String amplitude from QFT amplitude

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From strings to QFT

Perturbative string theory has many applications

- → Low energy effective actions . Connections to mathematics.

- It contains amplitudes in many field theories $(Yang-Mills, F^3, F^4, GR, R^2, R^3, Einstein-Yang-Mills\ldots)$ [Broedel, Dixon, He , Yong, Schlotterer *et al*]

- Shed light on new structures for QFT and quantum gravity

- e.g. string Kawai-Lewellen-Tye relations [Kawai, Lewellen, Tye, 1986] → field theory KLT [Bern *et al,* 1998] , $GR = YM \otimes YM$

Double copy a la BCJ

- New double copy relations, based on color-kinematic duality [Bern, Carrasco, Johansson, 0805] . At tree level, it is equivalent to KLT.

- Extended to many theories, $Born\!-\!Infeld = Non\!-\!Linear\!-\!Sigma\!-\!Model \otimes YM$
  $EYM = Yang\!-\!Mills\!-\!Scalar \otimes YM$ [Cachazo, He,Yuan, 1412] ,
  $Conformal\ Gravity = (DF)^2 \otimes YM$ [Azevedo, Engelund, 1711] …

- Extended to loop level [Bern, Carrasco, Johansson, 1004]…

- In this talk, we focus on tree level: BCJ double copy = KLT double copy

$$C = A \otimes B: \quad M^C = \sum_{\rho,\pi} M^A[\rho] S[\rho\,|\,\pi] M^B[\pi]$$

- Made manifest by Cachazo-He-Yuan formulas [Cachazo, He,Yuan, 1307]

From QFT to strings

- Double copy between field theory amplitude and massless string amplitude

$$open\ string = QFT \otimes Z\!-\!theory$$

- First discovered for $type\ I = SYM \otimes Z\!-\!theory$ [Mafra, Schlotterer, Stieberger, 1106]

- Recently generalized to bosonic and heterotic string [Huang , Schlotterer, Wen 1602] [Azevedo, Chiodaroli, Johansson, Schlotterer, 1803]

string $\otimes$ QFT	SYM	$(DF)^2 + \text{YM}$	$(DF)^2 + \text{YM} + \phi^3$
Z-theory	open superstring	open bosonic string	compactified open bosonic string
sv(open superstring)	closed superstring	heterotic (gravity)	heterotic (gauge/gravity)
sv(open bosonic string)	heterotic (gravity)	closed bosonic string	compactified closed bosonic string

- This talk: to reduce string integrand to logarithmic form using integration by part (IBP) and express QFT amplitudes using CHY formulas

CHY formulas

- CHY formulas express tree-level S-matrices of massless particles as integrals over the moduli space of punctured Riemann spheres,

$$M_n = \int d\mu_n^{CHY}\, \mathbf{I}_n(z, k, \cdots) \qquad with \quad d\mu_n^{CHY} = \frac{d^n z}{SL(2)} \prod_{i=1}^{n}{}' \delta(\underbrace{\sum_{j\neq i} \frac{s_{ij}}{z_i - z_j}}_{scattering\ equations,\ universal})$$

- $\mathbf{I}_n$ is always a product of two integrands. e.g.

$$\mathbf{I}_n^{GR} = \mathrm{Pf}'\Psi_n(\epsilon)\,\mathrm{Pf}'\Psi_n(\tilde{\epsilon}) \qquad \mathbf{I}_n^{YM} = PT(\rho)\,\mathrm{Pf}'\Psi_n(\epsilon) \qquad \mathbf{I}_n^{\phi^3} = \mathrm{PT}(\rho)\,\mathrm{PT}(\pi)$$

$$with\ \mathrm{PT}(1234) = \frac{1}{z_{12}z_{23}z_{34}z_{41}}, \quad z_{i,j} = z_i - z_j$$

- These CHY integrands can be obtained from string integrands by IBP

String integrand

- Any massless open string amplitude takes the form

$$A^{string} = \int_{z_1<\cdots<z_n} \frac{d^n z}{SL(2)} \prod_{i<j} z_{ij}^{\alpha' s_{ij}} \; I_n(\{k_i, \epsilon_i, z_i\})$$

- In I_n , there are terms containing only logarithmic singularities, such as PT(1,2,3,4). But there are terms that are non-logarithmic (i.e. with subcycles), like

$$\frac{1}{z_{12}z_{21}z_{34}z_{43}}, \quad \frac{1}{z_{12}z_{23}z_{31}z_{45}z_{56}z_{64}}$$

