Various Quantum Properties

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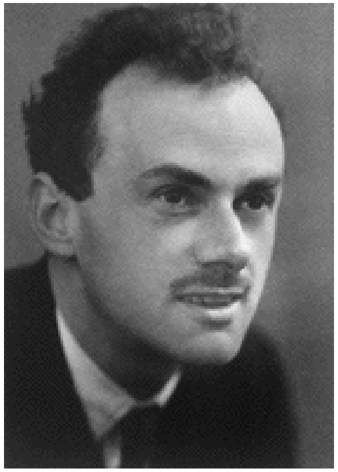
Nonclassical properties of quantum states

- Bell-type nonlocality
- Nonclassicality based on quasi-probability functions
- Macroscopic superpositions – ‘quantum macroscopicity’
- Coherence based on off-diagonal elements of the density matrix
- Entanglement, discord, steerability…

Quantum superposition principle

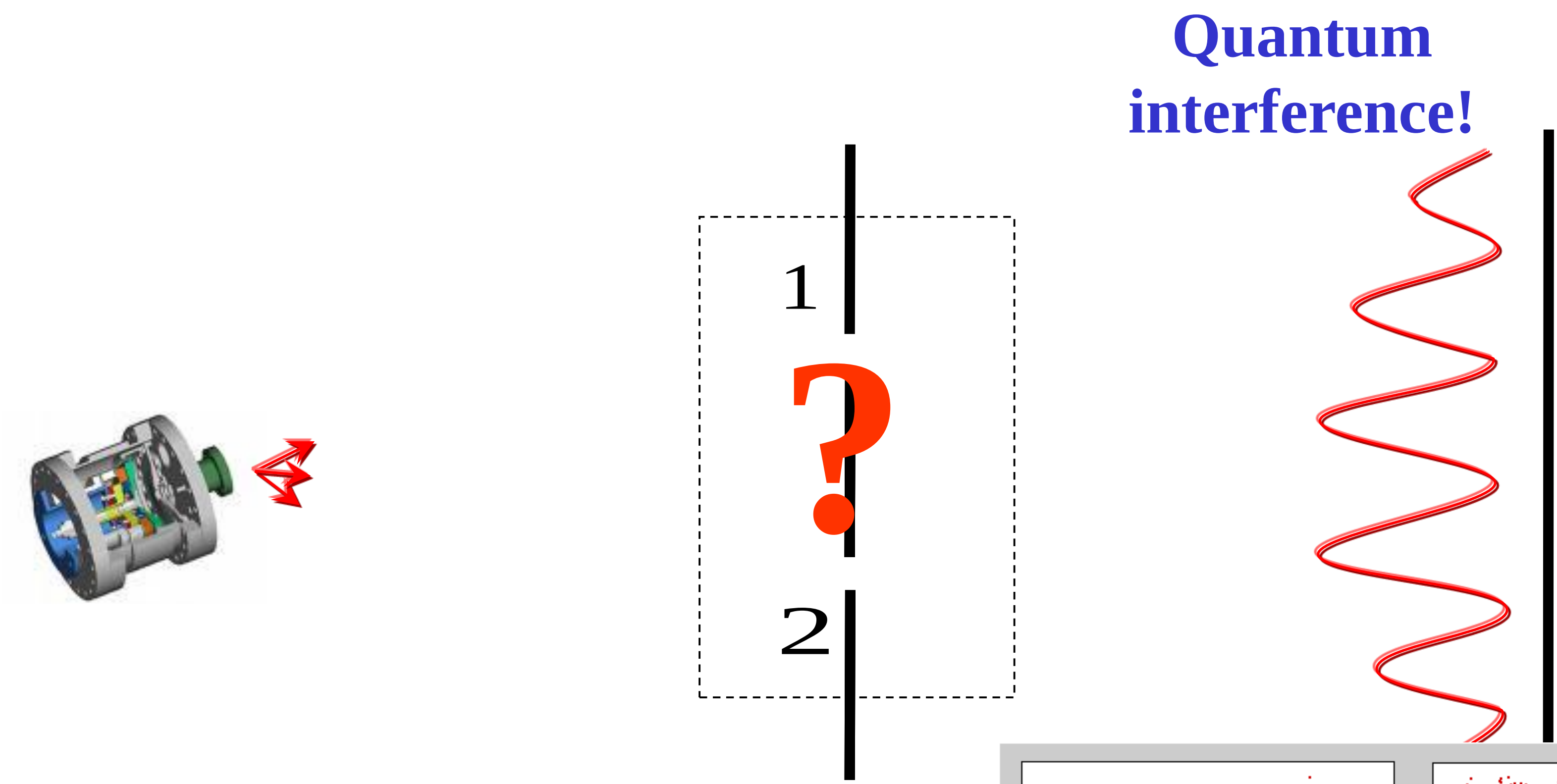
The principle of quantum superposition: Any two states may be superposed to give a new state.

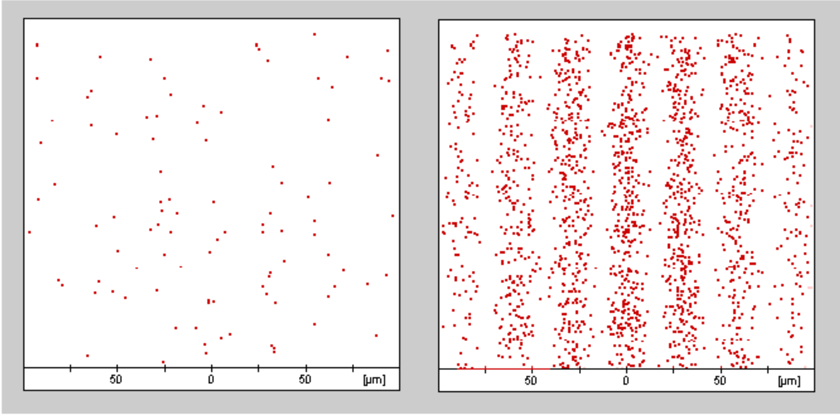


P. A. M. Dirac

"The superposition principle lies at the very heart of quantum mechanics."

Double split experiment with electrons





- Researchers led by Pier Giorgio Merli, the University of Milan (1974).
- Tonomura *et al*., Hitachi in Japan (1989).

By Charles Addams

Quantum entanglement

- Quantum superposition of two (or more than two) physical systems (e.g. for systems 1 and 2).

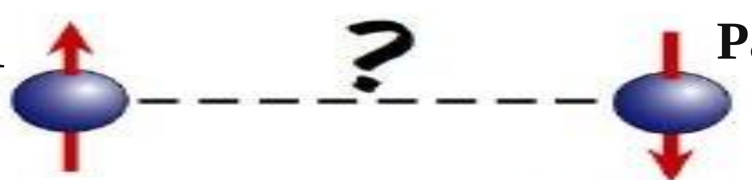
$$\left|\Psi\right\rangle = \left|\uparrow\right\rangle_1 \left|\downarrow\right\rangle_2 + \left|\downarrow\right\rangle_1 \left|\uparrow\right\rangle_2$$

- Such a state cannot be represented by a product of two separate states.

$$\left|\Psi\right\rangle \neq \left|A\right\rangle_1 \left|B\right\rangle_2$$

- We then say that the physical systems 1 and 2 are "entangled."
- "Spooky action at a distance" – Albert Einstein.

Particle 1



Particle 2

Observed here

Quantum entanglement

- Quantum superposition of two (or more than two) physical systems (e.g. for systems 1 and 2).

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Particle 1

Particle 2

Observed here $\Rightarrow$ Determined over there!

Quantum mechanics vs local realism

- Einstein-Podolsky-Rosen (EPR) paradox (1935): Is quantum mechanics a complete physical theory?
  [A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).]

- Local realism: ***locality* + *realism***

  - ***Locality***: "*Distant objects cannot have direct influence on one another.*"

  - ***Realism***: "*The results of observations are a consequence of properties carried by physical systems, i.e., an external reality exists independent of observation.*"

Violation of local realism

- Bell's inequality: an inequality which must be obeyed by any local-realistic theory [J. S. Bell, Physics 1, 195 (1964)].
- Bell's inequality is violated by quantum mechanics, *i.e.*, quantum mechanics is ***inconsistent*** with local realism.

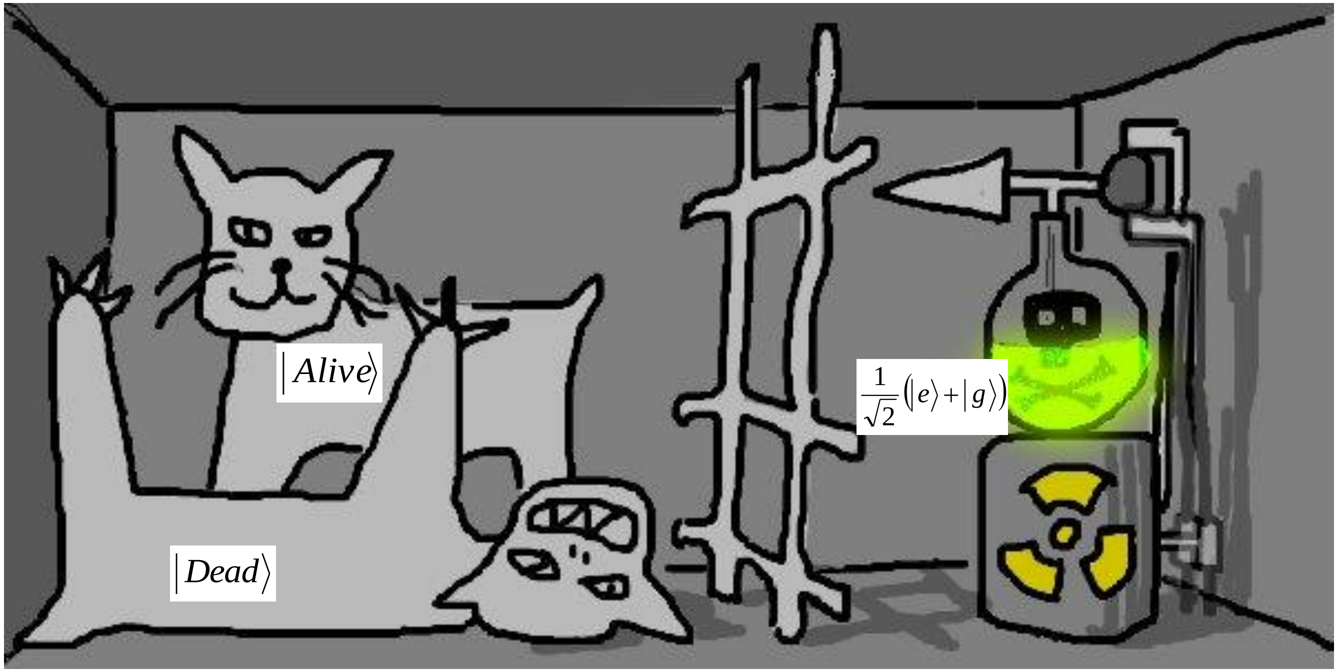
$$|C(a,b)+C(a',b)+C(a,b')-C(a',b')| \leq 2$$

- Loophole-free Bell violations were recently demonstrated [B. Hensen *et al.*, Nature 2005 *etc.*]

John S. Bell

Schrödinger's cat paradox

E. Schrödinger, *Naturwissenschaftern*. 23 (1935)

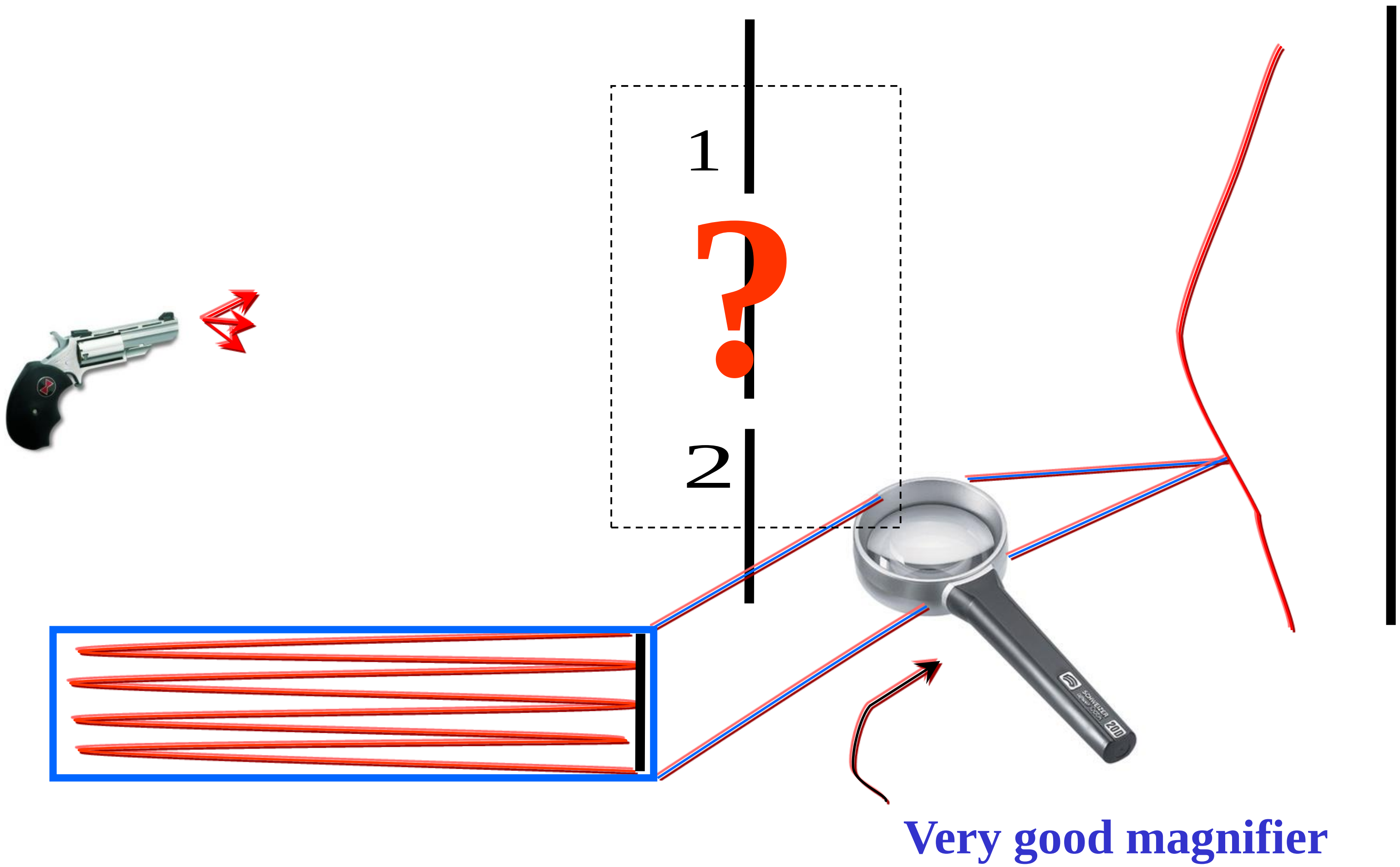


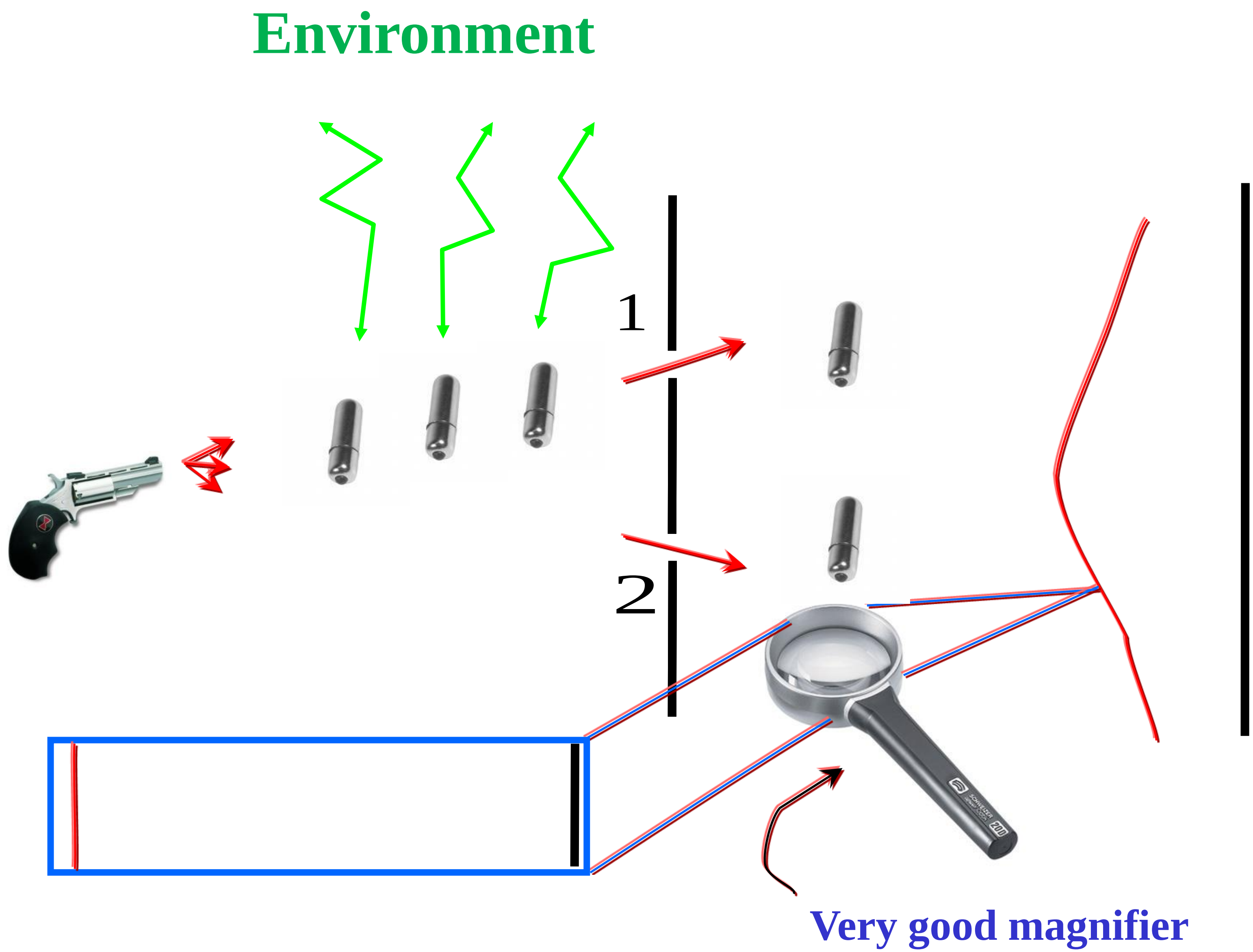
The Brussels Journal (29 October 2007)

$$\frac{1}{\sqrt{2}}(|e\rangle + |g\rangle)|Alive\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|e\rangle|Alive\rangle + |g\rangle|Dead\rangle\right)$$

Quantum superposition or entanglement of macroscopically distinguishable states

Classical gun – if we are really living in a *quantum* universe…




Environment
1
2
Very good magnifier

Decoherence

[W. H. Zurek, Rev. Mod. Phys. 75, 715 (2003) and references therein]

$$\boxed{\left|tot\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\psi\right\rangle + \left|\varphi\right\rangle\right)_a \left|E\right\rangle_b} \quad \Longrightarrow \quad \boxed{\left|tot'\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\psi'\right\rangle_a \left|E'\right\rangle_b + \left|\varphi'\right\rangle_a \left|E''\right\rangle_b\right)}$$

$$\boxed{\rho'_a = \mathrm{Tr}_b[\left|tot'\right\rangle\left\langle tot'\right|] = \frac{1}{2}\left(\left|\psi'\right\rangle\left\langle\psi'\right| + \left|\varphi'\right\rangle\left\langle\varphi'\right|\right)_a}$$

- ***It is difficult to isolate macroscopic objects from their environment.***
- ***Macroscopic quantum states decohere (lose their quantum properties) faster than microscopic states.***

Some examples with microscopic and macroscopic systems (Tegmark, 1993)

3 Decoherence Through Interaction with the Environment

Table 3.2. Some values of the localization rate Λ given by Tegmark (1993). T values given for the case of a dust particle agree well with those of earlier wc (Table 3.1).

