Large AdS black holes from QFT

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Black hole

- Perfect black body with a strong attraction force in classical gravity. Behave as a thermodynamic system with definite energy and entropy.

$T = \frac{\kappa}{2\pi}$ Hawking temperature $\qquad$ $S_{\rm BH} = \frac{A}{4G_N}$ Bekenstein-Hawking entropy

- Thermodynamic entropy has statistical, microscopic interpretation. String theory provides the microscopic interpretation of BH entropy.

[Strominger, Vafa]

$$\frac{A}{4G_N} = \log \Omega_{\rm micro}$$

- This talk: charged and rotating AdS_5 black holes via 4d N=4 SYM.

[Gutowski, Reall] [Kunduri, Lucietti, Reall] [Chong, Cvetic, Lu, Pope] [Cvetic, Gibbons, Lu, Pope] [Wu]

Superconformal index of N=4 SYM: [Kinney, Maldacena, Minwalla, Raju]
1/16 BPS operators at weak coupling: [Berkooz, Reichmann, Simon] [Grant, Grassi, S.Kim, Minwalla] [Chang, Yin]

The degeneracy captured was <u>not</u> sufficient to account for the $O(N^2)$ entropy.

“Index” of N=4 SYM

$$Z = \mathrm{Tr}\Big[e^{-\beta E}\prod_{I=1}^{3} e^{-\Delta_I Q_I}\prod_{i=1}^{2} e^{-\omega_i J_i}\Big] \quad \text{with} \quad \textstyle\sum_{I=1}^{3}\Delta_I = \sum_{i=1}^{2}\omega_i + 2\pi i$$

Q_1, Q_2, Q_3 : Cartan of SO(6) R-symmetry

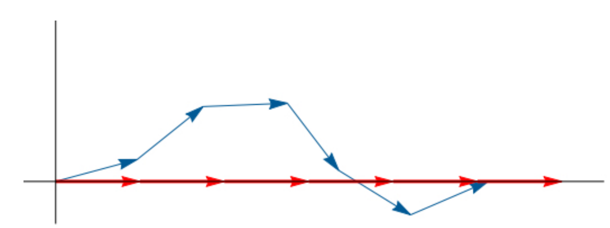
J_1, J_2 : Cartan of SO(4) rotation on S^3

“1/16 BPS index condition” at $\beta \to 0$

$$\{e^{-\Delta_I Q_I - \omega_i J_i}, \mathcal{Q}^{+++}_{--}\} = 0$$
$$\{e^{-\Delta_I Q_I - \omega_i J_i}, \mathcal{S}^{---}_{++}\} = 0$$

- Imaginary part of chemical potentials can be tuned as non-zero.

Phase factors in monomials can partially obstruct an “accidental” boson/fermion cancelation.



- SUSY-protected observable: by counting BPS letters at zero coupling.

$$Z = \oint [d\alpha]\cdot \exp\left[\sum_{a,b=1}^{N}\sum_{n=1}^{\infty}\frac{1}{n}\left(1 + \sum_{s_1,s_2,s_3=\pm 1}\frac{s_1 s_2 s_3 (-1)^{n-1} e^{\frac{n s_I \Delta_I}{2}}}{2\sinh\frac{n\omega_1}{2}\cdot 2\sinh\frac{n\omega_2}{2}}\right) e^{in\alpha_{ab}}\right]$$

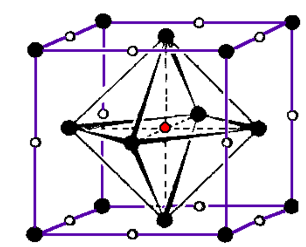
Asymptotic free energy

- Take the saddle point approximation in the Cardy limit $|\omega_{1,2}| \ll 1$.
  Maximally deconfining saddle point $\alpha_1 = \cdots = \alpha_N$ is most dominant.

$$\log Z = -\frac{N^2}{\omega_1\omega_2} \sum_{s_1 s_2 s_3 = +1} \left[\mathrm{Li}_3 \left(-e^{\frac{s_I \Delta_I}{2}} \right) - \mathrm{Li}_3 \left(-e^{-\frac{s_I \Delta_I}{2}} \right) \right] \sim \mathcal{O}(N^2)$$

In the canonical chamber:

$$-2\pi < \mathrm{Im}(+\Delta_1 + \Delta_2 + \Delta_3) < 2\pi, \quad -2\pi < \mathrm{Im}(+\Delta_1 - \Delta_2 - \Delta_3) < 2\pi$$
$$-2\pi < \mathrm{Im}(-\Delta_1 + \Delta_2 - \Delta_3) < 2\pi, \quad -2\pi < \mathrm{Im}(-\Delta_1 - \Delta_2 + \Delta_3) < 2\pi$$



$$\log Z = \frac{N^2 \Delta_1 \Delta_2 \Delta_3}{2\omega_1\omega_2} \quad \text{with} \quad \textstyle\sum_{I=1}^{3} \Delta_I = \sum_{i=1}^{2} \omega_i + 2\pi i$$

General expression respects the periodicity in imaginary part of chemical potentials.

- Alternative derivation: the S³ effective action of background fields.
  - Relation between free energy & chiral anomaly becomes manifest.
  - Applicable to non-Lagrangian theories, e.g., 6d (2,0) SCFT.

Entropy of AdS_5 black hole

- Since black holes are characterized by their energy and charges, take a Legendre transformation to the microcanonical ensemble.

$$S = \frac{N^2}{2}\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} + \sum_{I=1}^{3} Q_I\Delta_I + \sum_{i=1}^{2} J_i\omega_i \quad \text{with} \quad \sum_{I=1}^{3}\Delta_I = \sum_{i=1}^{2}\omega_i + 2\pi i$$

[Hosseini, Hristov, Zaffaroni]

- Known AdS_5 BPS black holes appear in the special case of Im[S] = 0.

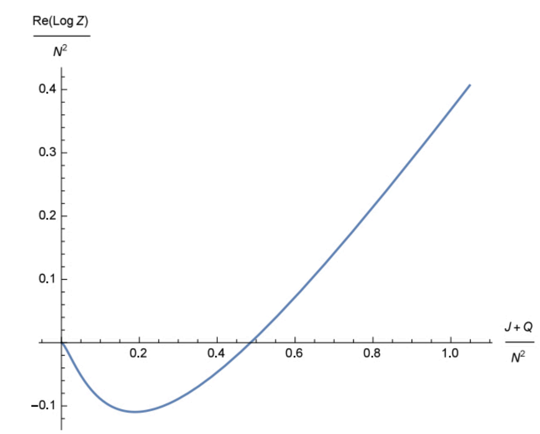
Entropy and chemical potential agree with those of Gutowski-Reall BH. [Cvetic, Gibbons, Lu, Pope]

"Im[S] = 0" imposes the charge relation of known black hole solution. [S.Kim, K.Lee]

$$S = \pi\left(Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \tfrac{1}{2}N^2(J_1+J_2)\right)^{1/2}$$

- Observation beyond the Cardy regime:

At small angular momentum + charge, the thermal graviton phase becomes more dominant than the current black hole saddle point.



Concluding remarks

- Our 1/16 BPS "index" captures the $O(N^2)$ entropy in the Cardy limit.
- It matches the BH entropy of the known Gutowski-Reall black hole, when the black hole charge relation is additionally imposed.
- We also reproduced the entropy of known AdS_7 black hole, with an additionally imposed charge relation. [Chong, Cvetic, Lu, Pope] [Chow]
- We numerically found many other saddle points from 1/8 BPS Macdonald sector. Interpretation as black holes?