East Asia Joint Workshop on Fields and Strings

@KIAS Nov.6, 2018

Holographic Spacetimes as Quantum Circuits of Path-Integrations

Tadashi Takayanagi

Yukawa Institute for Theoretical Physics (YITP), Kyoto Univ.

Based on 1808.09072

It from Qubit
Simons Collaboration

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Closely Related works

(1) ``Surface/State Correspondence''

Miyaji-TT,

PTEP (2015) no.7, 073B03 [arXiv:1503.03542].

(2)``Path-integral Optimization''

Caputa-Kundu-Miyaji -Watanabe-TT

PRL 119.071602 [arXiv:1703.00456]

(3) ``Tensor networks as path integral geometry''

Milsted-Vidal

arXiv:1805.12524

① Introduction

The AdS/CFT provides a formulation of quantum gravity on AdS in terms of CFTs.

[Maldacena 1997]

AdS/CFT

Gravity (String theory) on D+1 dim. AdS (anti de-Sitter space) = Conformal Field Theory (CFT) on D dim.Minkowski spacetime

↓ Classical limit

General relativity with $\Lambda < 0$

↓ Large N + Strong coupling

Strongly interacting Quantum Field Theories

Motivation 1

What is the basic mechanism of AdS/CFT ?

(AdS/CFT is still a Black Box !)

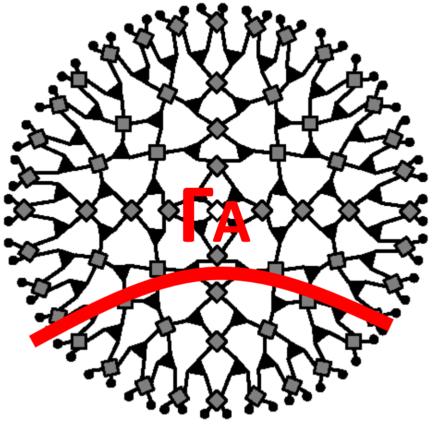
This is a very important question when we try to generalize the idea of holography to other spacetimes, such as de-Sitter space, flat space, and big-bang,.....

Note: In string theory, the holography looks the best framework to study quantum gravity.

A basic mechanism of AdS/CFT ?

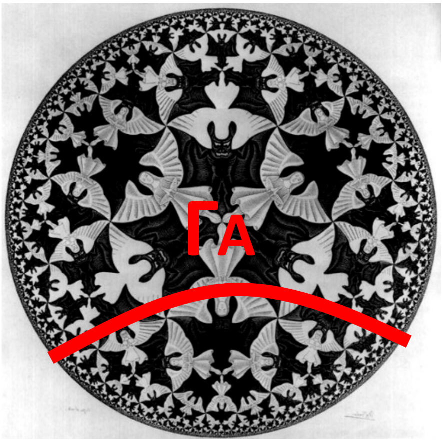
⇒ An interesting possibility is tensor networks (TNs) ! [Swingle 2009,....]

⇒``Emergent space from Quantum Entanglement''



MERA TN

≈



AdS

Tensor network = Network of Quantum entanglement

=``Geometry'' of Wave-functional in QFTs

Which surface in the AdS corresponds to the TN ?

⇒ This basic question has confused us quite a lot !!

Possibility 1 TN = Time Slice (hyperbolic space) in AdS ?

[Swingle 2009, Pastawski-Yoshida-Harlow-Preskill 2015,...]

⇒ Supported by HEE and QEC

But how the time coordinate emerges from TN ?

Possibility 2 TN = de Sitter space ?

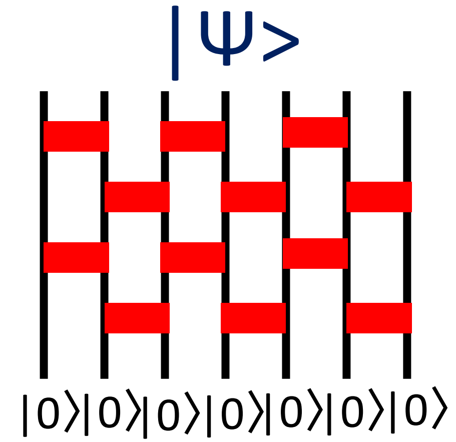
[Beny 2011, Czech-Lamprou-McCandlish-Sully 2015,....]

⇒Supported by Causal structure of MERA network

➡ Our new formulation can explain both of them !

Motivation 2 Holographic complexity ?

