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3rd East Asia Joint WS on Fields and Strings 2018

Towards solving the hierarchy problem in string theory

A "small" progress report after 1st East Asia Joint WS at Hufei, 2016

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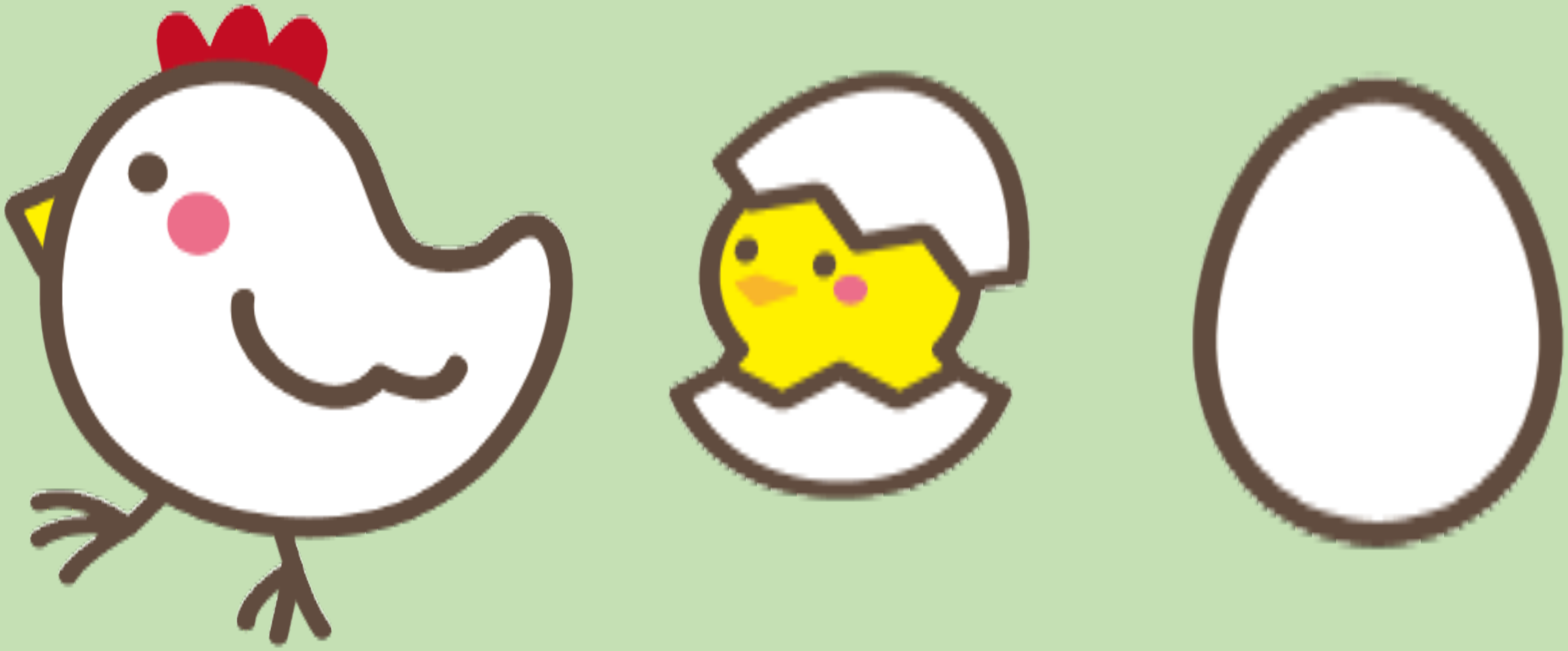
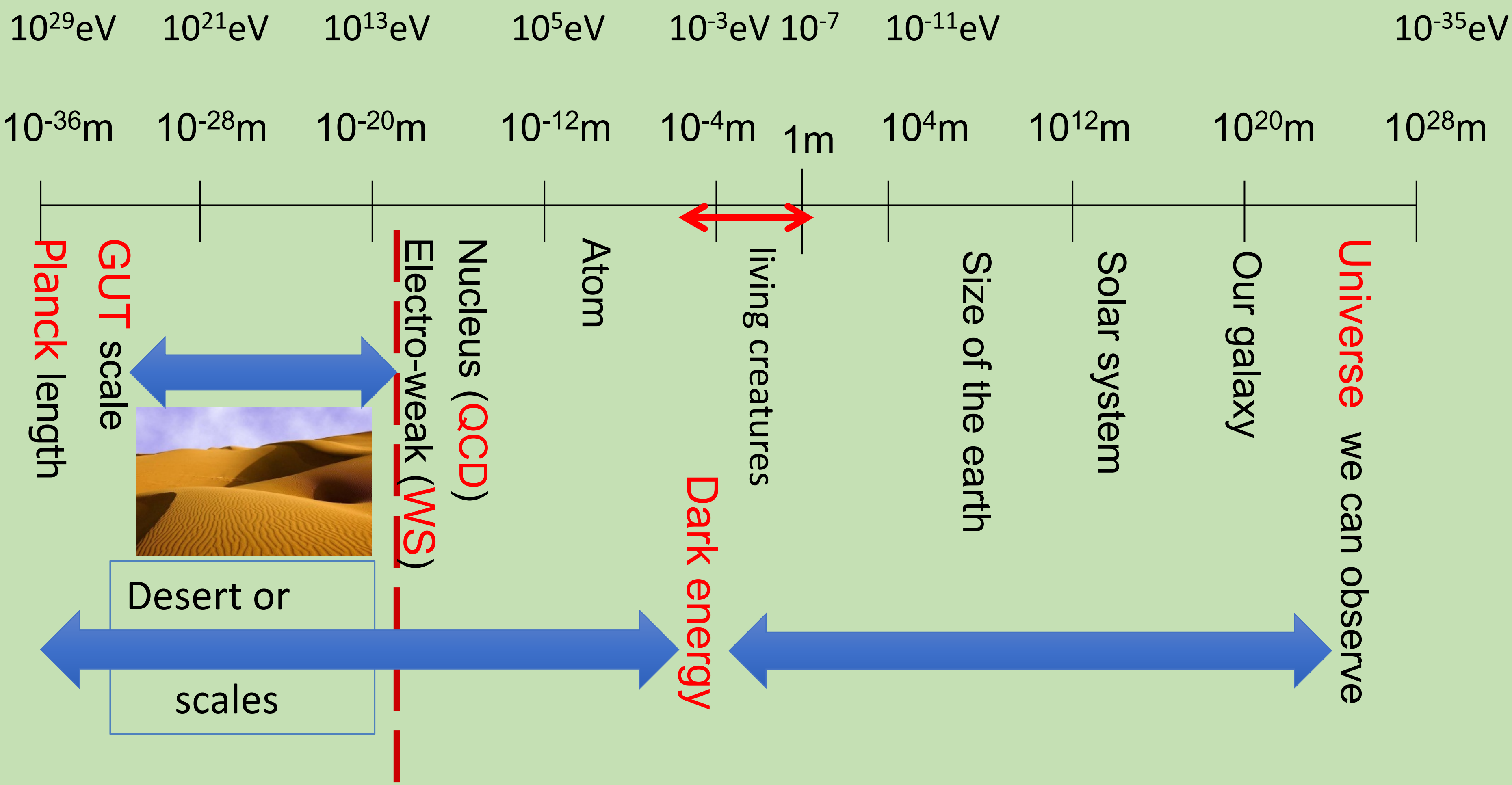
Which came first, the chicken or the egg ?



