Bootstrapping CFTs without using unitarity

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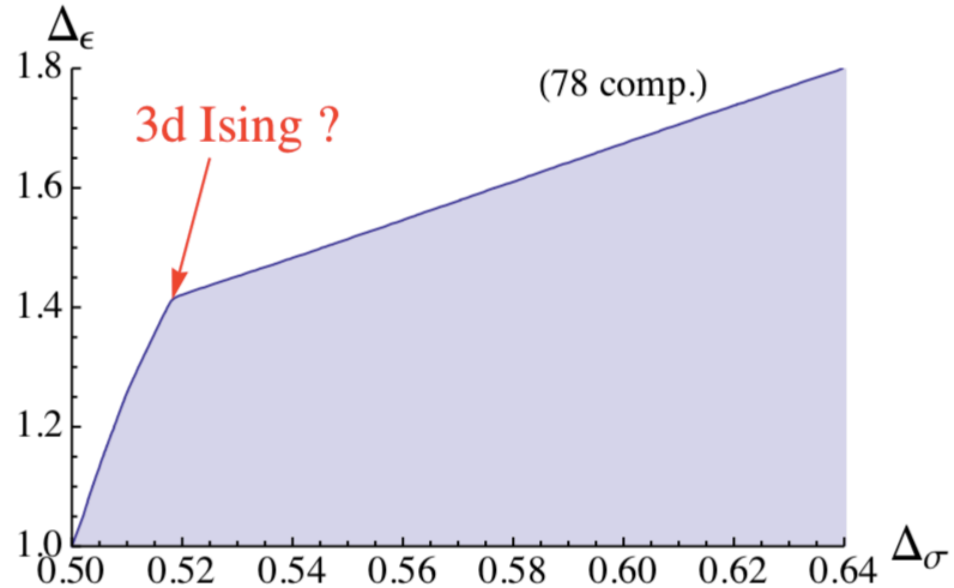
East Asia Joint Workshop on Fields and Strings 2018, KIAS

Based on 1706.04054 and 1711.09075

Unitarity is nice

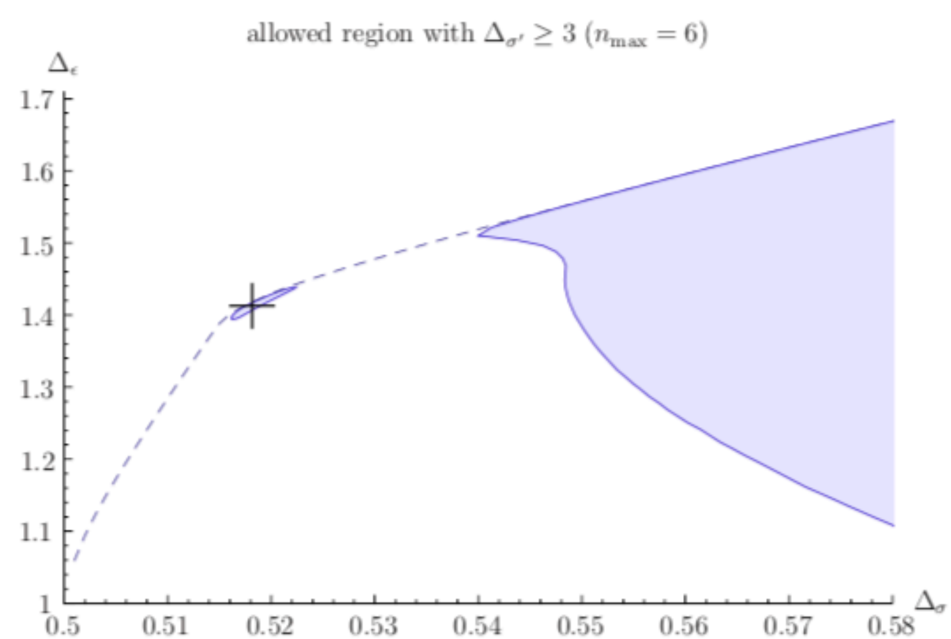
- Kink



El-Showk et al., 2012

Why at a kink?

