On dimensional reduction of N=1 Lagrangians for Argyres-Douglas theories

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Plan of the talk

- ❑ Argyres-Douglas (AD) theories and N=1 Lagrangians
- ❑ 3d reduction of the N=1 Lagrangians
- ❑ Summary and conclusion

Argyres-Douglas (AD) theories

- 4d N=2 Superconformal theories (SCFTs)

- Describe the low energy physics at special loci on the Coulomb branch of generic 4d N=2 theories [Argyres-Douglas `95] [Argyres-Plesser-Seiberg-Witten '95]

- At these special loci, magnetic monopoles and electrically charged particles simultaneously become massless

AD theories are Non-Lagrangian

- Impossible to write a manifestly Lorentz invariant Lagrangian with electrons as well as monopoles as elementary degrees of freedom

- Therefore AD theories are inherently non-perturbative

- Their Coulomb phase is well understood; much less is known about their conformal phase

- How to compute their partitions function on $S^4, \ S^3 \times S^1$ etc ?

- Partial answer : N=1 Lagrangians for (A_1, A_n) and (A_1, D_n) AD theories
  [Maruyoshi-Song`16] [PA-Maruyoshi-Song`16] [PA-Sciarappa-Song `17]

- Can use these to compute RG protected quantities such as the superconformal index

Generic features of these N=1 Lagrangians

- Many gauge invariant operators of the UV Lagrangian decouple as free fields in the IR

- The IR central charges a and c match exactly with the corresponding AD theories

- A subset of chiral operators give the Coulomb branch of the AD theory

3d reduction of N=1 Lagrangians

- Decoupling of an operator $\mathcal{O}$ can be automatically accounted for by including a flipping field $\beta_{\mathcal{O}}$

$$\delta W = \beta_{\mathcal{O}}\, \mathcal{O}$$

- Upon dim. red. to 3d, R-charges have to be fixed via Z- extremization

- Generically, extremization point is different if the flipping field β is included or not

- If $\beta_{\mathcal{O}}$ is not included, $\mathcal{O}$ may or may not decouple upon dimensional reduction

- Proposal: the flipping fields are necessary for correct dimensional reduction [Benvenuti-Giacomelli `17]
- (A_1, A_{2n-1}) Lagrangians : no SUSY enhancement in 3d without flipping fields [Benvenuti-Giacomelli `17]
- flow to the mirror of (A_1, A_{2n-1}) AD theory upon including the flipping fields [Benvenuti-Giacomelli `17]
- Let's study the expected necessity of including flipping fields further

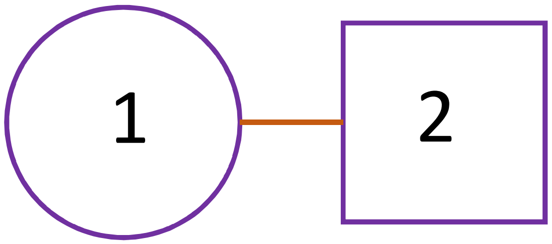
Lagrangian for the (A_1, D_3) AD theory

fields	$SU(2)_{\text{color}}$	$SO(3)_b$	$U(1)_T$	$U(1)_q$	$U(1)_R$	$U(1)_T - \frac{3}{2}U(1)_q$
q_1	$\mathbf{2}$	$\mathbf{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$1 - \frac{r_\phi}{2}$	$-\frac{1}{2}$
q_2	$\mathbf{2}$	$\mathbf{1}$	$-\frac{1}{4}$	$\frac{3}{2}$	$1 - \frac{r_{M_3}+r_\phi}{2}$	$-\frac{5}{2}$
ϕ	adj	$\mathbf{1}$	$-\frac{1}{2}$	-1	r_ϕ	1
M_3	$\mathbf{1}$	$\mathbf{1}$	1	-2	r_{M_3}	4
β	$\mathbf{1}$	$\mathbf{1}$	1	2	$2 - 2r_\phi$	-2

$$W = Tr\, q_1 \phi q_1 + M_3\, Tr\, q_2 \phi q_2 + \beta\, Tr \phi^2$$

In 4d, non-anomalous R-charge : $r_{M_3} - 4\, r_\phi = 0$

- The mirror of the (A_1, D_3) theory : $T[SU(2)]$ theory (self-mirror)