Genetic theory tells us that the egg came first
and many people may think so ....
Is this always the case ?
This is what I would like to discuss today.

Hierarchy of scales in nature



Naturalness problem = Hierarchy of various scales in nature

Today I will concentrate only on the EW scale.

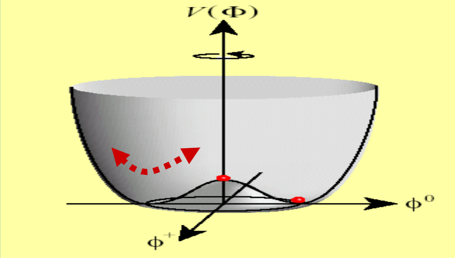
Why EW scale 100 GeV is much lower than UV scales

GUT scale 10^{16} GeV, if exists

Planck scale 10^{18} GeV

String scale 10^{17} GeV ?

Higgs potential $V = -\mu^2|H|^2 + \lambda(|H|^2)^2$



$$\delta V = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\mathrm{Str}\log(k^2 + M(\phi^2))$$

$$= \frac{\Lambda^2}{32\pi^2} Str M(\phi)^2 + Str\frac{M(\phi)^4}{64\pi^2}\ln\left(\frac{M^2}{\Lambda^2} - \frac{1}{2}\right)$$

quadratic divergence

→ $Str M(\phi)^2 = 0$ SUSY ?

Partial list of solutions to the hierarchy problem:

(1) Supersymmetry: cancellation of quadratic divergences

No SUSY particles are found, little hierarchy problem, ...

(2) Technicolor : dynamical generation of scales like QCD

Light Higgs difficult, big form factor (composite),

$$\Lambda_{TC} = M_{TC} \exp(-\frac{8\pi^2}{bg^2})$$

Higgs is a light pseudo-NG ? SO(4)/SO(3) etc.

(3) Multiverse / Anthropic ?

(4) Classical conformal : Coleman-Weinberg radiative breaking

$$V(\phi) \sim \frac{\lambda}{4}\phi^4 \left(\ln \frac{\phi^2}{M^2} - \frac{1}{2}\right) + V_0$$