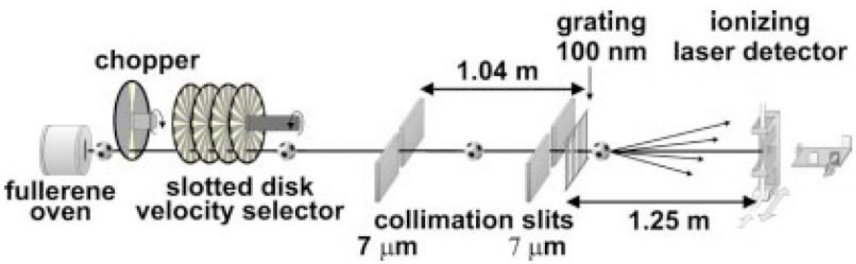
	Free electron	10^{-3} cm dust particle	Bowling ball $\boxed{cm^{-2}s^{-1}}$
300 K air at 1 atm pressure	10^{31}	10^{37}	10^{45}
300 K air in lab vacuum	10^{18}	10^{23}	10^{31}
Sunlight (on earth)	10^{1}	10^{20}	10^{28}
300 K photons	10^{0}	10^{19}	10^{27}
Background radioactivity	10^{-4}	10^{15}	10^{23}
Quantum gravity	10^{-25}	10^{10}	10^{22}
GRW effect	10^{-7}	10^{9}	10^{21}
Cosmic background radiation	10^{-10}	10^{6}	10^{17}
Solar neutrinos	10^{-15}	10^{1}	10^{13}

$$\boxed{t_{dec} \approx \frac{1}{\Lambda\,|x - x'|^2}}$$

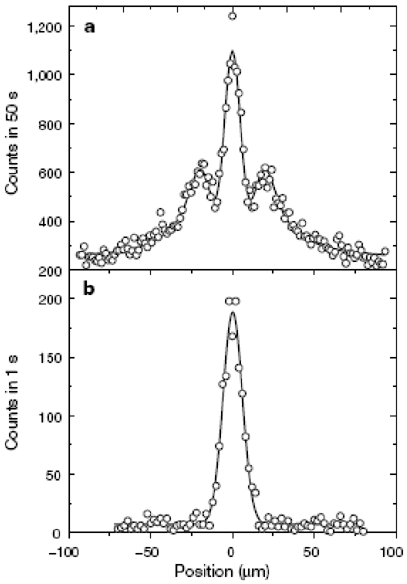
A tiny dust particle of the size of a virus (10^{-5}cm), scattering of air molecules leads to a decoherence time scale of the order of 10^{-13} s.

Macroscopic quantum interference

- Superconducting quantum interference device (SQUID)
  [J. R. Friedman *et al*., *Nature* 406, 43 (2000)]
- Interference with C60 molecules
  [M. Arndt *et al*., *Nature* 401, 680 (1999)]



- *"Schrödinger cat" states of light*
  [A. Ourjoumtsev *et al*., *Nature* 448, 784 (2007)]

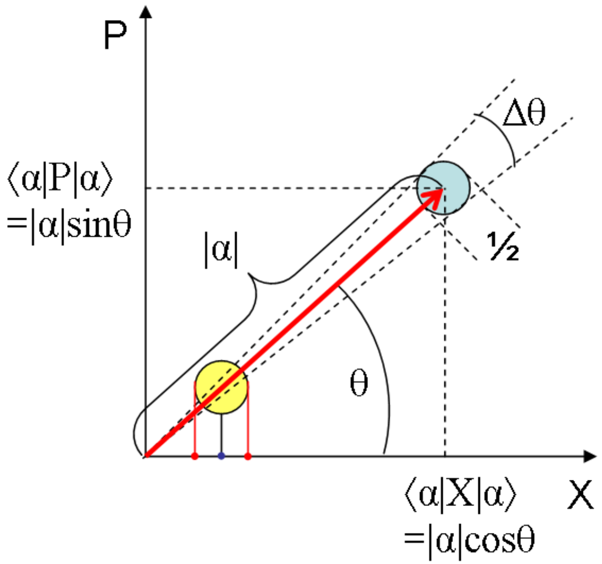


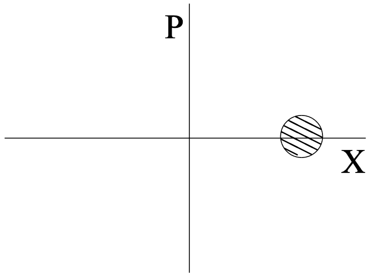
(Interference with C60 molecules, *Nature* 401, 680, 1999)

Glauber–Sudarshan *P* representation

- **Coherent state:** $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$

$$\boxed{\begin{aligned}\hat{X} &= \hat{a} + \hat{a}^{+} \\ \hat{P} &= -i(\hat{a} - \hat{a}^{+})\end{aligned}}$$





✓ Coherent states are most classical among all pure states: an analogy of ***classical*** point-particles in the quantum phase space [Schrödinger, *Naturwissenschaften* 14 (1926)] and most robust against decoherence.

Glauber–Sudarshan *P* representation

- **Coherent state:** $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

$$\hat{\rho} = \int P(\alpha) |\alpha\rangle\langle\alpha| \, d^2\alpha, \qquad d^2\alpha \equiv d\,\mathrm{Re}(\alpha)\, d\,\mathrm{Im}(\alpha).$$

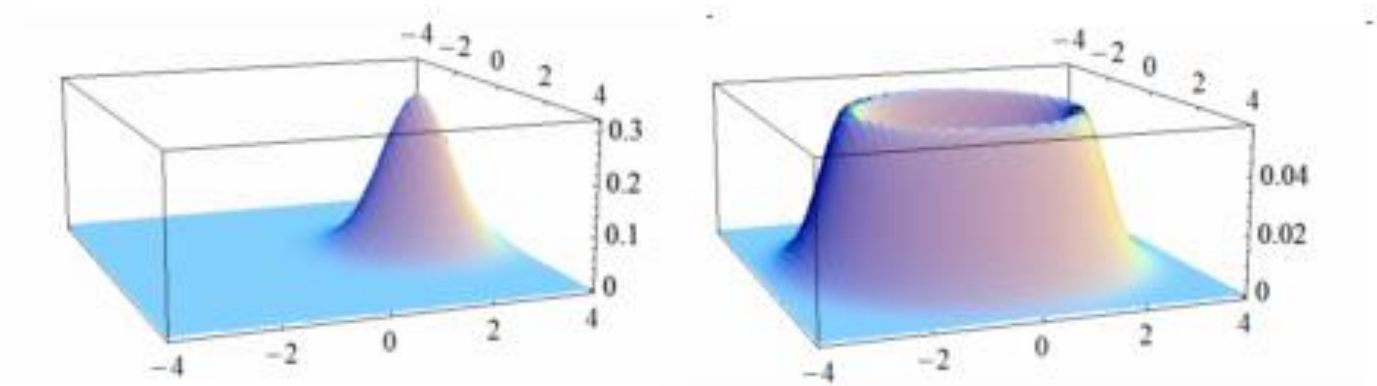
$$\chi_N(\beta) = \mathrm{tr}(\hat{\rho} \cdot e^{i\beta \cdot \hat{a}^\dagger} e^{i\beta^* \cdot \hat{a}})$$

$$P(\alpha) = \frac{1}{\pi^2} \int \chi_N(\beta) e^{-\beta\alpha^* + \beta^*\alpha} \, d^2\beta.$$

Q function (Husimi-Q distribution)

$$Q(\alpha) = \frac{1}{\pi}\langle\alpha|\rho|\alpha\rangle = \frac{1}{\pi}\mathrm{Tr}(|0\rangle\langle 0|\rho(-\alpha)) ,$$

$$\rho(-\alpha) = D(-\alpha)\rho D(\alpha) .$$



(Left) Five-photon coherent state and (Right) Five-photon Fock state

Picture from Haroche group's website

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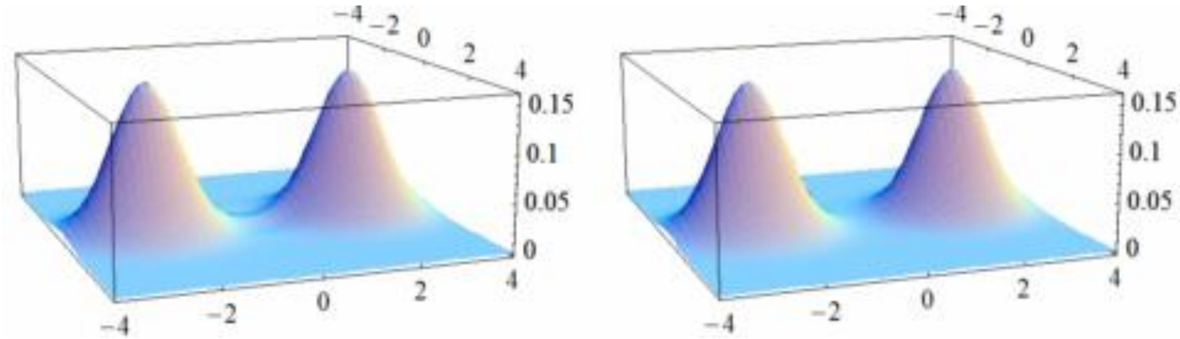


Limitation of Q function

(Left) Q functions for a 5 photon phase Schrödinger cat and (Right) a statistical mixutre of two five photons coherent states with opposite phases.

Picture from Haroche group's website

Wigner function

$$W(\beta) = \frac{1}{\pi^2} \int \exp[\xi^* \beta - \xi \beta^*] C(\xi) d^2 \xi$$

$$\boxed{C(\xi) = \mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]}$$

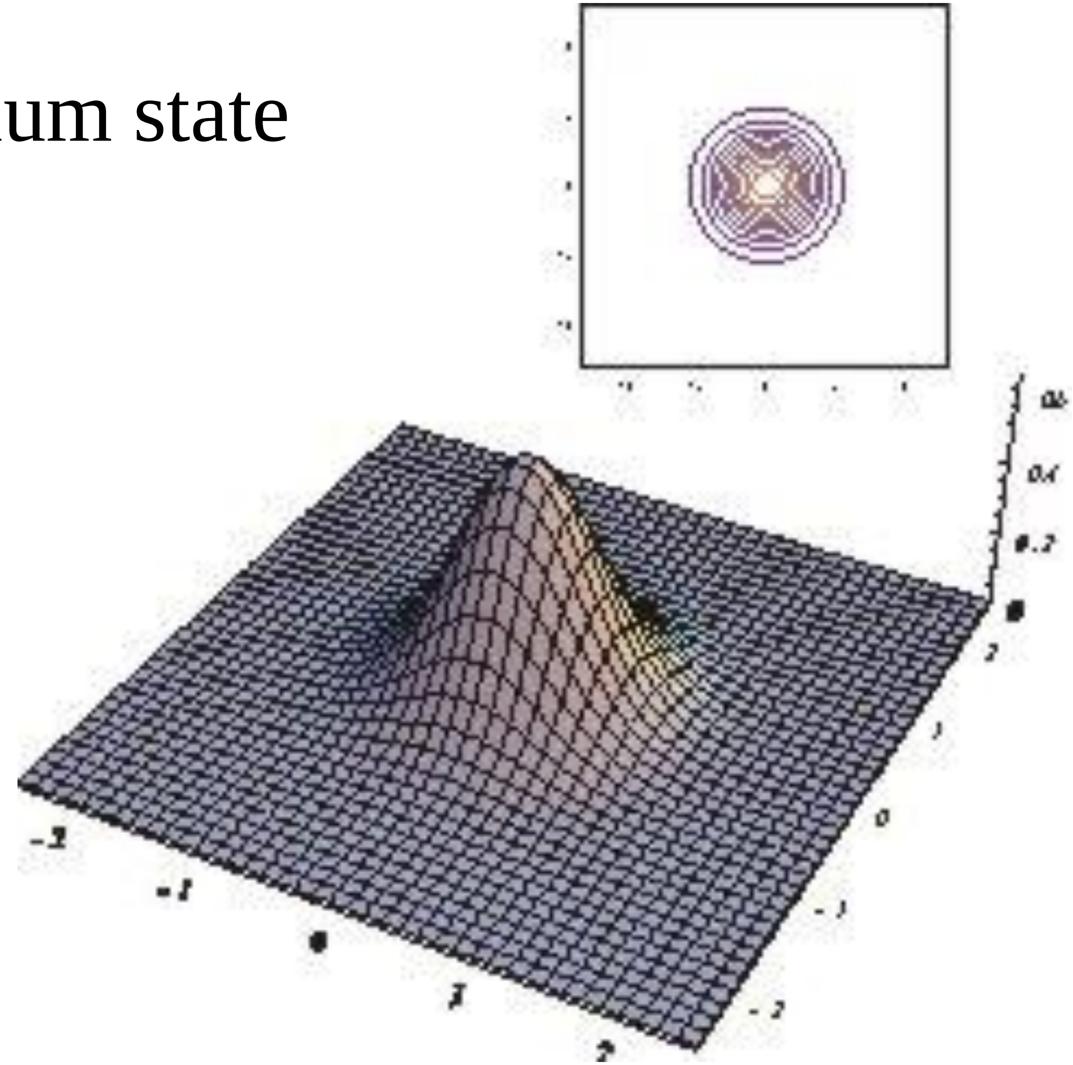
$$\boxed{\hat{D}(\xi) = e^{\xi \hat{a}^+ - \xi^* \hat{a}}}$$

$$\boxed{\beta = x + ip}$$

- The Wigner function is a quasi-probability distribution: an analogy of the classical probability distribution in the quantum phase space.
- Negative values in the Wigner function are a definite sign of non-classicality.

Wigner function

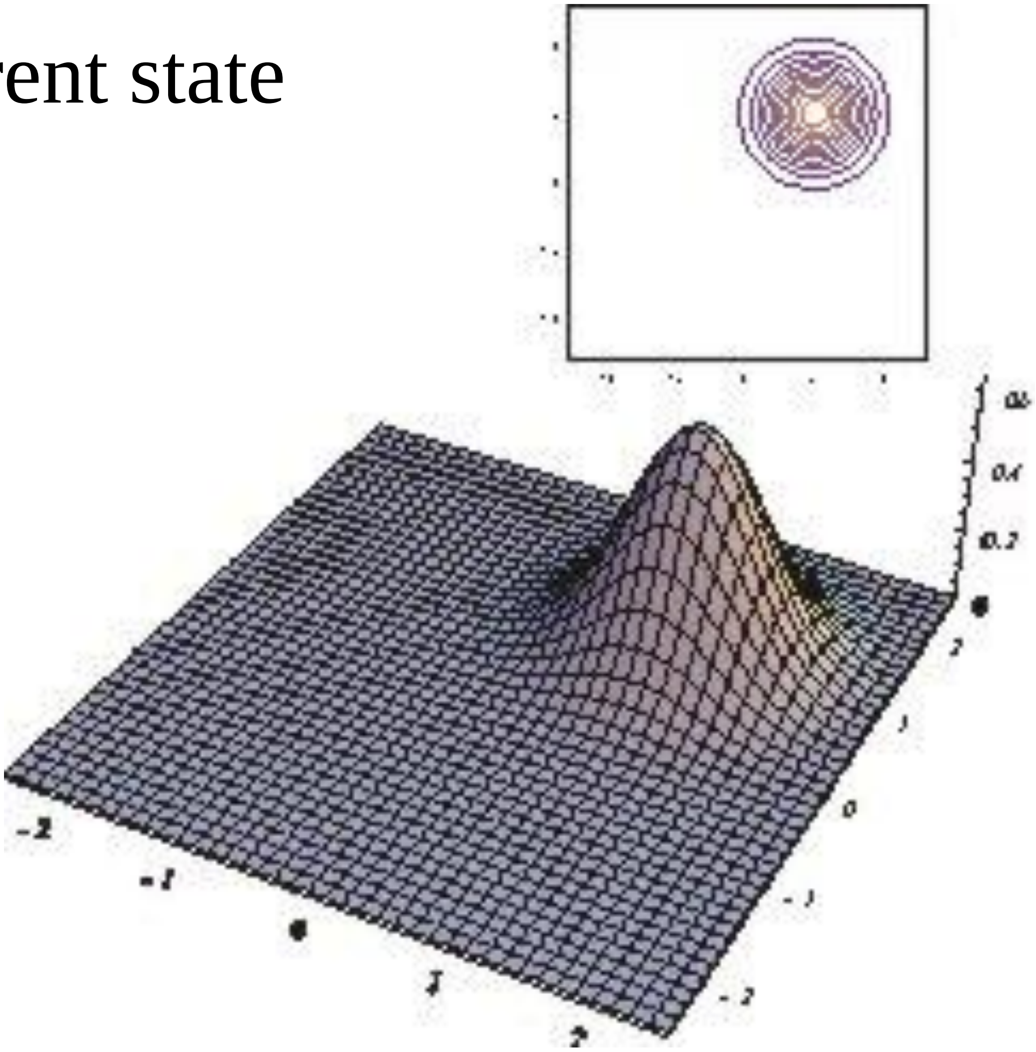
- Vacuum state



Picture from Lvovsky group’s website

Wigner function

- Coherent state

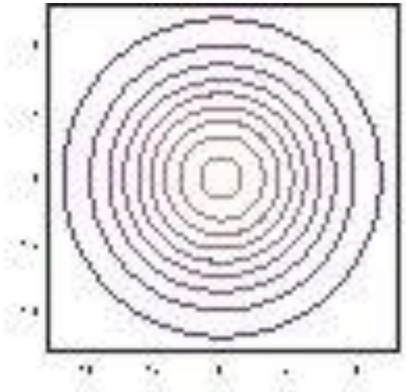


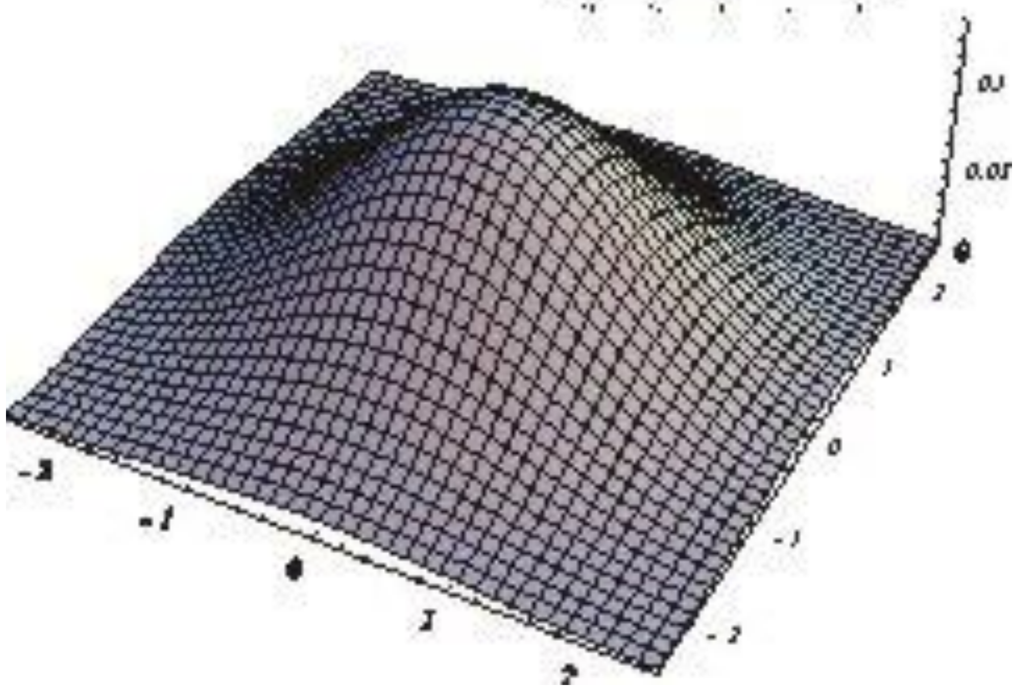
Picture from Lvovsky group's website

Wigner function

- Thermal state

$$\hat{\rho}_{th} = \frac{1}{\langle n \rangle + 1} \sum_{n=0}^{\infty} \left(\frac{\langle n \rangle}{\langle n \rangle + 1} \right)^n |n\rangle\langle n|$$