- Thus we expect the above Lagrangian to flow to the $T[SU(2)]$ theory upon 3d reduction

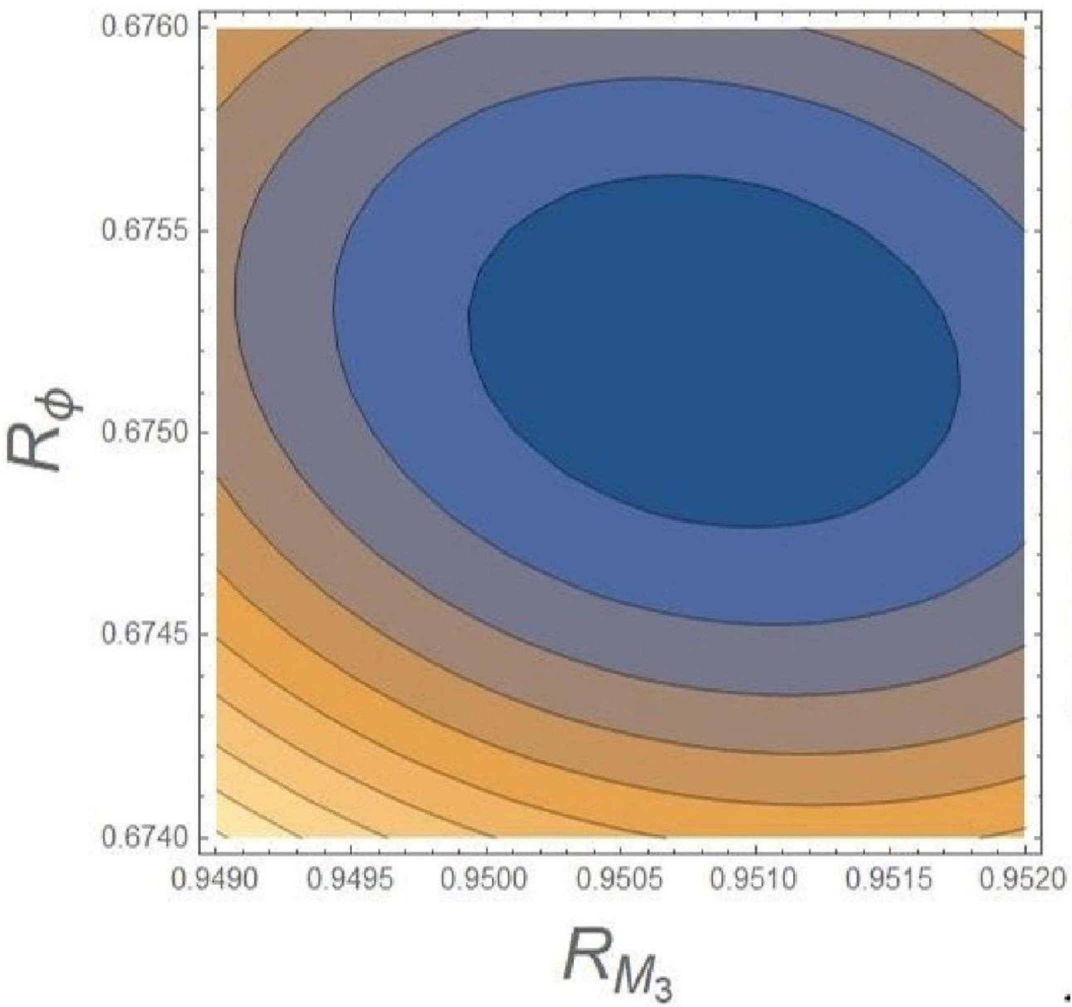
- Upon Z – extremization:

