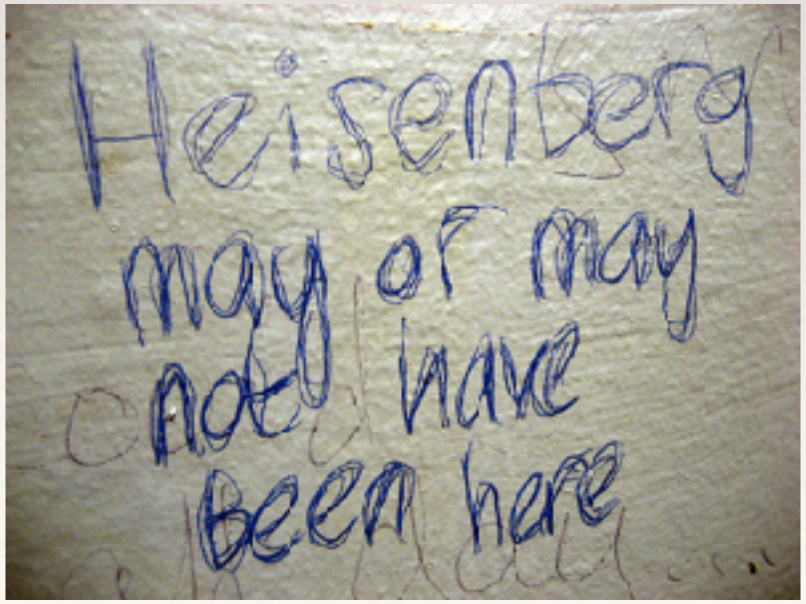
The 4th KIAS Workshop on Quantum Information and thermodynamics

Wonmin Son, Sogang Univ. sonwm@physics.org

[Un]known information in quantum uncertainty and quantum computation

What is known and unknown about the quantum uncertainty relation and quantum computation



Contents

Uncertainty of continuous variable quantum state with Entropy; Information in Quantum Optics

Question; [un]known information

- How different is the quantum uncertainty when it is expressed in entropy?
- How does the quantum state of light behave differently when it is optimized by Shannon entropy?

$$\sigma(A)\sigma(B) \geq \hbar/2|\langle[A,B]\rangle|$$

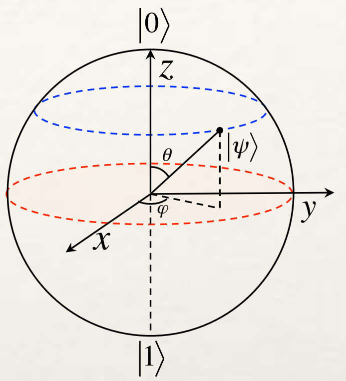
$$S(\rho) = \mathrm{Tr}[\rho \ln \rho]$$

$$H(A) + H(B) \geq -2\ln c$$

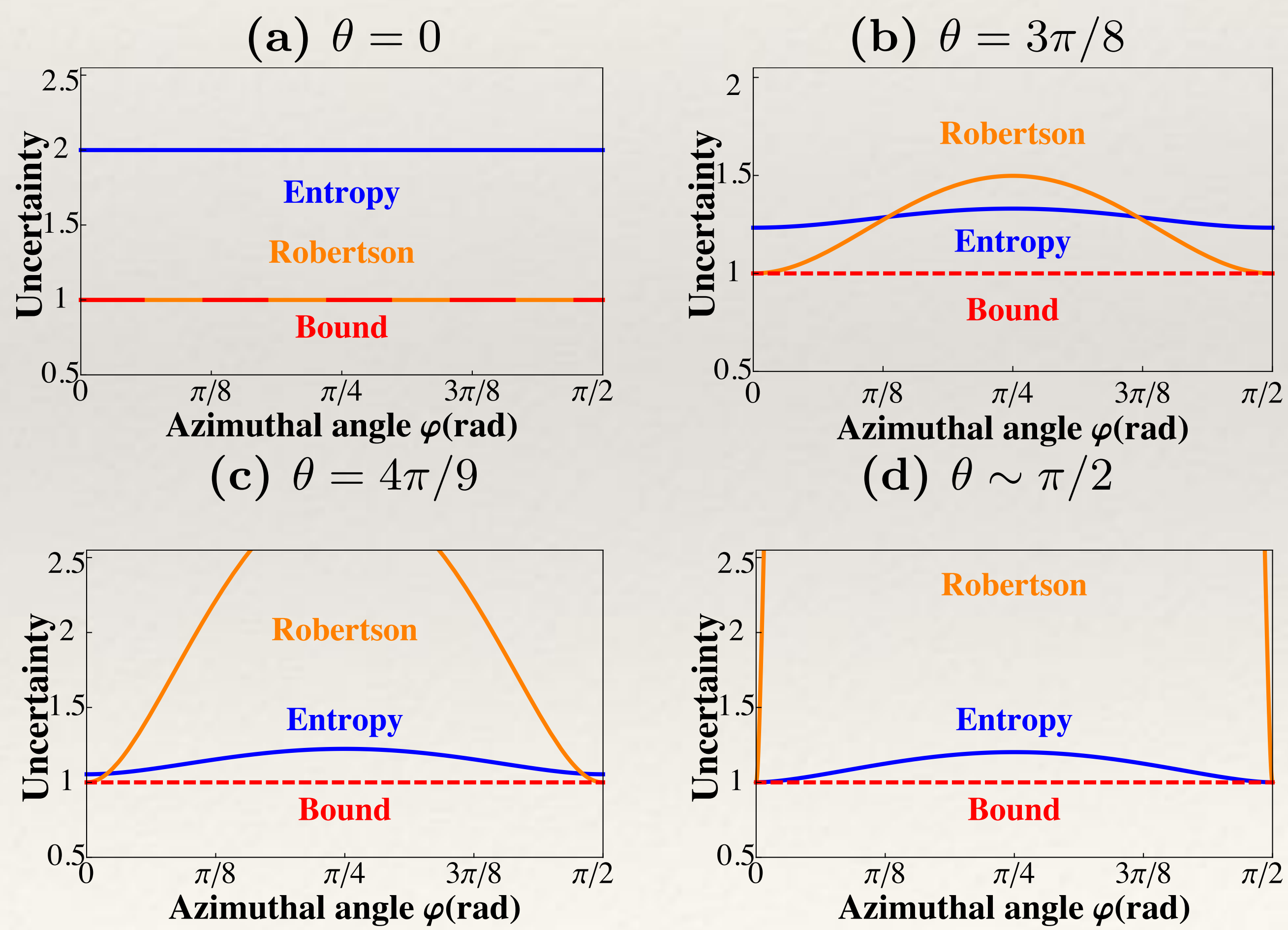
$$c(A,B) = \max_{ij}\langle a_i|b_j\rangle \ or \ 1/\sqrt{\pi e \hbar}$$

Comparison between the UR's I

$$\frac{H_{|\psi\rangle}(X) + H_{|\psi\rangle}(Y)}{-2\log c} \geq 1, \quad \frac{\delta(X)\delta(Y)}{|\langle[X,Y]\rangle|/2} \geq 1,$$



- The entropic uncertainty relation is optimized to characterize the state in the same plane of the measurements.
- The Heisenberg's uncertainty relation defines spin coherent state.