- Island

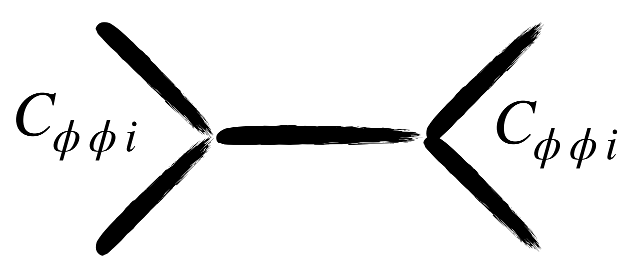


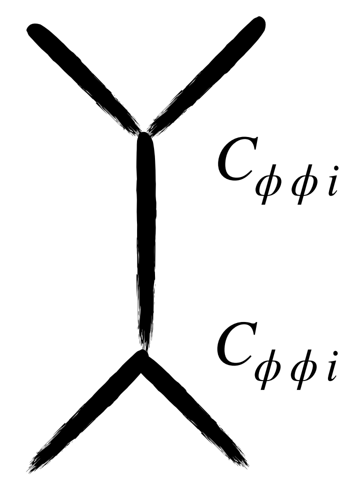
Kos et al. 2014

Why not using unitarity?

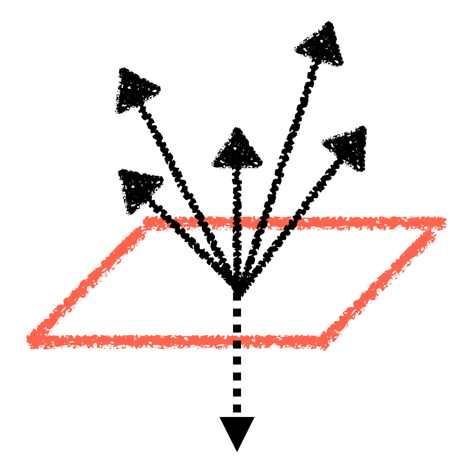
- But… not applicable to all interesting problems: 3 kinds.

- Let us see how unitarity is used
  Consider $< \phi\phi\phi\phi >$

$\sum_i \Big[$ 

$-$ 

$\Big] = 0$

$\left(C_{\phi\phi i}\right)^2$ + unitarity -> positivity constraints -> 

1. Non-unitary CFTs

- Statistical physics

  Lee-Yang edge singularity
  (imaginary coupling constant)

  polymers, percolation, disordered systems
  (logarithmic CFTs)

- Complex CFTs

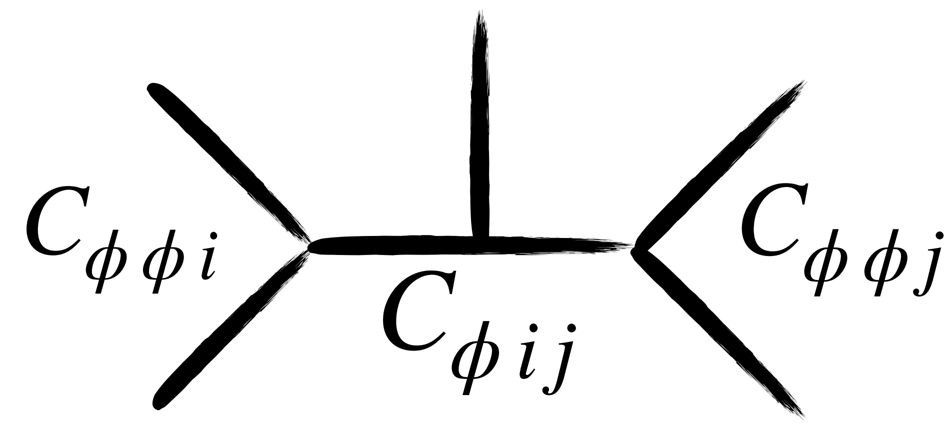
  scaling dimensions/OPE coefficients are complex numbers

