

Entropy and Fluctuation theorem

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Project 1: Brief review on stochastic thermodynamics

Most processes one can find in nature are nonequilibrium (NEQ) processes, which include key dynamic processes in biological cells and social networks besides usual physical phenomena. Nevertheless, our understanding on NEQ dynamic processes has been quite primitive, except near equilibrium (EQ). Recently, there has been a considerable progress on the issue of the thermodynamic second law, which is known as the law of entropy increase or irreversibility. In particular, a novel symmetry known as the Gallavotti-Cohen symmetry is found in nonequilibrium (NEQ) fluctuations, which leads to so-called fluctuation theorems. The thermodynamic second law is a simple corollary of fluctuation theorems, from which one can predict quantitatively how often NEQ processes violate the law of entropy increase. Violations disappear in the thermodynamic limit, but can be observed reasonably well in small systems. Various fluctuation theorems can be derived easily by the recently established *stochastic thermodynamics*.

Below are a few introductory review papers .

- (1) H. Park, J. Kor. Phys. Soc. **72**, 1413 (2018).
- (2) C. Jarzynski, Annu. Rev. Condens. Matter Phys., **2**, 329-351 (2011).
- (3) J. Kurchan, J. Stat. Mech., P07005 (2007).
- (4) U. Seifert, Eur. Phys. J. B **64**, 423-432 (2008).

Please read these review papers and give a brief review lecture at the end of the winter camp. Of course, it will be helpful if you listen carefully to my lectures and actively participate in questions and discussions.

Project 2: Conditional probability (propagator) in stochastic calculus

Consider an one-dimensional Brownian motion described by the Langevin equation as

$$\begin{aligned}\frac{dx_t}{dt} &= v_t, \\ m \frac{dv_t}{dt} &= f(x_t) - \gamma v_t + \xi_t,\end{aligned}\tag{1}$$

where m is the mass of the Brownian particle, v_t is its velocity at time t , f is the force exerted on the particle, and γ is the damping coefficient. The force f can be either conservative or non-conservative. ξ_t is the Gaussian white noise at t , satisfying $\langle \xi_t \rangle = 0$ and $\langle \xi_t \xi_{t'} \rangle = 2\gamma T \delta(t - t')$ with the reservoir temperature T . For convenience, here, we set the Boltzmann constant $k_B = 1$.

1. Calculate the conditional probability $\Pi[x_{t+dt}, v_{t+dt}; x_t, v_t]$ for an infinitesimal interval $[t, t + dt]$ with the choice of the Stratonovich calculus, such that $x_{t+dt} - x_t = v_{t+dt/2} dt = [v_t + v_{t+dt}]/2 dt$ and so on.
2. Heat energy absorbed by the particle is related to the log ratio of the conditional probabilities for the time-forward and time-reversed processes. Derive this relation explicitly. Note that $dQ_t/dt = v_t \circ (-\gamma v_t + \xi_t)$ with $\circ$ representing the Stratonovich product.
3. Discuss the case when $f = f(x_t, v_t)$ (for example, Lorenz force included).