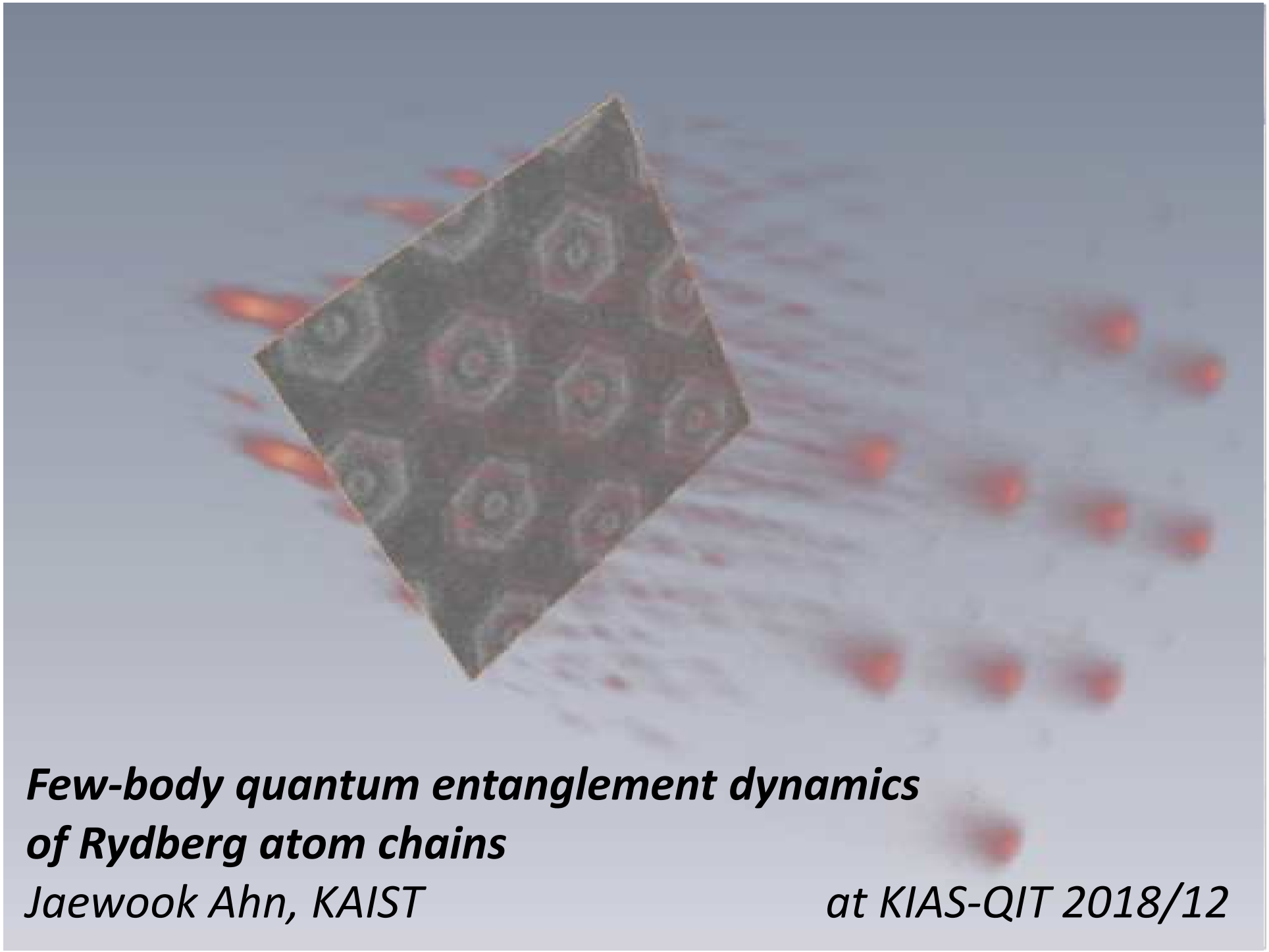


Few-body quantum entanglement dynamics of Rydberg atom chains

Jaewook Ahn, KAIST

at KIAS-QIT 2018/12

Dynamics of entangled Rydberg atoms

$$\hbar \frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle + \textit{dephasing and decays}$$

$$|\psi(t)\rangle = a_0|00000\cdots\rangle + a_1|10000\cdots\rangle + a_2|01000\cdots\rangle + a_3|11000\cdots\rangle + \cdots + a_{2^N-1}|11111\cdots\rangle$$
$$= \sum_{k=0}^{2^N-1} a_k(t)|k\rangle \quad : (2^N \textit{ microstate-bases})$$

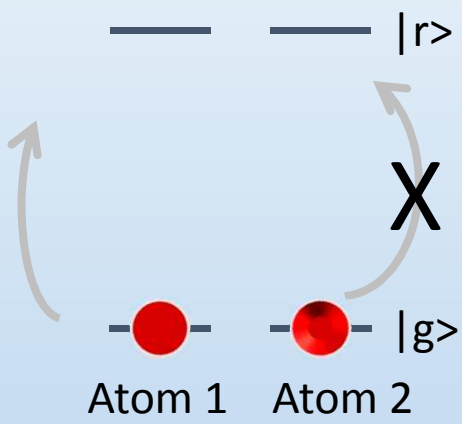
- *Passive dynamics (quenching + geometry) : Quantum Simulation*

  e.g.) $H = \sum_i \frac{\hbar\Omega}{2}\sigma_x^i + \sum_{i,j} V_{ij}\, n^i n^j$ *(quantum Ising)*

  $$|\psi(t)\rangle = a_0|W_0\rangle + a_1|W_1\rangle + a_2|W_2\rangle + a_3|W_3\rangle + \cdots + a_{N/2}\,|W_{N/2}\rangle$$
  $$= \sum_{k=0}^{N/2} a_k(t)|W_k\rangle \quad : (N/2 + 1 \;\; W\textit{-bases})$$

- *Controlled dynamics : e.g. Quantum Annealing*
  = Coherent Control of Entangled Systems

(Future?) Fully programmable dynamics : (Universal) Quantum Computation



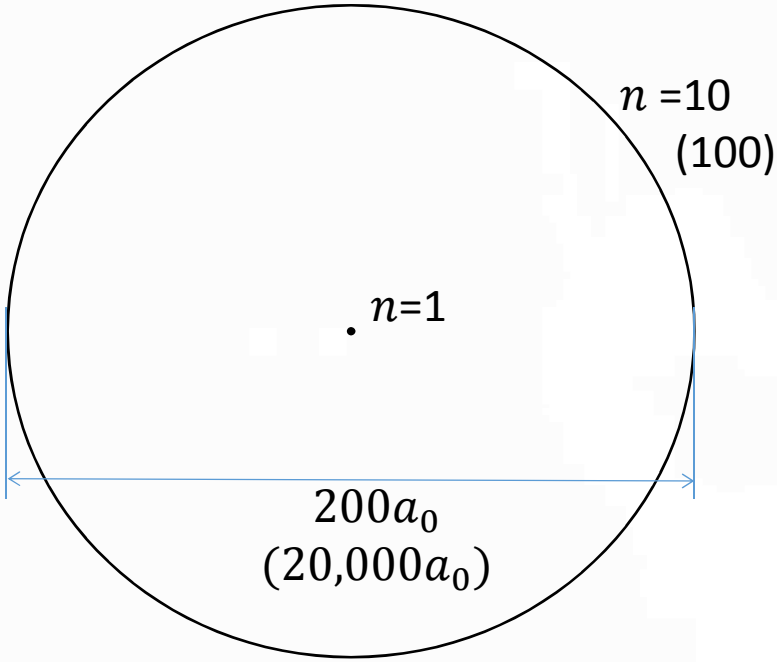
Rydberg-atom dipole-blockade effect

Quantum entanglement

$$|\psi_{12}\rangle = \sqrt{1/2}\,(|g\rangle_1|r\rangle_2 + |r\rangle_1|g\rangle_2)$$

Rydberg Atoms → huge interaction

Rydberg atoms are atoms in high n-states (n : principal quantum number).