  complexified deformations of real CFTs

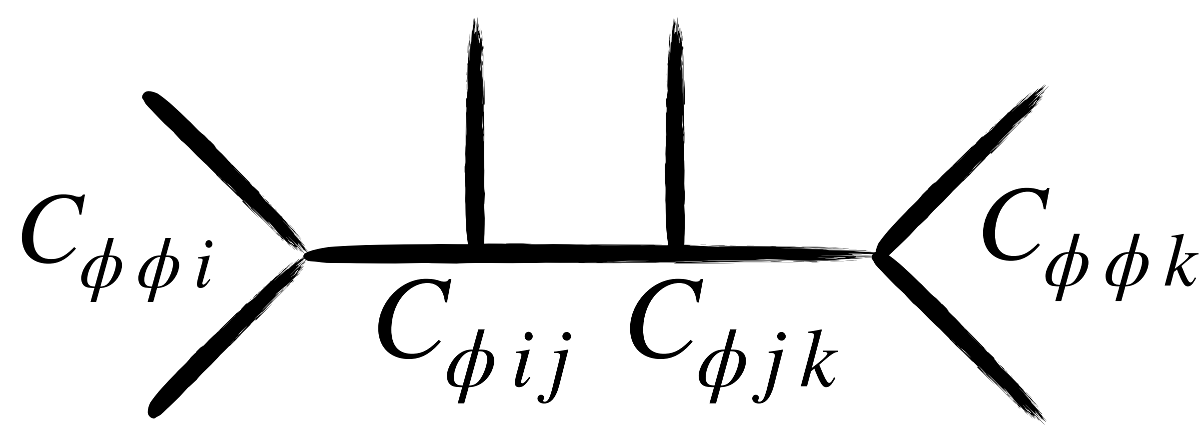
  walking in gauge theories ~ weakly first-order phase transition
  (Gorbenko-Rychkov-Zan, ’18)

2. Higher-point functions

- 5-point $C_{\phi\phi i} C_{\phi i j} C_{\phi\phi j} \neq (\ldots)^2$

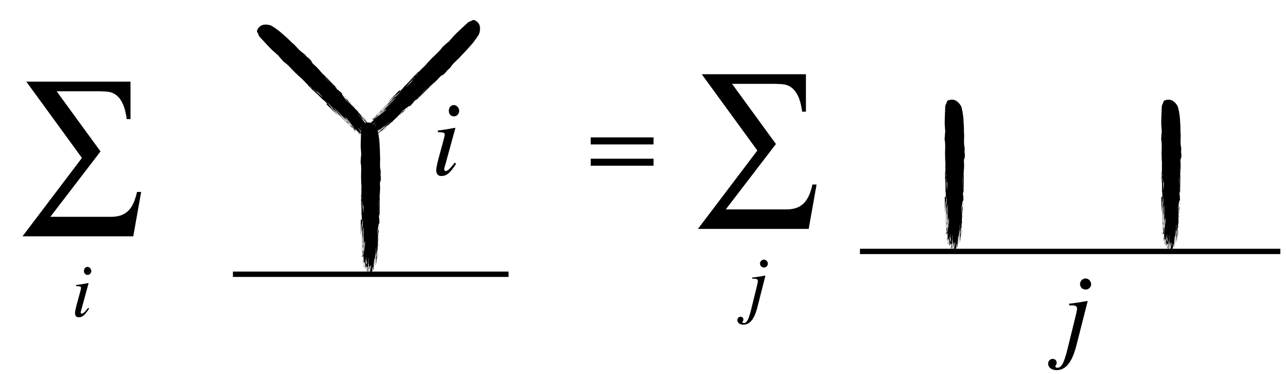


- 6-point $C_{\phi\phi i} C_{\phi i j} C_{\phi j k} C_{\phi\phi k} \neq (\ldots)^2$



3. When 1-point functions are non-zero

- 2-point functions are not diagonal. 4-point functions will not have squares.
- 2-point functions already lead to nontrivial consistency equations
- Boundary CFTs/Defect CFTs
  surface critical phenomena, entanglement entropy
  non-local probes/order parameters in gauge theories