Computational Complexity
of a quantum state |Ψ>
= Min [# of Quantum Gates]

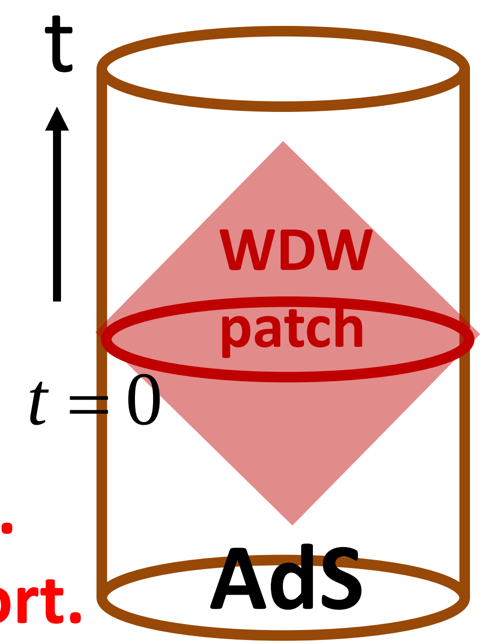


Unitary trf. between two adjacent spins

A Holographic Complexity Proposal

Hol. Complexity
= Gravity Action in Wheeler-DeWitt (WDW) patch of AdS

[Brown-Roberts-Susskind-Swingle-Zhao 15]

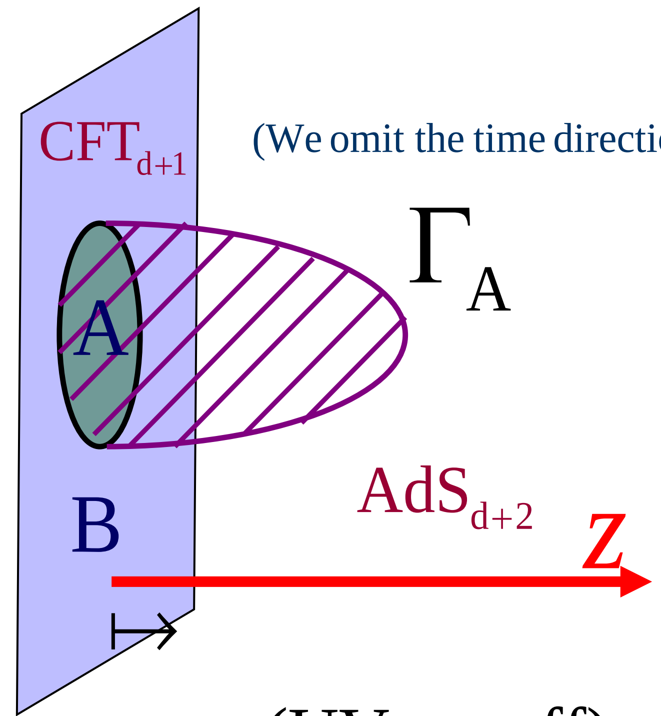


→ **So far no clear derivation of this formula.**
Our formulation provides a partial support.

Motivation 3 Holographic Entanglement Entropy (HEE)

[Ryu-TT 06, Hubeny-Rangamani-TT 07]

$$S_A = \operatorname*{Min}_{\substack{\partial\Gamma_A = \partial A \\ \Gamma_A \approx A}} \left[\frac{\text{Area}(\Gamma_A)}{4G_N} \right]$$



Γ_A is the area of codim.=2 **minimal space-like surface** such that $\partial A = \partial\gamma_A$ and $A \sim \gamma_A$.

homologous

$z > \varepsilon$ (UV cut off)

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

➡ What about areas of time-like surfaces ? (or Gtt)

Our formulation suggests their holographic duals.

Contents

② ``Spacetime = Path-integral Circuit'' Conjecture

(2-1) Surface/State duality [Miyaji-TT 2015]

Consider Einstein gravity on a d+2 dim. AdS.

The surface/state duality argues:

Σ : a d dim. convex space-like surface in AdS_{d+2}.