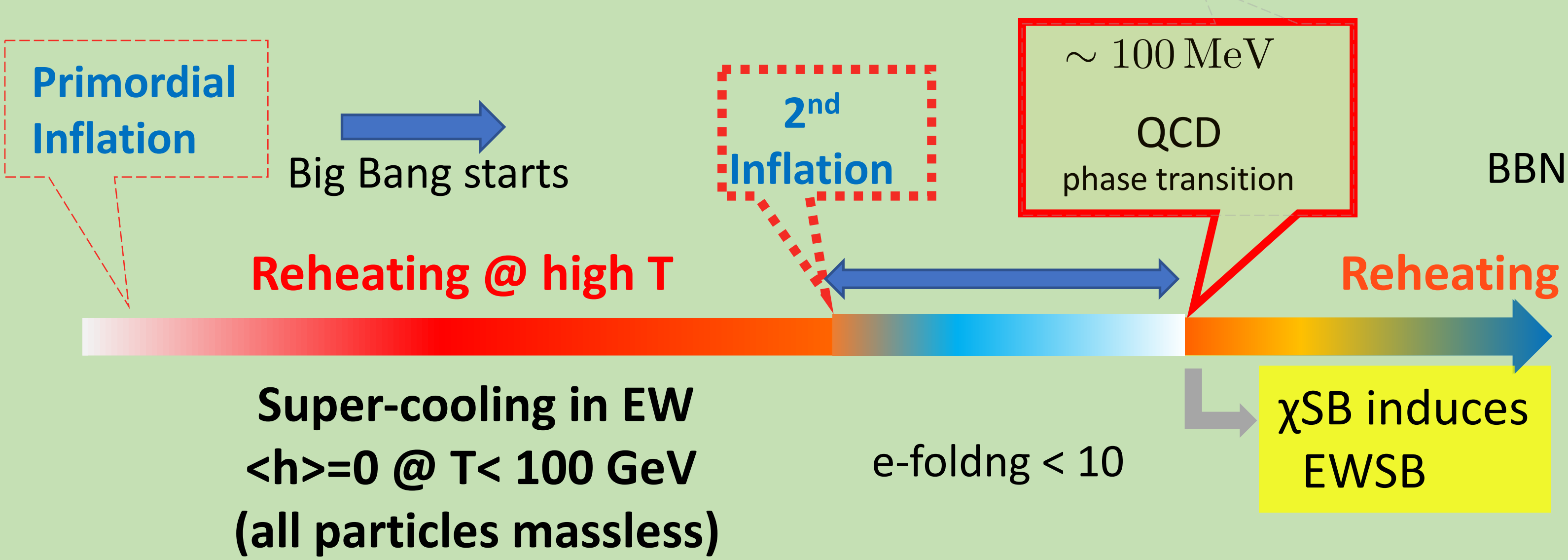
My favorite model

coupling to gravity -> quantum scale invariance ?

1-page summary of recent progress in the classical conformal pheno.

Super-cooled universe with the second (low scale) inflation in the classically conformal model

Serpico, Shimada, SI (2017)



DM produced → dilute → "super-cool DM"

(an appropriate number of e-folding)

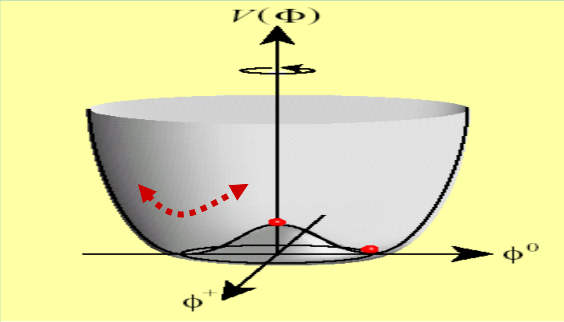
Thermal inflation starts at TeV scale and ends at 100 MeV !

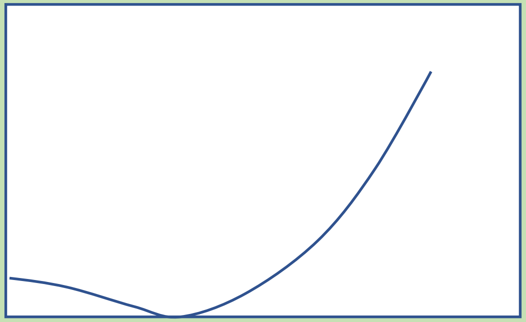
→ gravitational wave, PBH, Baryogenesis

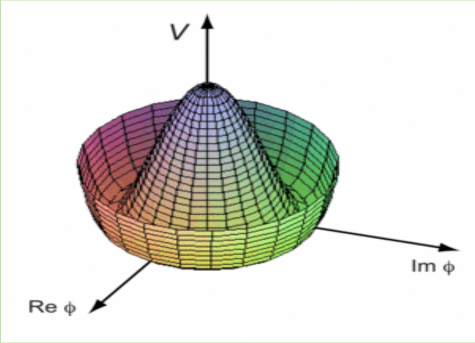
Anyway ....

There are many proposals to solve the hierarchy problem,
but there is one common basic assumption

"Calculate the Higgs potential first !"



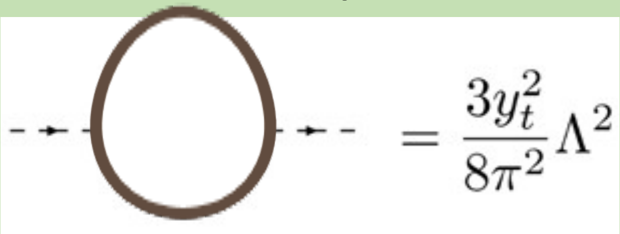




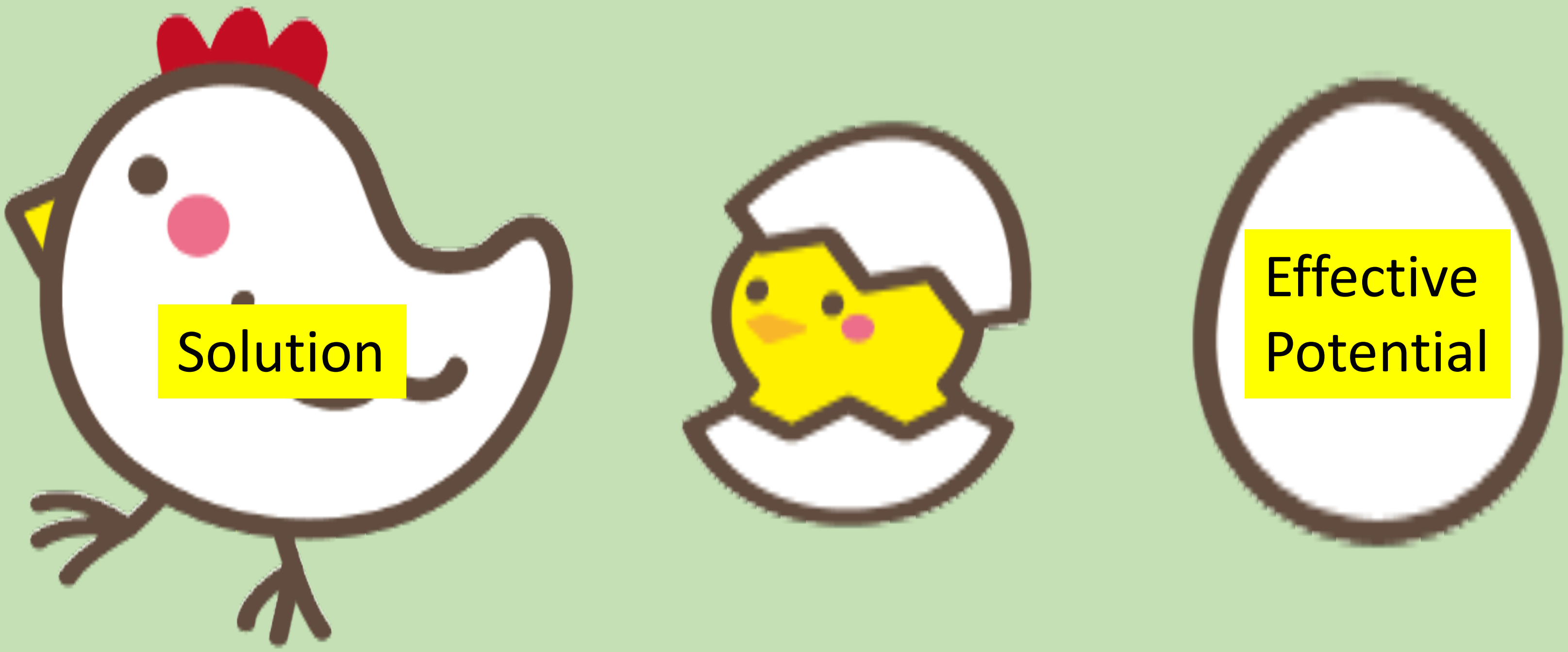
And then obtain the solution = minimum of the potential.

