Quantum vortices on M2-branes

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This talk is based on:

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"Quantum vortices and AdS_4 black holes"

Work in progress

M2 degrees of freedom

$N^{3/2}$:

- Entropy of black M2-branes [Klebanov, Tseytlin]
- $Z[S^3]$ ~ vacuum entanglement entropy [Drukker, Marino, Putrov]

[Herzog, Klebanov, Pufu, Tesileanu]

Understand this feature from "entropic/spectral" quantity, by counting certain states

Figure out the "origin" of this number

Deconfinement & AdS black holes

Hawking-Page transition in AdS gravity

- **Low temperature** phase:
  thermal gas of **gravitons**
- **High temperature** phase:
  large **AdS black holes**

Confinement-deconfinement transition in CFT dual [Witten]

- **Confined phase**: $F \sim O(N^0)$
- **Deconfined phase**: $F \sim O(N^2)$ of gluons(~matrices) in 4d Yang-Mills

In 3d, deconfined phase should exhibit $F \sim O(N^{3/2})$
→ **NOT** elementary d.o.f.

It is important to understand which d.o.f. deconfine in 3d.

QFT set-up

- Witten indices on **flat space**: counting BPS state with $(-1)^F$

- **Vortices** in Higgs branch of **mass-deformed CFT** (hard to directly study CFT)

  - Understand how $N^{3/2}$ d.o.f. emerge as we approach to small masses

  - Factorization of superconformal index to two vortex partition functions may suggest magnetic monopole operators deconfine in our 3d CFT.

[Beem, Dimofte, Pasquetti]
[C.Hwang, H.Kim, J.Park]

- "Mirror dual" of maximal SYM

  - Brane configuration: N D2-branes probing 1 D6-brane in flat space

  - UV: $\mathcal{N}=4$ SUSY, U(N) gauge theory w/ 1 fundamental & 1 adj. hypers

  - IR: $\mathcal{N}=8$ SCFT (same as maximal SYM & ABJM$_{k=1}$)

[Aharony, Bergman, Jafferis, Maldacena]

Vortex partition function

- $Z[R^2_\beta \times S^1]$ with suitable Higgs branch VEV at infinity

$$Z(q,t,z,\tilde{Q}) = \mathrm{Tr}\left[(-1)^F q^{R+r+2j} t^{R-r} z^{2L} \tilde{Q}^T\right]$$

R, r: $SO(4)$ R-charges ($q = e^{-\beta}$: Ω-deformation)

z: flavor sym. of adjoint hyper, $\tilde{Q}$: vortex charge, topological $U(1)_T$

→ Closely related to $Z[D_2 \times S^1]$ with suitable boundary conditions at $\partial D_2 = S^1$ ~ Dirichlet/Neumann b.c.

- **Contour integral expression** [Yoshida, Sugiyama]: $\tilde{Q} \equiv q^{4\pi r\zeta}$ $\quad (a;q)_\infty \equiv \prod_{n=0}^{\infty}(1-aq^n)$

$$Z = \frac{1}{N!}\oint \prod_{a=1}^{N}\left[\frac{ds_a}{2\pi i s_a} s_a^{-2\pi r\zeta}\right] \prod_{a=1}^{N} \frac{(s_a t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)_\infty}{(s_a t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)_\infty} \cdot \frac{\prod_{a\neq b}(s_a s_b^{-1}; q^2)_\infty}{\prod_{a,b=1}^{N}(s_a s_b^{-1} t^{-1} q; q^2)_\infty} \cdot \prod_{a,b=1}^{N} \frac{(s_a s_b^{-1} z t^{-\frac{1}{2}} q^{\frac{3}{2}}; q^2)_\infty}{(s_a s_b^{-1} z t^{\frac{1}{2}} q^{\frac{1}{2}}; q^2)_\infty}$$

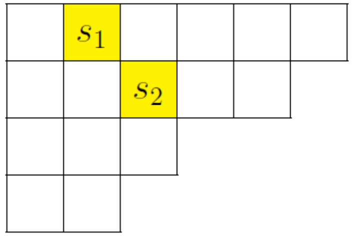
"Confining" spectrum

- When $\tilde{Q} \ll 1$ (heavy vortex), evaluate Z by a residue sum $Z = Z_{prefactor}\, Z_{pert}\, Z_{vortex}$

$$Z_{\text{prefactor}} = \frac{\left(u^N v^{\frac{N(N-1)}{2}}\right)^{2\pi r\zeta}}{(q^2;q^2)_\infty^N} \qquad Z_{\text{pert}} = \prod_{a=1}^{N} \frac{(u^{-2}v^a q^2;q^2)_\infty}{(v^a;q^2)_\infty} \qquad Z_{\text{vortex}} = \sum_{0\le k_1\le\cdots\le k_N} \tilde{Q}^{k_1+\cdots+k_N} Z_{k_1,\cdots,k_N}$$