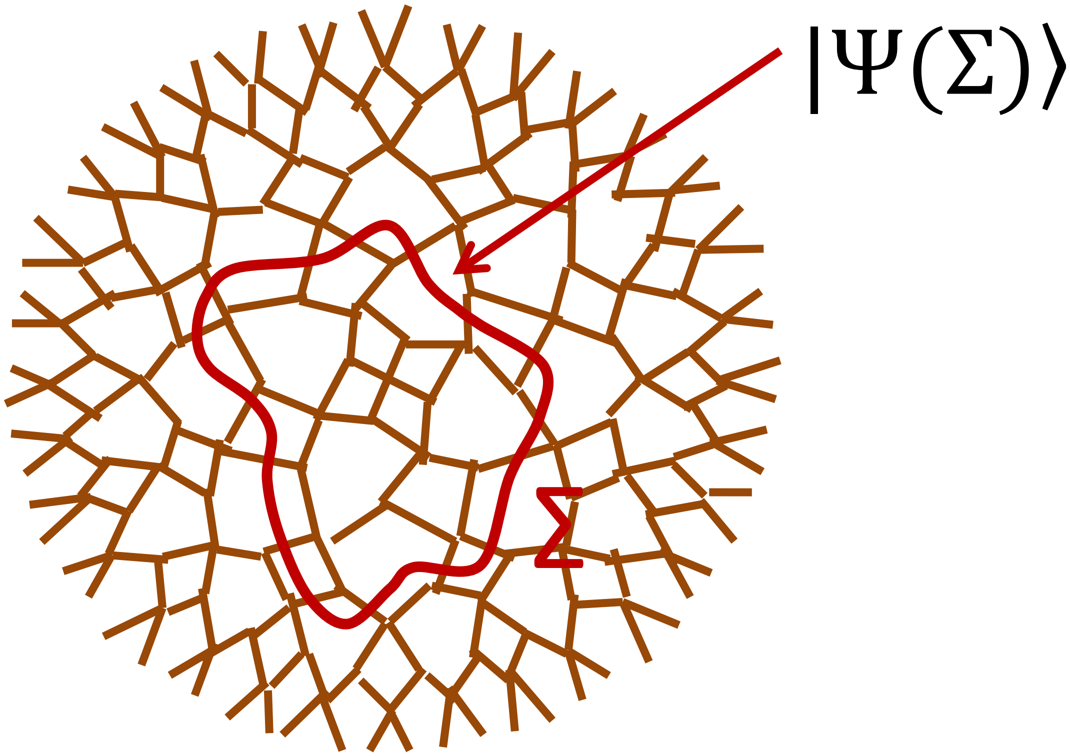
which is closed and homologically trivial

Dual

$$|\Psi(\Sigma)\rangle \in H_{CFT}$$

A pure state

The surface/state duality is motivated by the tensor network description of holography.



How to construct such states $|\Psi(\Sigma)\rangle$ in the continuum limit of CFTs ?

(2-2) ``Slice = Quantum Circuit'' Conjecture

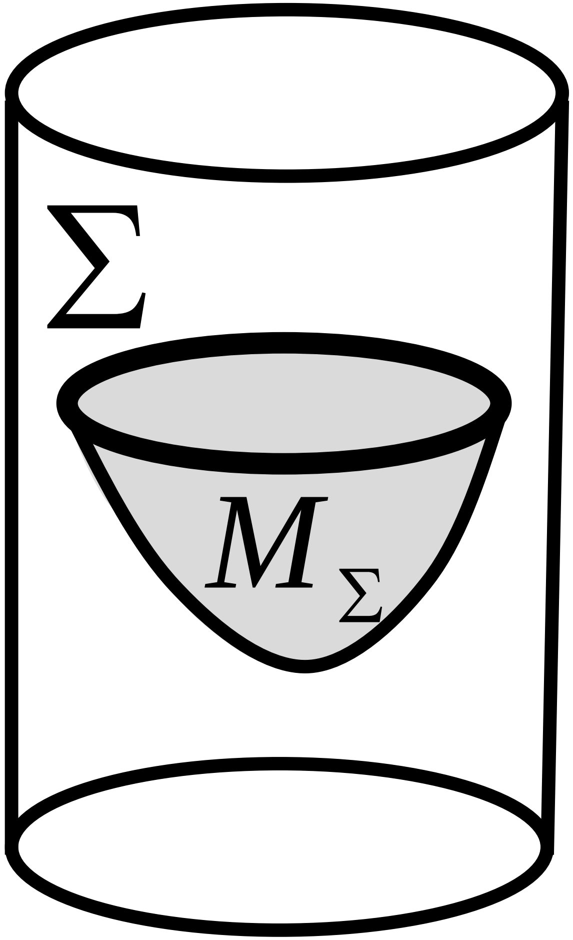
A (d+1) dim. slice M_Σ in AdS_{d+2}

⇕ **Dual**

A quantum circuit which creates the state $|\Psi(\Sigma)\rangle$ by path-integrals

$$\Psi(\Sigma)[\varphi_0] = \int_{M_\Sigma} D\varphi \, e^{-S_{CFT}[\varphi]} \delta[\varphi|_\Sigma - \varphi_0]$$

A CFT action on the curved space M_Σ with an appropriate coarse-graining s.t. z= lattice spacing



Euclidean AdS

(i) The path-integral circuit is not unitary.

(ii) $|\Psi(\Sigma)\rangle$ does not depend the choice of M_Σ owing to the conformal symmetry (up to normalization).

$$\Psi(M_\Sigma : \Sigma)[\varphi_0] = e^{C(M_\Sigma) - C(M'_\Sigma)} \cdot \Psi(M'_\Sigma : \Sigma)[\varphi_0]$$

We call the quantity $C(M_\Sigma)$ a path-integral complexity of the circuit M_Σ .