one solution to one Higgs potential

→ Also we are faced with the naturalness problem



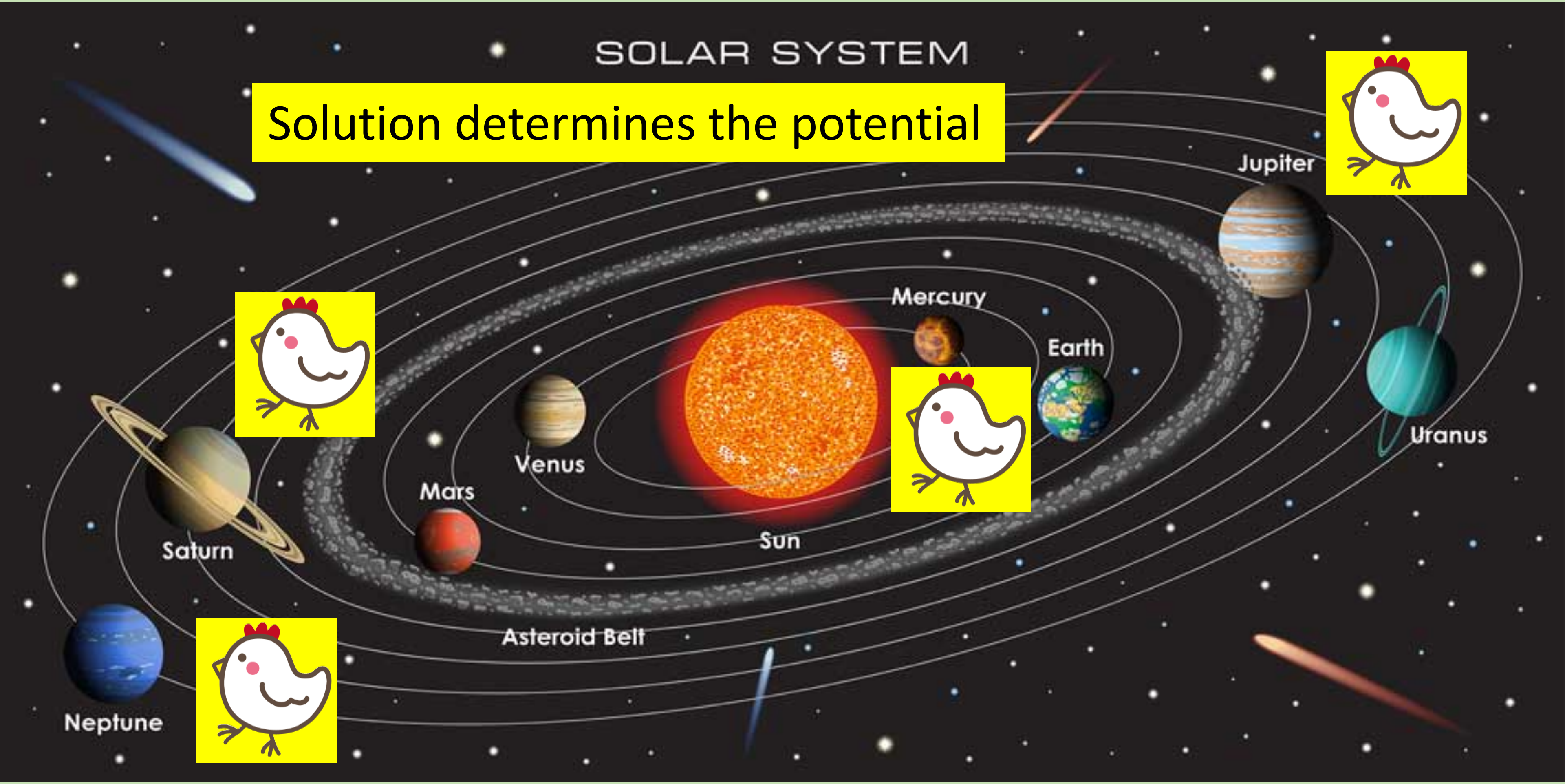
Which came first, the chicken or the egg ?