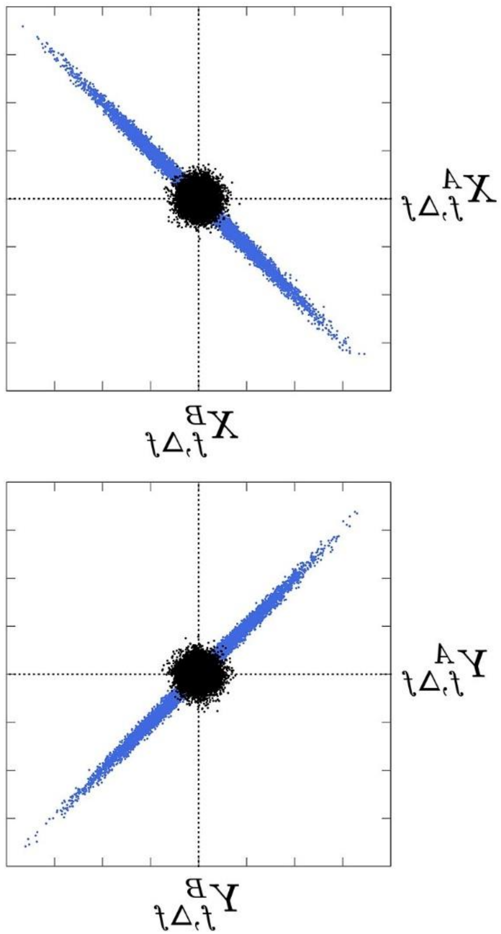


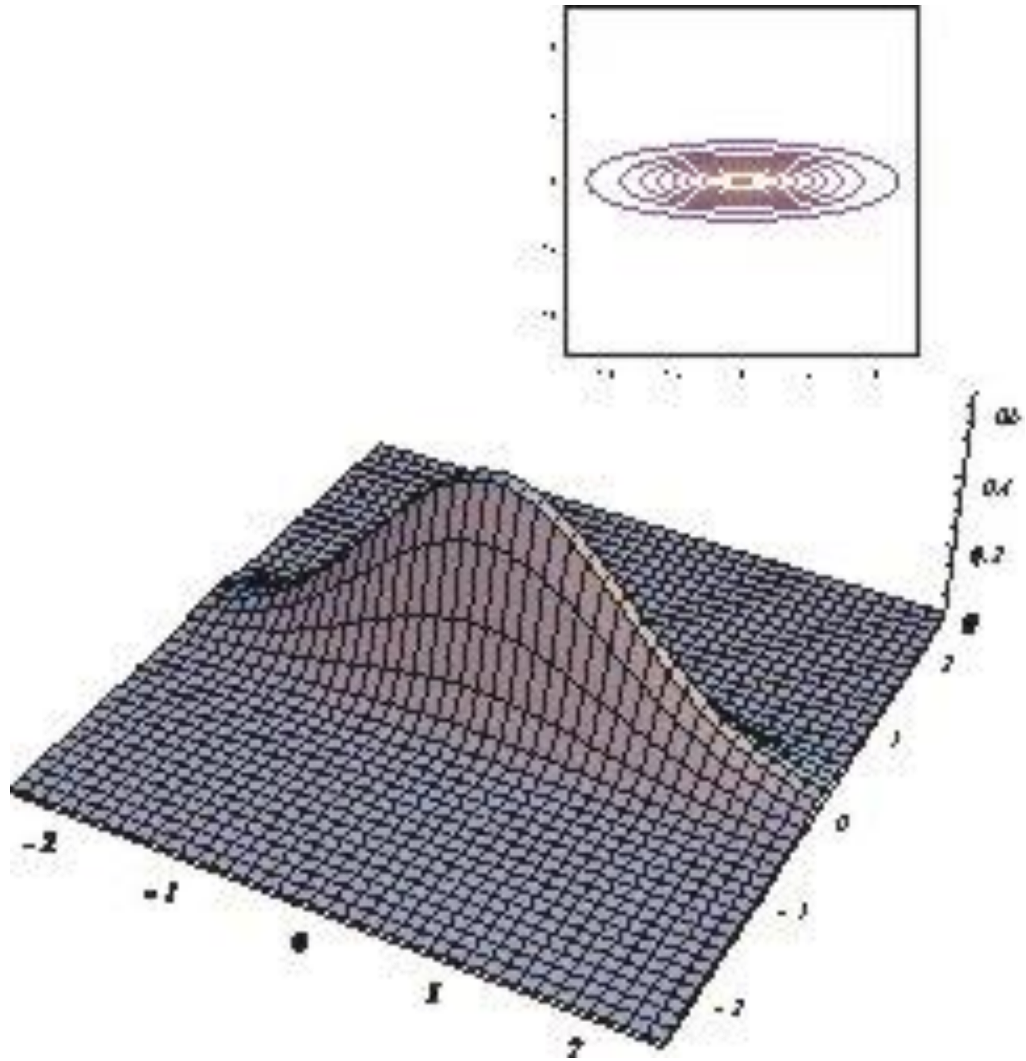


Picture from Lvovsky group's website

Wigner function

- Squeezed state

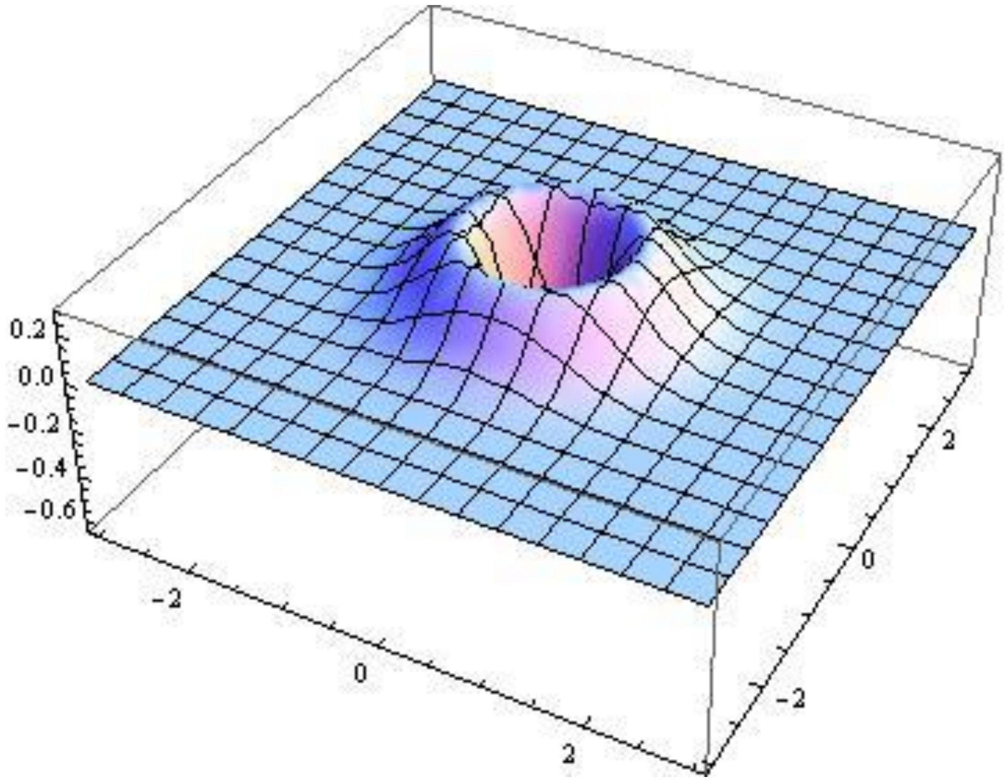




Picture from Lvovsky group's website

Wigner function

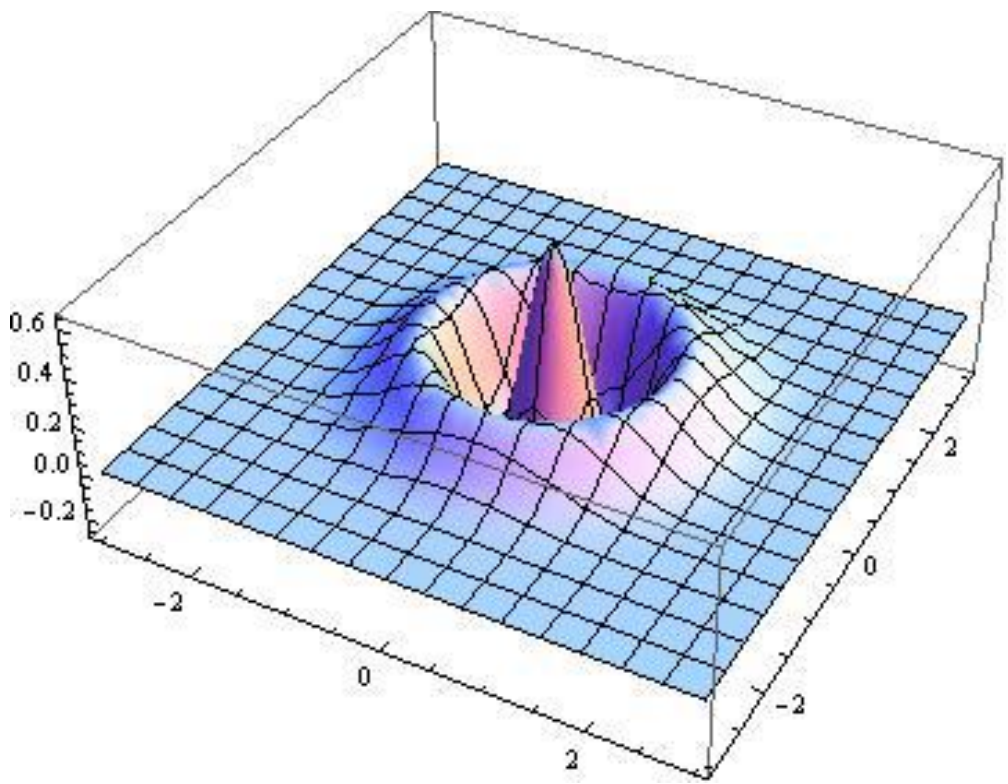
- n=1 Fock state



Picture from Tzu-Chieh Wei's website

Wigner function

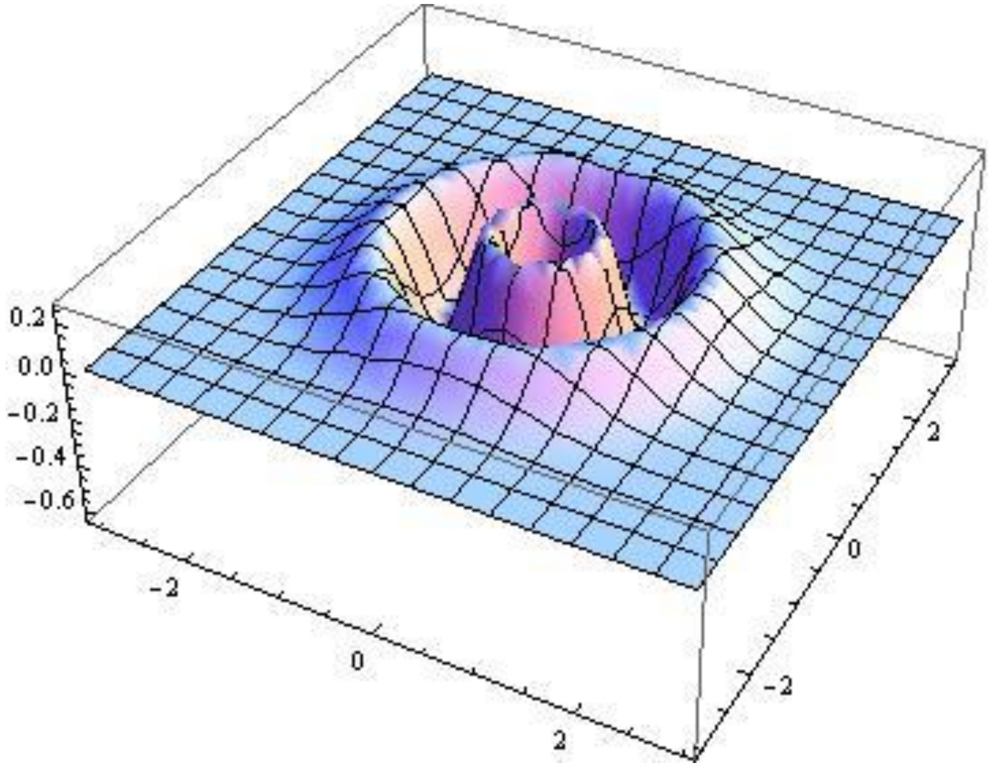
- n=2 Fock state



Picture from Tzu-Chieh Wei's website

Wigner function

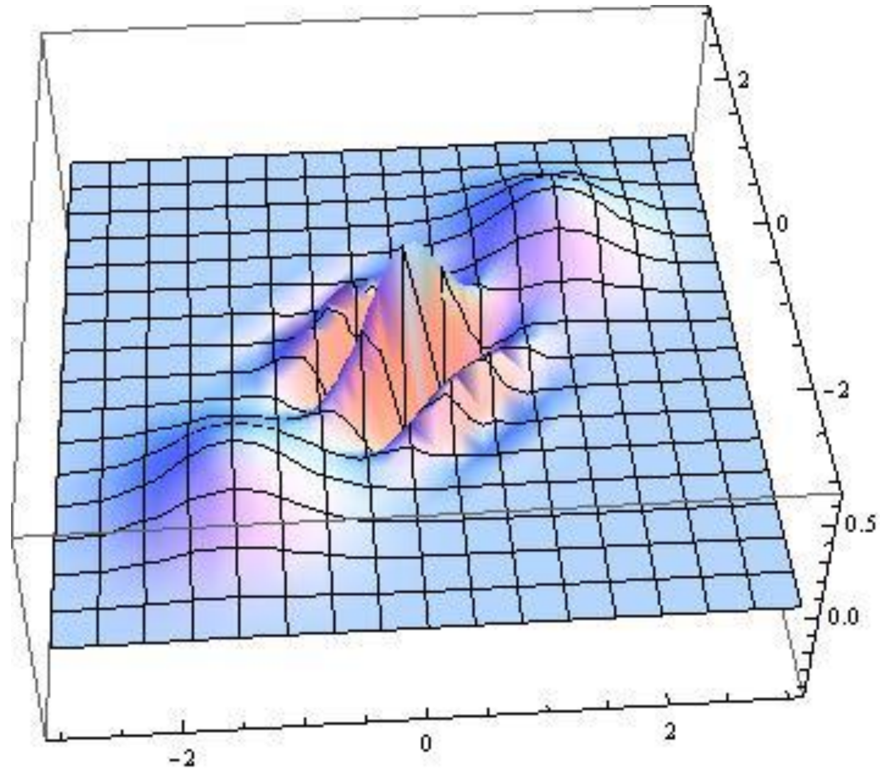
- n=3 Fock state



Picture from Tzu-Chieh Wei's website

- Schrodinger cat state (α=1.5+1.5i)

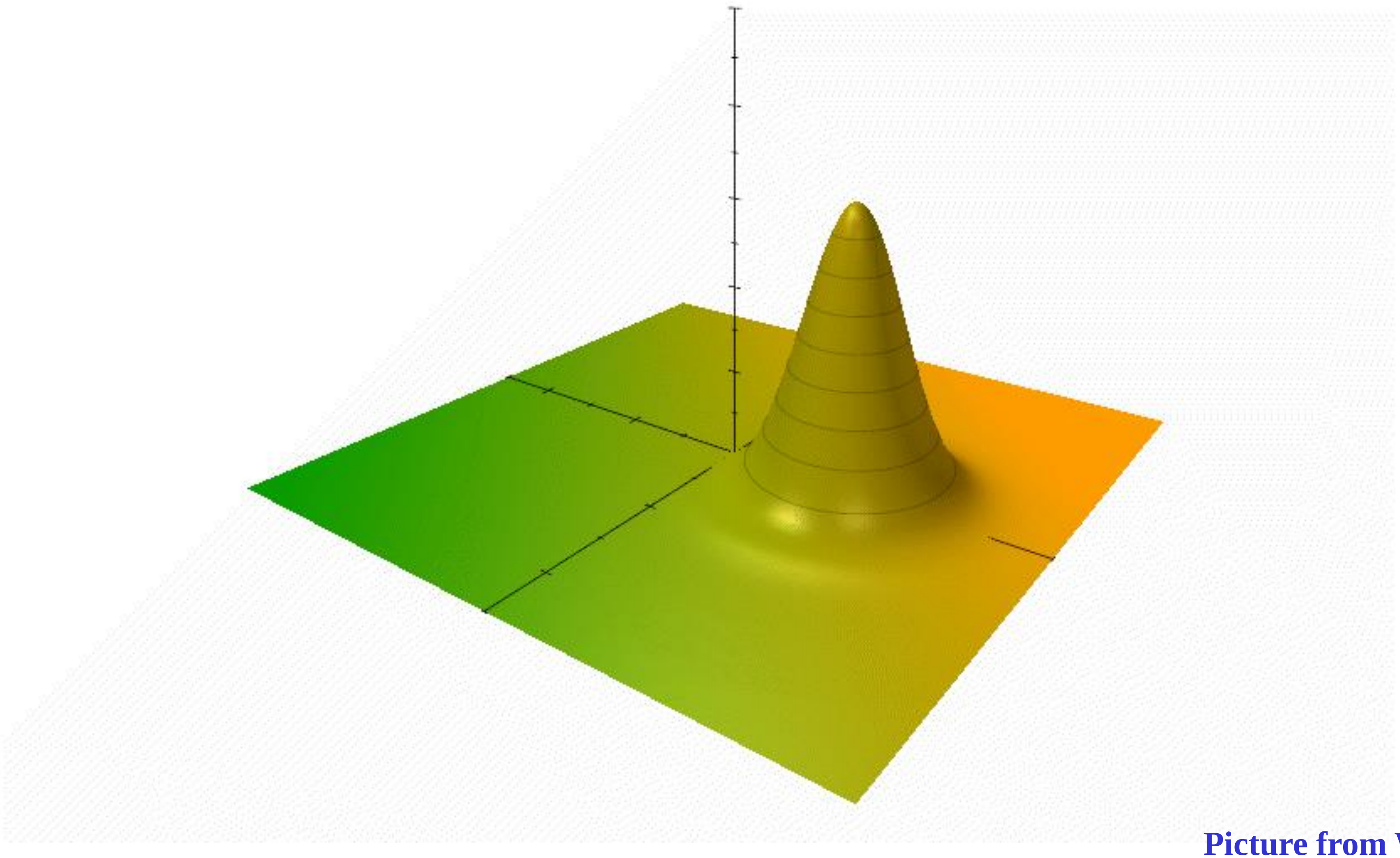
$$|cat\rangle = N\big(|\alpha\rangle + |-\alpha\rangle\big)$$