Path-integral Optimization and Complexity

Euclidean Path-integral Complexity

Minimize $C(M_\Sigma)$ by changing M_Σ for a given $|\Psi(\Sigma)\rangle$.

This defines the Euclidean path-integral complexity of the state: $C(\Psi_\Sigma) = \mathrm{Min}[C(M_\Sigma)]$.

This is equivalent to the path-integral optimization.

[Caputa-Kundu-Miyaji-Watanabe-TT 17]

Consider two dimensional CFTs. We write the metric:

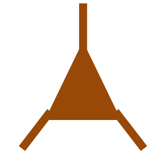
$$ds^2 = e^{2\phi(x,z)}(dx^2 + dz^2) \quad \text{with} \quad e^{2\phi}\big|_{z=\varepsilon} = \varepsilon^{-2}.$$

The rule of UV cut off: a lattice site for a unit area.

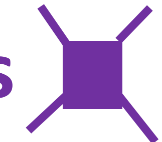
$$C(M_\Sigma) = \mathrm{Log}\left[\frac{\Psi_{g=e^{2\phi}\delta_{ab}}}{\Psi_{g=\delta_{ab}}}\right] = S_L[\phi],$$

Liouville Action

# of Isometries [Czech 17]



# of Unitaries



$$S_L[\phi] = \frac{c}{24\pi}\int dxdz\left[(\partial_x\phi)^2 + (\partial_z\phi)^2 + e^{2\phi}\right]$$

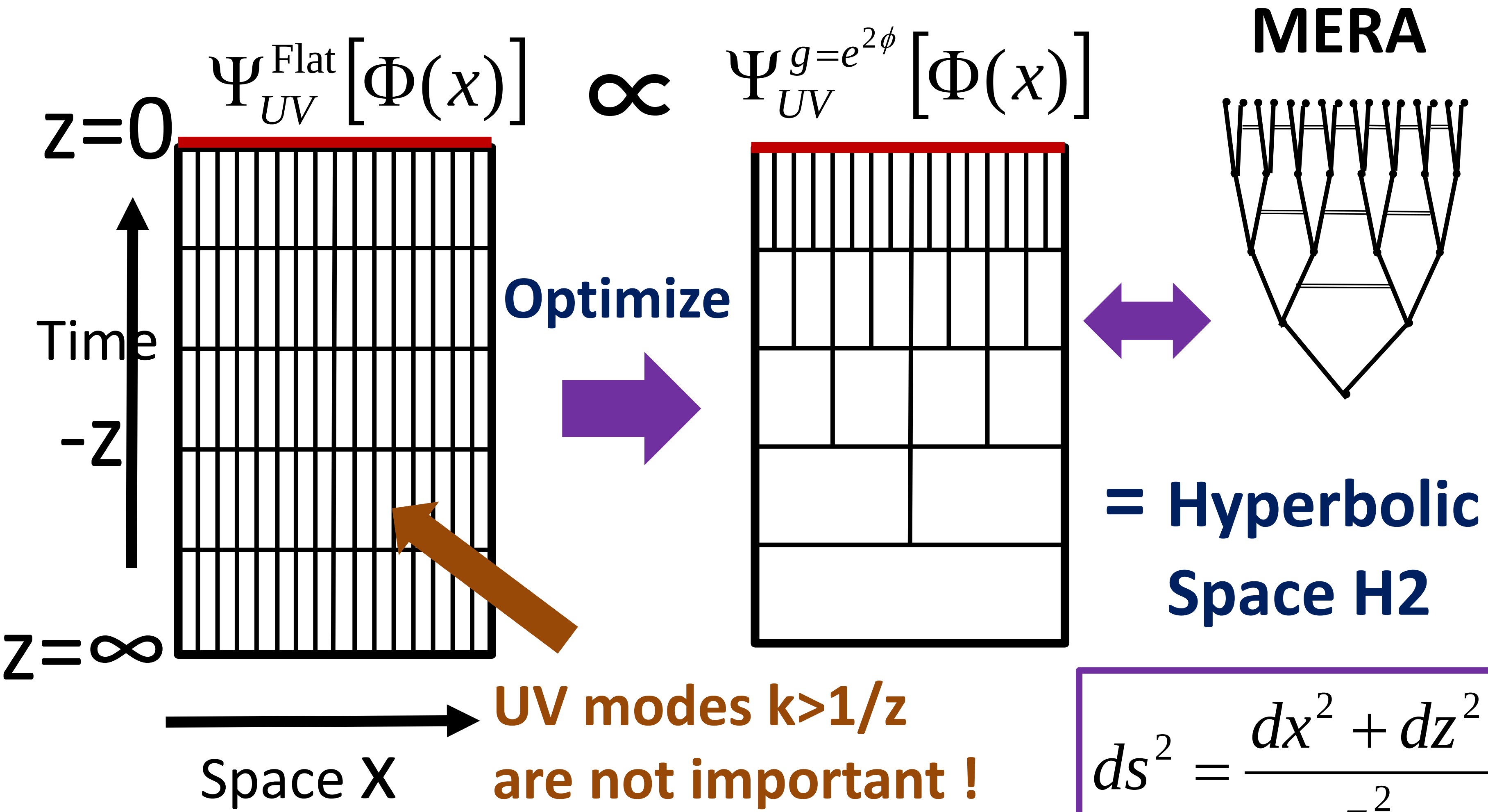
$$= \frac{c}{24\pi}\int dxdz\left[(\partial_x\phi)^2 + \left(\partial_z\phi + e^{\phi}\right)^2\right] + (\text{surface term})$$