Usually calculate the potential first, then obtain a solution.

Is it possible to obtain a solution first , then calculate potential?

A classical example of the Chicken first approach

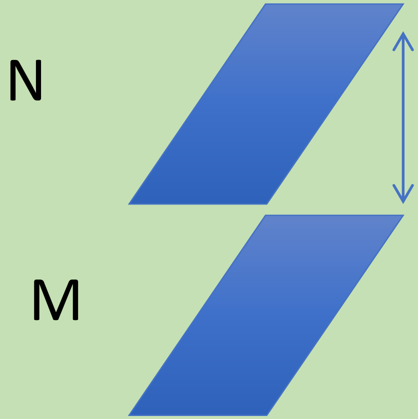


There are many orbits around the Sun: the Earth, the Jupiter, Mercury ..
each of the orbit is at the bottom of the corresponding potential.
But the underlying dynamics is the same.

A similar mechanism
to dynamically generate the EW scale.

I talked about the basic idea
at 1st East Asia Joint WS @ Huhei

In the D-brane model building,
the distance between branes (=moduli)
gives a vev of the scalar field.



U(N+M) → U(N) × U(M)

Distance ⇔ scale of the gauge symmetry breaking

$$L = \frac{M}{M_s^2}$$

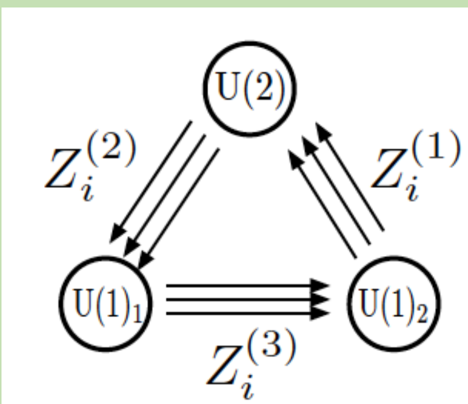
The natural scale of M should be
the string scale, not the EW scale.

Hierarchy problem in string theory
= Difficulty to generate the EW scale in string theory

Ex : D3s+anti-D7 on Z_3 orbifold $R^4 \times (T^2 \times T^2 \times T^2)/Z_3$

Put 4 D3-branes on a fixed point of T^6/Z_3

Assignment of Z_3 charge for D3s $\gamma_3 = \mathrm{diag}(\mathbf{1}_2, \alpha, \alpha^2)$

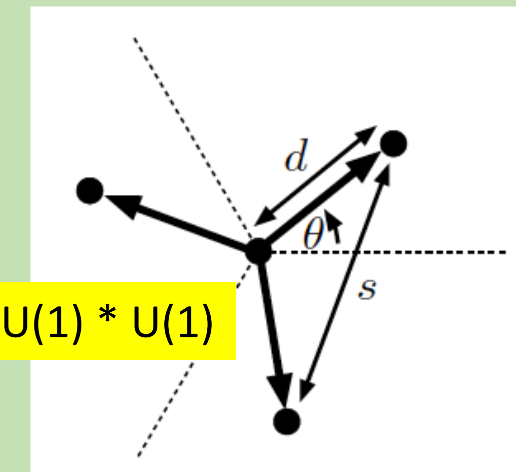


Quiver gauge theory

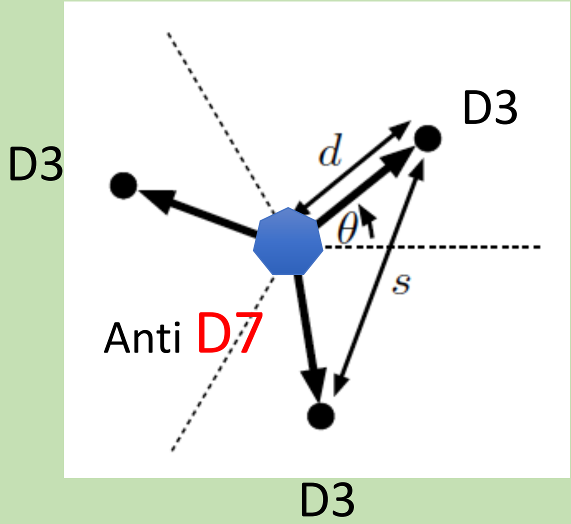
4

U(2) * U(1) * U(1)

vev →



U(1) * U(1)



Attractive force between D3s and anti-D7
due to open string 1-loop amplitudes

$$\mu^2 \sum_a |Z_3^{(a)}|^2$$

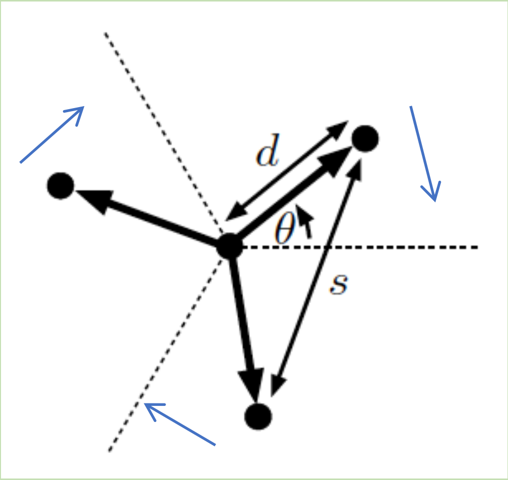
$$\mu^2 = \frac{1}{C^2}\frac{g^2}{16\pi^2}M_s^2$$

1-loop suppressed

Repulsive centrifugal force by revolution of D3s

Solution:
Hierarchy of EW scale

$$M \ll M_s$$



N.Kitazawa SI
PTEP,2015

High angular frequency $\omega = \mu \sim \frac{g}{4\pi}M_s$

Low velocity $v = \omega d \sim \frac{v_0}{M_s} \ll 1$

Merry-go-round scenario (named by Kimyeong)

It is possible to make a classically stable state with a short distance $r \ll l_{string}$.

