CONTROL OF VORTEX DOMINATED FLOWS THROUGH VORTEX DYNAMICS

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For incompressible fluid, the vorticity $\boldsymbol{\omega}$, which is defined by $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$ from its velocity field $\boldsymbol{u}$, is *frozen* on a fluid particle, and it thus behaves like a material particle. Since those vortex particles are apt to concentrate in localized regions, complex evolutions of high Reynolds number flows are often described in terms of those localized vortex structures and their mutual interactions. In this sense, vortex structures are often regarded as "sinews and muscles" of fluid flows. Hence, it is of a theoretical and practical significance to consider the stabilization of dynamics of vortex structures to maintain the kinematic function and effect brought by those vortex structures over the whole flow evolutions.

Generally, when the viscosity of the fluid is negligibly small, the vorticity structure in 3D space increases its own intensity greatly and changes its shape and topology in a complex manner every moment, it is very difficult to control its dynamics. However, on the other hand, under a certain circumstance, for instance, where an obstacle is embedded in the uniform flow, the flow is strongly restricted in this direction and its evolution is thus dominated by almost nearly 2D vortex structures that exist on the section. In 2D space, since the vorticity becomes a scalar function and conserved along the path of a fluid particle for inviscid flows, it is relatively easy to construct a theoretical model stabilizing the dynamics of 2D vortex structures. Utilizing this 2D stabilization model, one can not only understands how the vortex dynamics is stabilized, but its controlability is also examined.

In this talk, I will present our recent results on the stabilization and control of 2D vortex sturcutures. The first example is the linear feedback stabilization of a point vortex entrapped around an aerofoil with multiple wings [1]. Since the entrapped point vortex generates an additional lift acting on the wing, its stabilization gives rise to an efficient wing design. The stabilization model is constructed through the potential flow theory with point vortices on a multiply connected domain, in which the dynamics of the point vortex is stabilized by a blow/suction placed on the wing. The second example is the stabilization of a vortex sheet, which is an inviscid model of strong shear layers [2]. Stabilization of vortex sheets are extremely difficult since infinitesimal perturbations with wavenumber n on a flat vortex sheet grow exponentially in time at a rate proportional to $|n|$, which is known as the Kelvin-Helmholtz instability. This high-wavenumber instability gives rise to ill-posedness in the sense of Hadamard, but it also causes the formation of a finite-time curvature singularity in the sheet. We construct a closed-loop system stabilizing the vortex sheet with using an array of source/sink structures. This talk is based on the joint works with Dr. Rhodri Nelson at Imperial College London and Prof. Bartosz Protas at McMaster University.

References

[1] R. Nelson, B. Protas and T. Sakajo, *Linear feedback stabilization of point vortex equilibria near a Kasper Wing*, J. Fluid Mech., **827** 121–154 (2017) (doi:10.1017/jfm2017.484) [arXiv:1703.07720]

[2] B. Protas and T. Sakajo, *Harnessing the Kelvin-Helmholtz instability: feedback stabilization of an inviscid vortex sheet*, Journal of Fluid Mechanics, **852** 156–177 (2018) [arXiv:1711:03249]

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