$$\Rightarrow \text{Minimum}: \ e^{2\phi} = \frac{1}{z^2}.$$

Hyperbolic plane (H_2) = Time slice of AdS3

$$ds^2 = (dx^2 + dz^2)/z^2.$$

A Sketch: Optimization of Path-Integral



= Hyperbolic Space H2

$$ds^2 = \frac{dx^2 + dz^2}{z^2}$$

Lorentzian AdS

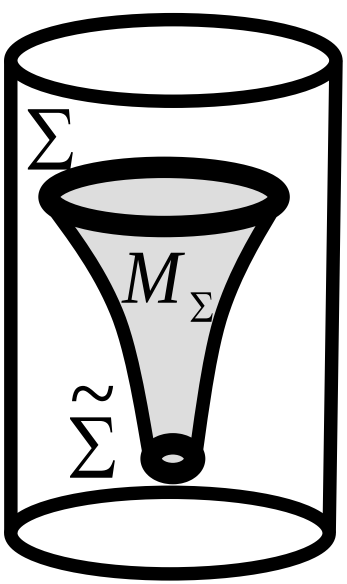
Case 1: M_Σ is space-like

$\Rightarrow$ The path-integral circuit is not unitary.

Case 2: M_Σ is time-like

$\Rightarrow$ The path-integral circuit is unitary.

$$\Psi(M_\Sigma : \Sigma)[\varphi_0] = e^{iC(M_\Sigma) - iC(M'_\Sigma)} \cdot \Psi(M'_\Sigma : \Sigma)[\varphi_0]$$



Unitary Transformation

$$U(\tilde{\Sigma} : \Sigma) = \mathrm{P} \cdot \exp\left(-i \int_{\Sigma}^{\tilde{\Sigma}} ds\, K(s) \right)$$

Depends only on $\tilde{\Sigma}$ and Σ

Depends on M_Σ

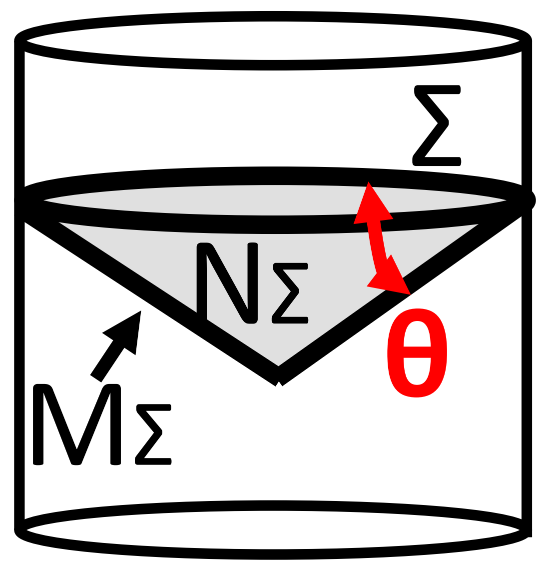
Lorentzian Path-integral Complexity

Again we can define the Lorentzian version of path-integral complexity: $C(\Psi_\Sigma)$ =Min[$C(M_\Sigma)$] .

In the gravity dual description, $C(M_\Sigma)$ is computed as The gravity action because ZG= $e^{iI_G} = e^{iC(M_\Sigma)}$.