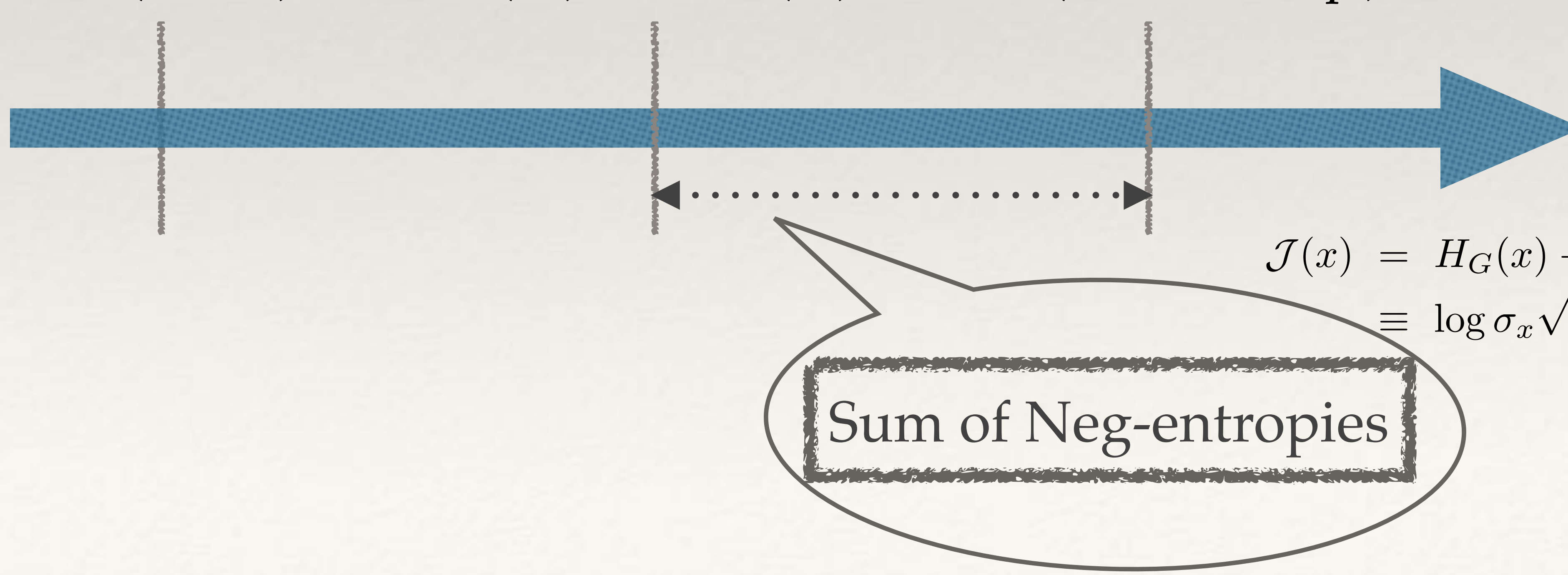
Comparison between the UR's II

- Continuous variable entropy is upper bounded by logarithm of the standard deviation & the inequality is saturated by Gaussian state.

$$H(x) \leq \ln(\sqrt{2\pi e}\sigma_x)$$

- The composite entropies are lower bounded by uncertainty and upper bounded by Gaussianity. [Entropic UR is general than Heisenberg UR]

$$\ln(\pi e \hbar) \leq H(x) + H(p) \leq \ln(2\pi e \sigma_x \sigma_p)$$

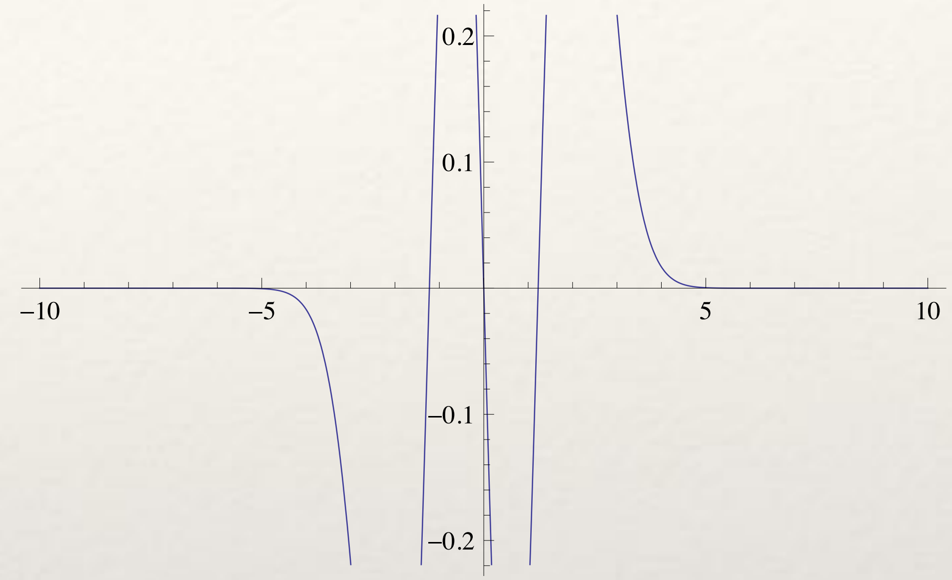


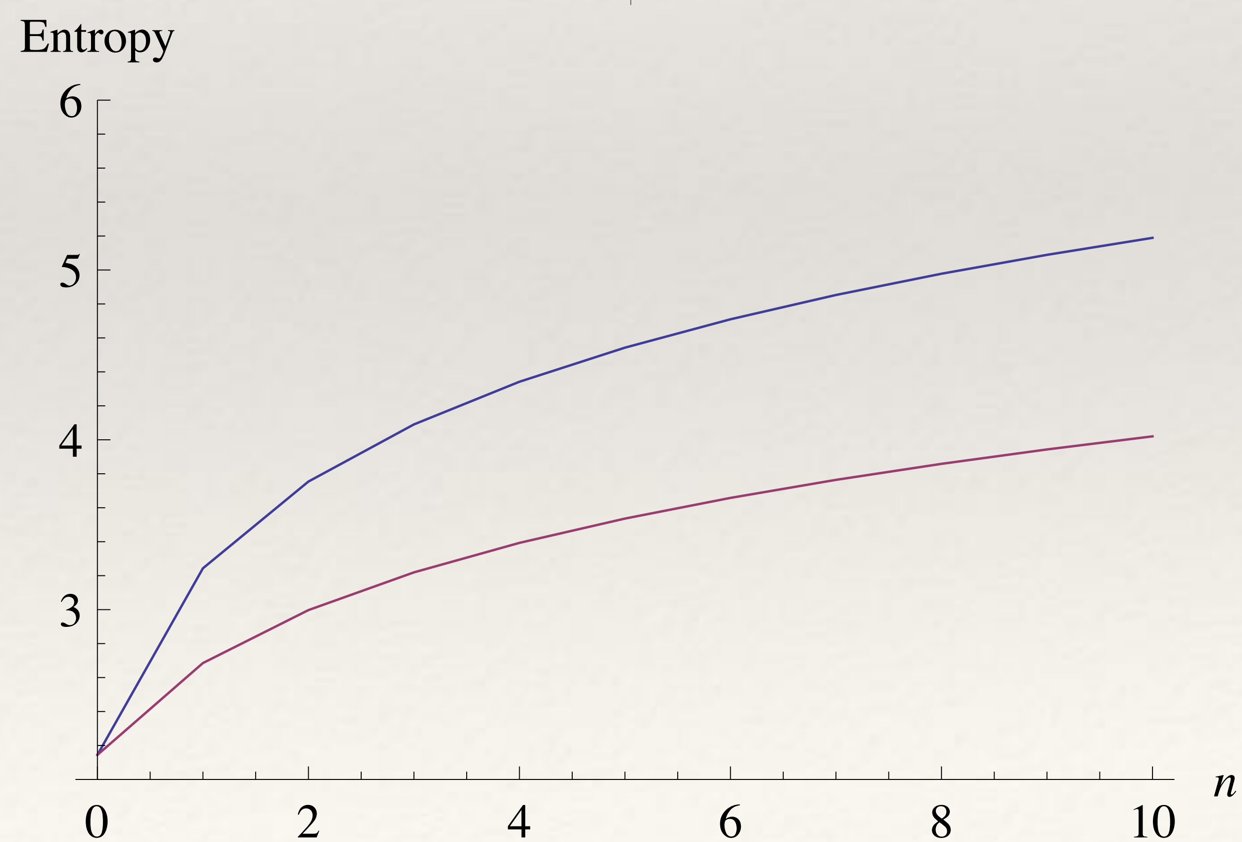
$$\begin{aligned} \mathcal{J}(x) &= H_G(x) - H(x) \\ &\equiv \log \sigma_x \sqrt{2\pi e} - H(x) \end{aligned}$$