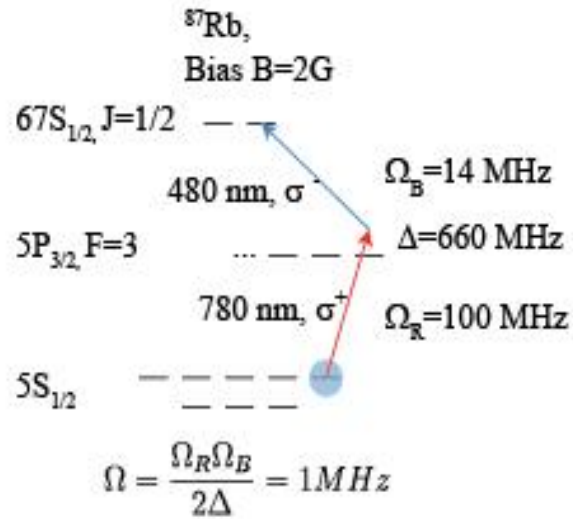
Properties (Rubidium)	scaling	5p	67	90
Orbital radius (μm)	n^2	0.0002	0.9	1.6
Radiative lifetime(μs)	n^3	0.026	250	618
Ryd-Ryd int. (GHz at 1μm)	n^{11}	NA	970	25000

^{87}Rb 67S state has 5MHz interaction @ 5μm distance.

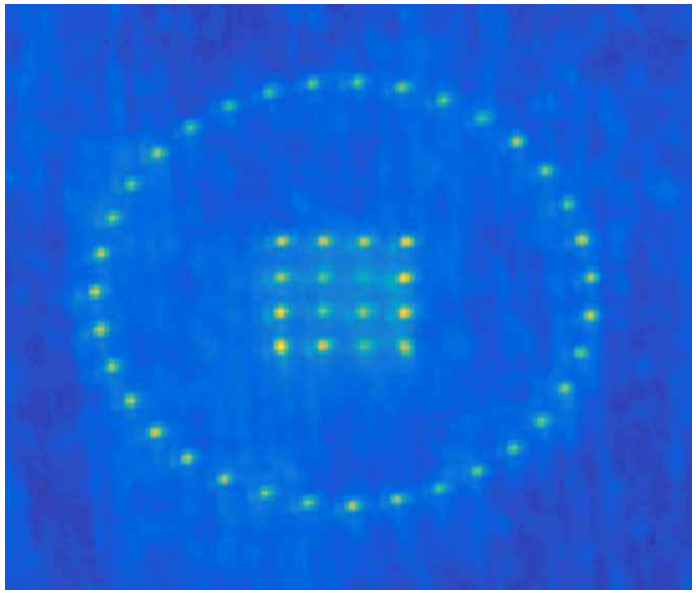
The interaction scales as $B = -C_6/R^6$.

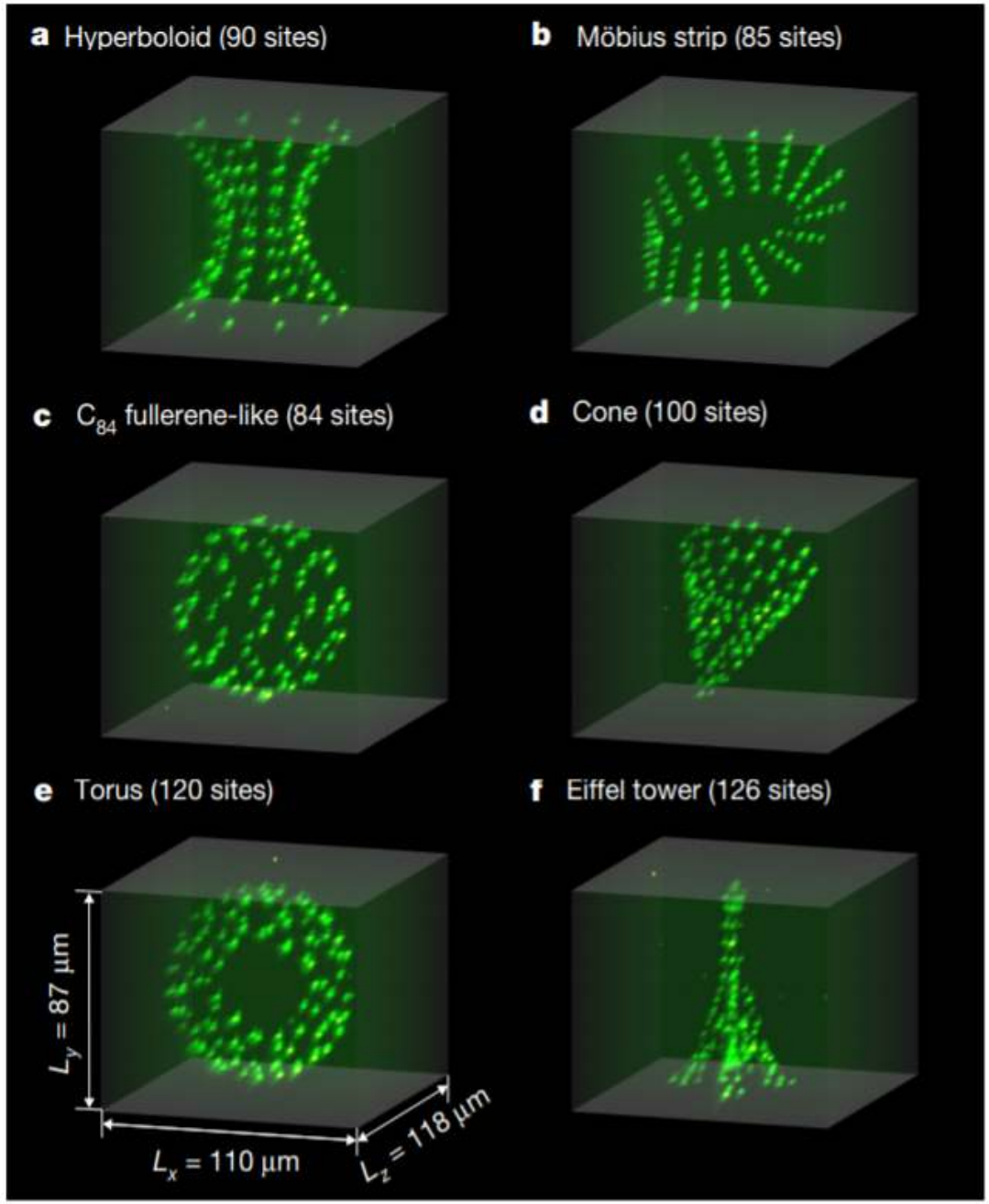
Two-photon Rabi oscillation: $\Omega = \frac{\Omega_{780}\,\Omega_{480}}{2\Delta_I} = (2\pi)MHz$