Picture from Tzu-Chieh Wei's website

Time evolution

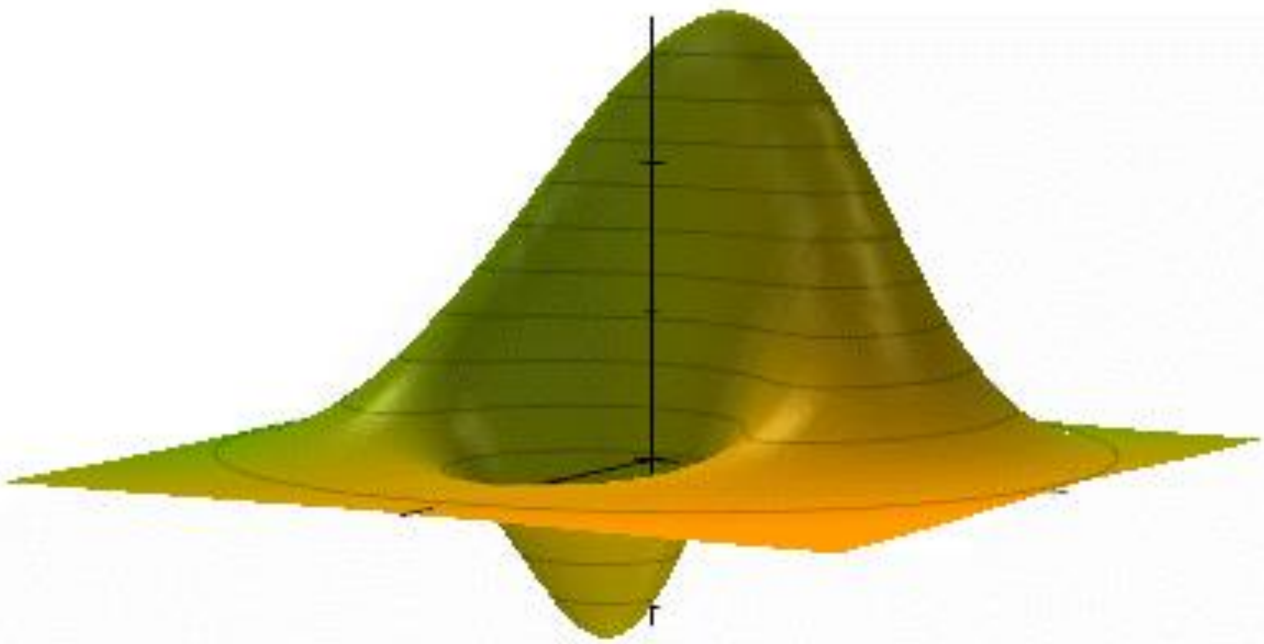
- Coherent state



Picture from Wikipedia

Time evolution

- Superposition of n=0 and n=1 Fock states



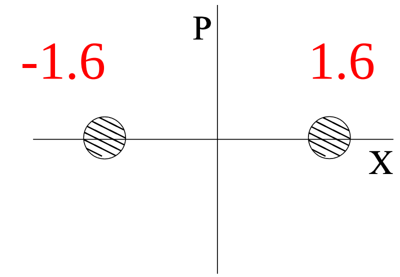
Picture from Wikipedia

Schrödinger cat states in optical fields

- **Schrödinger cat states:**

$$\boxed{|cat\rangle = N\left(|\alpha\rangle + e^{i\varphi}|-\alpha\rangle\right)} \quad \boxed{\alpha >> 1}$$

$$\boxed{\begin{aligned}\hat{X} &= \hat{a} + \hat{a}^{+} \\ \hat{P} &= -i(\hat{a} - \hat{a}^{+})\end{aligned}}$$



- **Coherent state:**

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

✓ Coherent states are most classical among all pure states: an analogy of ***classical*** point-particles in the quantum phase space [Schrödinger, *Naturwissenschaften* 14 (1926)] and most robust against decoherence.

✓ The two coherent states | α > and | -α > are “classically” (or macroscopically) distinguishable for α >>1, *i.e.*, they can be well discriminated by a homodyne measurement (HD) with ***limited*** efficiency.

(For 70% of HD efficiency: *D*≈99.7% for α=1.6 and *D*>99.9% for α=2.0.)

Wigner function

$$W(\beta)=\frac{1}{\pi^2}\int \exp[\xi^*\beta-\xi\beta^*]C(\xi)d^2\xi$$

$$\boxed{C(\xi)=\mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]}$$

$$\boxed{\hat{D}(\xi)=e^{\xi\hat{a}^+-\xi^*\hat{a}}}$$

$$\boxed{\beta=x+ip}$$

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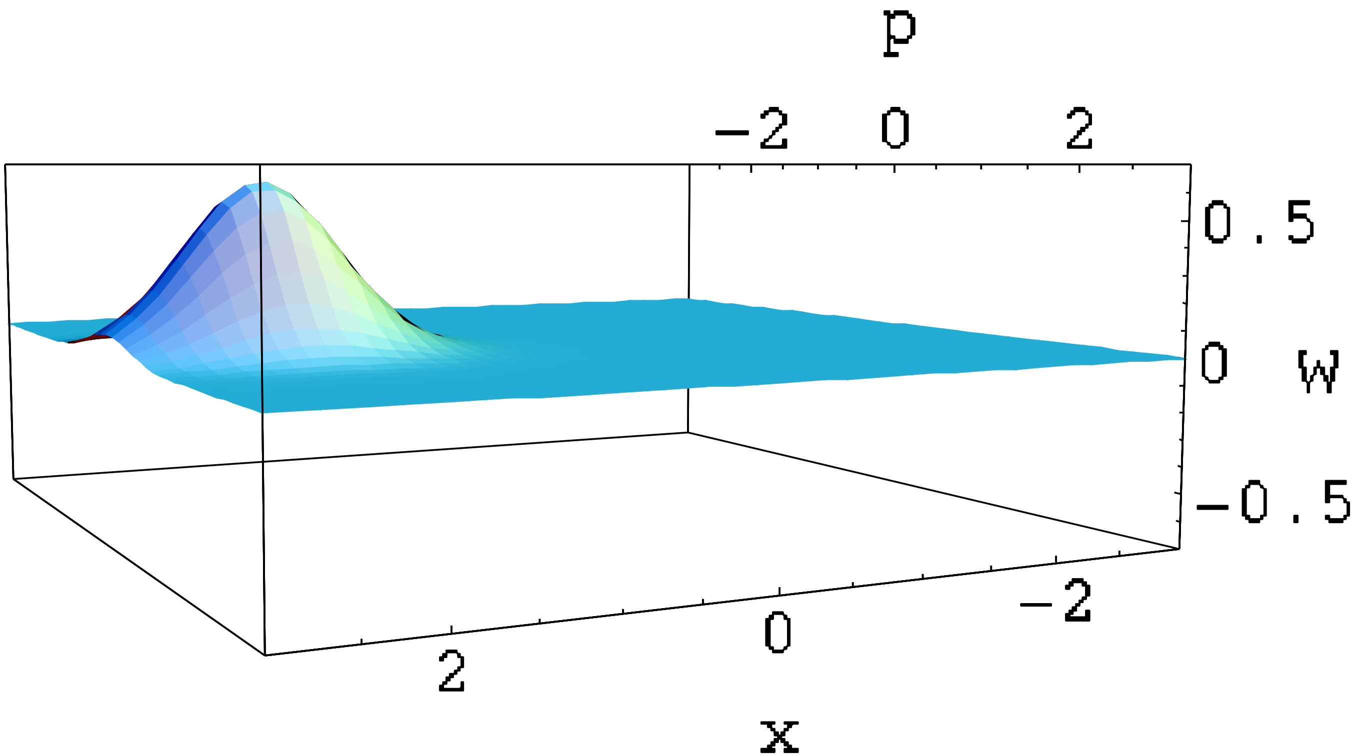
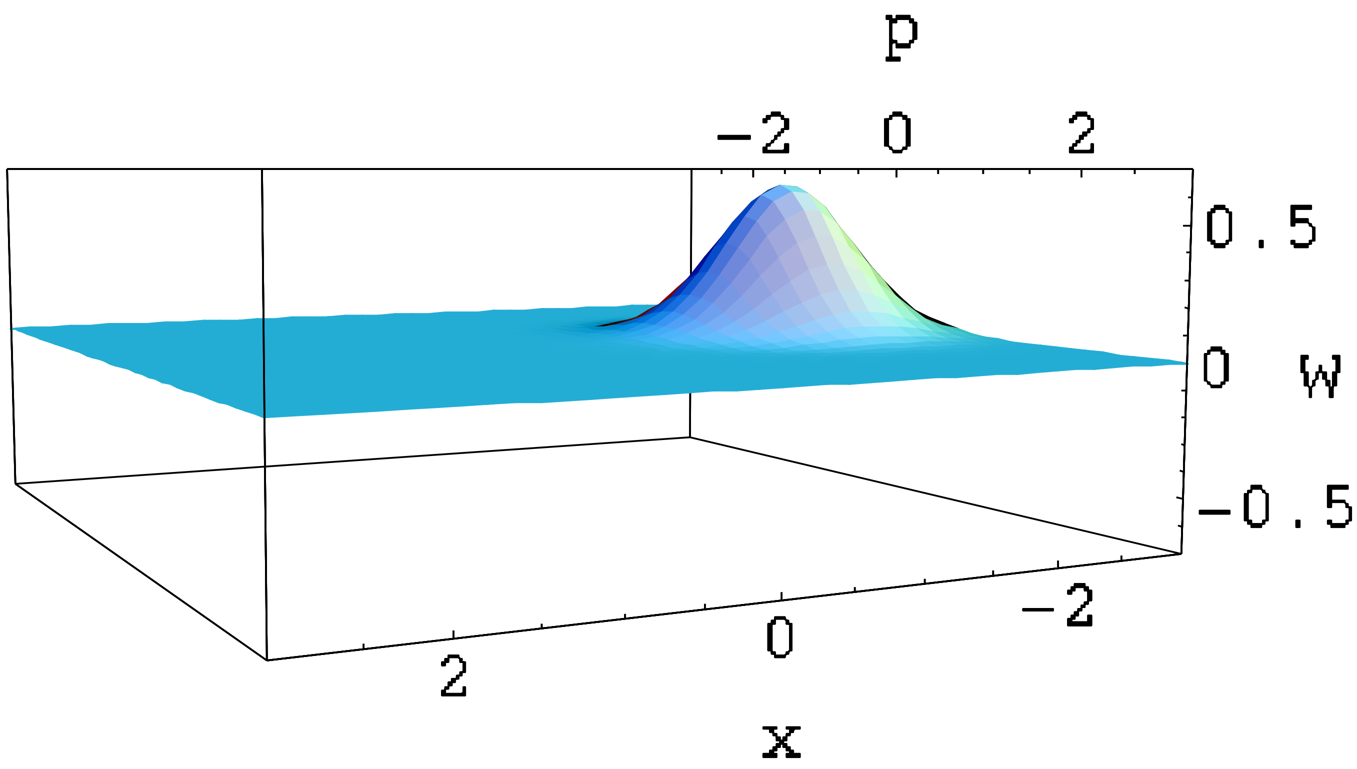
Coherent state α=2



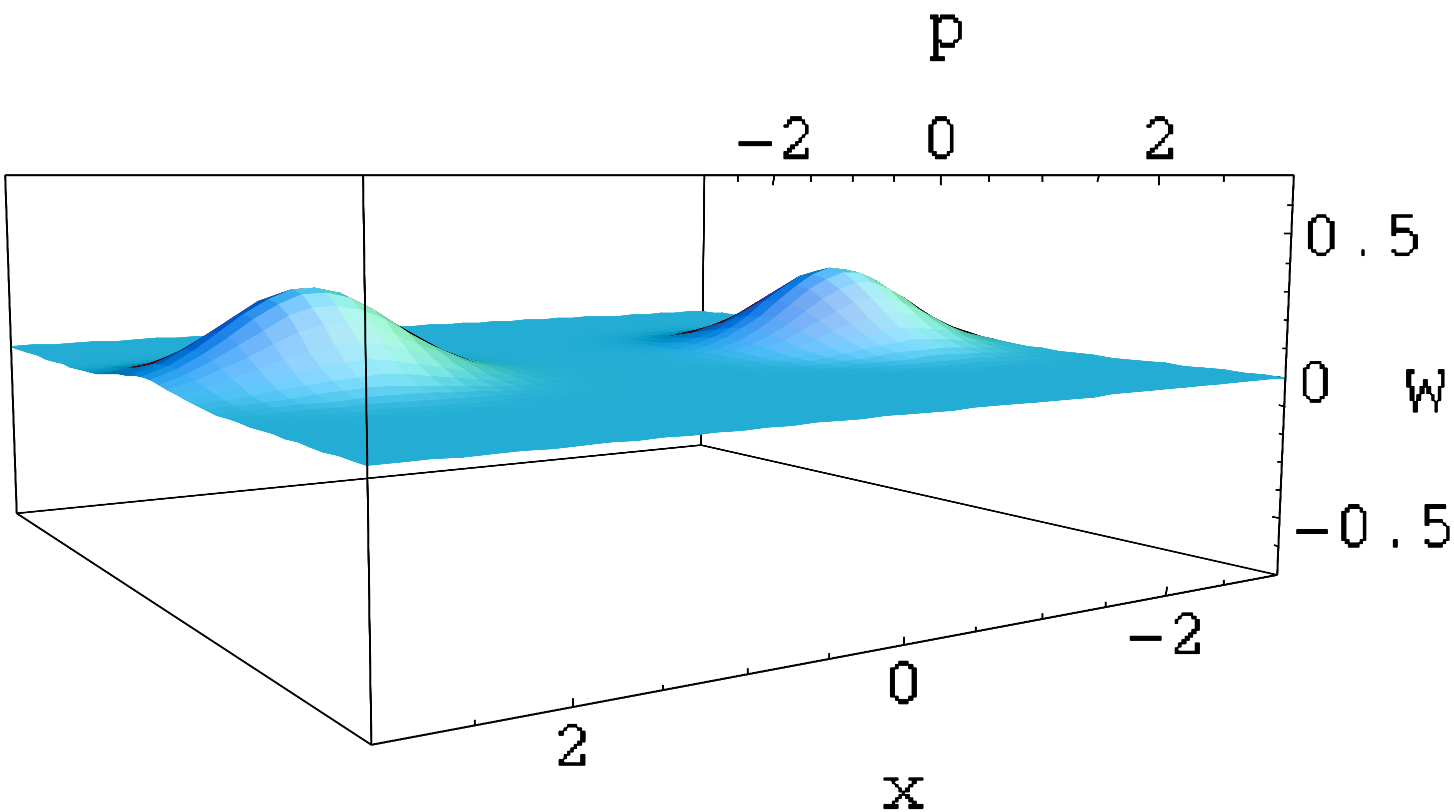
$$|\alpha\rangle \text{ with } \alpha = 2$$

Coherent state α=-2



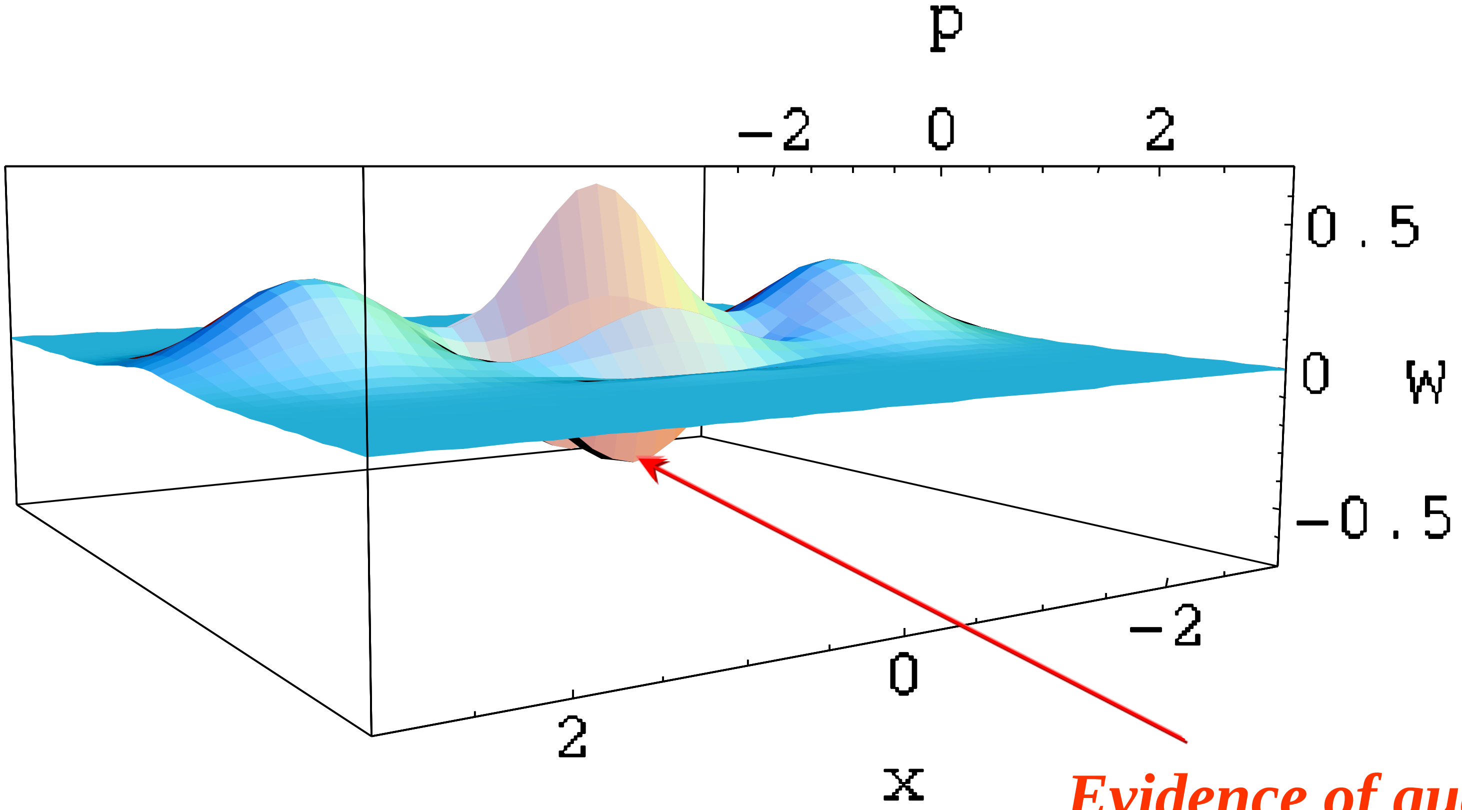
$$|\alpha\rangle \text{ with } \alpha = -2$$