But the large angular frequency

$$\omega = \mu \sim \frac{g}{4\pi} M_s$$

→ two problems

・ Dispersion relation of Higgs violates Lorentz symmetry (Coriolis force)

$$\omega^2 = \begin{cases} 2(p^2 + m^2) \\ \frac{\lambda v^2}{2\omega_0^2} p^2 \end{cases}$$

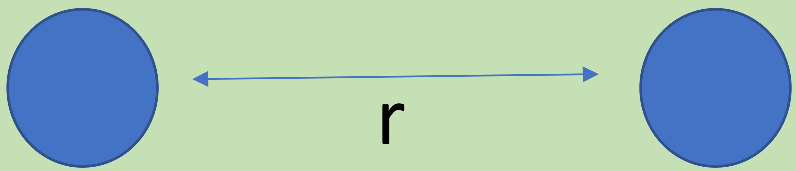
・ closed string emission → unstable

To avoid these problems,
it is necessary to make a bound (or resonant) state with $\omega << m_{str}$.

→ We need weak attractive force: $V << (m_{string})^3 \ r^2$

A simple way to avoid $\omega = m_{str}$ is to consider

Flat moduli such as Dp - Dp ,



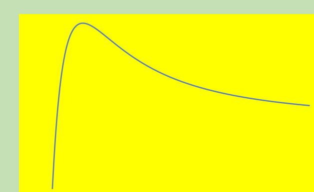
BPS = no interaction at rest

but

v^{-}-dependent attractive force is generated

when they are moving with a constant velocity

closed string picture



In the closed string region,

$$r \gg l_{str} \quad \frac{-Mv^4}{r^{7-p}} + \frac{J^2}{Mr^2}$$

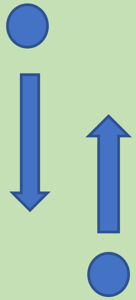
⇨ No minimum exists.

Potential barrier at $r < l_{str}$

In the closed-string dominated region ($r >> l_{str}$),
the repulsive force surpasses the attractive force.

However, "Beauty is Attractive"

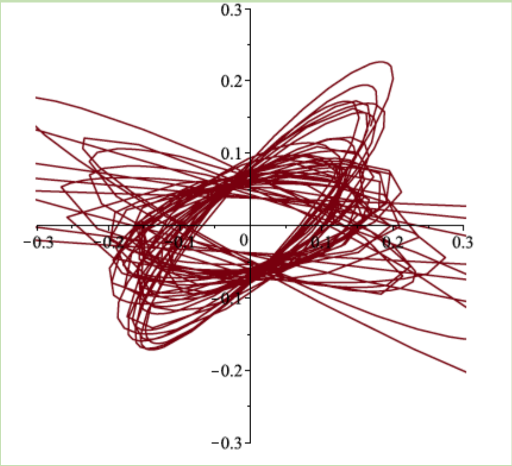
Kofman Linde Liu Malony
MaCllister Silverstein 2004



Mass changes rapidly. → non-adiabatic process

Open string particles are produced (= preheating).

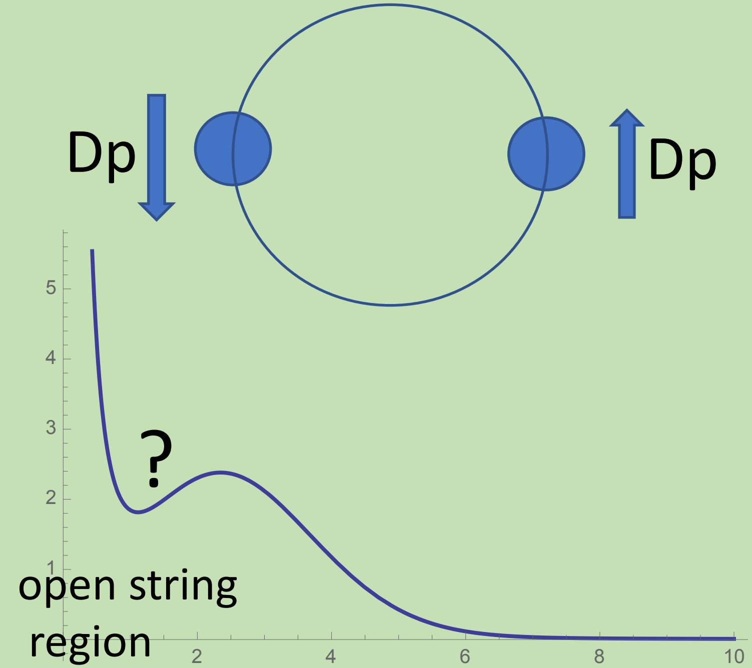
$$n_\chi = \frac{(gv)^{3/2}}{(2\pi)^3} e^{-\pi g \mu^2 / v}.$$



→ lose energy and the trajectory shrinks.

→ and when the trajectory becomes circular,
no more particle production occurs.

Is it possible to make
a "bound state" with $r \ll l_{str}$
between revolving D-branes?



