Microstates counting of magnetically charged AdS4 Black hole made of Wrapped M5-branes

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ArXiv : 1808.02797 with Nakwoo Kim (KyungHee U)
& WIP with Nakwoo Kim, Pando Zayas and James Liu (Michigan U)

A Magnetically charged AdS4 Black hole

Classically,

$$\mathrm{ds}^2 = -\left(\rho - \frac{1}{2\rho}\right)^2 \mathrm{dt}^2 + \left(\rho - \frac{1}{2\rho}\right)^{-2} \mathrm{d\rho}^2 + \rho^2 \mathrm{ds}^2(\Sigma_g)$$

$$F = \frac{\mathrm{dx}_1 \wedge dx_2}{{x_2}^2} \quad \text{(Magnetic flux for U(1) gauge field along Riemmann surface } \Sigma_{g>0}\text{)}$$

BPS Solution for 4D $\mathcal{N}$ =2 minimal gauged supergravity

$$I = \frac{1}{16\pi G_4} \int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2\right) + \text{(fermions)}$$

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In terms of AdS/CFT, the BH solution describes

RG : (3D $\boldsymbol{\mathcal{N}}$ =2 SCFT on $\boldsymbol{R}_t \times \Sigma_g$) ⟶ (1D SQM on $\boldsymbol{R}_t$)

With topological twisting : $\left(A^{(b.g)}\right)_R = -\omega(\Sigma_g)$

$\left(\partial + \left(\omega + \left(A^{(b.g)}\right)_R\right)\cdot\right)\epsilon = 0$ preserving 1/2 SUSY (2 supercharges)

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From semiclassical analysis [Bekenstein, Hawking]

$$S_{BH} = \frac{A}{4G_4} = \frac{(g-1)\pi}{2G_4} + \text{(subleadings in } G_4\text{)}$$

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If the BH solution (AdS4 supergravity) can be embedded into an UV complete Quantum Gravity,

We may give a non-perturbative definition of $\boldsymbol{d_{micro}(\#\ of\ micorstates\ of\ BH)}$, which should satisfy

1) $\boldsymbol{d_{micro}}(g, G_4)$ is an non-negative integer (after including all corrections)

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$F = \frac{dx_1 \wedge dx_2}{\ldots}$ (Magnetic flux for U(1) ... field along Riemann surface Σ_g)

In this talk,

- Embedding the BH into **M-theory** on AdS4x(…)
- $d_{micro}\,(g, G_4)$ using **AdS4/CFT3**
- Check of 1) integrality (at finite N)
  2) Bekenstein-Hawking + sub-leadings in large N

From

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A Magnetically charged AdS4 BH in M-theory

BH solution with asymptotically AdS4 ➡ Can be studied using AdS4/CFT3

A Magnetically charged AdS4 BH in M-theory

BH solution with asymptotically AdS4 → Can be studied using AdS4/CFT3

Two classes of well-established AdS4/CFT3 using M-theory

AdS4/CFT3 from M2-branes	AdS4/CFT3 from M5-branes
$R^{1,2}$ × Cone(Y_7) (Y_7 : Sasakian 7-manifold) with N M2-branes on $R^{1,2}$ → $T_N[Y_7]$ 3D $\mathcal{N}$=2 SCFT with global $U(1)_R \subset G$ = ISO(Y_7)	$R^{1,2}$x (T^*M)x R^2 (T^*M_3: cotangent-bundle of 3-manifold M_3) with N M5 branes on $R^{1,2}$x M_3 → $T_N[M_3]$ 3D $\mathcal{N}$=2 SCFT, with global $U(1)_R$
M-theory on AdS4xY_7 ↓ ($G_4 = \sqrt{\frac{27}{8N^3\pi^4}\text{Vol}(Y_7)}$) 4D $\mathcal{N}$=2 Gauged supergravity with G = ISO(Y_7) $S_{\text{BH}} = \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\text{Vol}(Y_7)}}N^{3/2}\pi^3$	M-Theory on Warped AdS4xM_3xS4 (for hyperbolic M_3) ↓ ($G_4 = \frac{3\pi^2}{2N^3 vol(M)}$) 4D $\mathcal{N}$ =2 Gauged supergravity with G =U(1) $S_{\text{BH}} = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)vol(M)}{3\pi}N^3$
Field theoretic description of $T_N[Y_7]$ [ABJM;08][HLLLP;09]…….. e.g) T_N[S7/Z_k]=ABJM model	Field theoretic description of $T_N[M_3]$ e.g) $T_{N=2}$[\`Thurston']=U(1) +Phi with k=-7/2