NG1-Number state

$|N\rangle$

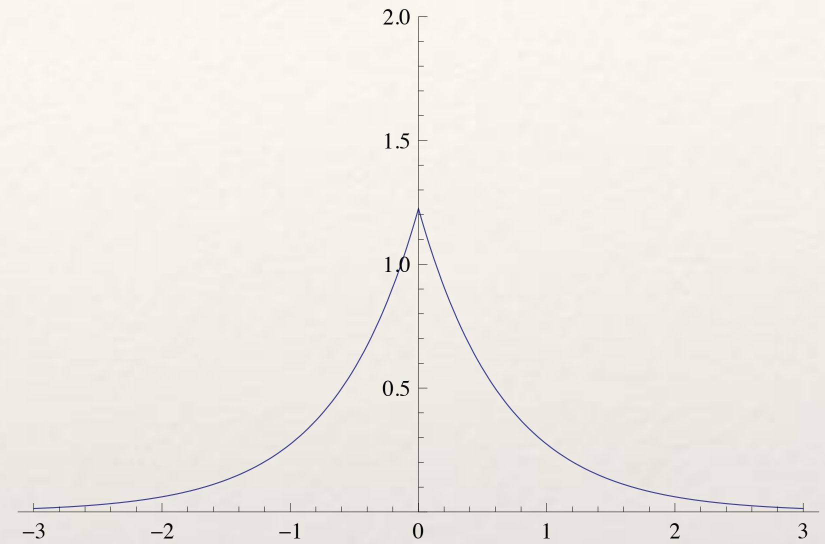
- The wave function for the quadrature is give by Gaussian times Hermit polynomial
- The Heisenberg uncertainty is linearly increased as N increased.
- Entropic uncertainty/Neg-entropy is also increasing log-linearly along N.



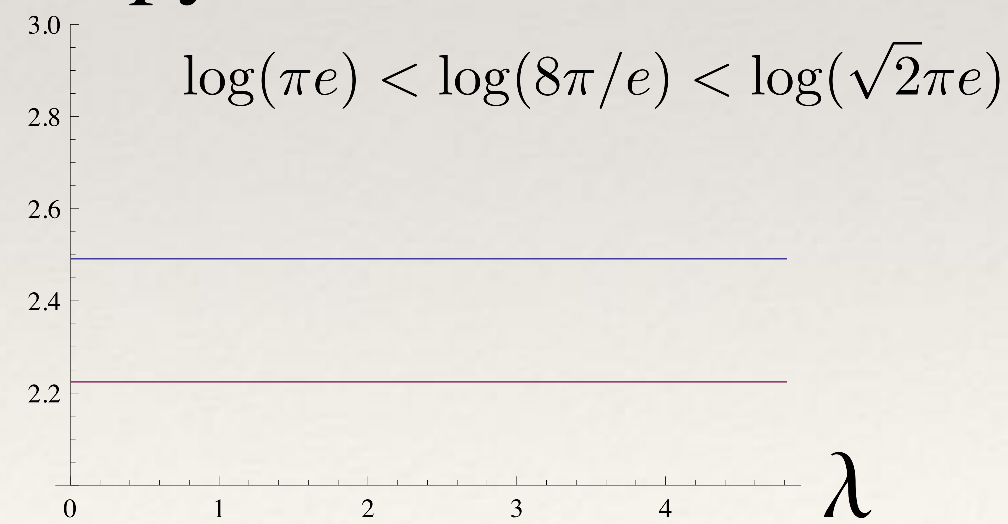


NG2-Laplace distribution

- Exp(-\lambda |x|)
- Fourier transformed to Lorenz distribution; 1/(p^2+\lambda^2)
- Quantum uncertainty is constant over \lambda
- Entropic uncertainty is quite saturated to the lower bound.

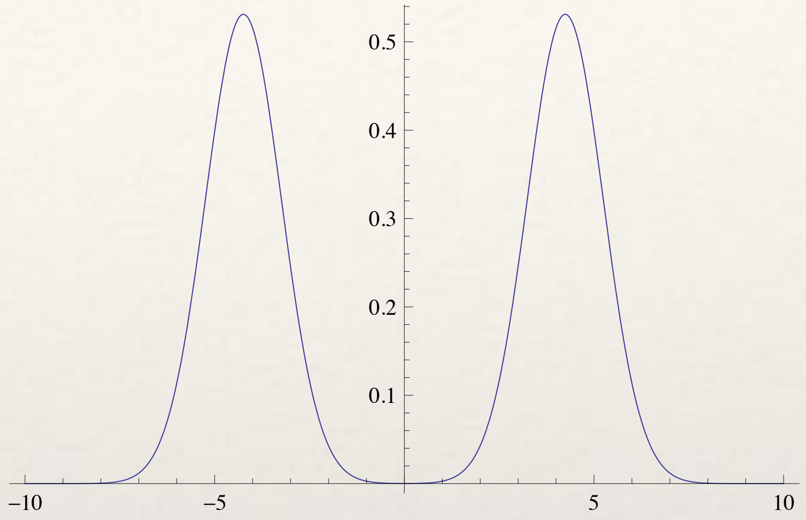


Entropy



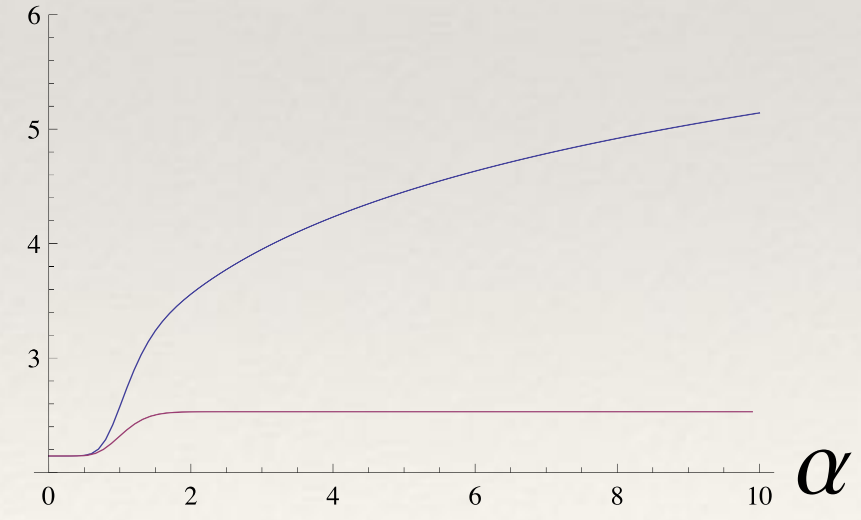
NG3-Schrodinger Cat state

$|\alpha\rangle + |-\alpha\rangle$



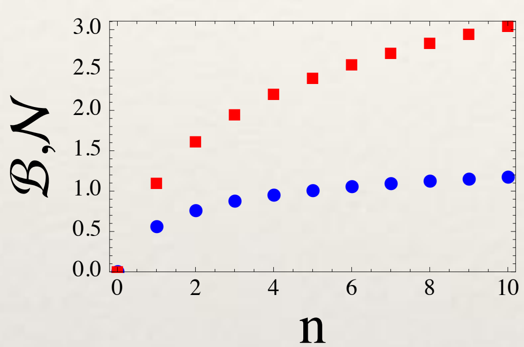
- Heisnberg uncertainty \delta_x\delta_p is linear increasing over |\alpha|.
- Entropic uncertainty H(x)+H(p) become saturated at large alpha.
- Neg-entropy is diverging.