Many works on D0-branes (N=16 SU(N) supersymmetric QM)

Witten index (Piljin Yi) $Tr(-1)^F = 1$ ---- threshold bound state
massless graviton

Quantum bound state (Kabat Pouliot) ---- resonant state $E > 0$

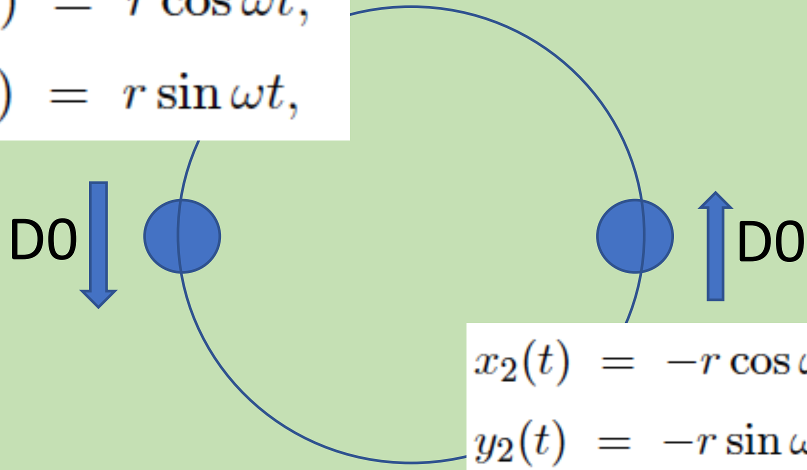
These states are near the ground state.

What we want is highly excited, but almost stable resonant states.
(like a solar system, not like a hydrogen atom)

Poor man's Calculation of attractive potential between revolving Dp, in particular p=0.

T. Suyama,
H. Ohta, SI
to appear soon

$$
\begin{aligned}
x_1(t) &= r\cos\omega t,\\
y_1(t) &= r\sin\omega t,
\end{aligned}
$$



$$
\begin{aligned}
x_2(t) &= -r\cos\omega t,\\
y_2(t) &= -r\sin\omega t.
\end{aligned}
$$

It is straightforward, but not so trivial because of the time-dependent boundary conditions:

In the rotational frame, the boundary condition becomes simple, but the system is interacting:

$$
\begin{aligned}
S &= -\frac{1}{4\pi\alpha'}\int d^2\sigma\Big[-\partial_\alpha\tilde{T}\partial^\alpha\tilde{T}+\partial_\alpha\tilde{X}\partial^\alpha\tilde{X}+\partial_\alpha\tilde{Y}\partial^\alpha\tilde{Y}+\partial_\alpha\tilde{X}^i\partial^\alpha\tilde{X}_i\\
&\qquad +2\omega\partial_\alpha\tilde{T}(\tilde{X}\partial^\alpha\tilde{Y}-\tilde{Y}\partial^\alpha\tilde{X})+\omega^2(\tilde{X}^2+\tilde{Y}^2)\partial_\alpha\tilde{T}\partial^\alpha\tilde{T}\Big].
\end{aligned}
$$

Furthermore, by taking variation with respect to the T fields,

$$\delta S\Big|_{bdy} = -\frac{r^2}{2\pi\alpha'}\delta\tilde{T}\left[-\partial_\sigma\tilde{T} + v(\tilde{X}\partial_\sigma\tilde{Y} - \tilde{Y}\partial_\sigma\tilde{X}) + v^2(\tilde{X}^2+\tilde{Y}^2)\partial_\sigma\tilde{T}\right]\Big|_{bdy}$$

Thus T – field must satisfy → at the boundaries:

$$(1-v^2)\partial_\sigma\tilde{T} - v\partial_\sigma\tilde{Y} = 0. \qquad \sigma = 0$$

$$(1-v^2)\partial_\sigma\tilde{T} + v\partial_\sigma\tilde{Y} = 0. \qquad \sigma = \pi$$

These conditions can be simplified by introducing a new variable

$$T := \tilde{T} - \frac{v}{1-v^2}x(\sigma)\tilde{Y}$$

With these (and a few more) changes of world sheet fields, the action of open strings stretched between revolving D0s becomes

Open string world sheet action between revolving D0s

$$
\begin{aligned}
S \;=\; & -\frac{r^2}{4\pi\alpha'}\int d^2\sigma\Bigg[-\dot{X}^2-\dot{Y}^2-(\dot{X}^i)^2+\left(X'-\frac{2}{\pi}\right)^2+(Y')^2+(X^{i\prime})^2 \\
& + \left[1-v^2\Big(\big(X+x(\sigma)\big)^2+Y^2\Big)\right]\left[\left(\dot{T}+\frac{vx(\sigma)}{1-v^2}\dot{Y}\right)^2-\left(T'+\frac{v}{1-v^2}\big(x(\sigma)Y'-\frac{2}{\pi}Y\big)\right)^2\right] \\
& - 2v\left(\dot{T}+\frac{vx(\sigma)}{1-v^2}\dot{Y}\right)\Big(\big(X+x(\sigma)\big)\dot{Y}-Y\dot{X}\Big) \\
& + 2v\left(T'+\frac{v}{1-v^2}\big(x(\sigma)Y'-\frac{2}{\pi}Y\big)\right)\Big(\big(X+x(\sigma)\big)Y'-Y\big(X'-\frac{2}{\pi}\big)\Big)\Bigg].
\end{aligned}
$$

boundary conditions

$$
X|_{\sigma=0,\pi}=0, \quad Y|_{\sigma=0,\pi}=0, \quad \partial_\sigma T|_{\sigma=0,\pi}=0, \quad X_i|_{\sigma=0,\pi}=0
$$

One-loop amplitude of open string between revolving D0s perturbation with respect to the angular velocity ω up to ω^2

$$
\begin{aligned}
Z &= \int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}\left[e^{-2\pi t(H_{rot}-\frac{1}{8})}\right]\left(\eta(it)\right)^{-21} \\
&= \int_0^\infty \frac{dt}{2t}\,\mathrm{Tr}\left[e^{-2\pi t(H_{rot}-1)}\right]\prod_{m=1}^{\infty}(1-e^{-2\pi m t})^{-21}
\end{aligned}
$$