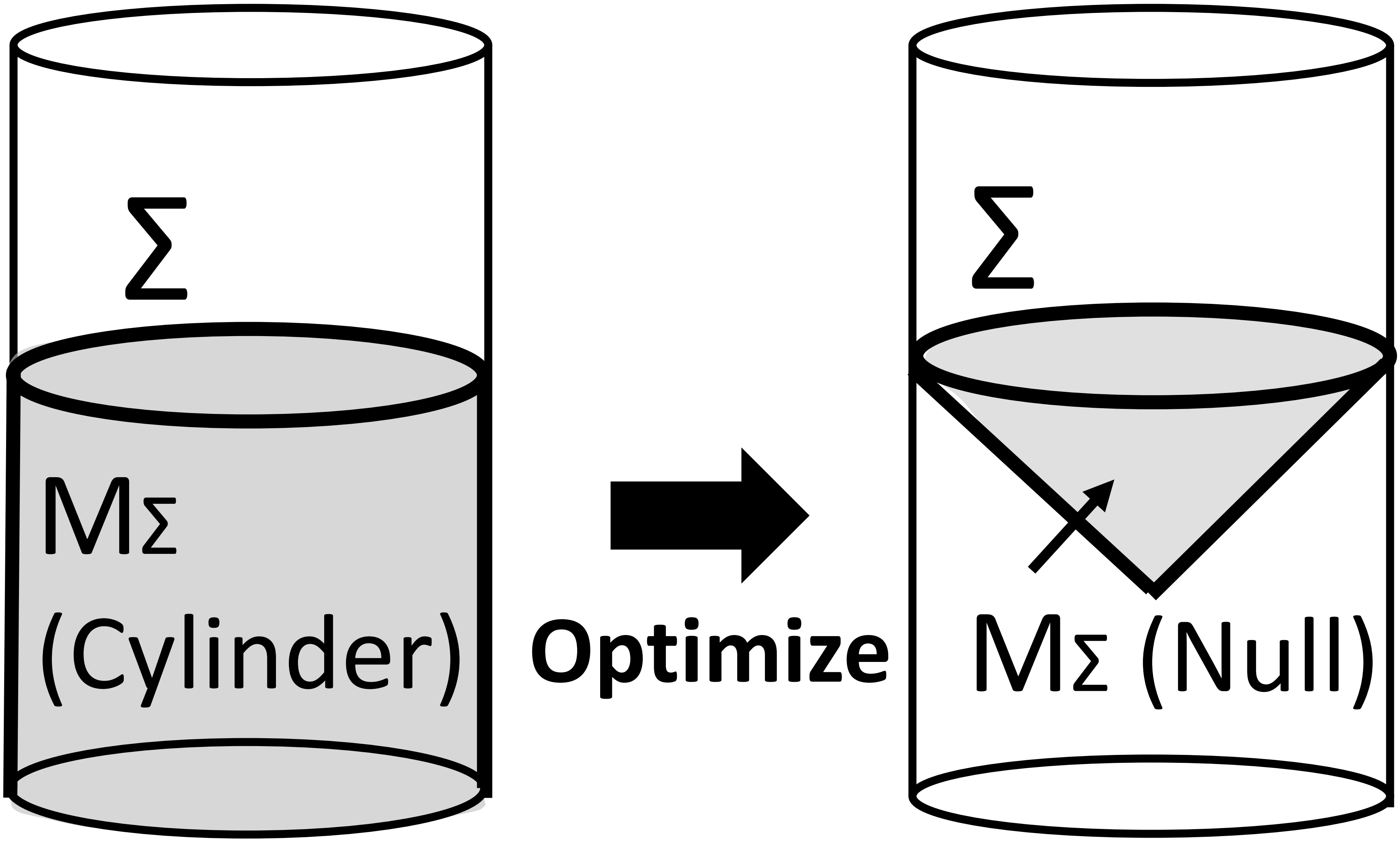
$$I_G = -\frac{1}{16\pi G}\int_{N_\Sigma}\sqrt{-g}(R-2\Lambda)$$

$$+\frac{1}{8\pi G}\int_{M_\Sigma}\sqrt{h}K - \frac{1}{8\pi G}\int_{\Sigma}\sqrt{\gamma}\theta$$



I_G is minimized at θ=∞, i.e. MΣ becomes light-like.

➡ The holographic complexity based on WDW patch !



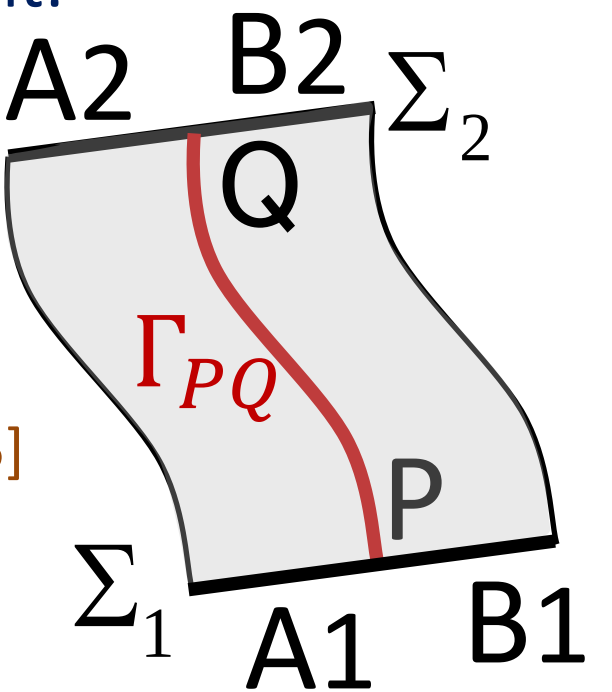
③ Time-like Surface and Quantum Entanglement