Entropy

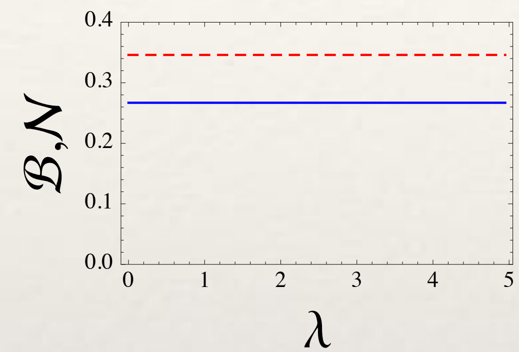


Non-Gaussian upper bound

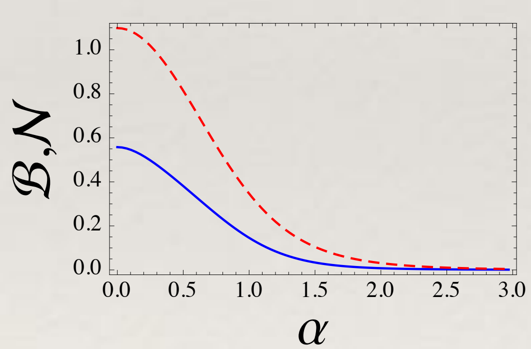
- Non-gaussian upper bound



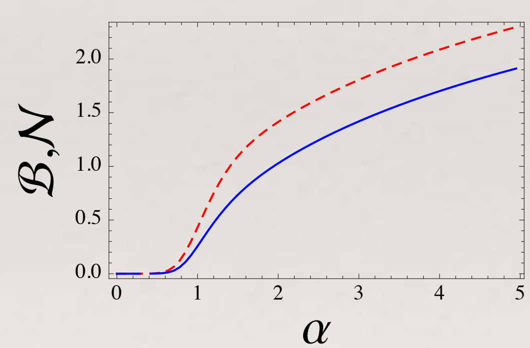
(a) Photon number state. The difference $\mathcal{B}-\mathcal{N}$ becomes larger as the photon number increased.



(b) Possion/Laplace state. $\mathcal{B}$ and $\mathcal{N}$ remain constant for any value of standard deviation.

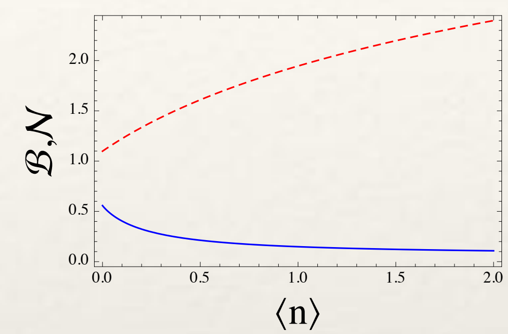


(c) Photon added coherent state. The bound becomes zero as the state approaches to gaussian state.

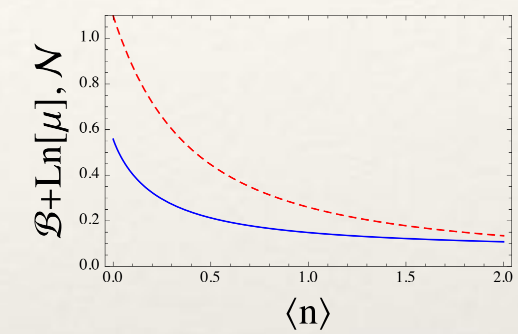


(d) Schrodinger Cat state. The difference between $\mathcal{B}$ and $\mathcal{N}$ becomes constant at the large α.

$$\mathcal{N} \equiv \mathcal{J}(X) + \mathcal{J}(P) \qquad \mathcal{J}(X) + \mathcal{J}(P) \leq \ln(2\sigma_x\sigma_p).$$



(a) Entropic uncertainty and upper bound



(b) The upper bound with purity correction

$$\mathcal{J}(X) + \mathcal{J}(P) \leq \ln(2\sigma_x\sigma_p) + \ln(\mu)$$

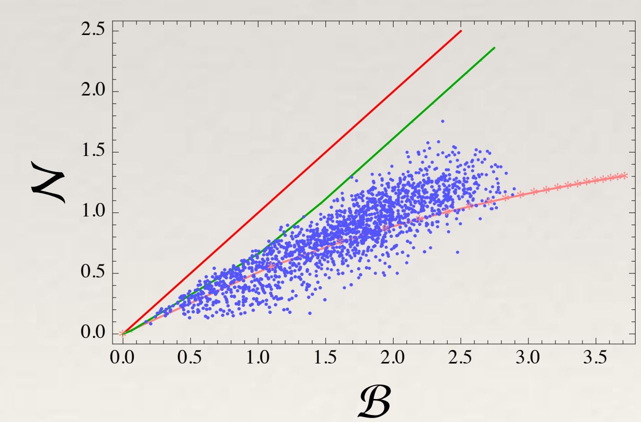


FIG. 3: Comparison of the total neg-entropy $\mathcal{N}$ versus the state-dependent bound $\mathcal{B}$ for different examples of quantum states together with the randomly generated pure state. It shows that the coherent superposition state behaves closely to the lower bounds while there exists more optimal state.

CV Entropic uncertainty & Gaussianity

PHYSICAL REVIEW A **89**, 032108 (2014)

Optimized entropic uncertainty for successive projective measurements

Kyunghyun Baek,[1] Tristan Farrow,[2,3] and Wonmin Son[1,*]
[1]*Department of Physics, Sogang University, Mapo-gu, Shinsu-dong, Seoul 121-742, Korea*
[2]*Atomic & Laser Physics, Clarendon Laboratory, University of Oxford, OX1 3PU, United Kingdom*
[3]*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543, Singapore*
(Received 4 December 2013; published 10 March 2014)

We focus here on the uncertainty of an observable Y caused by a precise measurement of X. We illustrate the effect by analyzing the general scenario of two successive measurements of spin components X and Y. We derive an optimized entropic uncertainty limit that quantifies the necessary amount of uncertainty observed in a subsequent measurement of Y. We compare this bound to recently derived error-disturbance relations and discuss how the bound quantifies the information of successive quantum measurements.