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Field theoretic description of $T_N[Y_7]$ [ABJM;08][HLLLP;09]........ e.g) T_N[S7/Z$_k$]=ABJM model	Field theoretic description of $T_N[M_3]$ [Dimoft-Gukov-Gaiotto;11][DG-Yonekura;18]........ e.g) $T_{N=2}$[ 5]=U(1) + Φ with k=-9/2

Non-perturbative definition of d_{micro} using AdS4/CFT3

$\boldsymbol{Question}$: $\textit{Which quantity in CFT3 corresponds to the } d_{\text{micro}} \textit{ of the BH ?}$

Hints:

BH : Asymptotic AdS_4 with $\partial(\text{AdS}_4) = \boldsymbol{R}_t \times \Sigma_g$ → Near horizon $\text{AdS}_2 \times \Sigma_g$,

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Natural Answer : *the number of ground states of 3d SCFT on* Σ_g

d_{micro}

$= \dim H^{E=0}(\text{3D } \boldsymbol{N} = 2 \text{ SCFT } on\ \Sigma_g)$

= # of supersymmetric ground states of (3D $\boldsymbol{N}$ =2 SCFT $on\ \Sigma_g$)

$cf)\ d_{\text{micro}}^{\text{SUSY}} := \text{Tr}_{H^{E=0}(\text{3D } \boldsymbol{N} = 2 \text{ SCFT } on\ \Sigma_g)} (-1)^R = \text{Tr}_{H(\text{3D } \boldsymbol{N} = 2 \text{ SCFT } on\ \Sigma_g)} (-1)^R e^{-\beta E}$

Twisted index

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d_{micro}

= dim $H^{E=0}$(3D $\boldsymbol{N}$ =2 SCFT on Σ_g)

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Twisted index

Recently people found that [Benini-Hristov-Zaffaroni ;'16].....

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[Y_7],g)) \xrightarrow{N \to \infty} \frac{(g-1)\pi}{2G_4} = (g-1)\sqrt{\frac{2}{27\text{Vol}(Y_7)}}N^{3/2}\pi^3 + \text{sub-leadings}$$

$d^{\text{SUSY}}_{\text{micro}}(T_N[Y_7]) = d_{\text{micro}}(T_N[Y_7])$??

Twisted index $d^{\text{SUSY}}_{\text{micro}}(g) = \text{Tr}_{H(3\text{D } \mathcal{N} = 2 \text{ SCFT on } \Sigma_g)}(-1)^R e^{-\beta E}$

For $g = 1$ $(\Sigma_g = T^2)$ case : It is just usual Witten index [Kim-Kim ;'10] [Seiberg-Intrilligator ;'12]

For $g = 0$ (S^2) case [Benini-Zaffaroni ;'15]

For general g [Closset-Kim ;'16] [Benini-Zaffaroni ;'16]

For general $3d$ $N = 2$ theory with gauge G,
the index can be written as finite sum over so called `Bethe vacua'

$$d^{\text{SUSY}}_{micro}(g) = \sum_{\alpha:\text{Bethe}-\text{vacua}} (H^\alpha)^{g-1},$$

[Closset-Kim-Willet ;'17]