Blockade radius: $r_B = (C_6/\Omega)^{1/6} = 6.6\mu m$

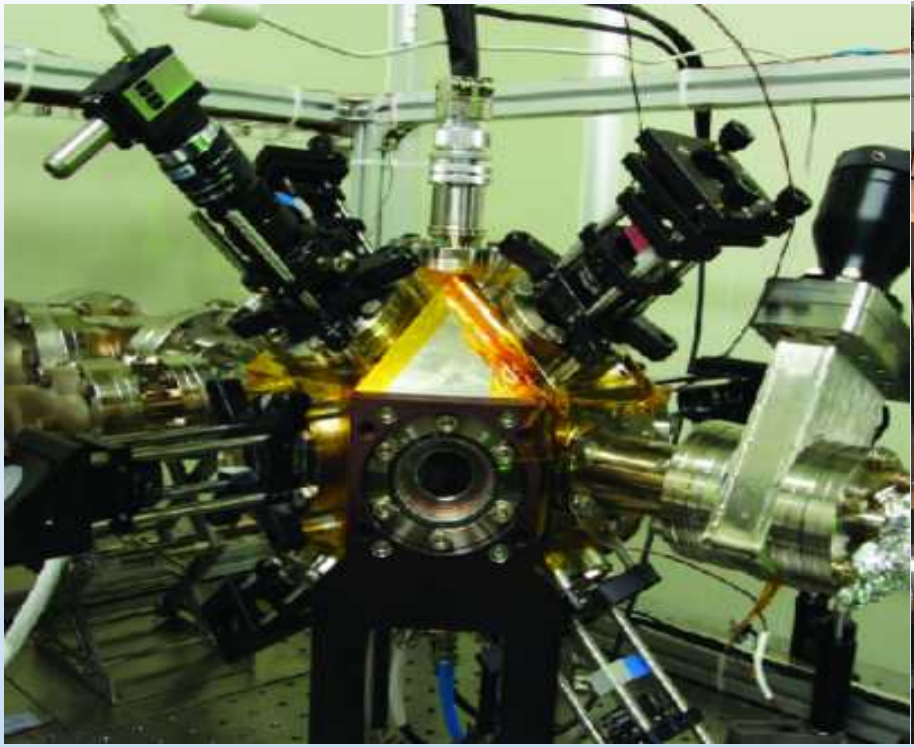


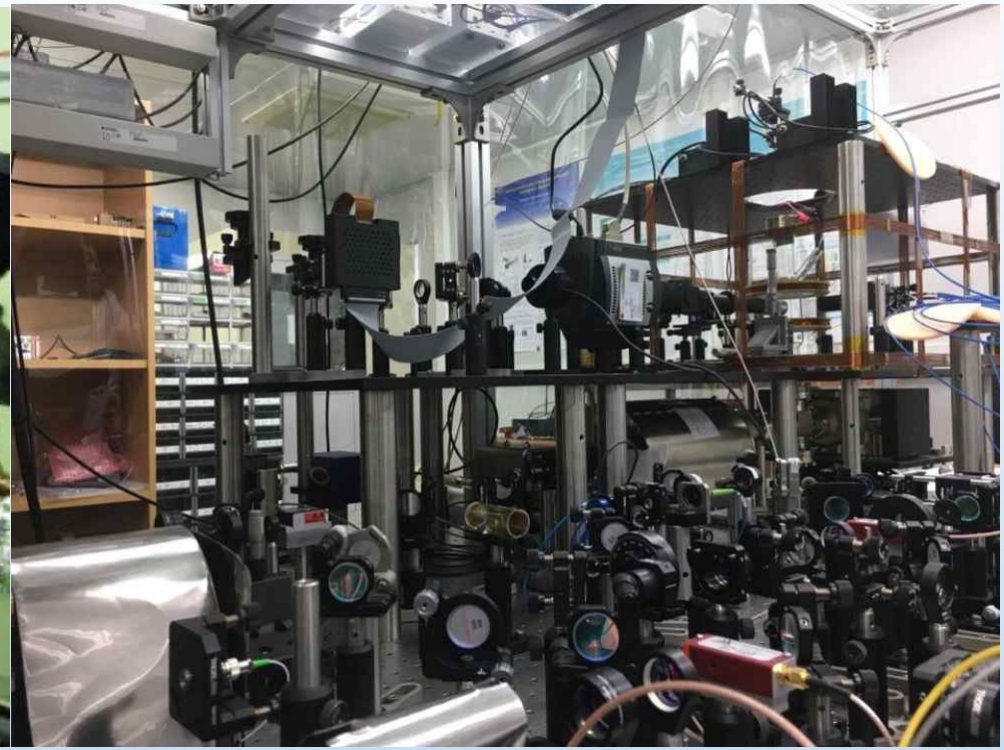
Lee et al., “3D rearrangement of single atoms using actively controlled optical microtraps,"Optics Express (2016).





Barredo et al., “Synthetic 3D atomic structures assembled atom by atom,” Nature (2018).





KAIST Rydberg quantum simulator

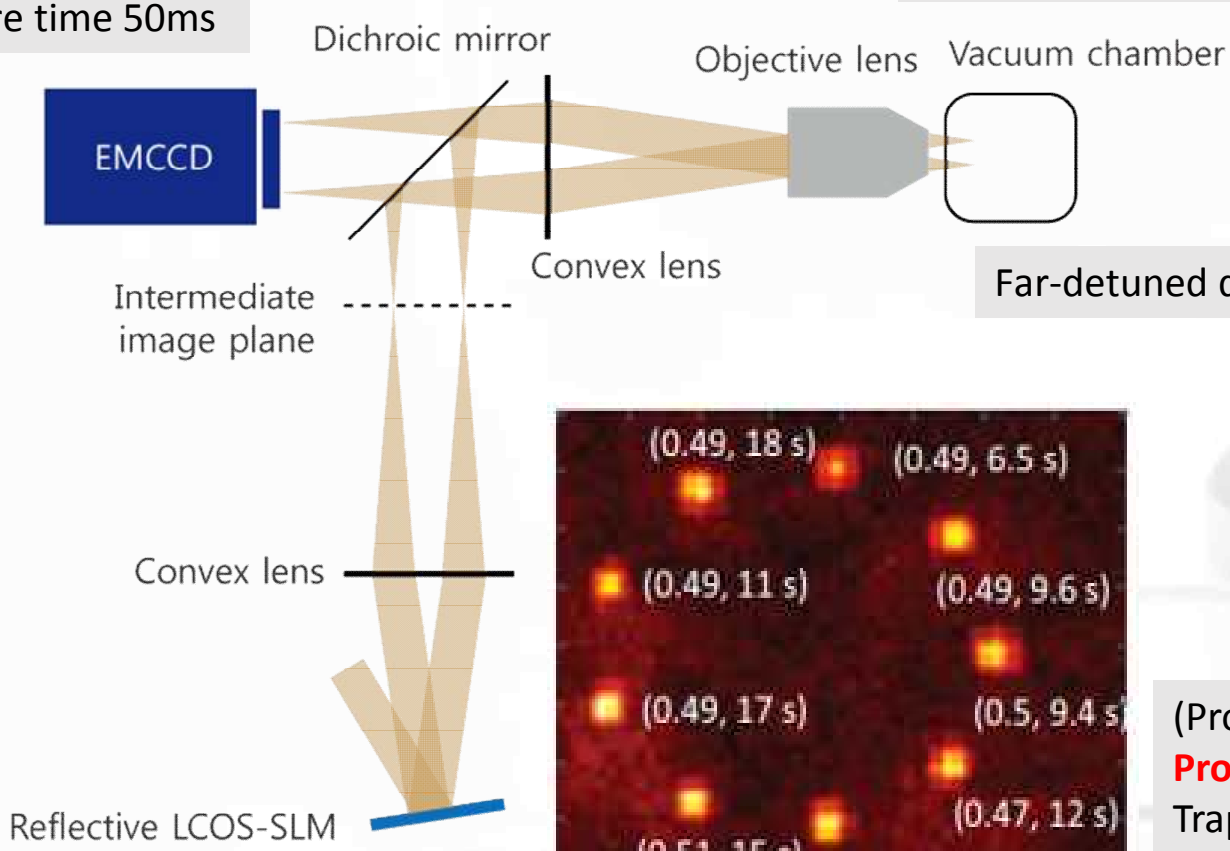
= 80 optical tweezers (fine position-control of 40 single atoms)
+ Laser cooling and trapping apparatus (Rb, 300K → 30μK)
+ high resolution microscope (single atom real-time imaging)
+ high-performance digital computer (tracking quantum dynamics)
+ various light sources and control electronics

Holographic microtraps (tweezer trap)

^{87}Rb atoms Pressure 3×10^{-10} Torr

Density 10^{10} cm^{-3} Temperature $100\ \mu\mathrm{K}$

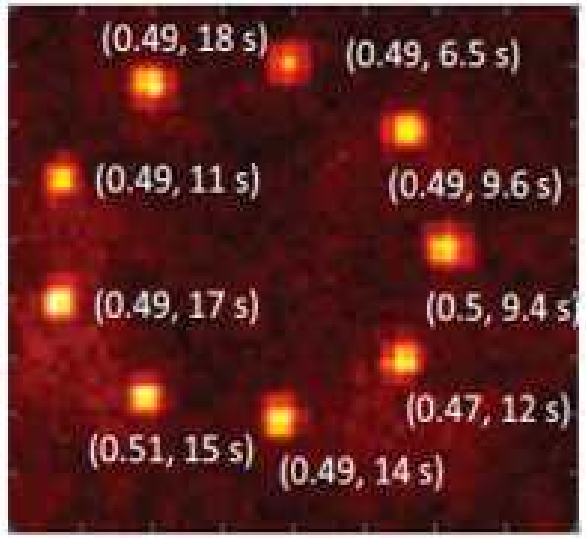
Exposure time 50ms



Far-detuned dipole trap (λ =820 nm)