DOI: 10.1103/PhysRevA.89.032108 PACS number(s): 03.65.Ta, 03.75.Dg, 42.50.Xa, 03.67.−a

PHYSICAL REVIEW A **92**, 012114 (2015)

Role of quantum non-Gaussian distance in entropic uncertainty relations

Wonmin Son[*]
Department of Physics, Sogang University, Mapo-gu, Shinsu-dong, Seoul 121-742, Korea and University of Oxford, Department of Physics, Parks Road, Oxford OX1 3PU, United Kingdom
(Received 20 April 2015; published 17 July 2015)

A Gaussian distribution of a quantum state with continuous spectra is known to maximize the Shannon entropy at a fixed variance. Applying it to a pair of canonically conjugate quantum observables $\hat{x}$ and $\hat{p}$, the quantum entropic uncertainty relation can take a suggestive form, where the standard deviations σ_x and σ_p are featured explicitly. From the construction of the entropic uncertainty relation, it follows in a transparent manner that (i) the entropic uncertainty relation implies the Kennard-Robertson uncertainty relation in a modified form, $\sigma_x\sigma_p \geqslant \hbar e^{\mathcal{N}}/2$; (ii) the additional factor $\mathcal{N}$ quantifies the quantum non-Gaussianity of the probability distributions of two observables; and (iii) the lower bound of the entropic uncertainty relation for a non-Gaussian continuous-variable (CV) mixed state becomes stronger with purity. The optimality of specific non-Gaussian CV states for the refined uncertainty relation has been investigated and the existence of a new class of CV quantum state is identified.

DOI: 10.1103/PhysRevA.92.012114 PACS number(s): 03.65.Ta, 03.67.−a, 42.50.−p

Remarks

- For the case of discrete variable system, the Heisenberg UR and Entropic UR characterizes uncertainty of the systems differently. (Provide state sensitive characterization.)

- For the case of continuous variable state, entropic UR provide stronger condition then Heisenberg UR. (i.e. The lower bound of the Heisenberg UR is needed to be changed when it is not Gaussian.)

$$\sigma_x \sigma_p \geq \frac{1}{2} \mathrm{e}^{J_x + J_p}$$

Further remarks

- As like coherent state is optimised for the Heisenberg uncertainty,

$$|\alpha\rangle \qquad \Delta_x \Delta_p \geq \hbar/2$$

- Schrodinger Cat state is optimised for the entropic uncertainty relation.

$$|\alpha\rangle + |-\alpha\rangle$$

$$H(A) + H(B) \geq -2\ln c$$

$$c(A,B) = \max_{ij}\langle a_i|b_j\rangle \ or \ 1/\sqrt{\pi e\hbar}$$

Quantifiable simulation of quantum computation beyond Stochastic ensemble computation; Quantum advantage in a calculation

The first (BIG) question(s)

- Benchmark of quantum advantage for the extra ordinary computation

- How much does the causality take part in at the process of computation?

Quantum computation vs Stochastic ensemble comp.

- Standard process for quantum computation

$$|\psi_0\rangle \to \hat{U}_1|\psi_1\rangle \to \hat{U}_2 \cdots \to \quad \hat{U}_L\,|\psi_L\rangle,$$

$$\hat{C}_{\text{comp}} = \hat{U}_L \cdots \hat{U}_2\hat{U}_1$$

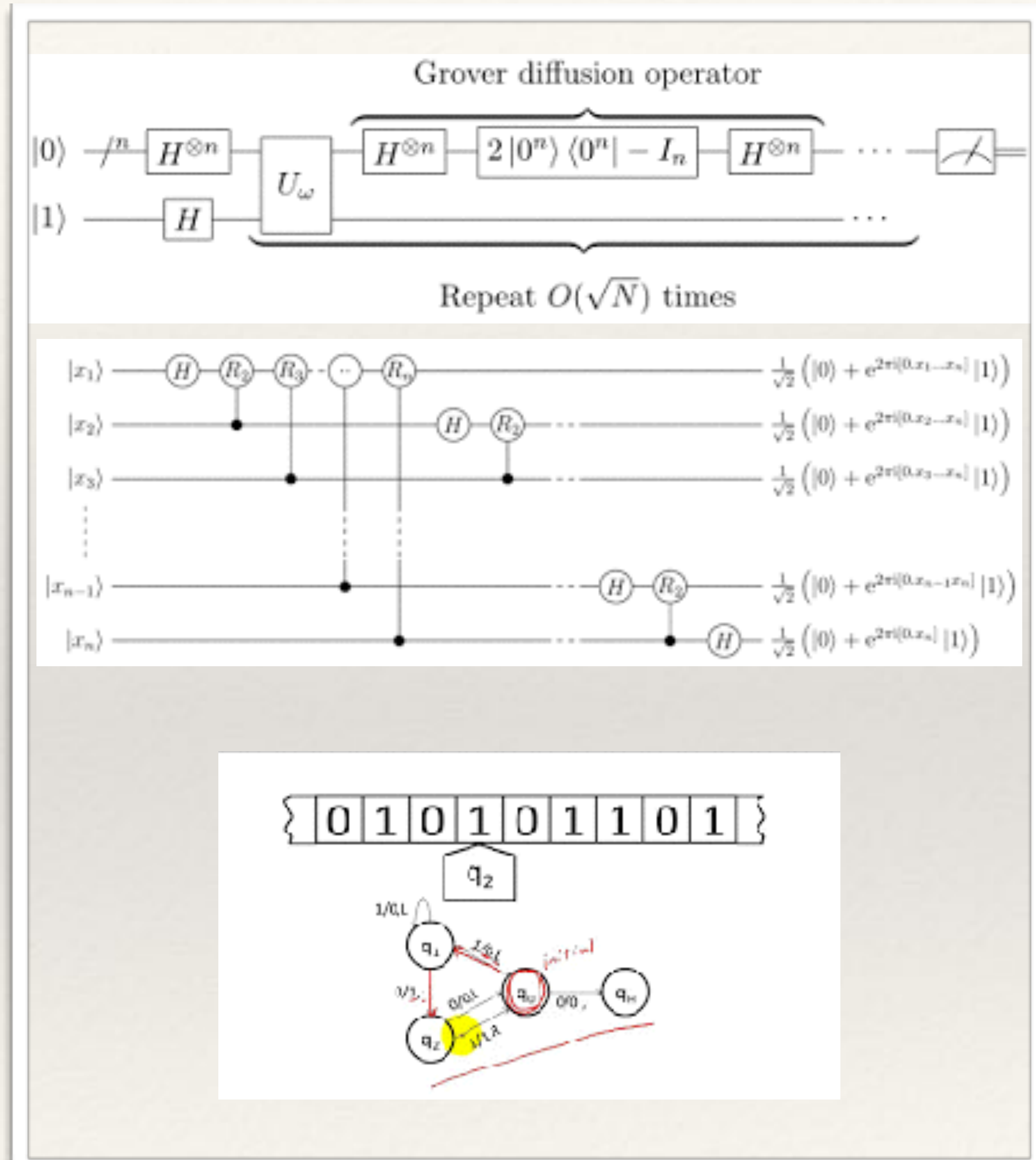
$$P_Q(m_j|\psi_j) = |\omega_{m_j}|^2 = |\langle m_j|\psi_j\rangle|^2$$

- Stochastic ensemble computation (Probabilitistic Turing machine)

$$s_k \in \mathcal{S} \quad \mathbf{s}_j = (s_1, s_2, \cdots)_j \quad m(s_k) \in \mathcal{M}$$

$$P(m_j|\mathbf{s}_j) = \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N}\delta_{m(s_k),m_j} \quad \sum_{m_j} P(m_j|\mathbf{s}_j) = 1$$

$$P(\mathbf{s}_0, \ldots, \mathbf{s}_j) = C(\mathbf{s}_j|\mathbf{s}_{j-1}, \ldots, \mathbf{s}_0)\cdots C(\mathbf{s}_1|\mathbf{s}_0)\,P(\mathbf{s}_0)$$



Basic assumptions and readout inequality

- Basic assumptions for the read out probabilities (Bayesian & Marginals + consistency hypothasis)

$$P(m_j|m_{j-1}) = \frac{P(m_{j-1}, m_j)}{P(m_{j-1})}$$

$$P(m_0, \ldots, m_J) = \sum_{\{s_j\}} P(s_0, \ldots, s_J) \prod_{l=0}^{J} P(m_l|s_l)$$