Bethe vacua: solutions of eqn $\exp\left(2\pi i z_i \frac{\partial W}{\partial z_i}\right) = 1$, for $i = 1,..,\text{rank}(G)$

$W(z_1, \ldots, z_{\text{rank}(G)})$: Twisted superpotential for $2d$ (2,2) theory obtained by S^1 reduction keeping all infinity KK$-$modes

$$\text{Chiral field : } \delta\text{W} = \text{Li}_2\left(\prod z_i^{-Q_i}\right), \qquad \text{CS term } \delta\text{W} = k_{\text{ij}}\text{Log}[z_i]\text{Log}[z_j]$$

$H^\alpha(z_1, \ldots, z_{\text{rank}(G)})$: `handle gluing operator' ,

$$\text{Log}[H] = -\log \text{Det}[\partial_{\log[z_i]}\partial_{\log[z_j]}\text{Log}[W]] + \sum_{\text{Chiral}} \text{Li}_1(z_i^{-Q_i})$$

Most recent studies on AdS4 BH microstates counting are about BH made of M2-branes

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BH made of M5-branes?

$$\text{Log}(d^{\text{SUSY}}_{\text{micro}}(T_N[M_3],\text{g})) \xrightarrow{N \to \infty} (g-1)\frac{N^3\text{vol}(M_3)}{3\pi} \ ??$$

Bad : UV Gauge theory description is very ugly, no matrix model

Good : we can use the power of **3d-3d relation**

(Computation of perturbative sub-leadings are doable)

$d_{micro}^{SUSY}(T_N[M_3],g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $R^2 \times S^1) \sim (\mathrm{SL}(N, C)$ Chern Simons theory on $M_3)$, **not duality but a relation**

[Yamazaki-Terashima ;'11][Dimoft-Gukov-Gaiotto;11]
[Yagi;13][Lee-Yamazaki;13][Cordova-Jafferis;13]

M-theoretic derivation : $6dA_{N-1}(2,0)$ theory on $(R^2 \times S^1) \times M_3$

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Dictionary :

$T_N[M_3]$ on $R^2 \times S^1$	$SL(N,C)$ Chern Simons theory on M_3
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Handle gluing operator H^α	1/N $\text{Exp}[-2S^\alpha(1)]$

$$dA^\alpha + A^\alpha \wedge A^\alpha = 0$$

$$\frac{\delta CS[A]}{\delta A} = dA + A \wedge A$$
$$CS[A] = \int_M \text{tr}\left(AdA + \frac{2}{3}A^2\right)$$

Recall that $d_{micro}^{SUSY}(g) = \sum_{\alpha:\text{Bethe-vacua}} (H^\alpha)^{g-1}$,

$S^\alpha(n)$: *n loop perturbative expansion coefficient around a flat connection* A^α

$$\log \int \frac{[d(\delta A)]}{(\text{gauge})} \text{Exp}[\frac{1}{2\hbar} CS[A^\alpha + \delta A]] \longrightarrow \frac{1}{\hbar} S^\alpha(0) + S^\alpha(0) + ..\hbar^{n-1} S^\alpha(n) + \ldots$$

$$S^\alpha(1) = \frac{1}{4} \text{Log}[\frac{\left(\det{}' \Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}' \Delta_1{}^{(\alpha)}\right)}]$$ (Ray-singer torsion)

$\Delta_n{}^{(\alpha)} = * d_A * d_A + d_A * d_A *$, $d_A = d + A^\alpha \wedge$
(Laplacian acting on n-form twisted by A^α)

$d^{SUSY}_{micro}(T_N[M_3], g)$ from 3d-3d relation

3d-3d relation : $(T_N[M_3]$ on $R^2 \times S^1) \sim$ (SL(N, C) Chern Simons theory on M_3) , not duality but a relation

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Derivation : From $M = \left(\bigcup_{i=1}^{k} \Delta_i \bigcup_i^m S_i\right)/\sim$, Δ: (*ideal tetrahedron*), S: *solid torus*