IBP $\to$ logarithmic form

- Using IBP, we can reduce them to logarithmic forms

$$\int_{0<z<1} \frac{dz}{z^2} z^{\alpha' s}(1-z)^{\alpha' t} = \int_{0<z<1} \frac{dz^{\alpha' s-1}}{\alpha' s - 1}(1-z)^{\alpha' t}$$

$$= \int_{0<z<1} dz\Big(\partial_z\big(\frac{z^{\alpha' s-1}}{\alpha' s-1}(1-z)^{\alpha' t}\big) - \frac{z^{\alpha' s-1}}{\alpha' s-1}\alpha' t(1-z)^{\alpha' t-1}\Big)$$

$$= \int_{0<z<1} \frac{\alpha' t}{1-\alpha' s}\frac{dz}{z(1-z)} z^{\alpha' s}(1-z)^{\alpha' t-1}$$

- We say $\displaystyle \frac{1}{z_{12}z_{21}z_{34}z_{43}} \overset{IBP}{\cong} \frac{\alpha' t}{1-\alpha' s}\frac{1}{z_{12}z_{23}z_{34}z_{41}}$

- We have a generic method to eliminate any subcycle.

String from QFT: double copy

- We can use logarithmic form as CHY half integrand. Its double copy with Z-theory gives string amplitude

$$A^{string} = \left(\int d\mu_n^{CHY} \, I'_n \, \mathrm{PT}(\rho) \right) \otimes Z_{12\cdots n}(\pi)$$

$$with \quad Z_{12\cdots n}(\pi) := \int_{z_1 < \cdots < z_n} \frac{d^n z}{SL(2)} \prod_{i<j} z_{i,j}^{\alpha' s_{i,j}} \, \mathrm{PT}(\pi)$$

- The prime example is open superstring, where all tachyon poles cancel.

$$I_n^{type\ I} \overset{IBP}{\cong} I_n^{\prime type\ I} \overset{SE}{\cong} \mathrm{Pf}'\Psi_n(\{k_i, \epsilon_i, z_i\})$$

Proof

- Any logarithmic form can be expanded in terms of PTs

$$I_n \overset{IBP}{\cong} I'_n = \sum_{\pi} N_\pi \mathrm{PT}(\pi)$$

- String amplitude is expanded onto Z-theories,

$$A^{string} = \sum_{\pi} N_\pi \underbrace{\int_{z_1<\cdots<z_n} \frac{d^n z}{SL(2)} \prod_{i<j} z_{i,j}^{\alpha' s_{i,j}} \mathrm{PT}(\pi)}_{Z_{12\cdots n}(\pi)}$$

- Inserting $1 = \sum_{\gamma \in S_{n-3}} \delta_{\gamma\pi}$ [Cachazo, He,Yuan, 1306] , we have

$$A^{string} = \sum_{\pi} \sum_{\rho} \int d\mu_n^{CHY} \underbrace{\sum_{\gamma} N_\gamma \mathrm{PT}(\gamma)}_{I'_n} \mathrm{PT}(\rho) S[\rho\,|\,\pi]\; Z_{12\cdots n}(\pi)$$

Bosonic and heterotic string

- We can do the same for bosonic open strings. Tachyon pole can't be cancelled. Similarly for heterotic strings.

- Heterotic string : One sector with SUSY. The other without SUSY, which behaves like (compactified) bosonic strings.

$$\mathscr{V}_i^{gauge}(z,\bar{z}) = \mathscr{J}^{a_i}(z)\, V_i^{SUSY}(\bar{z})\, e^{k_i\cdot X(z,\bar{z})}$$
$$\mathscr{V}_i^{gravity}(z,\bar{z}) = -\frac{1}{\alpha'}\epsilon_i^{\mu}\partial X_{\mu}(z)\, V_i^{SUSY}(\bar{z})\, e^{k_i\cdot X(z,\bar{z})}$$

Heterotic string integrand

- We first consider the integrand with n gravitons and 2 gluons in a trace

- One graviton: $I_{1+2} = -\sum_{i\neq 1} \frac{\epsilon_1 \cdot k_i}{z_{1i}} \frac{1}{z_{23}^2}$

- Two graviton: $$I_{2+2} = (\sum_{i\neq 1} \frac{\epsilon_1 \cdot k_i}{z_{1i}} \sum_{i\neq 2} \frac{\epsilon_2 \cdot k_i}{z_{2i}} + \frac{\epsilon_1 \cdot \epsilon_2}{\alpha' z_{12}^2}) \frac{1}{z_{34}^2}$$

- Performing IBP

$$\frac{1}{z_{12}z_{21}z_{34}z_{43}} \overset{IBP}{\cong} \frac{\alpha' t}{1-\alpha' s} \frac{1}{z_{12}z_{23}z_{34}z_{41}}$$

- Before presenting general results, let's recall the CHY formula of Einstein-Yang-Mills