H *There exists an SEnM such that it provides the conditional probabilities $P(m_j|m_{j-1})$ equal to $P_Q(m_j|m_{j-1})$ at each jth step*

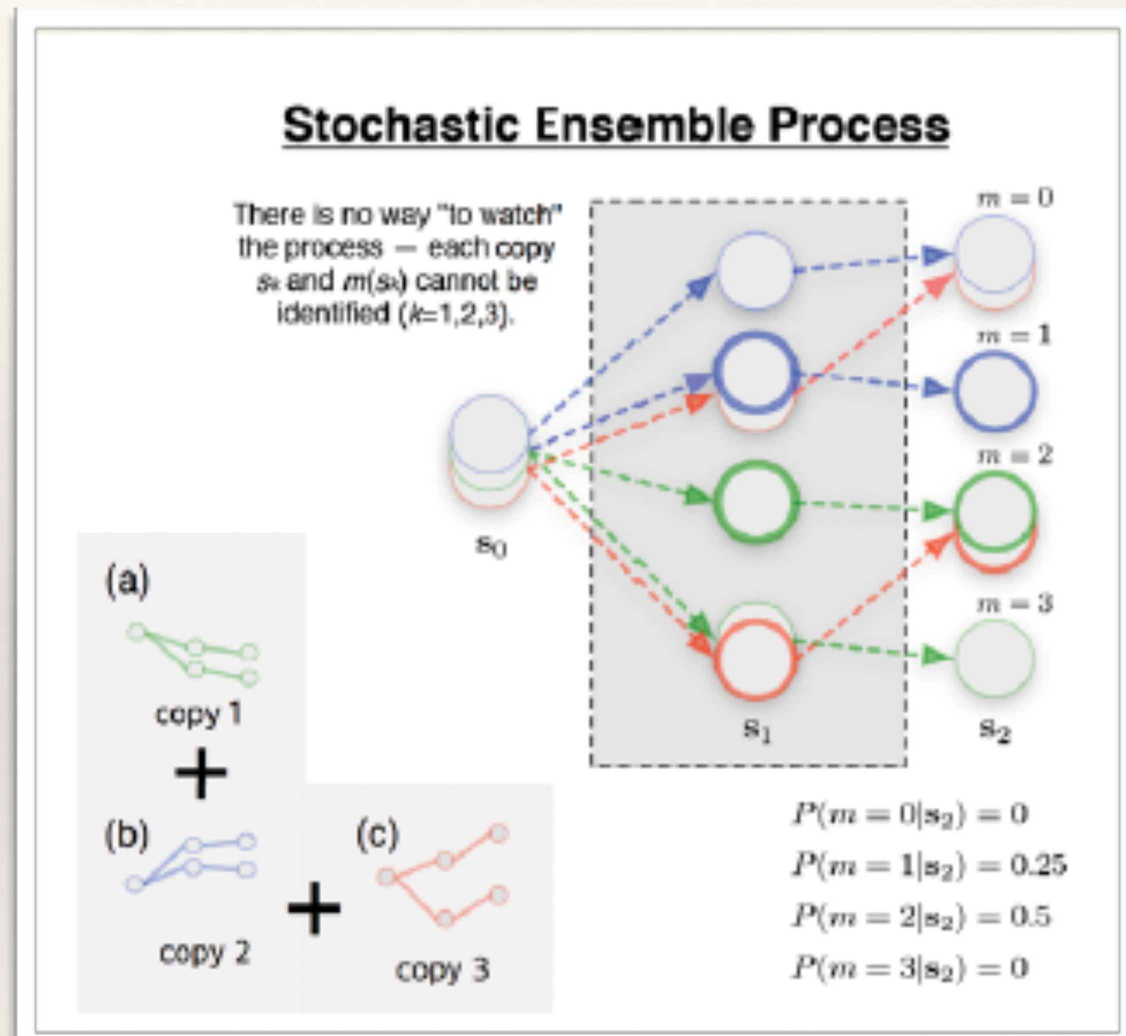
$$P(m_j|m_{j-1}) = P_Q(m_j|m_{j-1}) \quad (7)$$

for all $j = 1, \ldots, L$, and then, $P(m_L|m_0)$ for obtaining the final computation outcome m_L given the initial m_0 is equal to the quantum probability $P_Q(m_L|m_0)$

$$P(m_L|m_0) = P_Q(m_L|m_0) \quad (8)$$

- Read out inequality for causality (Macro realism)

$$H(M_L|M_0) \leq \sum_{j=1}^{L} H(M_j|M_{j-1})$$



Quantum violation of the read out inequalities

- Evolution of quantum state and its measurements

$$\hat{G}_g = -e^{i\Theta\hat{\sigma}_x}$$

$$\hat{C}_{\mathrm{comp}} = \hat{U}_L \cdots \hat{U}_2 \hat{U}_1 = \hat{G}_g^{n_d}$$

$$P_Q(m_j) = \mathrm{Tr}(\hat{\rho}_j \hat{\Pi}_{m_j})$$

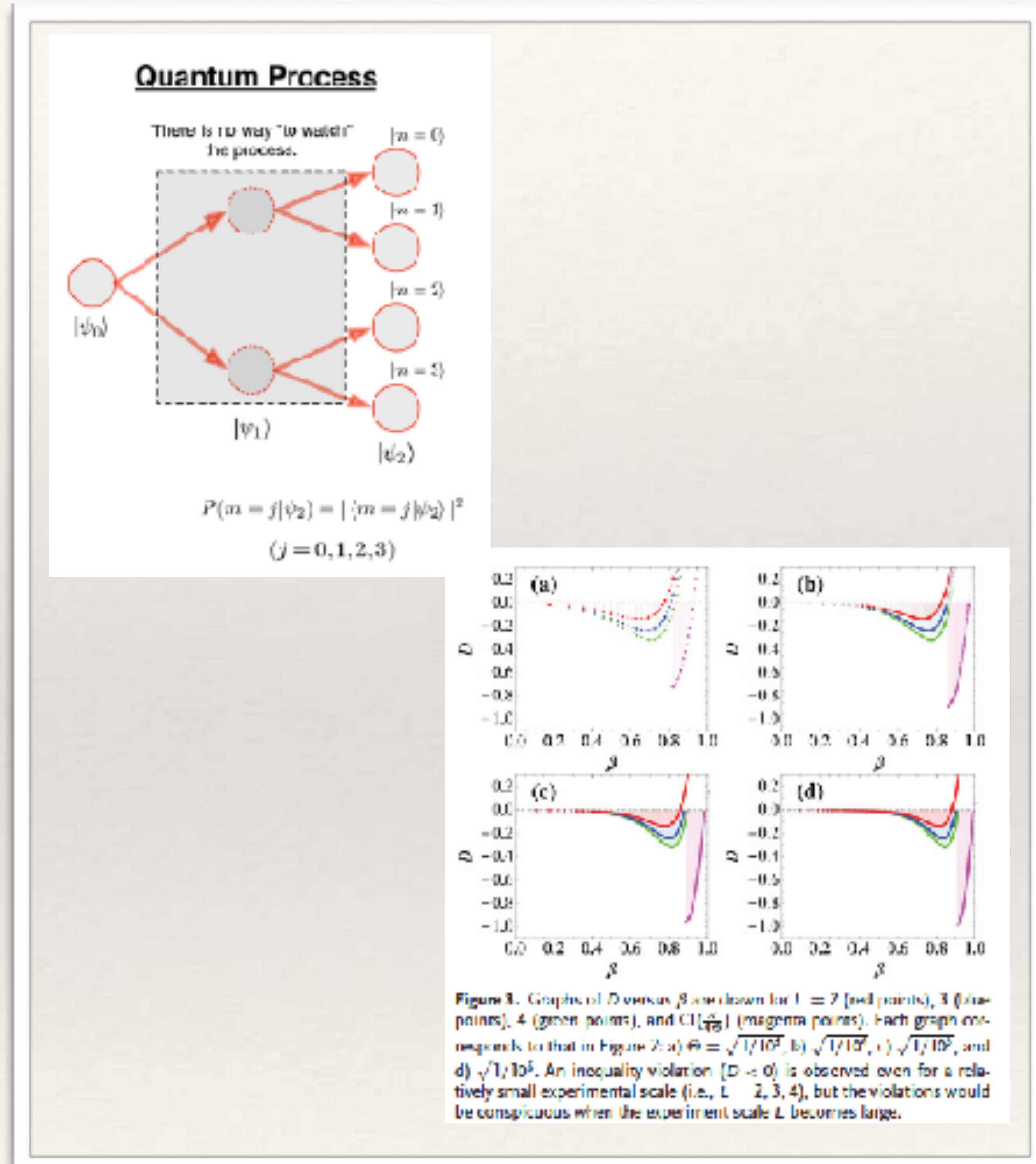
$$P_Q(m_j | m_{j-1}) = \left| \langle m_j | \hat{G}_g^{n_d/L} | m_{j-1} \rangle \right|^2$$