Trap depth 1.4 mK

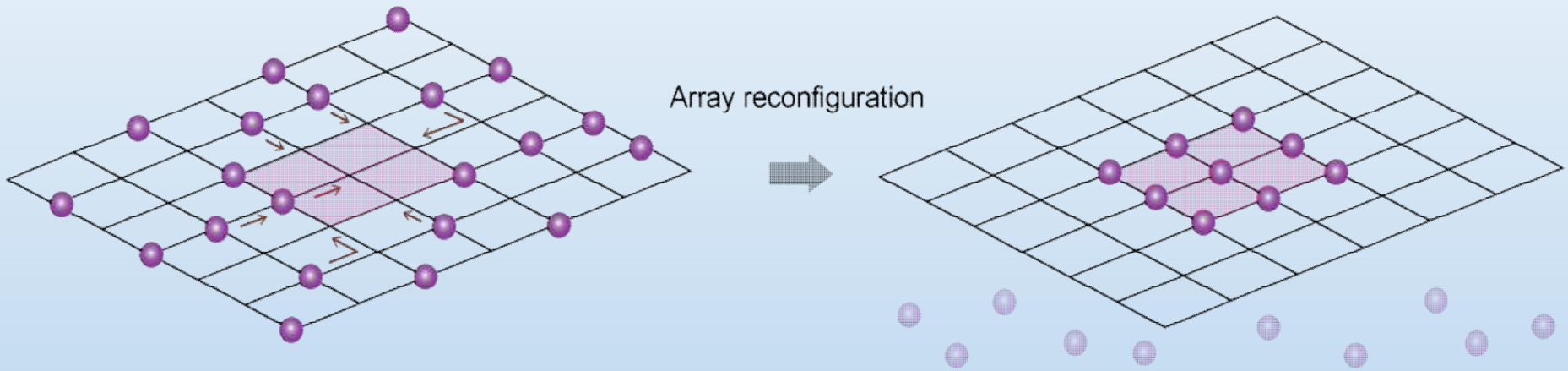
Focal radius 1 μm



(Probability, Trapping time)

Probability = 0.5

Trapping time = 10~40 s



Making a defect-free array of atoms "tweezer trap"

D. Weiss et al., "Another way to approach zero entropy for a finite system of atoms," PRA 70, 040302 (2004).

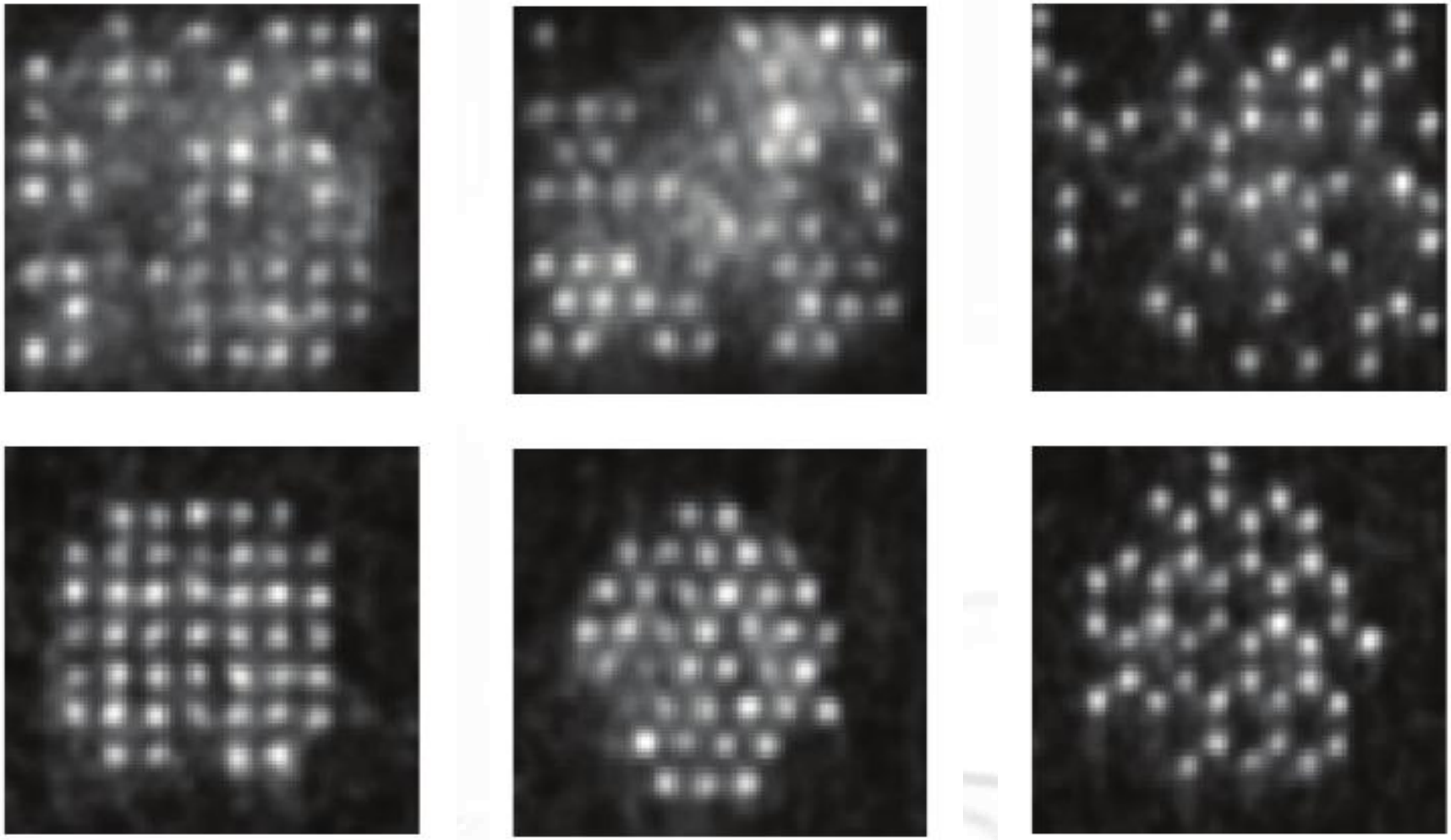
H. Kim and W. Lee et al., "In situ single-atom array synthesis by dynamic holographic optical tweezers," Nature Comm. 7, 13316 (2016).

D. Barredo et al., "An atom-by-atom assembler of defect-free arbitrary 2D atomic arrays," Science 354, 1021 (2016).

M. Endres et al., "Atom-by-atom assembly of defect-free one-dimensional cold atom arrays," Science 354, 1024 (2016).

Designer's atom lattice (80 → 40)

Square lattice | Triangular lattice | Kagome lattice



Modified Gerchberg-Saxton algorithm for hologram generation
+ Hungarian algorithm for atom-site matching: e.g. in PRA 95, 053424 (2017)

