<Complex variables approach to a free boundary problem of 2d vortex patch>

A free-boundary problem for two-dimensional Euler flows with uniform vorticity is considered in complex variables formulation on the surfaces of a plane and a sphere. Utilizing the stereographic projection and the argument principle, a circle turns out to be the unique solution for both cases, if the flow speed is constant on the boundary. Below are some references:

[1] Sun-Chul Kim, A free-boundary problem for Euler flows with constant vorticity on the sphere. J. Math. Anal. Appl. 465 (2018), no. 1, 703-711.

[2] Sun-Chul Kim, Some remarks on free boundaries of recirculating Euler flows with constant vorticity. Inverse problems and related topics (Kobe, 1998), 89–95, Chapman & Hall/CRC Res. Notes Math., 419, Chapman & Hall/CRC, Boca Raton, FL, 2000.

[3] Sun-Chul Kim, A free-boundary problem for Euler flows with constant vorticity. Appl. Math. Lett. 12 (1999), no. 4, 101–104.