On de Sitter Spacetime

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De Sitter Spacetime

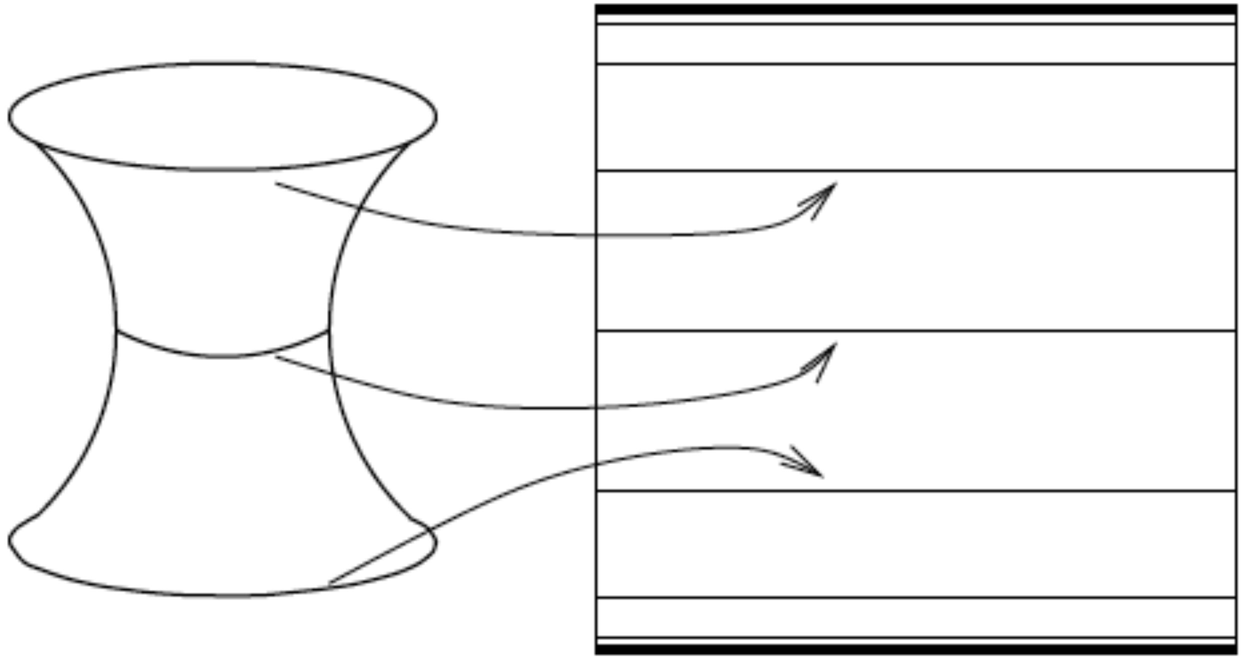


FIG. 1 de Sitter space as a hyperboloid. Time goes up.—Right: Penrose diagram. Horizontal lines represent three-spheres.

Cosmological horizon

$$\ell = \sqrt{\frac{3}{\Lambda}}.$$

$$\frac{ds^2}{\ell^2} = -d\tau^2 + \cosh^2\tau\, d\Omega_3^2$$

$$\frac{ds^2}{\ell^2} = -V(r)\, dt^2 + \frac{1}{V(r)} dr^2 + r^2 d\Omega_2^2,$$

$$V(r) = 1 - r^2,$$

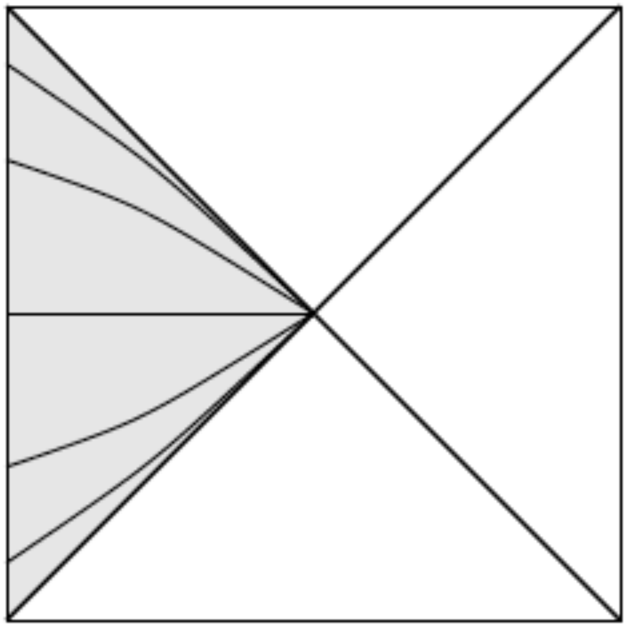


FIG. 2 Past and future event horizon (diagonal lines). The static slicing covers the interior of the cosmological horizon (shaded).

