_3)}{6\pi} + a(M_3)N + b(M_3) + o(e^{-N})$$

$$a(M_3) = \frac{19 \text{vol}(M_3)}{6\pi} + \text{Re}[S^{hyp}(1)[M_3, N=3]] - S^{hyp}(1)[M_3, N=1] - \sum_{[\gamma]} \sum_{s=1}^{\infty} \frac{1}{s} \left(\frac{e^{-3s\ell(\gamma)}}{1 - e^{-s\ell(\gamma)}} \right)$$

$$d_{micro}^{SUSY}(T_N[M_3], g) = \underbrace{2\text{Cos}[\theta_N[M_3]]}_{\text{(Relative phase)}} \exp \left(\underbrace{(g-1) \left(\frac{N^3 \text{vol}(M)}{3\pi} - 2aN - 2b - \log N + o(e^{-N}) \right)}_{\text{(Bekenstein-Hawking)}} \right) + o(e^{-N})$$

(match with gravity computation $\frac{(g-1)}{3} \log G_4$)

Summary and future directions

We study microstate counting $d_{micro}(T_N[M_3], g)$ for 4d magnetically charged BH made of wrapped N M5-branes on 3-manifold M_3

Using a 3d-3d relation, the counting to a mathematical problem

$$d_{micro}^{SUSY}(T_N[M_3], g) = N^{1-g} \sum_{\alpha} \exp(-2S^{\alpha}(1)[M_3; N])^{g-1}$$

Then, using known mathematical results

$$d_{micro}^{SUSY}(T_N[M_3], g) = 2\text{Cos}[\theta_N[M_3]] \exp\left((g-1) \left(\frac{N^3 \text{vol}(M)}{3\pi} - 2aN - 2b - \log N + o(e^{-N}) \right) \right) + o(e^{-N})$$

(Relative phase)
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Future work : 1) 6D derivation of the 3d-3d relation for twisted index

2) Understanding the perturbative corrections (contributions from intersecting M2?)

3) $d_{micro}^{SUSY}(T_N[M_3]) = d_{micro}(T_N[M_3])$??