(3-1) Holographic Entanglement for Quantum Circuit

As we argued, we can regard a codim.=1 slice MΣ as a (non-unitary or unitary) quantum circuit:

Space-like Time-like

$$V(\Sigma_2:\Sigma_1) = \mathrm{P}\cdot\exp\left(-i\int_{\Sigma_1}^{\Sigma_2} ds\, K(s)\right)$$



Channel-State duality

[e.g. Hosur-Qi-Roberts-Yoshida 15]

$$|\Psi_{\Sigma_2\Sigma_1}\rangle = \sum_i V(\Sigma_2:\Sigma_1)|i\rangle_{\Sigma_1}\otimes|i\rangle_{\Sigma_2}$$

➡ Quantum circuit V generates quantum entanglement.

New Proposal of Holographic Formula

$$\left(\Delta S^{S}_{A1A2}\right)^2 \text{-} \left(\Delta S^{T}_{A1A2}\right)^2 = \left(\frac{\Delta A(\Gamma_{PQ})}{4G}\right)^2$$

$\Delta S^{S}_{A1A2} =$ The increased amount of entanglement between A and B along ΓPQ due to **non-unitary (=space-like) quantum gates**.

$\Delta S^{T}_{A1A2} =$ The increased amount of entanglement between A and B along ΓPQ due to **unitary (=time-like) quantum gates**.

$A(\Gamma_{PQ}) = \int_{\Gamma_{PQ}} \sqrt{g}$ ➡ **A= Real when ΓPQ = space-like**
A= Imaginary when ΓPQ = time-like

More precisely, ΔS^{S}_{A1A2} counts only entanglement (= ♯ of gates) which scrambles between A and B.

$$\Delta S_{A1A2} \Rightarrow -\Delta I_3(A1\!:A2\!:B1)$$

Examples

Case 1: ΓPQ =Space-like extremal surface

⇒ Reduced to the Hol. EE. $(\Delta S^{T}_{A1A2}=0)$

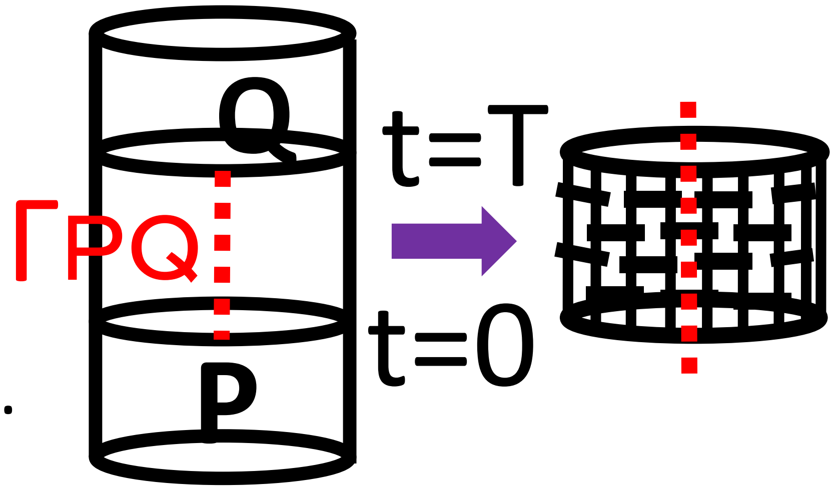
Case 2: ΓPQ = Time-like interval at the AdS bdy

