# On Finite Difference Approximations to the Incompressible Navier-Stokes equations

Byungjoon Lee*

## Abstract

Understanding fluid flow is one of the fundamental tasks in sciences. Among many mathematical models, Navier-Stokes equations have been served as the most successful tool for analyzing the behavior of fluids. Despite of intensive studies on the Navier-Stokes equations, we do not even know the existence of their global solution in mathematical sense [1]. For this reason, well-constructed numerical approximations of Navier-Stokes equations are ubiquitous for analyzing real phenemena related with fluids.

This talk considers on the following incompressible Navier-Stokes equations:

$$\begin{cases} \rho\left(U_t + (U\cdot\nabla)U\right) &= -p + \mu\nabla\cdot\left(\mu\left(\nabla U + \nabla U^T\right)\right) + \rho g \\ \nabla\cdot U &= 0 \end{cases} \text{ in } \Omega \tag{1}$$

In this talk, numerical approaches based on the finite difference method for (1) and will be presented. And also, a brief introduction on the Level Set method will be given for the purpose of describing the fluid's interface motion. Finally, an energy-stable numerical approximation to the Incompressible Navier-Stokes equations is presented with perspective on solving the millenium problem.

## References

[1] James A. Carlson, Arthur Jaffe, Andrew Wiles, The Millennium Prize Problems, American Mathematical Soc., 2006

[2] A.J. Chorin, A numerical method for solving incompressible viscous flow problems, J. Comput. Phys. 135(2), 1997, 118-125

[3] B.Lee, J. Darbon, S. Osher, M. Kang, Revisiting the Redistancing Problem using the Hopf-Lax Formula, J. Comput. Phys., 330, 2017, 268-281

[4] B.Lee, C. Min, An Energy-Stable method for solving the Incompressible Navier-Stokes Equations with Non-slip boundary condition, J. Comput. Phys., 360, 2018, 104-119

[5] C.K.Cho, B.Lee, S.Kim, Dual-Mesh Characteristics for Particle-Mesh Methods for the simulation of convection-dominated flows, SIAM J. Sci, Comput., 40(3), 2018, A1763-A1783

*Department of Mathematics, Catholic Univerisity of Korea (CUK), Republic of Korea
blee@catholic.ac.kr