The assumptions of our calculation are

$$r \ll l_{string} \qquad v = r\omega \ll 1$$

↓ low lying open string states dominate

↓ perturbation is valid

Effective potential induced by massive states

: Double expansion with respect to w^2 and r^2.

$$
\begin{aligned}
\mathcal{V}_2 \quad &\sim \quad m_{\rm str} \left(c_1 + c_2 \left(\frac{w}{m_{\rm str}} \right)^2 + \cdots \right) \left(\frac{r}{l_{\rm str}} \right)^2 \\
&+ \quad m_{\rm str} \left(c_3 + c_4 \left(\frac{w}{m_{\rm str}} \right)^2 + \cdots \right) \left(\frac{r}{l_{\rm str}} \right)^4 + \cdots
\end{aligned}
$$

Bosonic case: the first excited massive state

$$
\begin{aligned}
\mathcal{V}_2(r,\omega) \quad = \quad & \frac{1296 + 106\alpha'\omega^2}{8\sqrt{\alpha'}} + \frac{1296 + (-730 + 432\pi^2 + 1296\pi^2\epsilon_0)\alpha'\omega^2}{16\sqrt{\alpha'}} \left(\frac{r^2}{\pi^2\alpha'} \right) \\
& - \frac{1296 + (-2430 - 864\pi^2 + 2592\pi^2\epsilon_0)\alpha'\omega^2}{64\sqrt{\alpha'}} \left(\frac{r^2}{\pi^2\alpha'} \right)^2 + \mathcal{O}(\omega^4, r^6).
\end{aligned}
$$

In superstring $c_1 = c_3 = 0$

$C_2 = C_4 = 0$?

→ We are checking how it deviates from constant velocity case

Effective potential induced by massless states stretched bet. D0s

$$m_{\rm str}\sqrt{\left(\frac{r}{l_{\rm str}}\right)^2 + C_1\left(\frac{\omega}{m_{\rm str}}\right)^2 + C_2\left(\frac{r}{l_{\rm str}}\right)^2\left(\frac{\omega}{m_{\rm str}}\right)^2}$$

mass^2 acquired by the effect of revolution

c.f. Field theory calculation

$$-{\rm Tr}\log(\Delta + m^2)^{-1/2} = \int_\epsilon^\infty \frac{dt}{2t}\int \frac{d^{p+1}k}{(2\pi)^{p+1}} e^{-(k^2+m^2)t}$$

$$\sim \int_\epsilon^\infty \frac{dt}{2t} t^{-\frac{p+1}{2}} e^{-m^2 t} \sim \begin{cases} (m^2)^{\frac{p+1}{2}} & p = {\rm odd} \\ (m^2)^{\frac{p+1}{2}} \log m^2 & p = even. \end{cases}$$

Note that C_1 is negative for vectors (positive for KK scalar) which indicates that the system is unstable for large ω .

Closed string emission with spins is suppressed for $\omega << m_{string}$

angular momentum is conserved → r and ω are related:

$$J = Mr^2\omega = \frac{V}{g l_{str}^p} \left(\frac{r}{l_{str}}\right)^2 \left(\frac{\omega}{m_{str}}\right)$$

The total potential per unit volume for revolving Dp-brane is

$$\frac{m_{str}}{2g} \left(\frac{r}{l_{str}}\right)^2 \left(\frac{\omega}{m_{str}}\right)^2$$ + the following attractive potential

Dp-Dp potential (p=odd)

$$\sum_i (-1)^{F_i} n_i \left(\left(\frac{r}{l_{str}}\right)^2 + C_i \left(\frac{\omega}{m_{str}}\right)^2 \right)^{\frac{p+1}{2}} \log \left(\left(\frac{r}{l_{str}}\right)^2 + C_i \left(\frac{\omega}{m_{str}}\right)^2 \right)$$

$$\begin{aligned} \mathcal{V}_2 \quad &\sim \quad m_{str} \left(c_1 + c_2 \left(\frac{w}{m_{str}}\right)^2 + \cdots \right) \left(\frac{r}{l_{str}}\right)^2 \\ &+ \quad m_{str} \left(c_3 + c_4 \left(\frac{w}{m_{str}}\right)^2 + \cdots \right) \left(\frac{r}{l_{str}}\right)^4 + \cdots \end{aligned}$$

The potential has a minimum around

$$\frac{r}{l_{\rm str}} \sim \frac{\omega}{m_{\rm str}} \ll 1$$

consistent with the assumptions of our calculations

Total angular momentum

$$J = \frac{V}{g l_{\rm str}^p} \left(\frac{r}{l_{\rm str}}\right)^2 \left(\frac{\omega}{m_{\rm str}}\right)$$

For p=0, J << 1

→ only s-wave (ground state) is allowed.

Thus only threshold bound state exists.

For p>0 J can be larger than 1.

A resonant state with higher J may exist.

Summary

"The chicken-first approach" to hierarchy problem

Stationary solutions of D-branes

If resonant states exist,
it must be a hierarchical solution $\frac{r}{l_{str}} \sim \frac{\omega}{m_{str}} \ll 1$

which means $E_{EW} << m_{string}$.

We have calculated the effective potential
corresponding to the solution.

For p>0, there may exist a "classical" bound state satisfying

$\omega \ll m_{string}$ Lorentz violation, instability by radiation → OK

Future issues:

(1) Smarter calculation by D-brane EFT in t-dep background

(2) construct phenomenologically realistic models

(3) SUSY breaking: TeV SUSY? solution to little hierarchy?

관심을 가져 주셔서 감사합니다