$$\text{where } \hat{\rho}_j = |\psi_j\rangle\langle\psi_j| = \hat{G}_g^{(n_d/L)j} \hat{\rho}_0 \hat{G}_g^{(n_d/L)j\dagger}$$

- Relevant read out inequality and its violation

$$h(n_d\Theta) \leq L h\left(\frac{n_d}{L}\Theta\right)$$

$$h(x) = -(\cos^2 x)\log_2(\cos^2 x) - (\sin^2 x)\log_2(\sin^2 x)$$



FULL PAPER
Quantum Computation

ADVANCED QUANTUM TECHNOLOGIES
www.advquantumtech.com

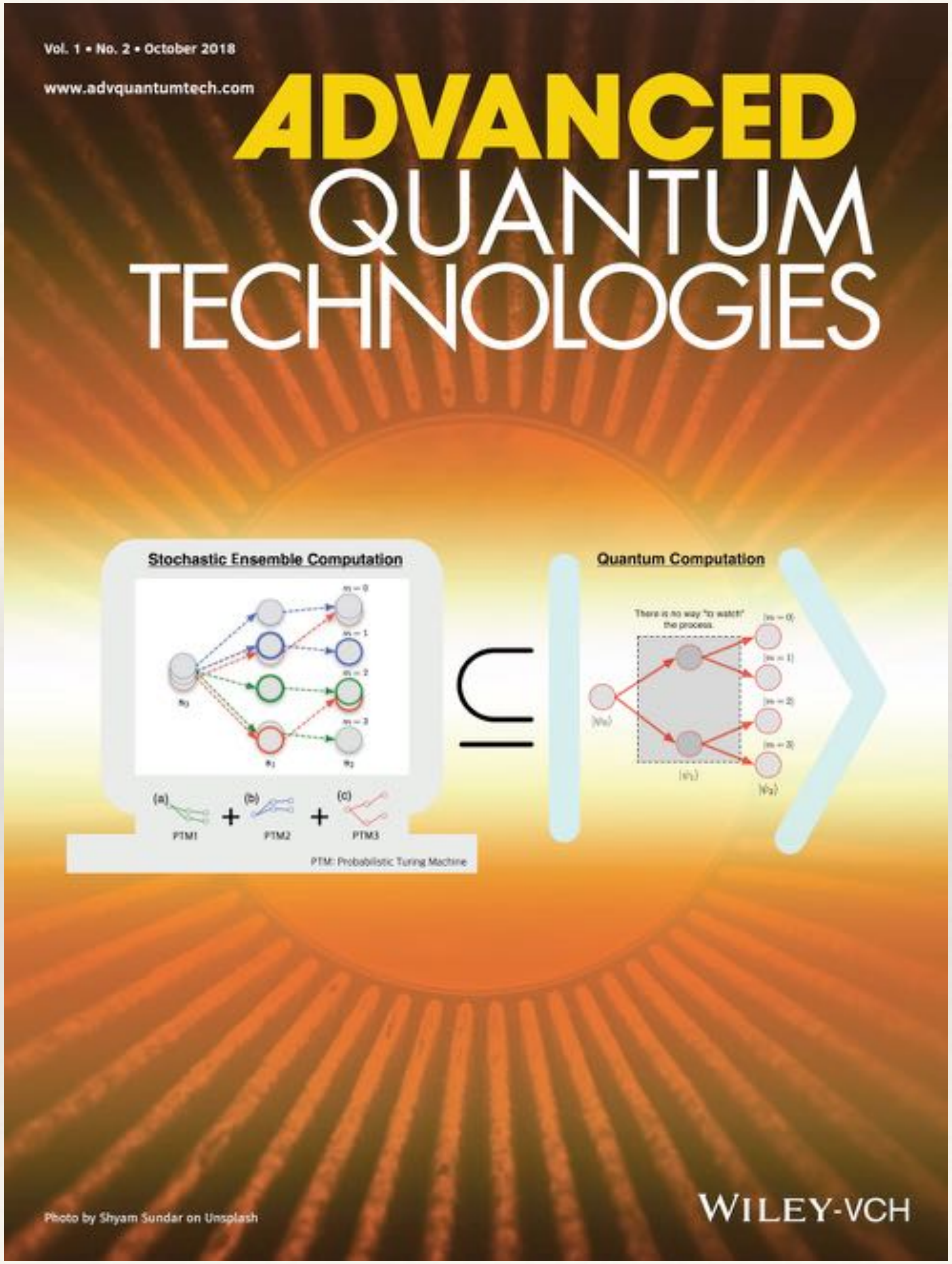
Quantifiable Simulation of Quantum Computation beyond Stochastic Ensemble Computation

Jeongho Bang, Junghee Ryu, Chang-Woo Lee, Ki Hyuk Yee, Jinhyoung Lee,* and Wonmin Son**

In this study, a distinctive feature of quantum computation (QC) is characterized. To this end, a seemingly-powerful classical computing model, called "stochastic ensemble machine (SEnM)," is considered. The SEnM runs with an ensemble consisting of finite copies of a single probabilistic machine, hence is as powerful as a probabilistic Turing machine (PTM). Then the hypothesis—that is, the SEnM can effectively simulate a general circuit model of QC—is tested by introducing an information-theoretic inequality, named readout inequality. The inequality is satisfied by the SEnM and imposes a critical condition: if the hypothesis holds, the inequality should be satisfied by the probing model of QC. However, it is shown that the above hypothesis is not generally accepted with the inequality violation; namely, such a simulation necessarily fails, implying that PTM $\subseteq$ QC.

a formal argument that a QTM can be more powerful than a PTM was provided,[4,5] and quantum computation (QC) has been investigated intently and more deeply with the advent of celebrated quantum algorithms.[6–8] Today, it is widely believed that QC solves hard problems much faster. However, skepticism toward QC still exists since the identification of a clear border between classical and quantum computations is still obscure.[9–11] This would arise without ruling out the potential of any classical probabilistic computation model that is believed to imitate the QC.[12–14]

With these open problems in mind, here we attempt to find a dissimilar aspect between classical versus quantum compu-