[Dimofte-Garoufalidis;12]

Field theoretic description of $T_N[M_3]$ [Dimofte-Gaiotto-Gukov;11][DG-Yonekura;18]

↓ (Localization)

$$\frac{1}{N}\text{Exp}[-2S^\alpha(1)] = H^\alpha$$

Observation, missing (?) M-theoretic derivation

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Field theoretic description of $T_N[M_3]$

↓ (Localization)

$$\frac{1}{N}\mathrm{Exp}[-2S^\alpha(1)] \;=\; H^\alpha$$

Observation, missing (?) M-theoretic derivation

The $1/N$ factor is crucial for

1) Integrality $d_{micro}(T_N[M_3],g)$
2) Correct subleading # log N of S_{BH}

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$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{1-g} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ; N]\ )^{g-1}$$

We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to to study BH entropy !!

$d^{SUSY}_{micro}(T_N[M_3],g)$ from 3d-3d relation

3d-3d relation : ($T_N[M_3]$ on $R^2 \times S^1$) ~ (SL(N, C) Chern Simons theory on M_3) , not duality but a relation

Dictionary :

$T_N[M_3]$ on $R^2 \times S^1$	SL(N, C) Chern Simons theory on M_3
Bethe vacuum α	SL(N,C) irreducible flat connection A^α
Handle gluing operator H^α	1/N $\text{Exp}[-2S^\alpha(1)]$

$$S^\alpha(1) = \frac{1}{4}\text{Log}\left[\frac{\left(\det\prime \Delta_0{}^{(\alpha)}\right)^3}{\left(\det\prime \Delta_1{}^{(\alpha)}\right)}\right]$$ (Ray-singer torsion)

$$d^{SUSY}_{micro}(T_N[M_3],g) = N^{1-g} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ; N]\ )^{g-1}$$

We reduce the microstates counting of BH to a mathematical problem !!
Use mathematical results to to study BH entropy !!

Using the expression, Let us check followings

1) $d^{SUSY}_{micro}(T_N[M_3],g)$ is an integer (after including all corrections)

2) $S_{\text{BH}} = \log d^{SUSY}_{micro}(T_N[M_3],g) = \frac{(g-1)\pi}{2G_4} = \frac{(g-1)vol(M)}{3\pi}N^3 + (\text{subleadings in } 1/N).$

Integrality of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

Irreducible flat connection : $dA^\alpha + A^\alpha \wedge A^\alpha = 0$

$$S^\alpha(1)[\mathrm{M};\mathrm{N}] = \frac{1}{4}\mathrm{Log}\left[\frac{\left(\det{}'\Delta_0{}^{(\alpha)}\right)^3}{\left(\det{}'\Delta_1{}^{(\alpha)}\right)}\right]$$ (Ray-singer torsion)

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Conjecture (?)

$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) \in \boldsymbol{Z}$

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Conjecture (?)

$d^{SUSY}_{micro}(T_N[M_3],g) \in \mathbb{Z}$

e.g) $M_3 =$  $_5$, $N=2$

$$\{\exp(-2S^\alpha(1)[M_3;N]\,)\}_{\alpha=1,2,3,4}$$

$$=\{-7.62152-2.27598\ i,\ -7.62152+2.27598\ i,\ 6.95966,\ 10.2834\}$$

$\Longrightarrow$ $\{d^{SUSY}_{micro}(T_N[M_3],g)\}_{g=0,1,2,..} = \{\ 0,\ 4,\ 1,\ 65,\ 97,\ 1045,\ldots\}$

Large N of $d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g})$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = N^{1-g} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ; N]\ )^{g-1}$$