Statistical mixture



$$|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|$$

Quantum superposition
(Schrödinger cat state)

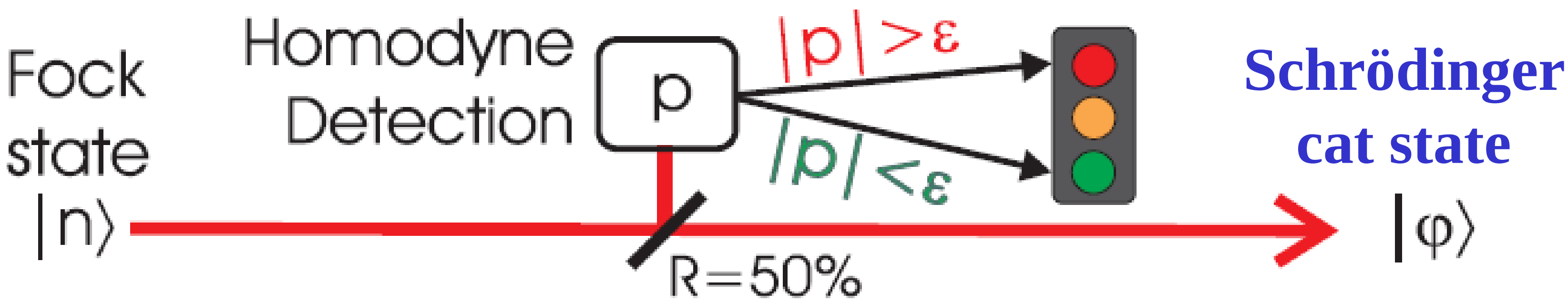


Evidence of quantum interference

$$|\alpha\rangle + |-\alpha\rangle$$

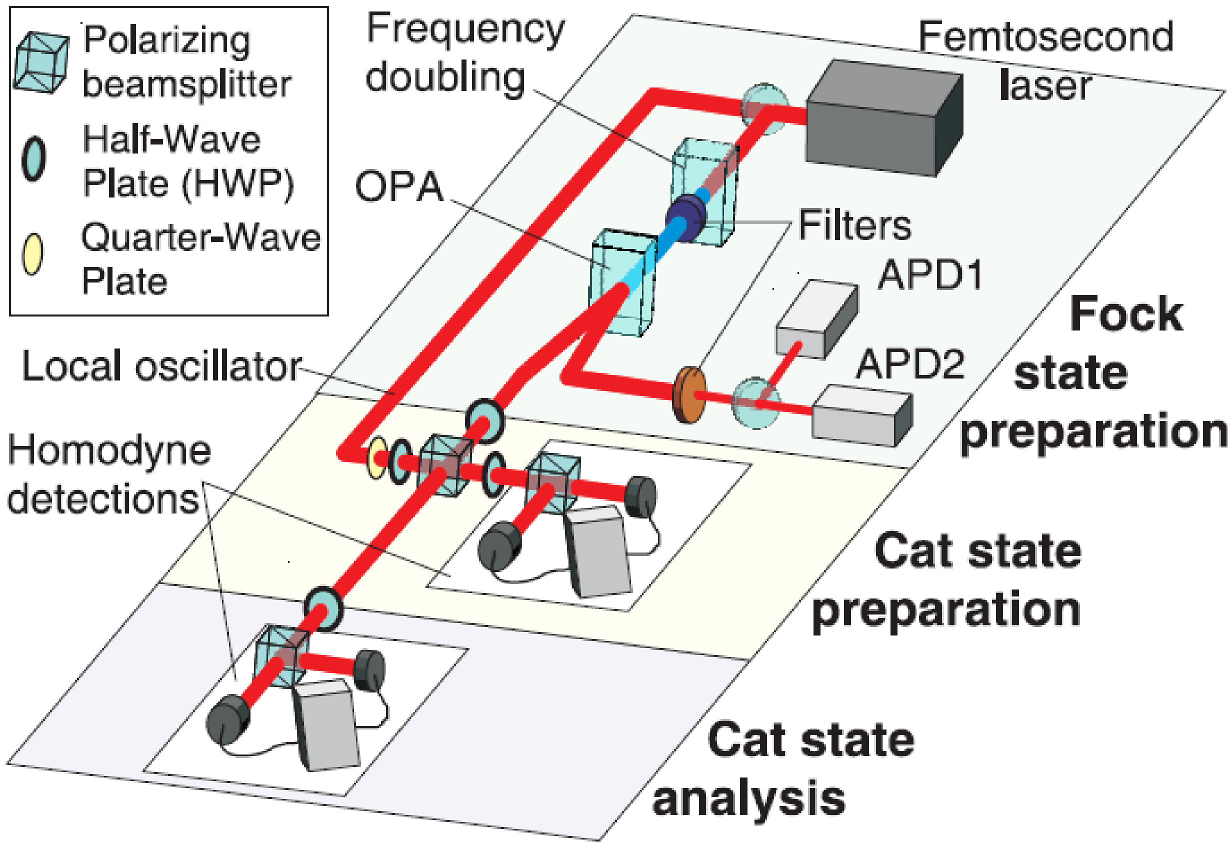
Generating a Schrödinger cat state using a two-photon state and homodyne detection

A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri and Ph. Grangier, *Nature* 448, 784 (2007)



- A two-photon state can be used to generate a Schrodinger cat state of α=1.6 with F=99%.

Experimental setup



A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri and Ph. Grangier, *Nature* 448, 784 (2007)

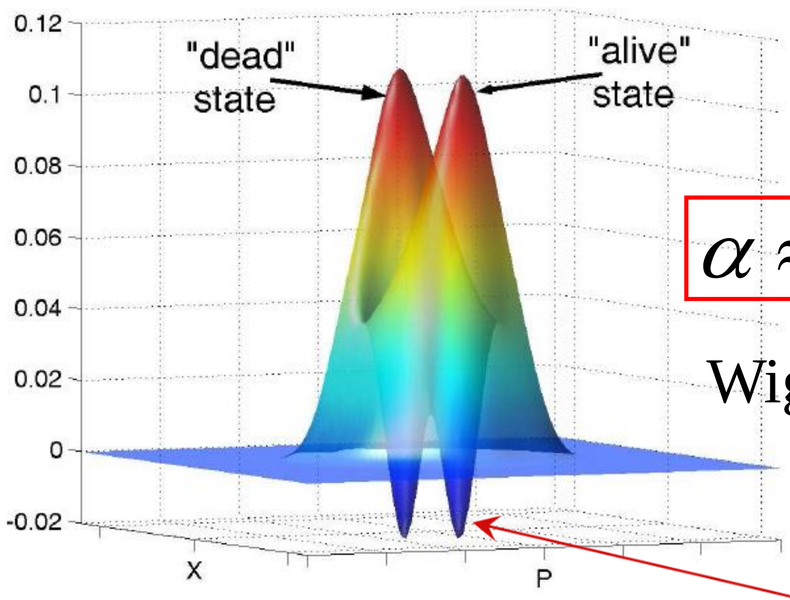
Experiment: n=2, $|\alpha|$=1.6, r=0.4, ε=0.1, 7.5% of success probability.

Homodyne efficiency: about 70%	Fidelity of the two photon sate: about 55%

Experimental Wigner function: Schrödinger cat state

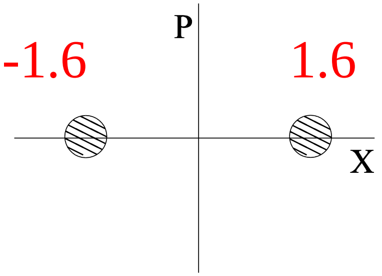
$$|cat\rangle = N\left(|\alpha\rangle + e^{i\varphi}|-\alpha\rangle\right)$$

$$\hat{X} = \hat{a} + \hat{a}^{+}$$
$$\hat{P} = -i(\hat{a} - \hat{a}^{+})$$



$$\alpha \approx 1.6$$

Wigner function



Success probability 7.5%

$(\varepsilon \approx 0.1)$

Evidence of quantum interference

A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri and Ph. Grangier, *Nature* 448, 784 (2007)

Nonclassical properties of quantum states

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- Nonclassicality based on quasi-probability functions
- Macroscopic superpositions – ‘quantum macroscopicity’
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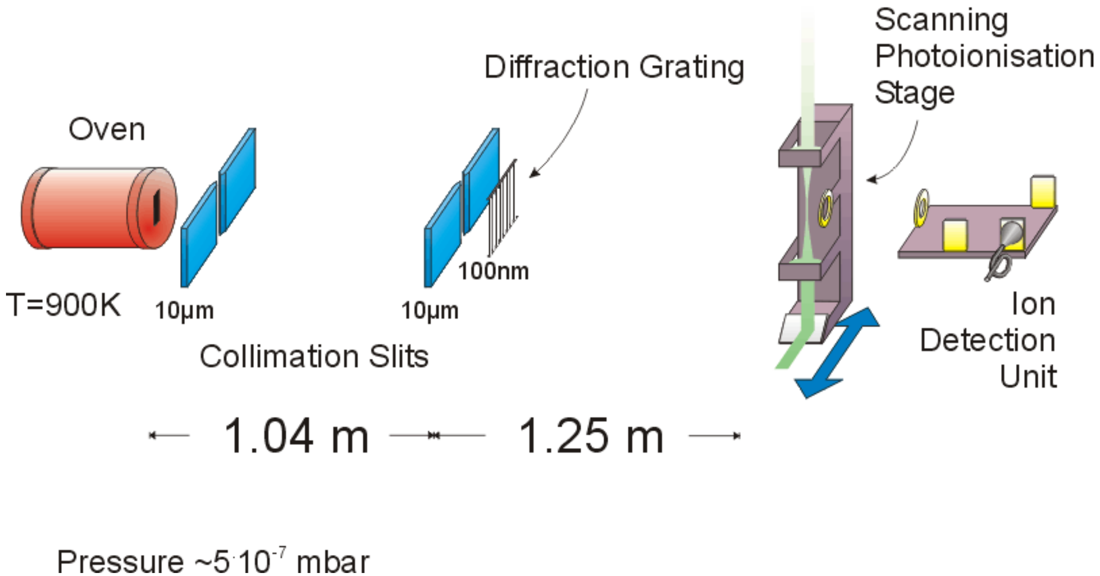
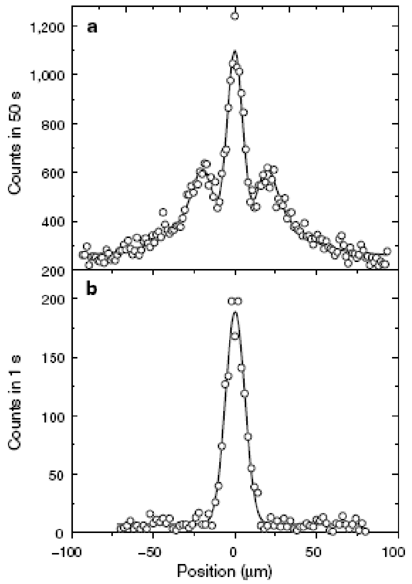
Macroscopic ***and*** quantum?

The Brussels Journal (29 October 2007)

E. Schrödinger, *Naturwissenschaftern*. 23 (1935)

Interference of large molecules



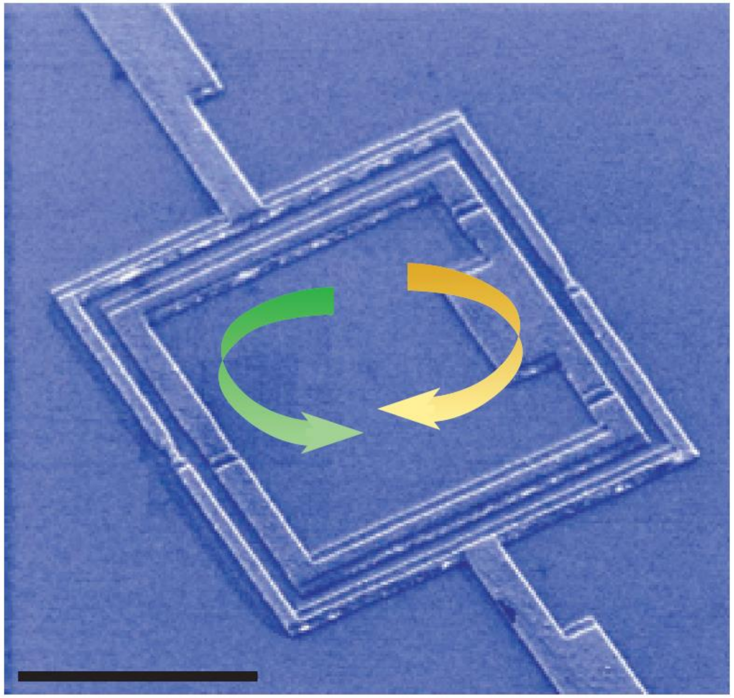


https://www.univie.ac.at/qfp/research/matterwave/c60/

(Interference with C60 molecules, *Nature* 401, 680, 1999)

- Interference with C60 molecules [M. Arndt et al., Nature 401, 680 (1999)]

Superposition of supercurrents



Copyright: C. Kohstall and R. Grimm, University of Innsbruck

- Quantum superposition of left- and right- circulating supercurrents [R. Friedman *et al.*, *Nature* 406, 43 (2000)]

Schrödinger cat states of light

A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri and Ph. Grangier, *Nature* 448, 784 (2007)

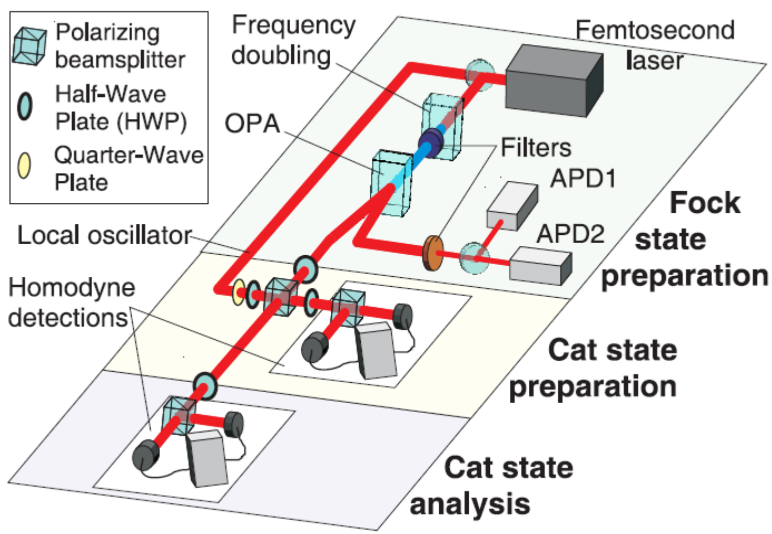
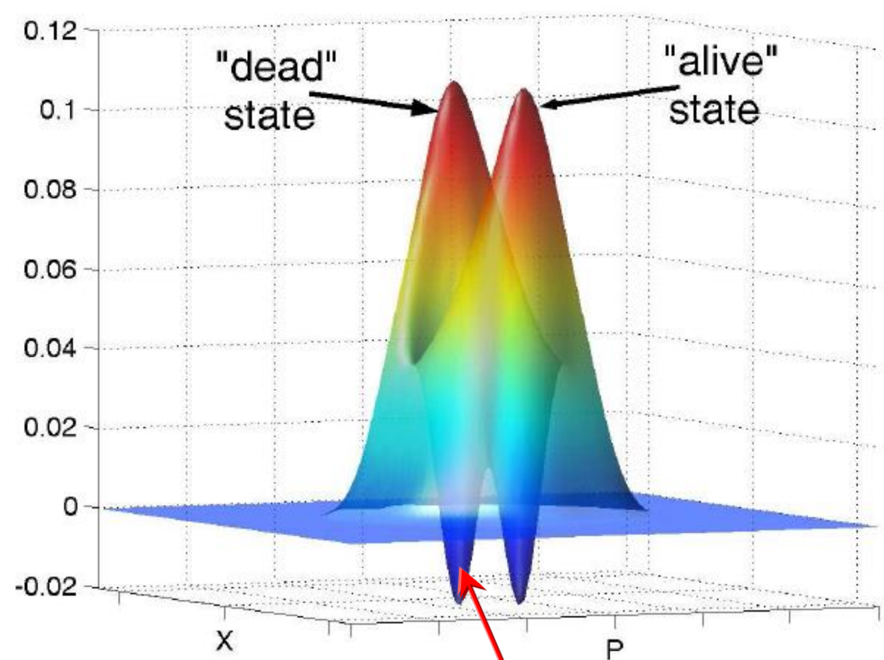


FIG. 3: Experimental setup.