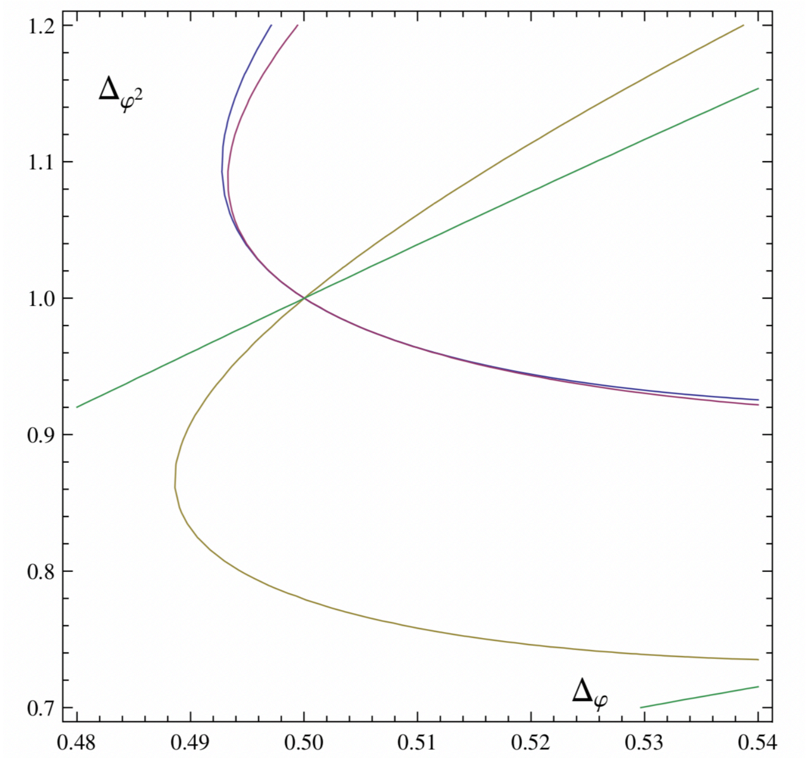
- Thermal bootstrap (CFT on $S^1 \times \mathbb{R}^{d-1}$)
  Kubo-Martin-Schwinger condition for thermal 2-point functions
  (Iliesiu-Kologlu-Mahajan-Perlmutter-Simmons-Duffin, '18)

Need a different approach

- In 2013, F. Gliozzi proposed an alternative method:
  "determinant method".
  (Gliozzi-Rago '14, Gliozzi-Liendo-Meineri-Rago '15, Nakayama '16, Gliozzi '16, Esterlis-Fitzpatrick-Ramirez '16, Hikami '17 & '18, LeClair-Squires '18)

- OPEs are truncated. Unitarity is not used.

- For an overdetermined system, determinants of minors should vanishes.
  ex. 3d free scalar CFT
  $\phi \times \phi = I + [\Delta_{\phi^2}, 0] + [\Delta_T = 3, 2] + [\Delta_{\phi^2} + 4, 4]$



- It is subtle to estimate the errors in the prediction, because OPE truncation errors are omitted.

How to tame the truncation error?

Source of discrepancy

Gliozzi-Rago method sets truncation error to zero, while we know that it's small but nonzero