- Z_{vortex} can be expressed in terms of Young diagram

$$Z_{\text{vortex}} = \sum_{Y} \tilde{Q}^{|Y|} \prod_{s\in Y} \frac{(1-u^{-2}q^{-2a(s)}v^{-l(s)})(1-u^{-2}vq^2q^{2a(s)}v^{l(s)})(1-v^Nq^{2x(s)}v^{-y(s)})}{(1-q^{-2}q^{-2a(s)}v^{-l(s)})(1-vq^{2a(s)}v^{l(s)})(1-u^{-2}q^2v^Nq^{2x(s)}v^{-y(s)})} \qquad \begin{aligned} u &= (qt)^{\frac{1}{2}} \\ v &= z(qt)^{\frac{1}{2}} \end{aligned}$$



$$\begin{cases} a(s): & \text{arm (horizontal) length} = \text{number of boxes to the right of } s \\ l(s): & \text{leg (vertical) length} = \text{number of the boxes below } s \\ x(s): & \text{horizontal position} = \text{number of boxes to the left of } s \\ y(s): & \text{vertical position} = \text{number of the boxes above } s \end{cases}$$

$$\Rightarrow \begin{aligned} &a(s_1) = 4,\ l(s_1) = 3,\ x(s_1) = 1,\ y(s_1) = 0, \\ &a(s_2) = 2,\ l(s_2) = 1,\ x(s_1) = 2,\ y(s_1) = 1. \end{aligned}$$

- Admits **smooth large N limit** when **vortex is heavier** than critical value

$$Z_{\text{vortex}}^{N\to\infty} = PE\left[\frac{(1-u^{-2})(1-u^{-2}vq^2)}{(1-q^{-2})(1-v)} \frac{\tilde{Q}}{1-q^2\tilde{Q}u^{-2}}\right]$$

- Agrees with 'suitably projected' **graviton index on $AdS_4 \times S^7$**

"Deconfining" spectrum

- Evaluate Z by the saddle point method at $\beta \to 0$ ("large volume", "high T") & large N

$$Z \sim \frac{1}{N!}\int \prod_{a=1}^{N} \frac{ds_a}{2\pi i s_a} \exp\left[-\frac{W(s,\cdots)}{2\beta}\right]$$

$$W = N\left[\mathrm{Li}_2(zt^{-\frac{1}{2}}) - \mathrm{Li}_2(zt^{\frac{1}{2}}) - \mathrm{Li}_2(t^{-1})\right] + \sum_{a=1}^{N}\left[\xi \log s_a + \mathrm{Li}_2(s_a t^{-\frac{1}{2}}) - \mathrm{Li}_2(s_a t^{\frac{1}{2}})\right] + \sum_{a\neq b}\left[\mathrm{Li}_2(s_a s_b^{-1}) - \mathrm{Li}_2(s_a s_b^{-1} t^{-1}) + \mathrm{Li}_2(s_a s_b^{-1} z t^{-\frac{1}{2}}) - \mathrm{Li}_2(s_a s_b^{-1} z t^{\frac{1}{2}})\right] .$$

Real fugacities → eigenvalues distribute on real axis $s_a = s_0 \exp\left[N^\alpha x_a\right] \quad (x_1 < x_a < x_2)$

- The **large N free energy** when **vortex is lighter** than critical value $(\tilde{Q} \sim 1)$
(saddle point exists only when the quantity inside square-root is positive)

$$\log Z \sim \mathrm{sgn}(\xi_{\mathrm{ren}} - T/2)\frac{\sqrt{2}N^{\frac{3}{2}}}{3\beta}\sqrt{-T_1 T_2 T_3 T_4}$$

$$Q \equiv q^{\frac{1}{2}} t^{-\frac{1}{2}} \tilde{Q} = e^{-\xi - \frac{T}{2} - \frac{\beta}{2}} \equiv e^{-\xi_{\mathrm{ren}}} \text{ is the canonical parameter for the } SO(8)$$

$$(T_1, T_2, T_3, T_4) = \left(\frac{T}{2} + f, \frac{T}{2} - f, -\frac{T}{2} + \xi_{\mathrm{ren}}, -\frac{T}{2} - \xi_{\mathrm{ren}}\right)$$

- A small test: **reproduces $Z[S^3]$ at large N**, computed from ABJM
(highly squashed $S^3_{b\ll 1} \sim R^2_\beta \times S^1$)

Summary

- We studied **two aspects** of the **vortex partition function** of **M2-brane QFT**.

- **Confining spectrum**: when **vortex is heavier** than critical value, it agrees with suitably projected **graviton index on $\mathrm{AdS}_4 \times S^7$ ($\log Z \sim N^0$)**.

- **Deconfining spectrum**: when **vortex is lighter** than critical value, **asymptotic free energy $\log Z \sim N^{3/2}$**, at large temperature-like parameter.

Future plans : SUSY AdS_4 black holes

- **Superconformal index**(SCI) is factorized into two **vortex partition functions**.

  [Beem, Dimofte, Pasquetti] [C.Hwang, H.Kim, J.Park]

- **Asymptotic free energy of SCI** **does** capture the entropy of large **supersymmetric AdS black holes**.

  (explored in Joonho's talk for 4d QFT) [SC, Joonho Kim, Seok Kim, June Nahmgoong]

- Our **asymptotic free energy** is curiously related to the **entropy function of SUSY black holes in** $\mathbf{AdS_4 \times S^7}$.

  [SC, Chiung Hwang, Seok Kim, June Nahmgoong]

Thank you for listening.