Wigner function

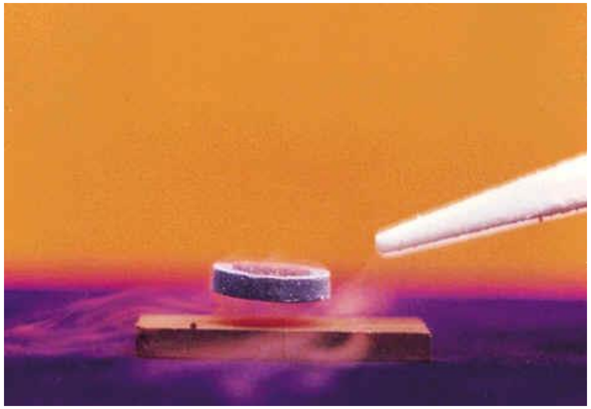


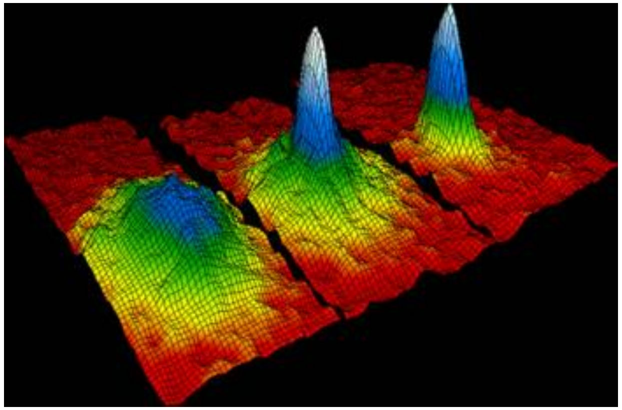
Evidence of quantum interference

- Cavity field: S. Deléglise *et al*., Nature 455, 510-514 (2008)
- Cavity field (circuit QED): Vlastakis *et al*. Science 342, 607 (2013)
- **And many more... (atomic systems, mechanical systems *etc*...)**

Macroscopic quantum phenomena

- Superconductivity
- Superfluidity
- Bose-Einstein Condensates





Macroscopic quantum phenomena

- Superconductivity
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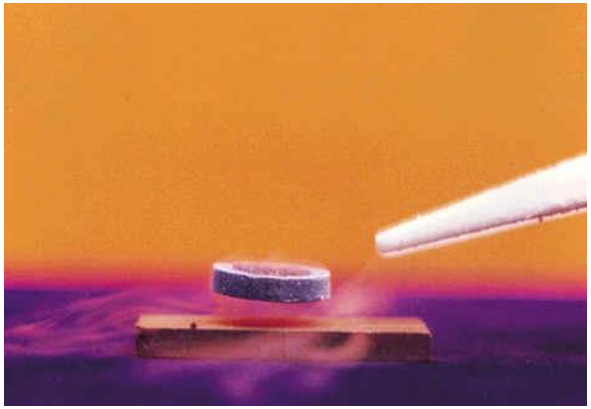  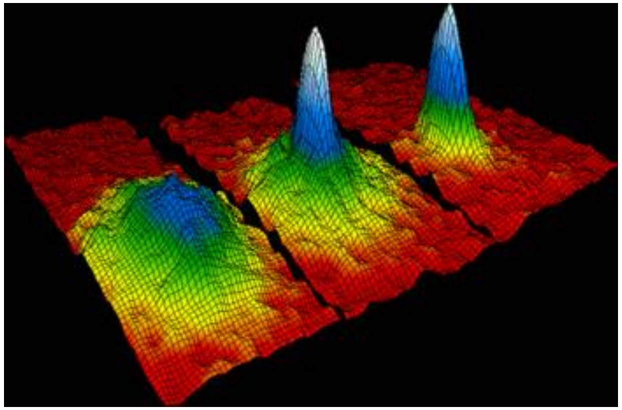

- However, these are ***not*** macroscopic superpositions nor macroscopic entanglement.

Can we quantify ‘Schrödinger’s-cattiness’ or ‘macroscopic quantumness’?

“*What is the correct measure of ‘Schrödinger’s-cattiness’?*

Ideally, one would like ***a quantitative measure which corresponds to our intuitive sense****; I shall attempt one below, but would emphasize that the choice between this and a number of similar and perhaps equally plausible definitions is, with one important exception (see below),* ***very much a matter of personal taste****, and that I very much doubt that 50 years from now anything of importance will be seen to have hung on it*.”

(A. J. Leggett, J. Phys.: Condens. Matter 14 (2002) R415–R451)

Previous studies

- Number of effective particles that involve the superposition (e.g. Leggett (1980); Dur, Simon and Cirac, PRL 2002)
- "Distance" or "distinguishability" between the component states (e.g. Bjork and Mana, J. Opt. B 2004; Korsbakken *et al*., PRA 2007)

[3] A.J. Leggett, J. Phys.: Condens. Matter 14 (2002) R415.
[4] A.J. Leggett, Prog. Theor. Phys. Suppl. 69 (1980) 80.
[5] W. Dür, C. Simon, J.I. Cirac, Phys. Rev. Lett. 89 (2002) 210402.
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[11] F. Marquardt, B. Abel, J. von Delft, Phys. Rev. A 78 (2008) 012109.
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[16] P. Sekatski, N. Sangouard, N. Gisin, Phys. Rev. A 89 (2014) 012116.

References taken from
H. Jeong, M. Kang and H. Kwon, Special Issue on Macroscopic Quantumness, Optics Communications 337, 12 (2015) (Review Article).

Previous studies

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- "Distance" or "distinguishability" between the component states (e.g. Bjork and Mana, J. Opt. B 2004; Korsbakken *et al*., PRA 2007)

Measures for macroscopic superpositions and quantum states

A. Summary of measures

1. Leggett (1980, 2002)
2. Dür *et al.* (2002)
3. Shimizu and Miyadera (2002) and followups
4. Björk and Mana (2004)
5. Cavalcanti and Reid (2006, 2008)
6. Korsbakken *et al.* (2007)
7. Marquardt *et al.* (2008)
8. Lee and Jeong (2011) and Park *et al.* (2016)
9. Fröwis and Dür (2012b) and followups
10. Nimmrichter and Hornberger (2013)
11. Sekatski *et al.* (2014c, 2017b)
12. Laghaout *et al.* (2015)
13. Yadin and Vedral (2015)
14. Kwon *et al.* (2017)

F. Fröwis *et al*., Rev. Mod. Phys. 90, 025004 (2018)

Disconnectivity

A. J. Leggett, Prog. Theor. Phys. Suppl. 69, 80 (1980); J. Phys. 14, R415 (2002)

- D quantifies genuine multipartite quantum correlation such as:

$$|\phi\rangle^{\otimes N} + |\phi^{\perp}\rangle^{\otimes N}$$

- D for ρ_N is defined as the largest number n that makes δ_n the smallest where

$$\delta_n = \frac{S_n}{\min_m(S_m + S_{n-m})} \qquad S_n = -\mathrm{Tr}[\rho_n \ln \rho_n]$$

  and ρ_n $(n < N)$ is a reduced density operator from ρ_N.
- Leggett pointed out that so-called "macroscopic quantum phenomena" such as superconductivity or superfluidity do not require the existence of a high-D state.
- Superfluidity can be explained by a product of identical bosonic states of which disconnectivity is obviously 1.
- A superconducting system described by Cooper pairs also shows a small value of D.

Disconnectivity

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  and ρ_n $(n < N)$ is a reduced density operator from ρ_N.

- Applicable only to certain types of pure states.

Distinguishability-based measure

J.I. Korsbakken, K.B. Whaley, J. Dubois, J.I. Cirac, Phys.Rev.A75, 042106 (2007).

- For an *N*-partite superposition state |A>+|B>
- n_{min}: number of measurements (with limited efficiency δ) required to distinguish between |A> and |B>.

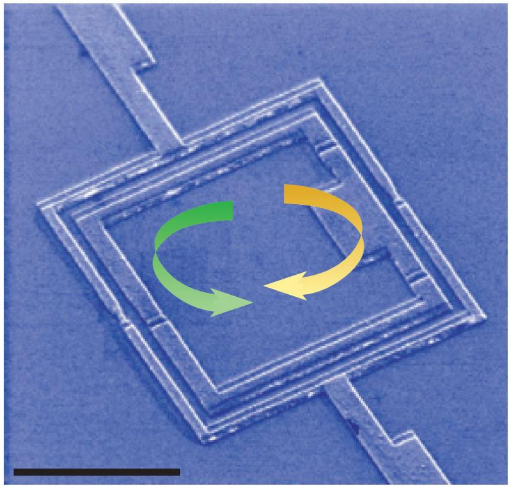
$$C_\delta(|\psi_N\rangle) = \frac{N}{n_{\min}}$$

Effective size of N-particle superposition state

J.I. Korsbakken, K.B. Whaley, J. Dubois, J.I. Cirac, Phys.Rev.A75, 042106 (2007).

$$C_\delta(|\psi_N\rangle) = \frac{N}{n_{\min}}$$

- Example: the effective size of the flux qubits, as a genuine macroscopic superposition, is surprisingly (but not trivially) ***small*** despite the apparent large difference in macroscopic observables with billions of electrons.



Just another "kitten"…

Distinguishability-based measure

J.I. Korsbakken, K.B. Whaley, J. Dubois, J.I. Cirac, Phys.Rev.A75, 042106 (2007).

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- n_{min}: number of measurements (with limited efficiency δ) required to distinguish between |A> and |B>.

$$C_\delta(|\psi_N\rangle) = \frac{N}{n_{\min}}$$

- *C* does not distinguish between a pure superposition and a classical mixture.
- *C* is decomposition-dependent.

General measure?

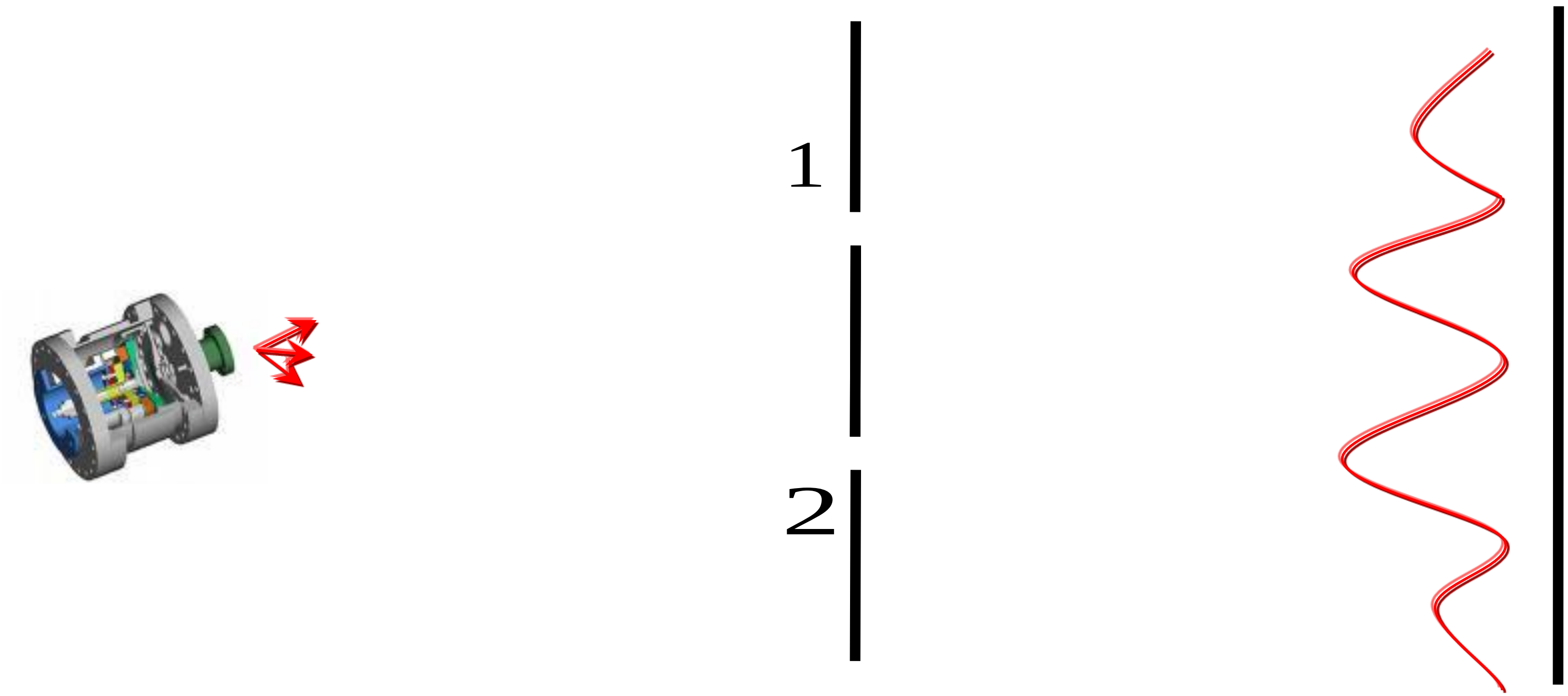
- *It should be applicable to a wide range of states, not limited to a specific type of states.*
- *It should be able to quantify the degree of a genuine superposition against a classical mixture together with its effective size factor.*

General (and useful) measure?

- *It should be applicable to a wide range of states, not limited to a specific type of states.*
- *It should be able to quantify the degree of a genuine superposition against a classical mixture together with its effective size factor.*
- *Independent of decomposition of the superposition*
- *Easy to calculate*
- *Experimentally measurable (without full tomography)*