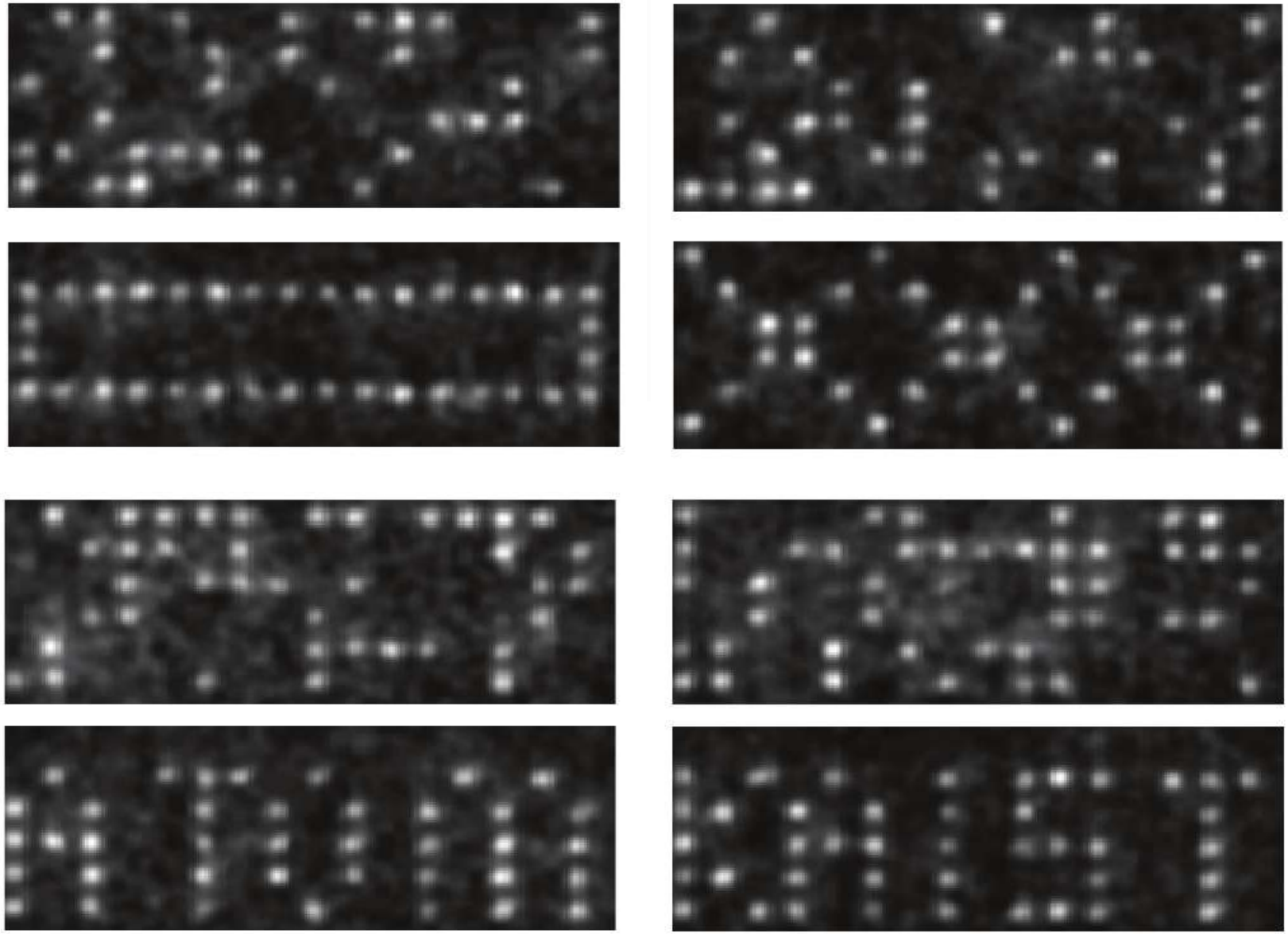
More examples (80→40) : power-limited

before

after

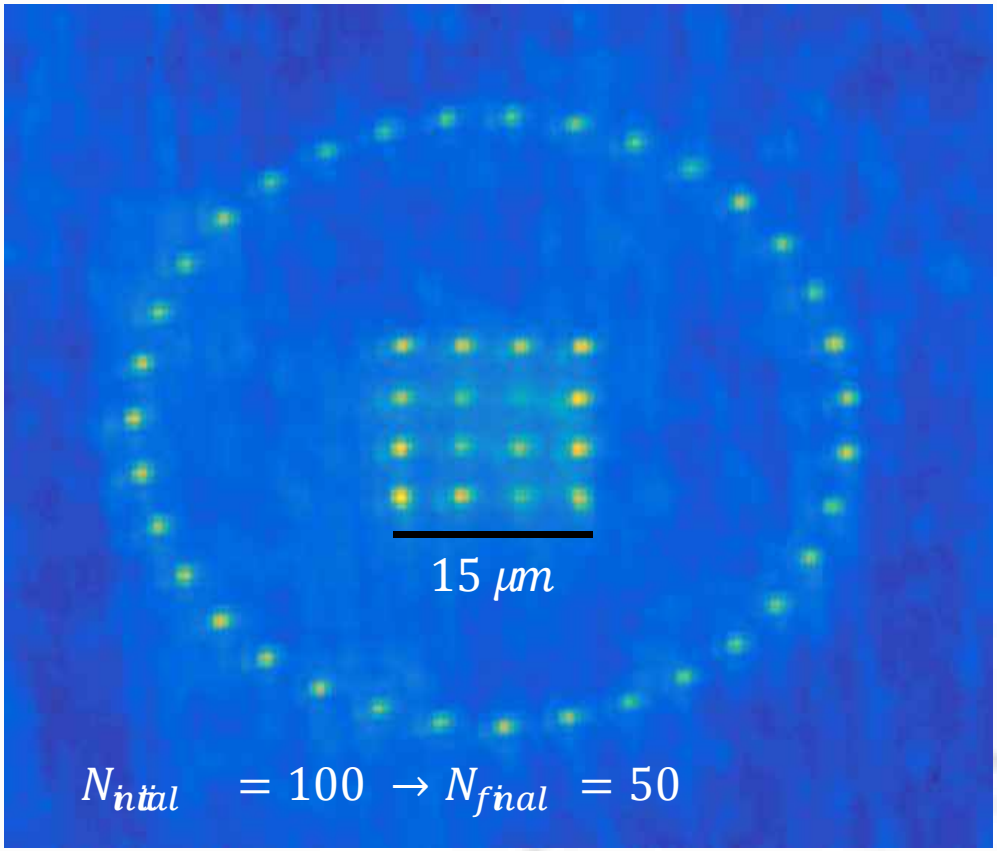
before

after



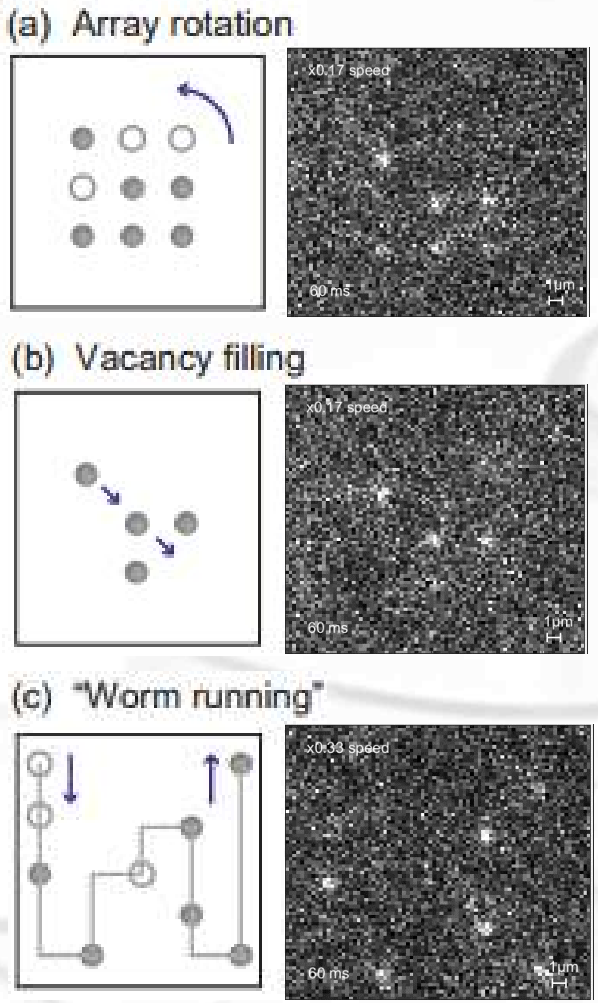
Atomic qubit platform :"tweezer trap"

