On Finite Difference Approximations to the Incompressible Navier-Stokes equations

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Abstract
Understanding fluid flow is one of the fundamental tasks in sciences. Among many mathematical models, Navier-Stokes equations have been served as the most successful tool for analyzing the behavior of fluids. Despite of intensive studies on the Navier-Stokes equations, we do not even know the existence of their global solution in mathematical sense [1]. For this reason, well-constructed numerical approximations of Navier-Stokes equations are ubiquitous for analyzing real phenomena related with fluids.

This talk considers on the following incompressible Navier-Stokes equations:

\[
\begin{aligned}
\rho U_t + (U \cdot \nabla) U &= - \nabla p + \mu \nabla \cdot (\mu (\nabla U + \nabla U^T)) + \rho g \\
\nabla \cdot U &= 0
\end{aligned}
\] in \( \Omega \) \hspace{1cm} (1)

In this talk, numerical approaches based on the finite difference method for (1) and will be presented. And also, a brief introduction on the Level Set method will be given for the purpose of describing the fluid's interface motion. Finally, an energy-stable numerical approximation to the Incompressible Navier-Stokes equations is presented with perspective on solving the millenium problem.

References

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