Entropy and Temperature of Event Horizons

A. Black holes Bekenshtein Hawking

$$T_{\rm hor} = \frac{\kappa}{2\pi}.$$

$$\kappa = (4M)^{-1}$$

the first law of thermodynamics

$$\frac{1}{T_{\rm hor}} = \frac{\partial S_{\rm hor}}{\partial M},$$

$$S_{\rm hor} = \frac{A}{4}.$$

B. de Sitter space

Gibbons Hawking

$$A_0 = 4\pi\ell^2,$$

$$\ell = \sqrt{\frac{3}{\Lambda}}$$

surface gravity $\kappa = 1/\ell$

$$T_{\mathrm{dS}} = \frac{1}{2\pi\ell},$$

$$S_0 = \frac{A_0}{4} = \frac{3\pi}{\Lambda}.$$

Swampland

Ooguri Vafa

$$|\nabla V| \geq \frac{c}{M_p} \cdot V \ ,$$

$$\min\left(\nabla_i \nabla_j V\right) \leq -\frac{c'}{M_p^2} \cdot V \ ,$$

$$\epsilon_V = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_V = M_{\rm Pl}^2\left(\frac{V''}{V}\right),$$

$$[\epsilon]: \ \epsilon_V \geq \frac{c^2}{2} \quad \text{or} \quad [\eta]: \ \eta_V \leq -c'.$$

Cosmological Constant

2D gravity $$g_{\mu\nu} = e^{\phi}\hat{g}_{\mu\nu},$$

$$\int \sqrt{-\hat{g}} d^2x [- \frac{25-c}{96\pi}(\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2\phi\tilde{R}) - \Lambda e^{(1+\frac{\gamma}{2})\phi}].$$

$$\frac{25-c}{48\pi}\nabla^2\phi - \Lambda e^{\phi} = 0,$$

$$e^{\phi_c} = (\frac{1}{-H\tau})^2, \quad H^2 = \frac{24\pi}{c-25}\Lambda.$$

IR logarithmic effects

$$\langle e^{\phi} \rangle \sim e^{\phi_c(t) + \frac{1}{2}\langle \phi^2 \rangle}.$$

$$\langle \phi^2 \rangle = -\frac{24}{c-25} \int_{P_{\min}}^{P_{\max}} \frac{dP}{P}.$$

$$\langle \phi^2 \rangle \sim -\frac{12}{c-25} \phi_c(t) \;\Rightarrow\; \langle e^{\phi} \rangle \sim e^{(1-\frac{6}{c-25})\phi_c(t)}.$$

$$H_{\text{eff}}^2 \sim H^2 e^{-\frac{6}{c-25}\phi_c(t)} \sim H^2 a(t)^{-\frac{12}{c-25}}.$$

Geometric Entropy

$$\begin{aligned}\delta S &= -\delta I_0 \\ &= \delta\phi(\frac{\epsilon}{2G} - \frac{25-c}{48\pi})\mu^{\epsilon}[\int_M R\,\sqrt{\hat{g}}\;d^D x - 2\int_{\partial M} K\,\sqrt{\hat{\gamma}}\;d^{D-1}x]\end{aligned}$$

$\delta S/\delta\phi = (C-25)/12$

$S = (C-25)/12\ \phi(t) = (C-25)/6\ Ht$

Cosmological constant decreases as entropy increases

Infrared logarithmic effects to scalar perturbation

$$\epsilon = \frac{\kappa^2}{4}(\frac{\dot{\hat{\varphi}}}{H})^2 = \frac{1}{\kappa^2}(\frac{V'(\hat{\varphi})}{V(\hat{\varphi})})^2.$$

$$\int d^4xa^4(\tau)V'(\hat{\varphi})\tilde{\varphi}\{1 - \frac{11}{32}\frac{\kappa^2H^2}{4\pi^2}\log a(\tau)\}.$$