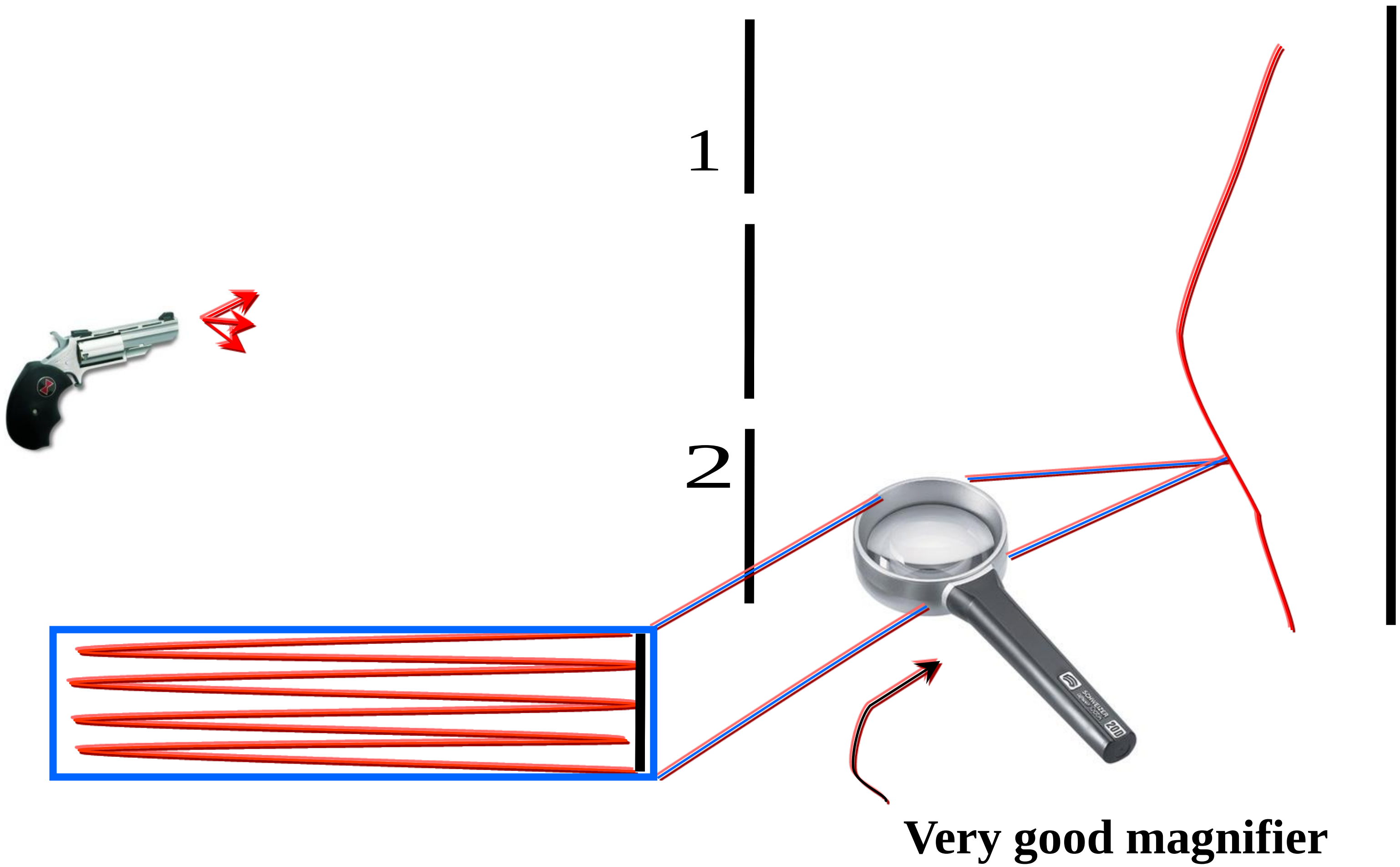
Double slit experiment

Feynman, *Lectures on Physics*, *Volume 3*

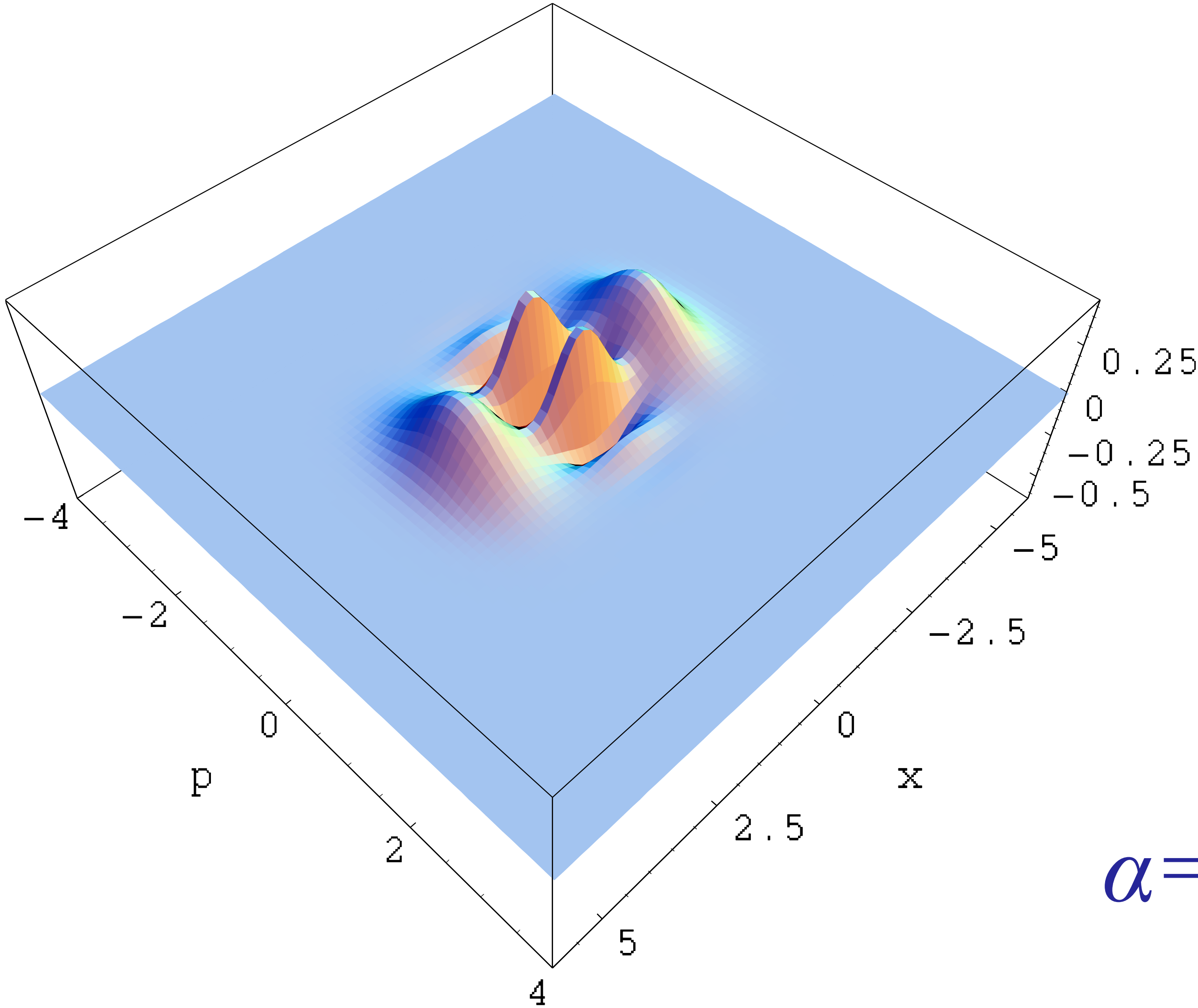


Double slit experiment

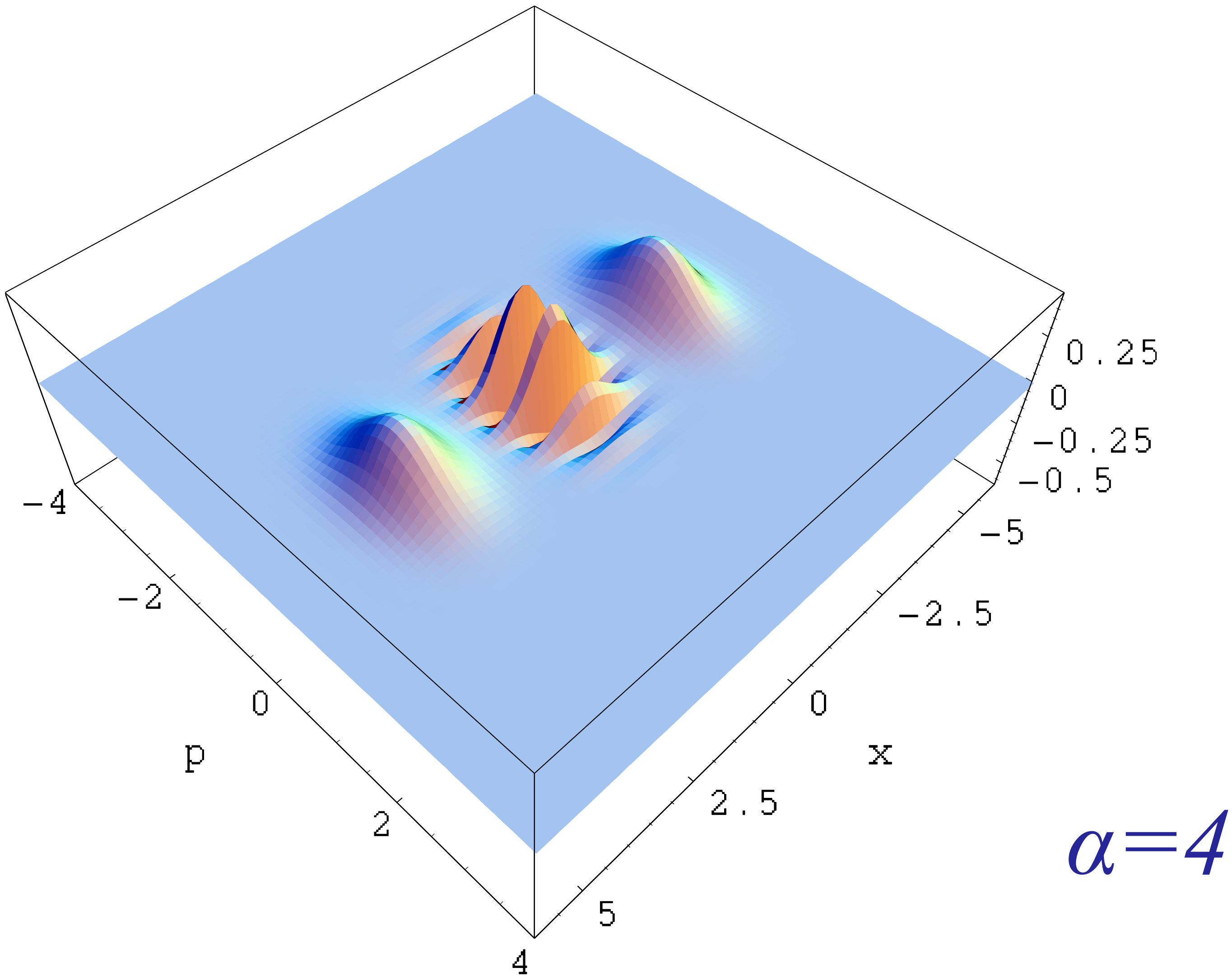
Feynman, *Lectures on Physics*, *Volume 3*



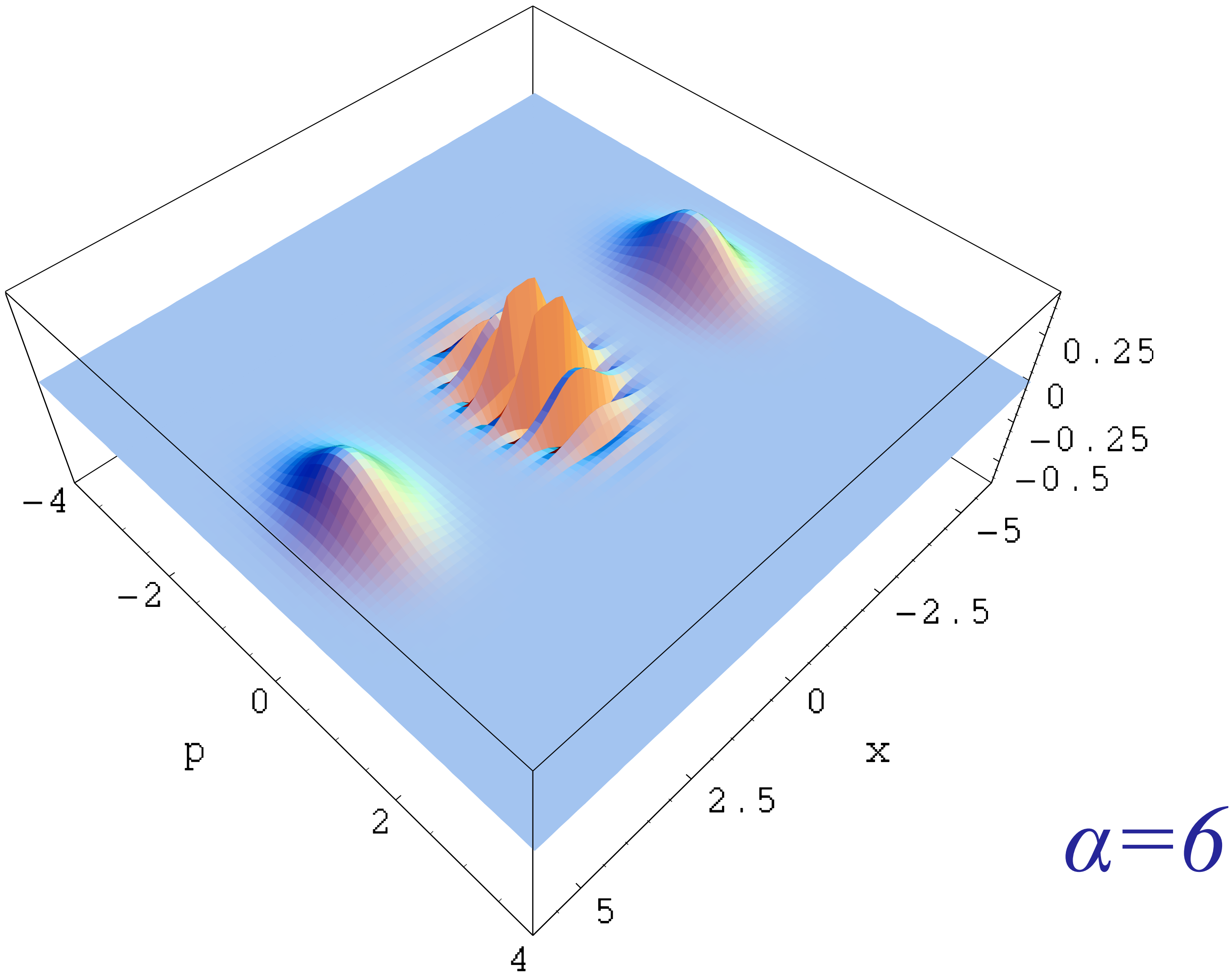
Wigner function of |α>+|-α>



α=2



0.25
0
−0.25
−0.5
−4
−2
0
2
4
p
−5
−2.5
0
2.5
5
x
α=4


0.25
0
-0.25
-0.5
-4
-2
0
p
2
4
-5
-2.5
0
x
2.5
5
$\alpha=6$

Interference-based general measure for bosonic systems

C.-W. Lee and H. Jeong, *Phys. Rev. Lett.* 106, 220401 (2011)

$$\mathcal{I}(\rho) \sim \int d^2\xi \left(\xi_r^2 + \xi_i^2\right) |\chi(\xi)|^2$$

$$\chi(\xi) = \mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]$$

$$\hat{D}(\xi) = e^{\xi \hat{a}^+ - \xi^* \hat{a}}$$

$$W(\beta) = \frac{1}{\pi^2} \int \exp[\xi^* \beta - \xi \beta^*] \chi(\xi) d^2\xi$$

General measure for bosonic systems

C.-W. Lee and H. Jeong, *Phys. Rev. Lett.* 106, 220401 (2011)

$$\int d^2\xi \left(\xi_r^2 + \xi_i^2\right) \left|\chi\left(\xi\right)\right|^2$$

$$\chi(\xi) = \mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]$$

- For an ***arbitrary*** state, it simultaneously quantifies (1) *how far-separate the component states of the superposition are* ***and*** (2) *the degree of genuine quantum coherence between the component states against their classical mixture*.
- It can be applied to *any* harmonic oscillator systems such as light fields.
- Independent of the decomposition of the component states: easy to calculate.

General measure for bosonic systems

C.-W. Lee and H. Jeong, *Phys. Rev. Lett.* 106, 220401 (2011)

$$\mathcal{I}(\rho) = \frac{1}{2\pi^M}\int d^2\boldsymbol{\xi} \sum_{m=1}^{M} \left[|\xi_m|^2 - 1\right] |\chi(\boldsymbol{\xi})|^2$$

$$= \frac{\pi^M}{2}\int d^2\boldsymbol{\alpha}\, W(\boldsymbol{\alpha}) \sum_{m=1}^{M} \left[-\frac{\partial^2}{\partial\alpha_m \partial\alpha_m^*} - 1\right] W(\boldsymbol{\alpha})$$

- $\mathcal{I}(\rho)$ is equivalent to the purity decay rate of the state as

$$\mathcal{I}(\rho) = -\frac{1}{2}\frac{d\mathcal{P}(\rho)}{d\tau}$$

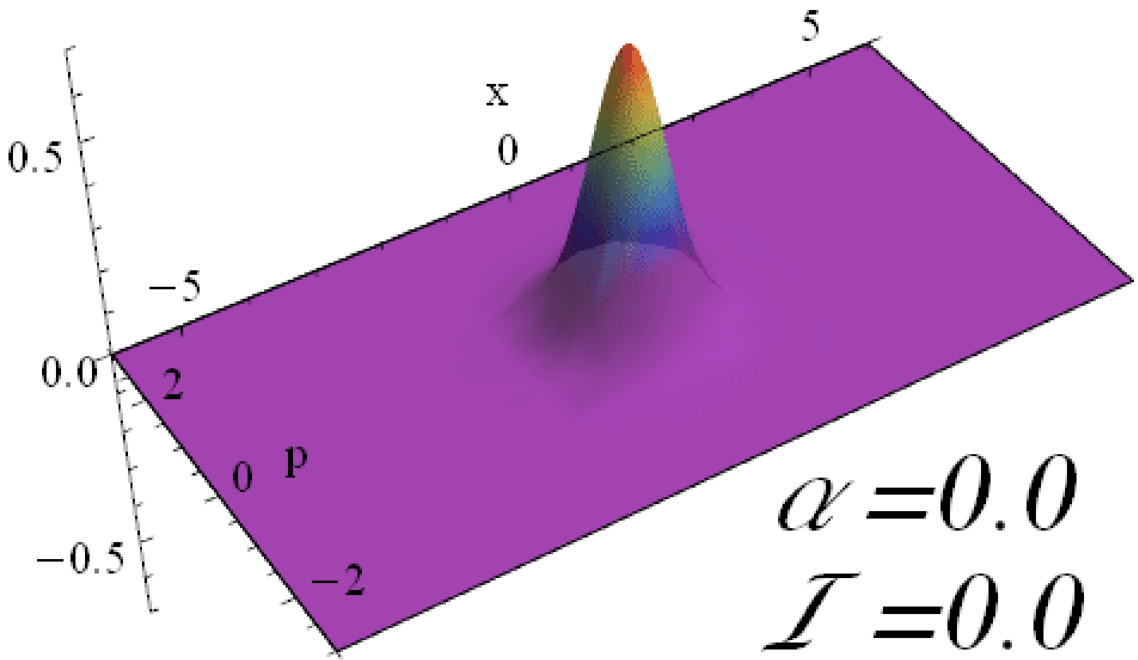
where $\mathcal{P}(\rho) = \mathrm{Tr}[\rho^2]$

for a standard decoherence model (loss for a photonic system).

✓ Approximately measurable without full tomography (Jeong *et al*., J. Opt. Soc. Am. 2014)

- $\mathcal{I}(\rho)$ can be applied to arbitrary spin systems with some modifications (C.-Y. Park *et al*., Phys. Rev. A 94, 052105 (2016)).

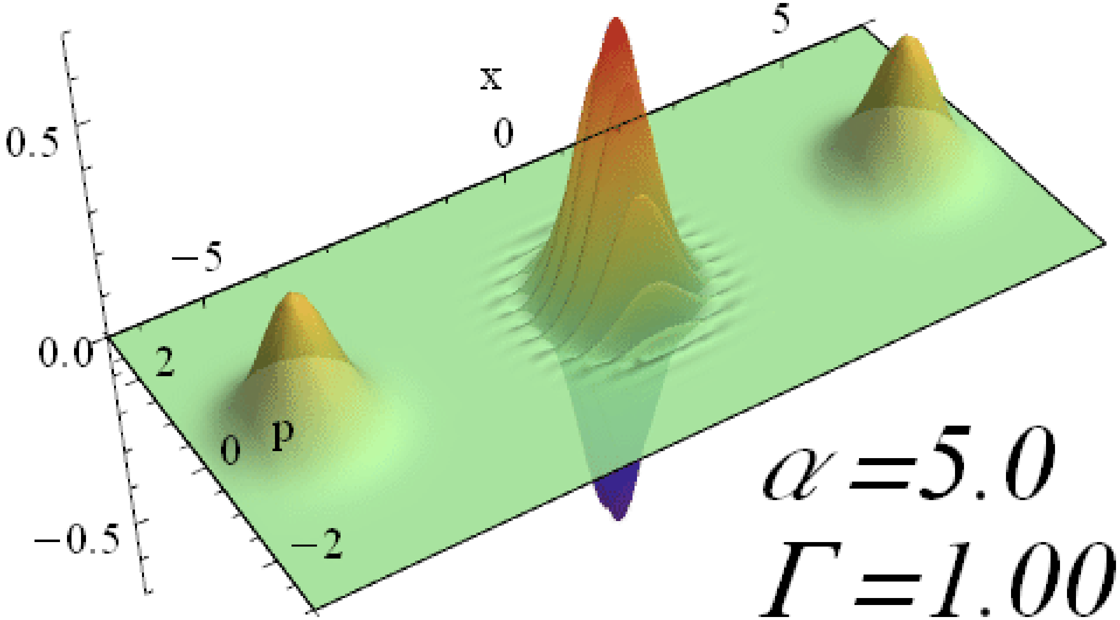
Macroscopic quantumness $\mathcal{I}$ of $|\alpha\rangle+|-\alpha\rangle$ with increasing α



Wigner function

Macroscopic quantumness $\mathcal{I}$ of

$$\rho_{\rm scs} = N_\Gamma[|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha| + \Gamma(|\alpha\rangle\langle-\alpha| + |-\alpha\rangle\langle\alpha|)]$$



$\alpha = 5.0$

$\Gamma = 1.00$

$\mathcal{I} = 25.0$

Wigner function

Macroscopically quantum?

- Coherent state: $I(|\alpha\rangle) = 0$ regardless of the value of α.
- *Invariant under passive linear optics operations such as the displacement operations and phase shifts.*

- Well known states in the "Schrödinger-cat family" with the maximum values of "quantum macroscopicity" *I*, i.e., the average photon number of the corresponding state:
  - ✓ Superposition of coherent states: $|\alpha\rangle + |-\alpha\rangle$
  - ✓ NOON state: $|n\rangle|0\rangle + |0\rangle|n\rangle$
  - ✓ GHZ state: $|H\rangle^N + |V\rangle^N$
  - ✓ Hybrid entanglement: $\frac{1}{\sqrt{2}}\left(|H\rangle|\alpha\rangle + |V\rangle|-\alpha\rangle\right)$

General measure for spin systems

F. Fröwis, W. Dür, New J. Phys. 14 (2012) 093039

- A quantum system is 'macroscopic' ***if*** there exists $\hat{A}$ such that

$$\max_{\hat{A}\in\mathcal{A}} F(\rho,\hat{A}) = O(N^2)$$

where F is quantum Fisher information and $\hat{A}$ is an additive operator $A = \sum_{i=1}^{N} A^{(i)}$.

$$F(\rho,\hat{A}) = 2\sum_{i,j}\frac{(\lambda_i-\lambda_j)^2}{\lambda_i+\lambda_j}\left|\langle i|\,\hat{A}\,|j\rangle\right|^2, \quad \text{where} \quad \rho = \sum_i \lambda_i |i\rangle\langle i|$$

✓ $F=O(N)$ for a product state $|\psi^-\rangle^{\otimes N/2}$

✓ $F=O(N^2)$ for a GHZ state $|\mathrm{GHZ}_L\rangle = \frac{1}{\sqrt{2}}\left(|0_L\rangle^{\otimes N} + |1_L\rangle^{\otimes N}\right)$

- Applicable to arbitrary spin systems.
- Operational meaning in relation to quantum metrology.

General measures

- *It should be applicable to a wide range of states, not limited to a specific type of states.*
- *It should be able to quantify the degree of a genuine superposition against a classical mixture together with its effective size factor.*

[13] C.-W. Lee, H. Jeong, Phys. Rev. Lett. 106 (2011) 220401.
[14] F. Fröwis, W. Dür, New J. Phys. 14 (2012) 093039.

- [13] corresponds to sensitivity to decoherence while [14] is sensitivity to phase shifts.
- For pure states, these two measures become ‘identical’ – the ***maximum variance*** of an arbitrarily chosen observable ***A***.