$(\Delta S^{S}_{A1A2}=0)$

Our formula leads to

$$\Delta S^{T}_{A1A2} \approx c \cdot \frac{V_{d-1}T}{\varepsilon^{d}}.$$

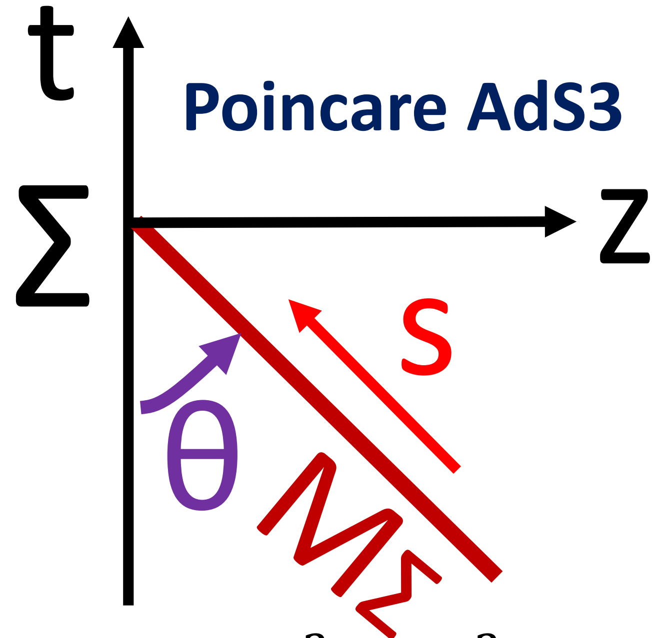
This agrees with 2d CFT results.



⇒ Gtt emerges from the Hamiltonian gates in CFT !

2d CFT Examples

$$V(M_\Sigma) = \mathrm{P}\cdot\exp\left(-i\int_{-\infty}^{0} ds\, K(s)\right)$$



$$K(s) = H + \sinh\theta \cdot D$$

$D \equiv$ dilatation in 2d CFT

$$ds^2 = \frac{dz^2 - dt^2 + dx^2}{z^2}$$

The null limit corresponds to θ=∞

⇒ **The quantum circuit is the dilatation !**

[see also Milsted-Vidal 2018]

Case 3: ΓPQ = Light-like surface

⇒ Non-scrambling Unitary gates

$\Delta I_3(A1{:}A2{:}B1)$=0 !

④ Gravitational Force from Quantum Circuits

Consider a massive particle in general relativity.

It is dual to a localized excitation in quantum circuits.

A basic idea: The presence of many quantum gates becomes an obstruction for the massive particle propagation because the excitation will be scattered off.

Our proposed formula (assume AdS3/CFT2) leads to

[♯ of scrambling gates] = $\frac{1}{4G}\int ds\sqrt{-g}$

➡ A particle moves toward the direction with a lower gravitational potential.

The wave function Ψ_m which corresponds to the excitation in the quantum circuits can be estimated:

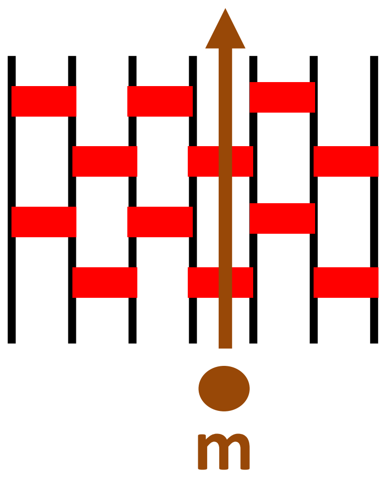
$$\Psi_m \approx e^{-i\Delta\theta \cdot \int ds\sqrt{-g}}.$$

# of Quantum gates

Universal Phase Shift for each quantum gate

$$\boldsymbol{\Delta\theta \approx E(=\Delta) = m}$$

[as expected from Lloyd's conjecture 2000]



This explains the particle action in curved spacetime and gravitational force !

$$S_m = -m \int ds\,\sqrt{-g}$$

⑤ Conclusions

In AdS/CFT , we argued the following correspondence:

1. A codim.2 space-like surface $\Sigma \Leftrightarrow$ A quantum state $|\Psi(\Sigma)\rangle$
2. A codim.1 slice $M\Sigma \Leftrightarrow$ A path-integral quantum circuit $V\Sigma$
3. The gravitational action surround by $M\Sigma$

   = The path-integral complexity → Hol. Complexity
4. The area of codim.2 surface Γ = ♯ of scrambling gates

➡ **Gtt and Gravitational force emerges from the density of unitary gates.**

Future Problems

Derivation of Einstein equation ?, Sub AdS locality ? ,
dS/CFT ? Flat space holography ?, ...

http://www2.yukawa.kyoto-u.ac.jp/~qist2019/index.php

YITP long-term workshop

Quantum Information and String Theory 2019

May 27 - June 28, 2019
Yukawa Institute for Theoretical Physics, Kyoto University

It from Qubit school/workshop

June 17 - June 28, 2019

YITP It from Qubit

Registration is open now !
Deadline: Feb.10th, 2019

Organizers:
Pawel Caputa (YITP), Tadashi Takayanagi (YITP, co-chair), Tatsuma Nishioka (Tokyo), Yasuaki Hikida (YITP), Keisuke Fujii (Kyoto), Tomoyuki Morimae (YITP, co-chair), Beni Yoshida (Perimeter), Yu Watanabe (YITP)

Thank you very much !