$$\frac{1}{\kappa^2}\frac{1}{V(\hat{\varphi})^2}\{1 + \frac{9}{4}\frac{\kappa^2H^2}{4\pi^2}\log a(\tau)\}.$$

$$\{1 + \frac{25}{16}\frac{\kappa^2H_*^2}{4\pi^2}\log a(\tau_*)\}\epsilon_*.$$

$$\eta = \frac{2V''(\hat{\varphi})}{\kappa^2 V(\hat{\varphi})}.$$

$$\int d^4x\ a^4 V''(\hat{\varphi})\tilde{\varphi}^2\{1 - \frac{3}{4}\frac{\kappa^2 H^2}{4\pi^2}\log a(\tau)\}.$$

$$\frac{1}{\kappa^2 V(\hat{\varphi})}\{1 + \frac{3}{4}\frac{\kappa^2 H^2}{4\pi^2}\log a(\tau)\}$$

Cancelation of IR effects to this parameter!

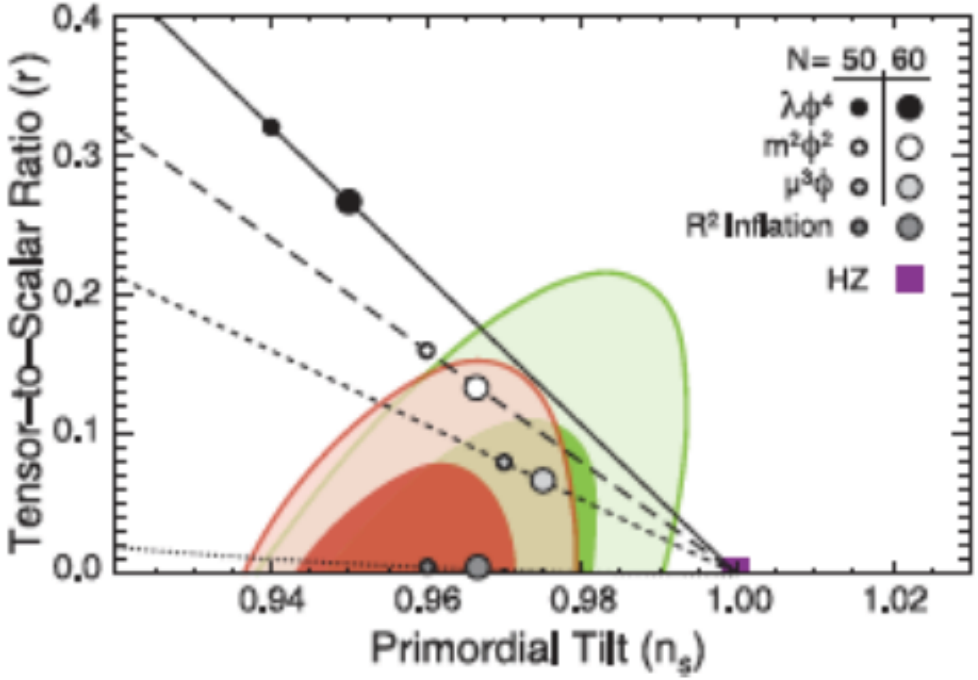
Generation of linear potential due to IR effects

$$\frac{1}{2H}\frac{\dot{\epsilon}}{\epsilon} = -\eta + 2\epsilon + \frac{25}{32}\frac{\kappa^2 H^2}{4\pi^2}.$$

$$\epsilon \sim \epsilon_0 e^{\frac{25}{16}\frac{\kappa^2 H_0^2}{4\pi^2} H_0 t},$$

$$V = \frac{6H_0^2}{\kappa^2}(1 - \sqrt{\epsilon}\kappa\hat{\varphi}).$$

We must observe tensor modes soon



Tensor-to-Scalar Ratio (r)
Primordial Tilt (n_s)
0.0
0.1
0.2
0.3
0.4
0.94
0.96
0.98
1.00
1.02
N= 50 60
$\lambda\phi^4$
$m^2\phi^2$
$\mu^3\phi$
R^2 Inflation
HZ

Conclusions

de Sitter spacetime may be a swampland in string theory

Cosmological constant decreases as entropy increases

$$S_0 = \frac{A_0}{4} = \frac{3\pi}{\Lambda}.$$

Evolusion of our universe suggests deep connection between cosmological constant and entropy