$$R_{M_3} \sim 0.95\,, R_\phi \sim 0.67$$

- The $SU(2)$ monopole operator decouples

- Remove the monopole operator contribution and re-extremize

$$R_{M_3} \sim 0.92\,, R_\phi \sim 0.68$$

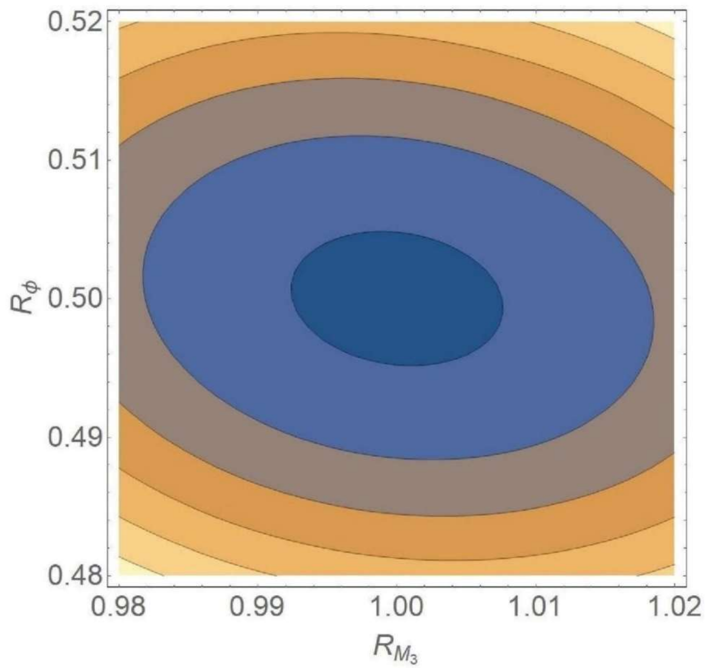


- No SUSY enhancement to N=4 !

- How can we fix this?

- The short answer: remove the flipping field β from the Lagrangian

Z – extremization without the flipping field



The $SU(2)$ monopole operator has dimension ½ and decouples as a free field

- The superconformal Index also matches with $T[SU(2)]$

- There is a systematic way of understanding why adding the flipping field β spoils the expected match.

- The $T[SU(2)]$ theory has an $SU(2)_T \times SU(2)_b$, global symmetry

- The 3d N=4 current multiplets of $SU(2)_T \times SU(2)_b$ contain chiral scalar operators called the momemt map

- In (A_1, D_3) Lagrangian, the $SU(2)_b$ moment map corresponds to

$$q_1^i q_2 \quad i = 1,2,3$$

- The $SU(2)_T$ moment map is generated by

$$M_3,\ \{\mathfrak{m}\phi\} \text{ and } Tr\phi^2$$

- Including β in the Lagrangian, removes $Tr\phi^2$ from the chiral ring
- This stops the (A_1, D_3) Lagrangian from flowing to $T[SU(2)]$
- (A_1, D_3) contradicts the prescription to include “flipping fields”

Summary and Conclusion

- Argyres – Douglas theories are simplest N=2 SCFTs
- Their non-Lagrangianity poses a major hurdle in understanding their conformal phase
- We have been successful in constructing N=1 Lagrangians whose IR fixed points describe AD theories
- Can use these to compute RG protected quantities such as the superconformal index

- It is also interesting to study dimensional reduction of these Lagrangians

- For (A_1, A_{2n-1}) type cases, correct dimensional reduction requires flipping fields

- However, including the flipping field does not always work

- (A_1, D_3) Lagrangian is a counter example to this expected necessity

- Is there a uniform way to understand when to include the flipping fields ?

- Need to understand the caveats which arise due non-commutation of the RG flow and dimensional reduction

THANK YOU!