Atoms are freely moved onto a designer's lattice with d = 2-10 μm, Δx < 0.5 μm

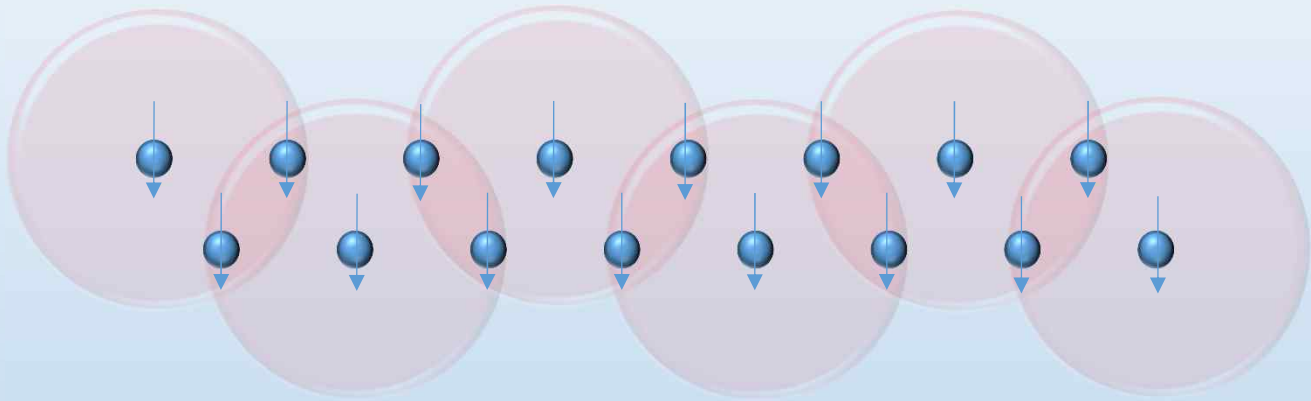


"Tweezer trap" atom lattice

Atom Tweezers TETRIS



W. Lee, H. Kim, JA, Optics Express **24**, 9816 (2016).
H. Kim and W. Lee et al., Nature Comm. 7, 13316 (2016).



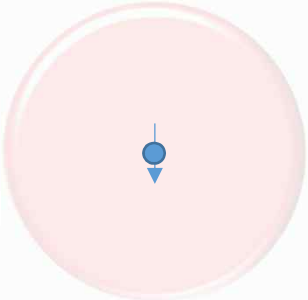
Quantum simulation

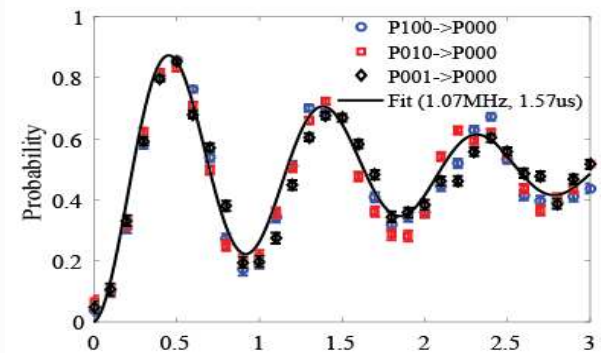
$$H = \sum_i \frac{\hbar\Omega}{2}\sigma_x^i + \sum_{i,j} V_{ij}\, n^i n^j \quad \textit{(quantum Ising-like problem)}$$

N=30 rubidium atoms are arranged as a zig-zag or linear chain
They are entangled through Rydberg excitation at the dipole blockade regime.
The dynamics is governed by quantum Ising-like model.
: Measurement of dynamic behavior of an N entangled system (quantum simulator).

Entanglement of Rydberg atoms

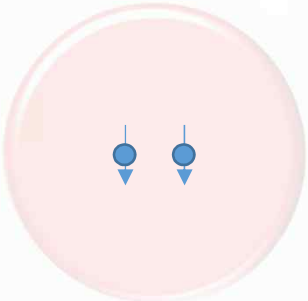
Entanglement via Rydberg blockade

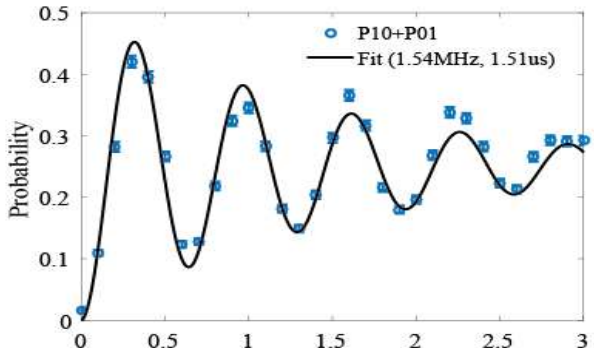




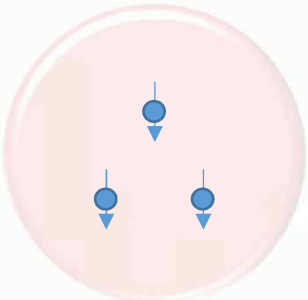
$$|0\rangle = |ground\ \ \rangle$$
$$|1\rangle = |Rydberg\ \ \rangle$$

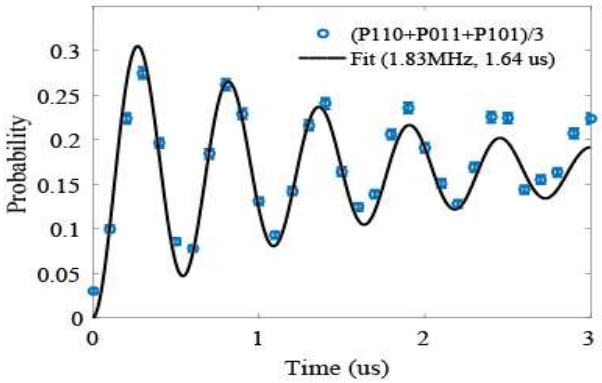
$$|\psi(\mathrm{t})\rangle = \cos\Omega t\,|0\rangle + i\sin\Omega t\,|1\rangle$$



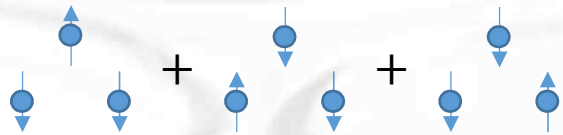


$$|\psi(\mathrm{t})\rangle = \cos\sqrt{2}\Omega t\,|00\rangle + i\sin\sqrt{2}\Omega t\,|B\rangle$$
$$|\mathrm{B}\rangle = \frac{|01\rangle+|10\rangle}{\sqrt{2}}$$ (pair-wise entanglement)



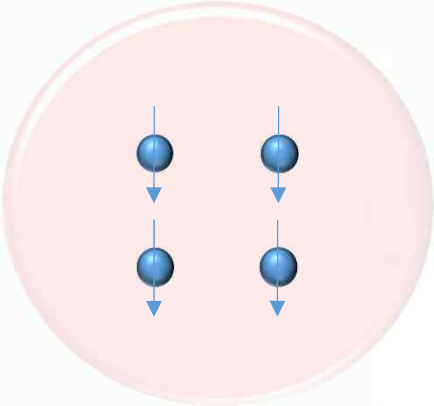


$$|\psi(\mathrm{t})\rangle = \cos\sqrt{3}\Omega t\,|000\rangle + i\sin\sqrt{3}\Omega t\,|W\rangle$$
$$|\mathrm{W}\rangle = \frac{|001\rangle+|010\rangle+|100\rangle}{\sqrt{3}}$$
(entangled single-excited state)



Entanglement of Rydberg atoms

Entanglement with full blockade



$$|\psi(\mathrm{t})\rangle = \cos\sqrt{4}\Omega t\,|0000\rangle + i\sin\sqrt{4}\Omega t\,|W\rangle$$

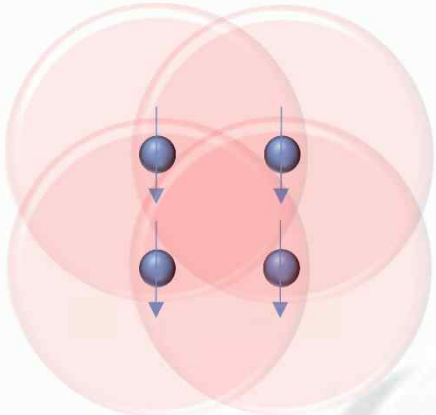
$$|\mathrm{W}\rangle = \frac{|0001\,\rangle+|0010\,\rangle+|0100\,\rangle+|1000\,\rangle}{\sqrt{4}}$$

$$H = \sum_i \frac{\hbar\Omega}{2}\sigma_x^i + \sum_{i,j} V_{ij}\, n^i n^j$$

Entanglement with pair-wise blockade

$$H = \sum_i \frac{\hbar\Omega}{2}\sigma_x^i + \sum_{(i,j)_{nn}} V_{ij}\, n^i n^j$$

(Quantum Ising w/ transverse field)