=> need a modification which would handle truncation error consistently

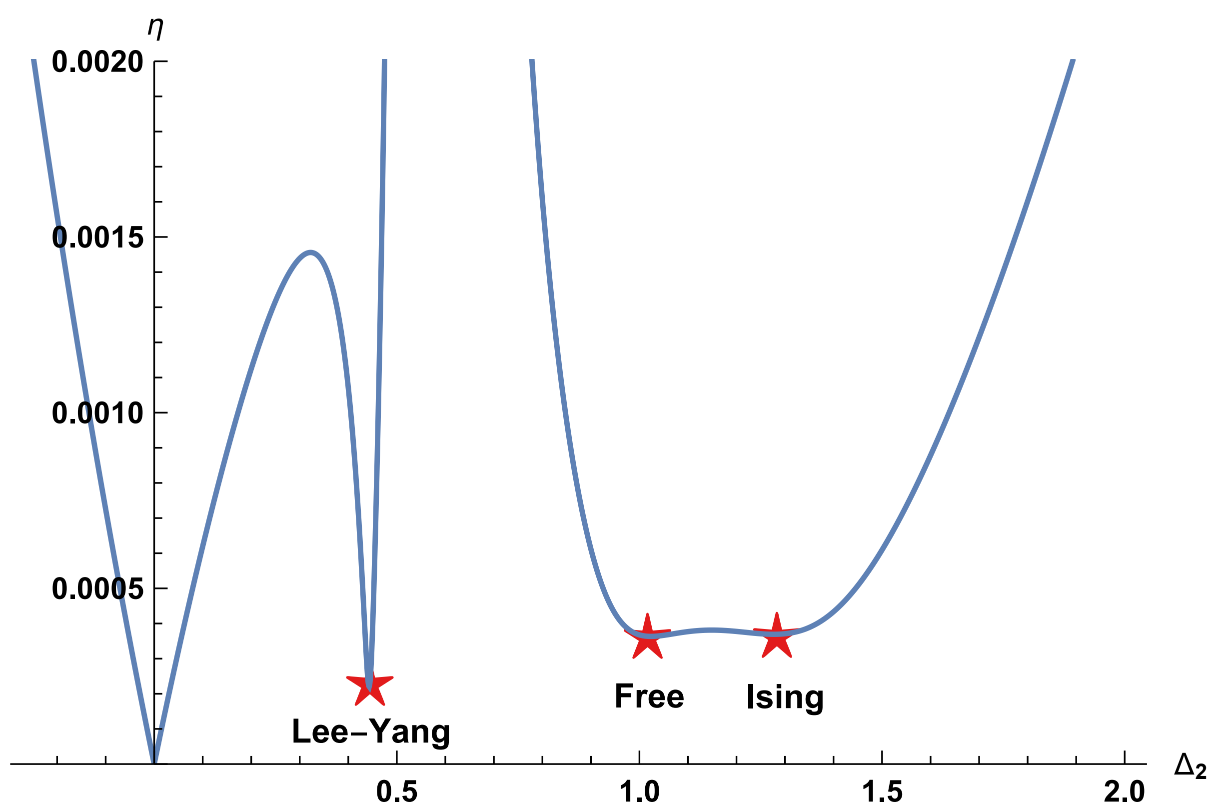
From a review talk by Slava Rychkov in 2015

Error minimization

- Modification
  OPE truncation error is minimized

1711.09075

- "Error function" = $\sqrt{\sum (\textbf{truncated crossing eqn})^2}$ ~ || truncated OPE ||



$$\phi_1 \times \phi_1 = I + \phi_2 + T$$

d=3 $\quad \Delta_1 = \frac{1}{2}$

Is OPE truncation a good approximation?

- Why is Ising at the kink?
  Something dramatic must happen

- Many operators decouple (El-Showk et al., 2014)
  spectrum is very sparse/minimal
  non-perturbative equations of motion
  (d>2 minimal models?)

- OPE convergence is particularly rapid
  OPE truncation error is particularly small

- Unitarity is not crucial
  In fact, 2d Lee-Yang is sparser than 2d Ising

Analytic results

1706.04054

- $\phi_1 \times \phi_1 = I + \phi_2$ → $P_2 \approx \dfrac{4\Delta_1}{3\Delta_2 - 4\Delta_1}$

P_2	free	ϕ^4 WF	2d LY	2d Ising	3d LY	3d Ising
estimate	2	$2-\epsilon$	-4	0.2	-4	1.0
exact/numerical	2	$2-2\epsilon/3$	-3.7	0.25	-3.9	1.1

- $\phi_1 \times \phi_1 = I + \phi_2 + T$ → d-dependent rational function

P_2	free	ϕ^4 WF	2d LY	2d Ising	3d LY	3d Ising
estimate	2	2-0.66ϵ	-3.63	0.254	-3.90/-3.88	1.12
exact/numerical	2	2-0.67ϵ	-3.65	0.25	-3.91/-3.88	1.11

universal, approximate relations for CFT data

What is more?

- Perturb the minima
  add subleading operators
  more precise results

still under development

- Strongly-coupled theories also have small expansion parameters:
  OPE coefficients of the subheading operators!

- Interplay between numerical & analytic results
  quantitative & qualitative understanding
  ex. double-twist spectrum & larger spin perturbations

Summary

- Lesson from the kink
  sparse spectrum due to operator decoupling
  rapid OPE convergence & small OPE truncation error

- New bootstrap method: error minimization
  Perturbation theory in small OPE coefficients

$$\phi_1 \times \phi_1 = \left(I + \phi_2 + T\right) + \left(\mathcal{O}_3 + \mathcal{O}_4 + \ldots\right)$$

non-perturbative **perturbations**

- Shifman-Vainshtein-Zakharov sum rules in QCD

Thank you