Two canonical irreducible flat connections A^{hyp} *and* $A^{\overline{hyp}}$

$A^{hyp} = \rho_N[\omega + ie]\ ,\quad A^{\overline{hyp}} = \rho_N[\omega - ie]$

ρ_N: su(2) → su(N), N − dimensionalirredrepresentation

ω: spin connection
e: vielbein

for unique hyperbolic metric on M satisfying $R_{\mu\nu} = -2g_{\mu\nu}$

Both of them can be locally considered as so(3) valued 1 forms

$\omega \pm ie$: *sl(2, C) valued 1 form satisyfing flat connection equation* $dA + A \wedge A$

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Using following mathematical facts [Muller;14]

$$\text{Re}[S^{\text{hyp}}(1)[M_3;N]] \xrightarrow[N\to\infty]{} -\frac{N^3\text{vol}(M_3)}{6\pi} + \text{a}(M_3)N \;+ \text{b}(M_3) + \text{o}(\text{e}^{-N})$$

$$a(M_3) = \frac{19\text{vol}(M_3)}{6\pi} + \text{Re}[S^{\text{hyp}}(1)[M_3, N=3] - S^{\text{hyp}}(1)[M_3, N=1] - \sum_{[\gamma]}\sum_{s=1}^{\infty}\frac{1}{s}\left(\frac{e^{-3s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}}\right)]$$

$$d^{\text{SUSY}}_{micro}(T_N[M_3],\text{g}) = \underbrace{2\text{Cos}[\theta_N[M_3]]}_{\text{(Relative phase)}}\exp\left(\underbrace{(g-1)(\frac{N^3\text{vol}(M)}{3\pi}}_{\text{(Bekenstein-Hawking)}} - 2\,a\,N - 2b \underbrace{-\log N}_{\text{(match with gravity computation } \frac{(g-1)}{3}\log G_4)} + \text{o}(\text{e}^{-N}))\right) + \text{o}(\text{e}^{-N})$$

(Relative phase) (Bekenstein-Hawking) (match with gravity computation $\frac{(g-1)}{3}\log G_4$)

Summary and future directions

We study microstate counting $d_{micro}(T_N[M_3],g)$ for 4d magnetically charged BH made of wrapped N M5-branes on 3-manifold M_3

$T_N[M_3]$ on $R^2 \times S^1$	$SL(N,C)$ Chern Simons $\times$ theory on M_3
Bethe vacuum α	SL(N,C) irreducible flat connection A^α
Handle gluing operator H^α	1/N $\text{Exp}[-2S^\alpha(1)]$

Using a 3d-3d relation, the counting to a mathematical problem

$$d^{SUSY}_{micro}(T_N[M_3],g) = N^{1-g} \sum_\alpha \exp(-2S^\alpha(1)[M_3\ ; N]\ )^{g-1}$$

Then, using known mathematical results

$$d^{SUSY}_{micro}(T_N[M_3],g) = 2\text{Cos}[\theta_N[M_3]]\exp\left((g-1)\left(\frac{N^3\text{vol}(M)}{3\pi} - 2\,a\,N - 2b - \log N + o(e^{-N})\right)\right) + o(e^{-N})$$

(Relative phase) (Bekenstein-Hawking) (match with gravity computation $\frac{(g-1)}{3}\log G_4$)

$$a(M_3) = \frac{19\text{vol}(M_3)}{6\pi} + \text{Re}[S^{\text{hyp}}(1)[M_3, N=3] - S^{\text{hyp}}(1)[M_3, N=1] - \sum_{[\gamma]}\sum_{s=1}^{\infty}\frac{1}{s}\left(\frac{e^{-3s\ell_{\mathbb{C}}(\gamma)}}{1-e^{-s\ell_{\mathbb{C}}(\gamma)}}\right)\ ]$$

Future work : 1) 6D derivation of the 3d-3d relation for twisted index

2) Understanding the perturbative corrections (contributions from intersecting M2?)

3) $d^{SUSY}_{\text{micro}}(T_N[M_3]) = d_{\text{micro}}(T_N[M_3])$??