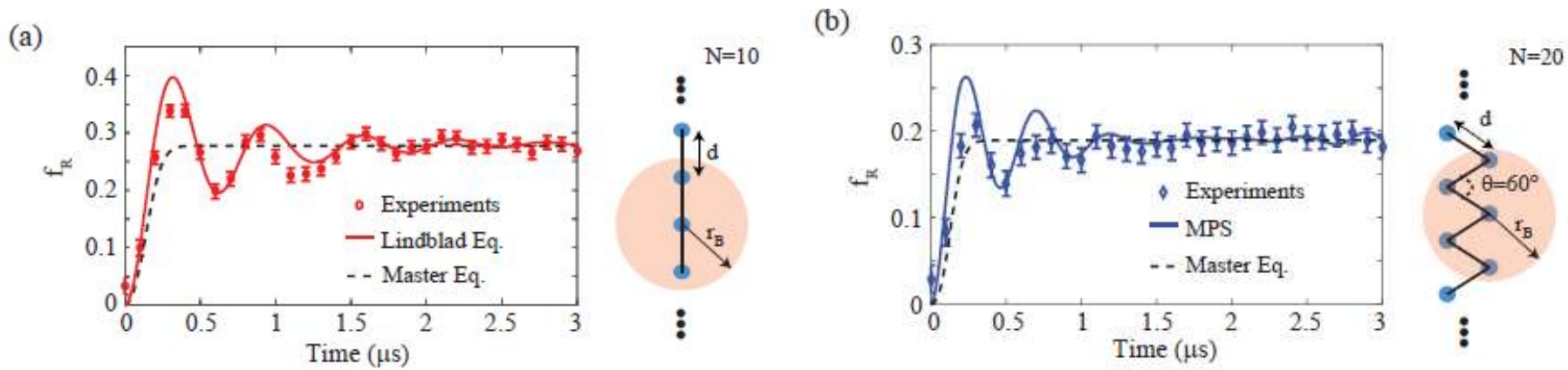


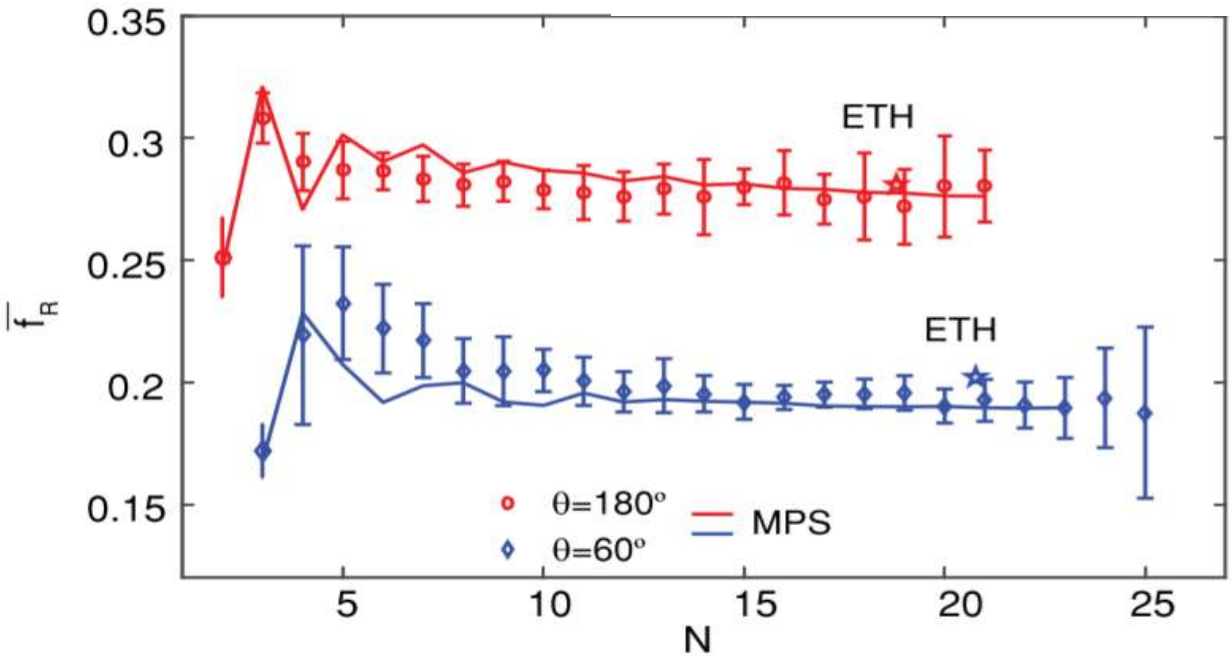
$$|\psi(\mathrm{t})\rangle = \frac{1+2\cos\sqrt{2}\Omega t}{3}|0000\rangle + \frac{\sqrt{2}(1-\cos\sqrt{2}\Omega t)}{3}|W_2\rangle + \frac{2i}{\sqrt{6}}\sin\sqrt{2}\Omega t\,|W\rangle$$

$$|\mathrm{W}\rangle = \frac{|0001\,\rangle+|0010\,\rangle+|0100\,\rangle+|1000\,\rangle}{\sqrt{4}}$$

$$|\mathrm{W}_2\rangle = \frac{|0101\rangle+|1010\rangle}{\sqrt{2}}$$ (doubly excited states)

Quantum simulation with N=25 qubits

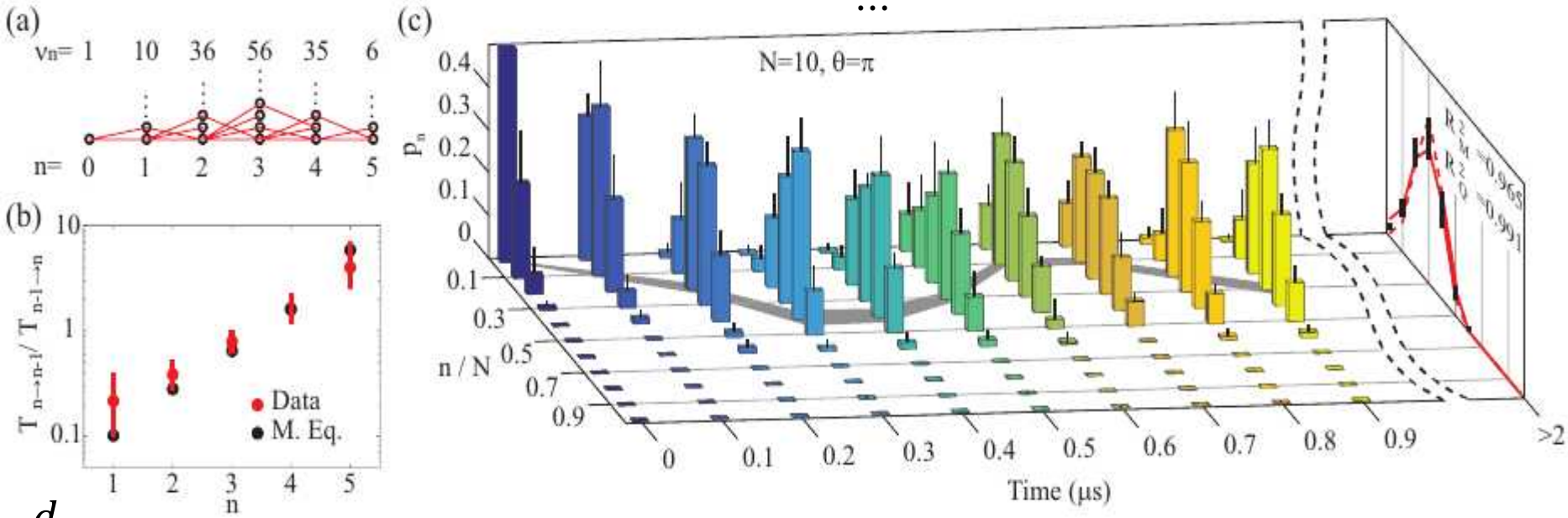




H. Kim, et al., "Detailed balance of thermalization dynamics in Rydberg-atom quantum simulators," Phys. Rev. Lett. 120, 180502 (2018).

Detailed balance at saturation

$$p_0(t) = |\langle\downarrow\downarrow\downarrow\cdots|\psi(t)\rangle|^2 = |\langle W_0|\psi(t)\rangle|^2 = |\langle J,-J|\psi(t)\rangle|^2$$
$$p_1(t) = |\langle\uparrow\downarrow\downarrow\cdots|\psi(t)\rangle|^2 + |\langle\downarrow\uparrow\downarrow\cdots|\psi(t)\rangle|^2 + \cdots = |\langle W_1|\psi(t)\rangle|^2 = |\langle J,m=1-J|\psi(t)\rangle|^2$$
$$p_2(t) = |\langle\uparrow\downarrow\uparrow\cdots|\psi(t)\rangle|^2 + \cdots = |\langle W_2|\psi(t)\rangle|^2 \approx |\langle J,m=2-J|\psi(t)\rangle|^2$$
$$\cdots$$