General framework for quantum macroscopicity

B. Yadin and V. Vedral, Phys. Rev. A **93**, 022122 (2016)

- Assume ρ in terms of eigenbasis $|i\rangle$ of observable $A = \sum_i a_i \, |i\rangle\langle i|$
- δ-coherence: $$\rho^{(\delta)} := \sum_{i,j:\, a_i - a_j = \delta} \rho_{i,j} \, |i\rangle\langle j|$$
- Free-operation: $\mathcal{E}(\rho)^{(\delta)} = \mathcal{E}(\rho^{(\delta)}) \; \forall \delta \in \Delta$
- Conditions for a macroscopic coherence measure $M(\rho)$

(M1): $M(\rho) \geq 0$ and $M(\rho) = 0 \iff \rho = \rho^{(0)}$.
(M2a): $M(\rho) \geq M(\mathcal{E}(\rho))$ for a trace-preserving free operation $\mathcal{E}$.
(M2b): $M(\rho) \geq \sum_\alpha p_\alpha M(\mathcal{E}_\alpha(\rho)/p_\alpha)$, where $\mathrm{Tr}\mathcal{E}_\alpha(\rho) = p_\alpha$.
(M3): $M(\sum_i p_i \rho_i) \leq \sum_i p_i M(\rho_i)$.
(M4): $M(|i\rangle + |j\rangle) > M(|k\rangle + |l\rangle)$, when $|a_i - a_j| > |a_k - a_l|$.

- ✓ (M1, M2a, M2b): requirements for a monotone
- ✓ (M3): convexity
- ✓ (M4): requirement for a ***macroscopicity*** measure

General framework for quantum macroscopicity

B. Yadin and V. Vedral, Phys. Rev. A **93**, 022122 (2016)

- When (M1-M4) are applied to quantum macroscopicity measure $\mathcal{I}(\rho)$, the (M2a) criterion is not satisfied.

  - $\mathcal{I}(\rho)$ could increase under certain free operations

    $\rho \otimes \sigma \to \rho \otimes \tilde{\sigma}$ with $\mathrm{Tr}\sigma^2 < \mathrm{Tr}\tilde{\sigma}^2$ can increase $\mathcal{I}(\rho)$

Question: Are these criteria sufficient?

Disturbance-based measure of macroscopic coherence

H. Kwon, C.-Y. Park, K. C. Tan, H. Jeong, New J. Physics 19, 043024 (2017)

For given observable $\hat{A} = \sum_i a_i |i\rangle\langle i|$,

- ✓ Coarse-grained measurement: $\hat{Q}_x^\sigma = \sum_i \sqrt{q_i^\sigma(x)}\, |i\rangle\langle i|$

  where $q_i^\sigma(x) := \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(a_i - x)^2}{2\sigma^2}}$

- ✓ Quantum state after the measurement: $\Phi_\sigma(\rho) = \int_{-\infty}^{\infty} \hat{Q}_x^\sigma \rho \hat{Q}_x^\sigma dx$

- ✓ Quantum state disturbance by coarse-grained measurement

$$M_\sigma(\rho) := D(\rho, \Phi_\sigma(\rho))$$

for the Bures distance $D_B(\rho,\tau) = 2 - 2\sqrt{\mathcal{F}(\rho,\tau)}$

and quantum relative entropy $D_R(\rho,\tau) = S(\rho||\tau)$

Disturbance-based measure of macroscopic coherence

H. Kwon, C.-Y. Park, K. C. Tan, H. Jeong, New J. Physics 19, 043024 (2017)

- Quantum state after measurement:

$$|k\rangle + |l\rangle \xrightarrow{\text{Measurement}} |k\rangle\langle k| + |l\rangle\langle l| + e^{-\frac{(a_k-a_l)^2}{8\sigma^2}} \left(|k\rangle\langle l| + |l\rangle\langle k|\right)$$

$$[\Phi_\sigma(\hat{\rho})]_{ij} = \int_{-\infty}^{\infty} \hat{Q}_x^\sigma \hat{\rho} \hat{Q}_x^\sigma = \sum_{i,j} e^{-\frac{(a_i-a_j)^2}{8\sigma^2}} \rho_{ij}$$

Macroscopic coherence $|a_i - a_j| \gg \sigma$ **vanishes**

microscopic coherence $|a_i - a_j| \ll \sigma$ **remains**

- State disturbance after measurement = The amount of macroscopic coherence initially contained in the state

✓Disturbance based measure of macroscopic coherence:

$$\mathcal{M}_\sigma(\hat{\rho}) = D(\hat{\rho}, \Phi_\sigma(\hat{\rho}))$$

(e.g.) Bures distance: $D_B(\hat{\rho}, \hat{\tau}) = 2 - 2\sqrt{\mathcal{F}(\hat{\rho}, \hat{\tau})}$

Quantum relative entropy: $S(\hat{\rho}||\hat{\tau}) = \mathrm{Tr}\hat{\rho}\log\hat{\rho} - \mathrm{Tr}\hat{\rho}\log\hat{\tau}$

Disturbance-based measure of macroscopic coherence

H. Kwon, C.-Y. Park, K. C. Tan, H. Jeong, New J. Physics 19, 043024 (2017)

For given observable $\hat{A} = \sum_i a_i |i\rangle\langle i|$,

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$$M_\sigma(\rho) := D(\rho, \Phi_\sigma(\rho))$$

for the Bures distance $D_B(\rho,\tau) = 2 - 2\sqrt{\mathcal{F}(\rho,\tau)}$

and quantum relative entropy $D_R(\rho,\tau) = S(\rho||\tau)$

satisfies all the conditions (M1) – (M4) for every σ>0.

Disturbance-based measure of macroscopic coherence

H. Kwon, C.-Y. Park, K. C. Tan, H. Jeong, New J. Physics 19, 043024 (2017)

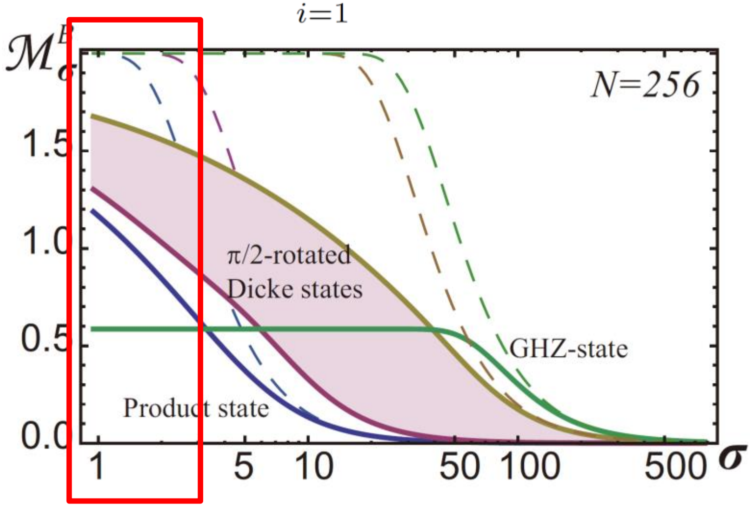
- Examples

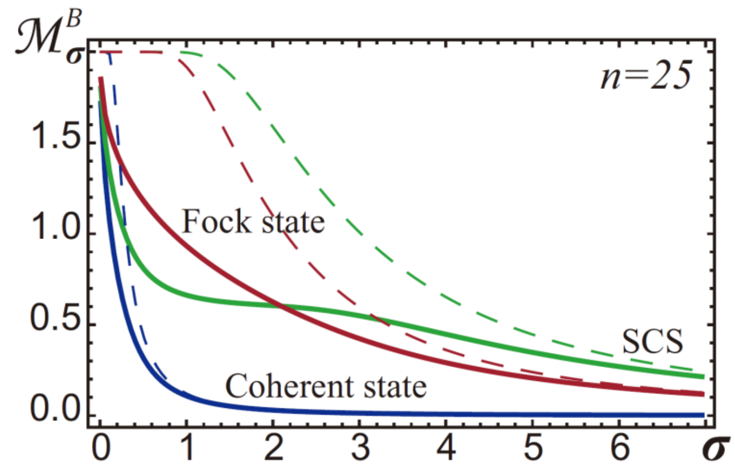
1) N-particle spin system with total spin measurement:

$$\hat{A} = \sum_{i=1}^{N} \vec{\alpha}^{(i)} \cdot \hat{\sigma}^{(i)}$$

2) Bosonic system with quadrature measurement:

$$\hat{X} = (\hat{a} + \hat{a}^{\dagger})/2$$





※ For precise measurement ($\sigma \rightarrow 0$), product states have larger values of M than the GHZ-state

Is a product state $\left(|0\rangle + |1\rangle\right)^{\otimes N}$ more 'macroscopically quantum' than a GHZ state $|0\rangle^{\otimes N} + |1\rangle^{\otimes N}$?

Disturbance-based measure of macroscopic coherence

H. Kwon, C.-Y. Park, K. C. Tan, H. Jeong, New J. Physics 19, 043024 (2017)

- Examples

1) N-particle spin system with total spin measurement:

$$\hat{A} = \sum_{i=1}^{N} \vec{\alpha}^{(i)} \cdot \hat{\sigma}^{(i)}$$

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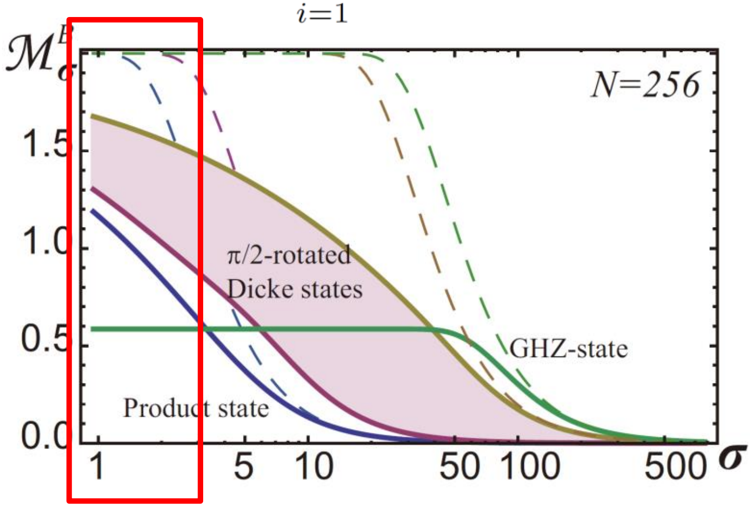
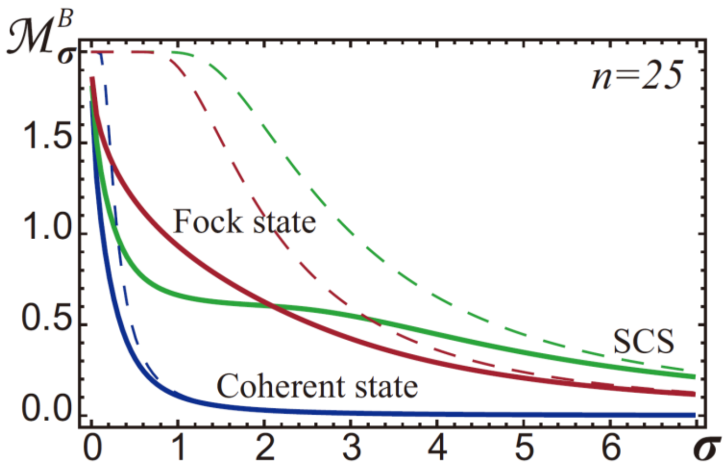




※ For precise measurement ($\sigma \to 0$), product states have larger values of M than the GHZ-state

→ ***The criteria of macroscopic coherence by Yadin and Vedral, (M1)-(M4), are insufficient.***

✓ Our solution: Take the coarse-graining scale to the classical measurement regime, $\sigma \gg \sqrt{N}$

➔ Quantifying the size of a superposition between (classically) distinct states.

Unification of the coherence and nonclassicality

- **Two kinds of quantum properties:**

Nonclassicality of Light

- Old: Roy Gauber *et al*. in 1960's
- A quantum state is expressed as a distribution over $\{|\alpha\rangle\}$, the eigenstates of the annihilation operator

$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

- Identified by negativity in the quasi-probability function (P function)
- Continuous Variables

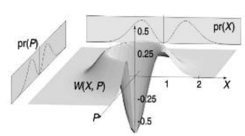
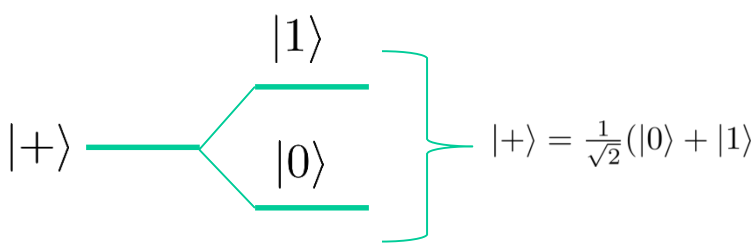


Figure 22: Wigner function of a single photon Fock state [from Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001)]

Coherence

- New: Martin Plenio *et al*. in 2014
- Identified by nonzero off-diagonal elements of the density matrix
- Discrete Variables
- Based on a resource theory (with 'free states' and 'free operations')



Unification of the coherence and nonclassicality

- **Two kinds of quantum properties:**

Nonclassicality of Light

- Old: Roy Gauber *et al*. in 1960's
- A quantum state is expressed as a distribution over $\{|\alpha\rangle\}$, the eigenstates of the annihilation operator

$$\rho = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$$

- Identified by negativity in the quasi-probability function (P function)
- Continuous Variables

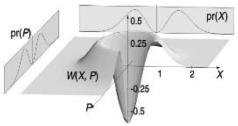


Figure 22: Wigner function of a single photon Fock state [from Lvovsky et al., Phys. Rev. Lett. 87, 050402 (2001)]

Coherence

- Quantum states are expressed as a density matrix, for a given basis $\{|i\rangle\}$ which is called the incoherent basis.

- The state is nonclassical when it cannot be expressed as a diagonal state of the form $\delta = \sum_i \delta_i |i\rangle\langle i|$, implying some superposition of incoherent states within the system.

Unification of the coherence and nonclassicality

K.-C. Tan, T. Volkoff, H. Kwon, and HJ, Phys. Rev. Lett. 119, 190405 (2017)

- The coherence is quantified in the coherent-state basis: α-coherence
- The α-coherence is zero iff the state is classical (the P function is positive definite); the coherence and nonclassicality are identical resources.
- Nonclassicality of light is a form of the coherence in a particular basis.
- Numerical examples show that the α-coherence is consistent with known phy sics and properties of nonclassical light.

Unification of the coherence and nonclassicality

K.-C. Tan, T. Volkoff, H. Kwon, and HJ, Phys. Rev. Lett. 119, 190405 (2017)

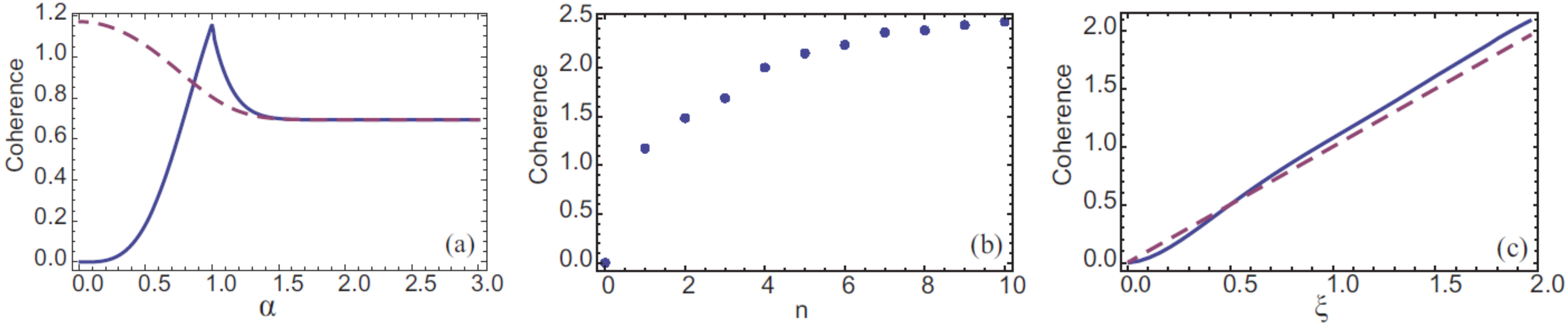


FIG. 1. α-coherence for photonic states. (a) Even(solid line) and odd(dotted line) cat states $|\alpha\rangle \pm |-\alpha\rangle$, (b) Fock states $|n\rangle$, and (c) squeezed states $S(\xi)|0\rangle$ are compared. For cat states, α-coherence approaches to $\log 2$, which is the maximum coherence for qubit states when α approaches infinity. α-coherence for Fock states and squeezed states increases as a photon number n and squeezing parameter ξ increase.

- **The ‘coherence’ and ‘quantum macroscopicity’ are different quantities although they are somehow related.**

Unification of the coherence and nonclassicality

K.-C. Tan, T. Volkoff, H. Kwon, and HJ, Phys. Rev. Lett. 119, 190405 (2017)

- Linear optical operations (passive linear optics operations and the displacement operation) do not increase the α-coherence.
- Nonlinear optical effects are required to increase the α-coherence (which is consistent with the fact that nonlinearities are required to generate nonclassical light.)
- ***These observations lead to a resource theory of liner optics***

  - Coherent states: free states

  - Linear optical operations: free operations

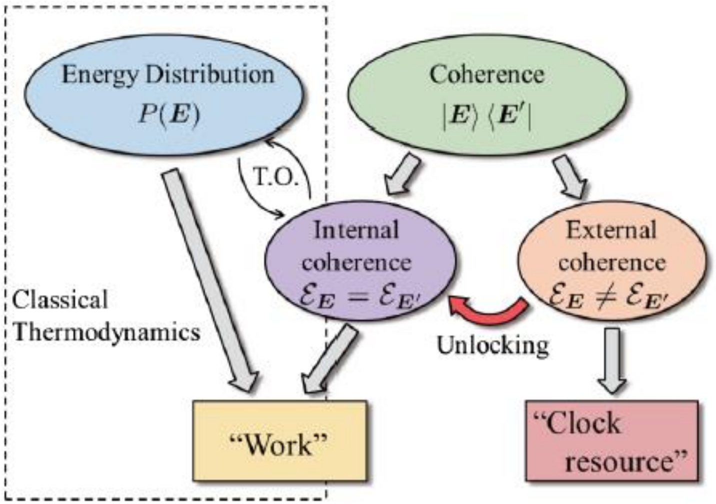
Ongoing work

- ***Resource for what? - We showed that any pure state with negativity in the P function (i.e. nonzero α-coherence) is useful for quantum metrology.***

Clock–Work Trade-Off Relation for Coherence in Quantum Thermodynamics

H. Kwon, H. Jeong, D. Jennings, B. Yadin, and M. S. Kim, Phys. Rev. Lett. 120, 150602 (2018)

- **Thermodynamic coherence**
  - ✓ **"Internal" coherence that admits an energetic value in terms of thermodynamic work**
  - ✓ **"External" coherence that does not have energetic value but that may be used as a "clock resource"**



THANK YOU