Further studies: Contextuality-LHV model (KCBS and Bell - CHSH)

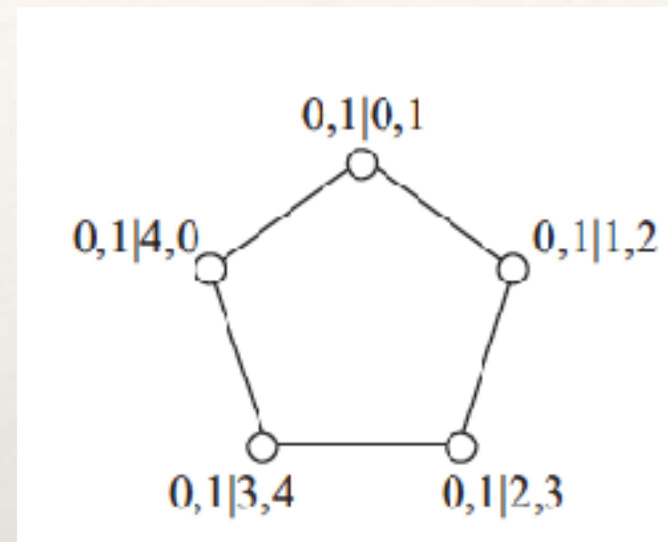
- Sum of joint probabilities for the KCBS scenario

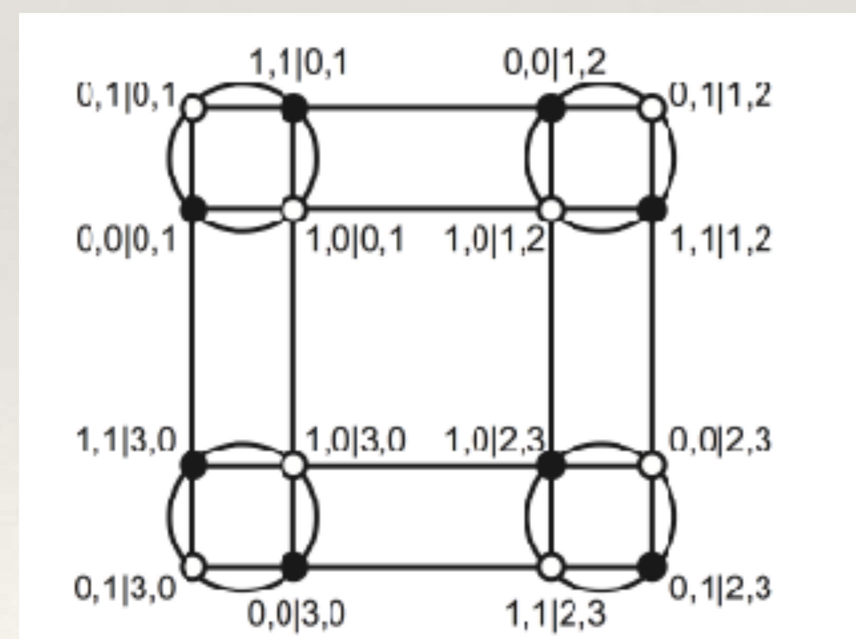
$$\sum_{i=0}^{4} P(0,1|i,i+1) \overset{\text{NCHV}}{\leq} 2 \overset{\text{QM}}{\leq} \sqrt{5} \overset{\text{E}}{\leq} \frac{5}{2},$$

- Bell-CHSH graph from the joint probabilities

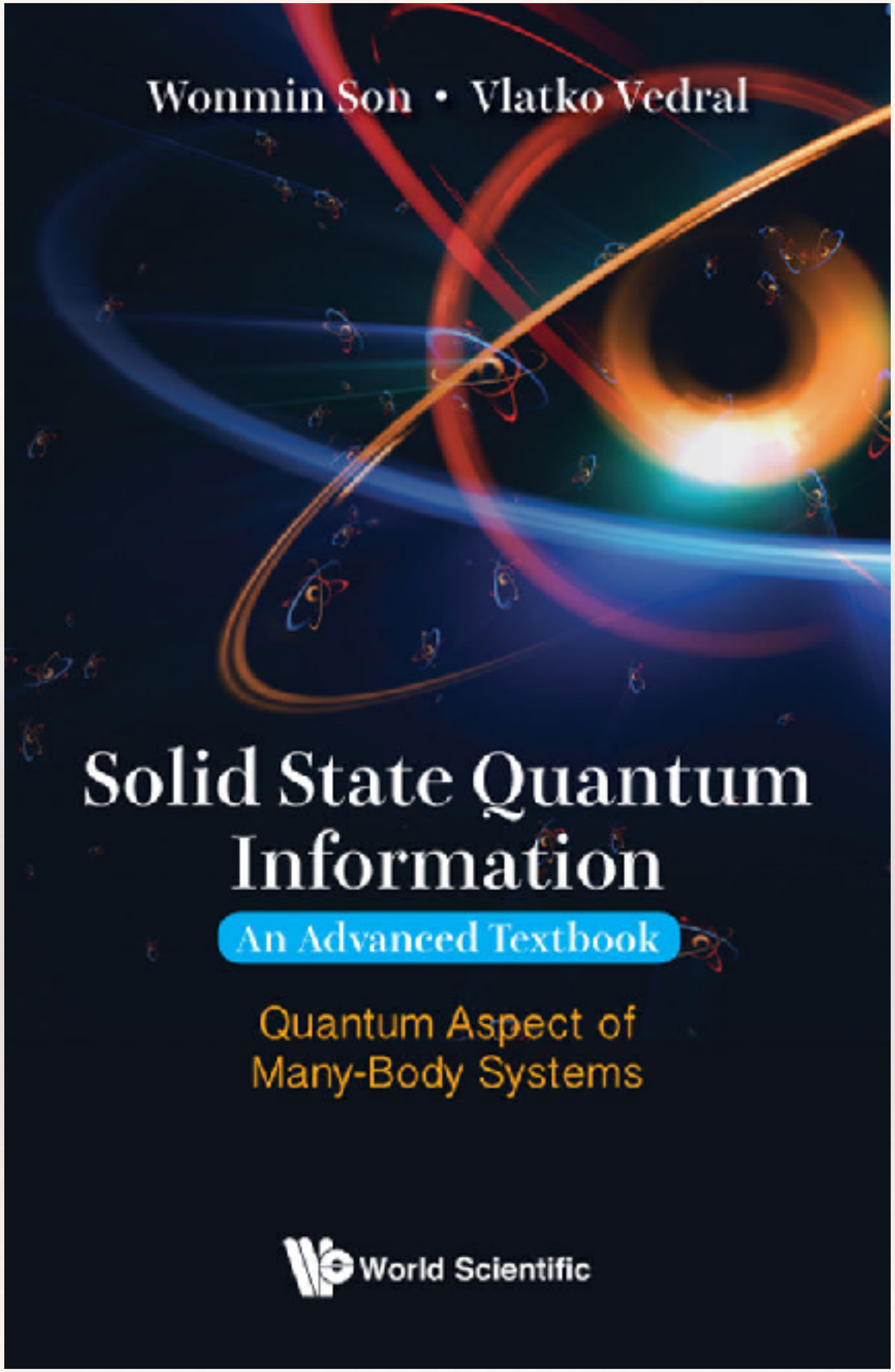
$$\sum P(a,b|x,y) \overset{\text{NCHV,LHV}}{\leq} 3 \overset{\text{QM}}{\leq} 2+\sqrt{2} \overset{\text{E,NS}}{\leq} 4,$$

- Generalization for the arbitrary graph





Recent arrival of an advanced textbook



Thanks for your attention!!!