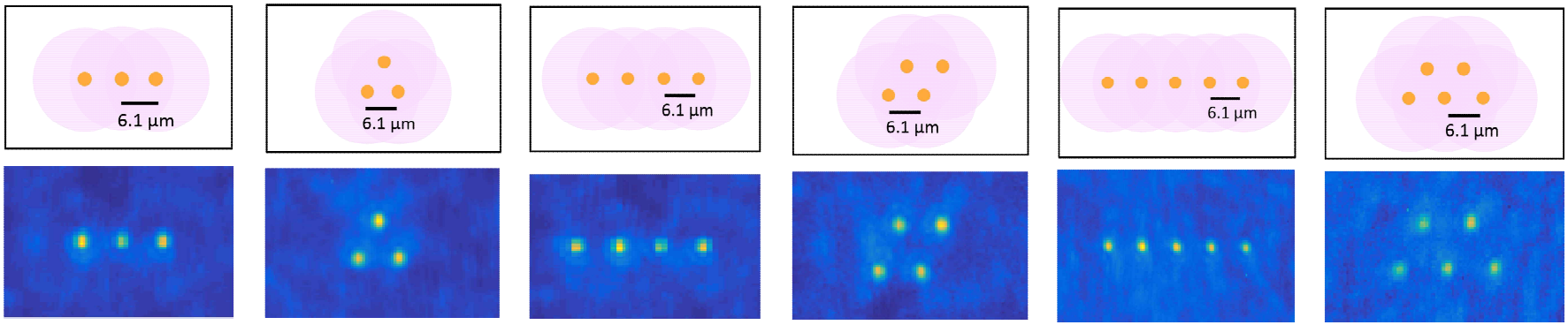
$$\frac{d}{dt}p_n = -T_{n\to n\pm1}p_n + T_{n\pm1\to n}p_{n\pm1}$$

% Fokker-Planck formalism
C. Ates, Phys. Rev. Lett. 108, 110603 (2012).

The $p_n(t)$ packet diffuses and oscillate.
Then settles down to the equilibrium.
Detailed balance gives the rate equation.
% Wigner rotation among Dicke states fails.

H. Kim, et al., "Detailed balance of thermalization dynamics in Rydberg-atom quantum simulators," Phys. Rev. Lett. 120, 180502 (2018).

(Latest result)
Few-body dynamics of Rydberg atoms

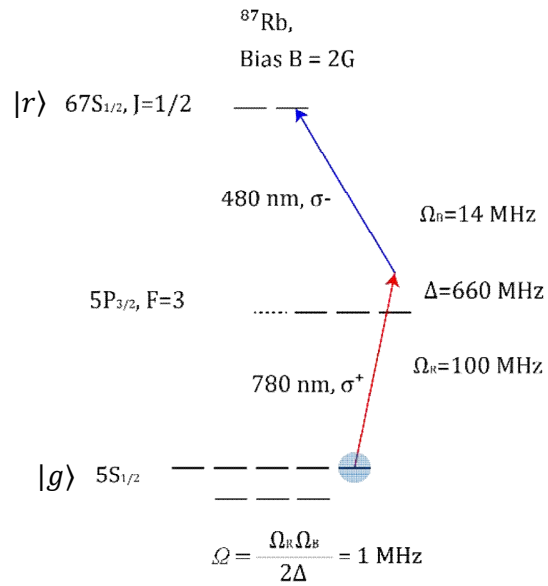


N-atom interaction via Rydberg blockade

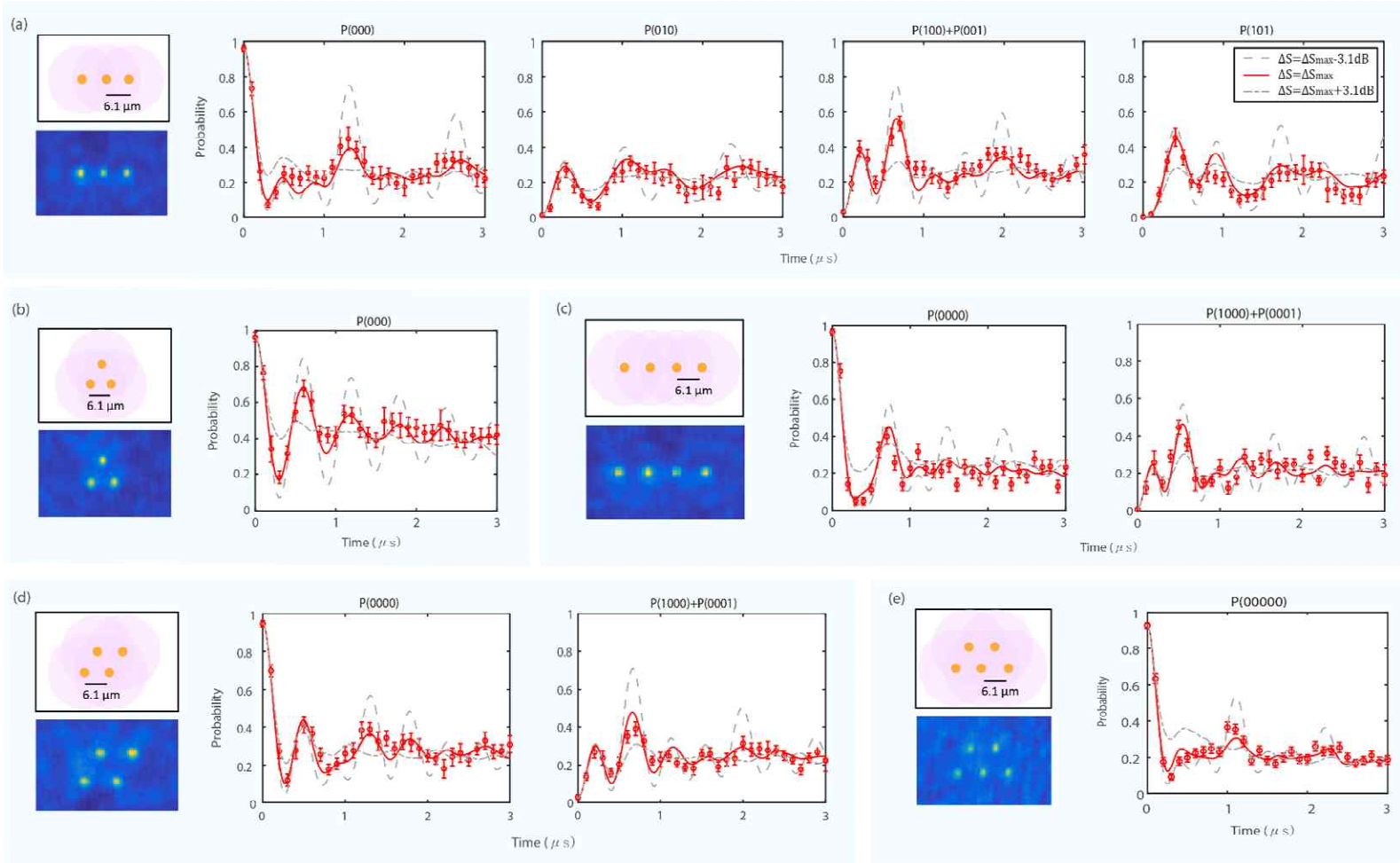
$$H = \sum_m \frac{\hbar\Omega}{2}\sigma_x^m + \sum_{m<l} V_{m\,l} n_m n_l$$

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \sum_p \left[L_p \rho L_p^\dagger - \frac{1}{2}\{L_p^\dagger L_p, \rho\}\right]$$

where V is a Van der Waals interaction $|V(R)| \propto \frac{1}{R^6}$



$$H = \sum_m \frac{\hbar\Omega}{2}\sigma_x^m + \sum_{m<l} V_{m\,l} n_m n_l \quad \dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \sum_p \left[L_p \rho L_p^\dagger - \frac{1}{2}\{L_p^\dagger L_p, \rho\} \right]$$



Peak value in scan → Ω uncertainty = $\pm 5\%$, ΔS uncertainty =$\pm 3.1\ dB$

KAIST team for atomic quantum computing

1. Ultrafast quantum gates (THz-clock speed computing)
2. Quantum simulation (entangled N Rydberg-atom dynamics)
3. Coherent control of entangled N-qubit system
4. Quantum computing (circuit-model, high-fidelity quantum gates)

Members: **Woojun Lee**, Yunheung Song,
Minhyuk Kim, Hanlae Cho, Woojeong Byun

Ex-members:
Hyosub Kim (LANL, USA)
Geol Moon (CNU)
Han-gyeol Lee (Kassel, Germany)
Jongseok Lim (Imperial College London)
Sangkyung Lee (ADD)
Kanghee Lee, Daehoon Han, Minwoo Yi, etc.

Collaboration:
C. Y. Park (KRISS), H. S. Sim (KAIST)
S. Guerin (Bourgorgne), R.

Financial Support from SAMSUNG 삼성미래기술육성재단