EYM

- n gravtions plus m gluons in a trace [Cachazo, He,Yuan, 1412]:

$$I^{EYM}_{n+m} = \mathrm{PT}(n+1,\cdots,n+m)\mathrm{Pf}\,\Psi_n$$

$$\begin{array}{c|cccc|cccc}
 & k_1 & \cdots & & k_n & \epsilon_1 & \cdots & & \epsilon_n \\
\hline
k_1 & 0 & & & \frac{k_i\cdot k_j}{\sigma_{ij}} & -C_{ii} & & & \frac{k_i\cdot\epsilon_j}{\sigma_{ij}} \\
\vdots & & 0 & & & & \ddots & & \\
k_n & \frac{k_j\cdot k_i}{\sigma_{ji}} & & & 0 & \frac{k_j\cdot\epsilon_i}{\sigma_{ji}} & & & -C_{jj} \\
\hline
\epsilon_1 & C_{ii} & & & \frac{\epsilon_i\cdot k_j}{\sigma_{ij}} & 0 & & & \frac{\epsilon_i\cdot\epsilon_j}{\sigma_{ij}} \\
\vdots & & \ddots & & & & 0 & & \\
\epsilon_n & \frac{\epsilon_j\cdot k_i}{\sigma_{ji}} & & & C_{jj} & \frac{\epsilon_j\cdot\epsilon_i}{\sigma_{ji}} & & & 0
\end{array}$$

with $C_{ii} := -\sum_{j\neq i} C_{ij} = -\sum_{j\neq i} \frac{\epsilon_i\cdot k_j}{\sigma_{ij}}$

- Cycle expansion

$$I^{EYM}_{n+m} = \mathrm{PT}(n+1,\cdots,n+m)\sum_{p\in S_n}\Psi_I\Psi_J\cdots\Psi_K$$

$$with\quad \Psi_{(i)} = \sum_{j\neq i}\frac{\epsilon_i\cdot k_j}{\sigma_{ij}} \qquad \Psi_{(i\cdots j)} = -\frac{1}{2}tr(f_i\cdots f_j)\mathrm{PT}(i\cdots j) \qquad f_i^{\mu\nu} = k_i^\mu\epsilon_i^\nu - \epsilon_i^\mu k_i^\nu$$

- For example

$$I^{EYM}_{2+3} = (\Psi_{(1)}\Psi_{(2)} + \Psi_{(12)})\mathrm{PT}(345)$$

$$I^{EYM}_{3+2} = (\Psi_{(1)}\Psi_{(2)}\Psi_{(3)} + \Psi_{(12)}\Psi_{(3)} + \Psi_{(13)}\Psi_{(2)} + \Psi_{(23)}\Psi_{(1)} + \Psi_{(123)} + \Psi_{(132)})\mathrm{PT}(45)$$

General result

- For heterotic string, all we need to do is to replace tr with $\tilde{tr}$, which depends on α' .

- All tachyon poles are contained in $\tilde{tr}$. We have a closed formula for most general cases. Some example:

$$\tilde{tr}(f_1 f_2) = \frac{tr(f_1 f_2)}{1 - \alpha' s_{12}}$$

$$\tilde{tr}(f_1 f_2 f_3) = \frac{tr(f_1 f_2 f_3) + \left(\frac{1}{2}\alpha' \tilde{tr}(f_1 f_2) k_2 \cdot f_3 \cdot k_1 + 2\ cyc\right)}{1 - \alpha' s_{123}}$$

$$\tilde{tr}(f_1 f_2 f_3 f_4) = \frac{tr(f_1 f_2 f_3 f_4) + \left(\alpha' \tilde{tr}(f_1 f_2 f_3) k_3 \cdot f_4 \cdot k_1 + \frac{1}{2}\alpha' \tilde{tr}(f_1 f_2) k_2 \cdot f_3 f_4 \cdot k_1 + 3\ cyc\right)}{1 - \alpha' s_{1234}}$$

$$+ \frac{\frac{1}{4}\alpha'^2 \tilde{tr}(f_1 f_2) s_{23} \tilde{tr}(f_3 f_4) s_{41} + 1\ cyc}{1 - \alpha' s_{1234}}$$

Summary

- We show $open\ string = QFT \otimes Z{-}theory$ and a natural formula for QFT after using IBP.

- We obtained a closed formula for heterotic string (also for multi trace and pure gravity). This has very interesting limit: at $\alpha' \to 0$, EYM; at $\alpha' \to \infty$, conformal gravity.

- Help to understand the integrated string amplitude, cancellation of tachyon pole in heterotic string amplitude

- Is there a string theory for any CHY integrand?

Thank you!