Black holes in AdS/CFT 1

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Plan

Lecture 1 (today):

- Introduction
- AdS black holes & gauge theories
- Supersymmetric AdS_5 black holes

- Brief summary of analytic/numerical solutions
- Index of SCFT & its Cardy limit
- Black hole thermodynamics from QFT

Lecture 2 (tomorrow):

- Evidence of new BH saddle points
- Deconfinement & Hawking-Page transitions
- AdS black holes in diverse dimensions

- AdS_5 and AdS_7: ‘t Hooft anomaly & background field method
- AdS_6 from CFT_5 & AdS_4 from CFT_3

Black hole thermodynamics

- Thermodynamic "analogies" of classical black hole physics

- 1st law: perturb stationary BH

$$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ$$

- 2nd law: "area theorem"

  (Hawking 1971)

$$\frac{dA}{dt} \geq 0$$

- Area ~ entropy carried by BH (cloaked behind event horizon): Bekenstein (1973)
- Hawking radiation (1974): Strengthens the statistical interpretation, also fixing

$$S_{\rm BH} = \frac{kA}{4\ell_{\rm P}^2} \qquad \ell_{\rm P} = \sqrt{G\hbar/c^3}$$

- A striking emergent aspect of gravity, encoding BH's quantum & thermal info on its geometry. May be a good clue for QG itself. Holography, entropic force, …

Statistical approaches to BH

- Want to engineer BH's w/ microscopic descriptions, e.g. D-branes.
- It turns out that the emergent gravitational description of BH thermodynamics (large charge, etc.) demands strong coupling QM/QFT analysis, making quantitative studies difficult.
- BPS black holes: protected physics, e.g. entropy ~ spectrum

- 5d BPS BHs from D1-D5-P: entropy from Cardy formula of 2d CFT

[Strominger, Vafa] (1996)

$$Z(\tau) \sim \mathrm{Tr}\left[e^{2\pi i\tau L_0}\right] \sim \exp\left[\frac{\pi i c}{12\tau}\right] \quad \text{at } \tau \to i0^+$$

$$e^{S(P,c)} = \oint d\tau Z(\tau) e^{-2\pi i\tau P} \sim \exp\left[2\pi\sqrt{\frac{cP}{6}}\right] \quad \text{at } P \gg 1 \qquad \text{at } c = 6Q_1Q_5$$

- Accounts for the Bekenstein-Hawking entropy $S_{\mathrm{BH}} = 2\pi\sqrt{Q_1Q_5P}$ of BH's.

- Various studies of similar sort: 4d & 5d BH's in asymptotically flat spacetime
- BH's in: heterotic strings, M-theory on CY_3, F-theory on CY_3, toroidal compactifications, orbifold compactifications, … ; small BH's ; some limited studies on non-BPS, ……

Many black holes. Too many.

- Further studies: other types of BH's than those first encountered & studied. e.g.
- 4d N=2 compactifications: single-center, multi-centered [Denef et.al, …], wall-crossings, …
- 5d: single-center, multi-center & threshold bounds [Vafa,Vandoren,Haghighat,Klemm,Murthy, …], black rings [Elvang-Emparan-Mateos-Reall, Bena-Warner, Gauntlett-Gutowski], hairs & "microstate geometry" [Bena, Warner, Shigemori, de Boer, …], …

- Certain black holes are simple (e.g. admit simple Cardy formula counting)
- Certain black holes are very tricky: e.g.
- Become dominant saddle points only in non-Cardy regimes
- Some BH's are even argued to be invisible in the index, …

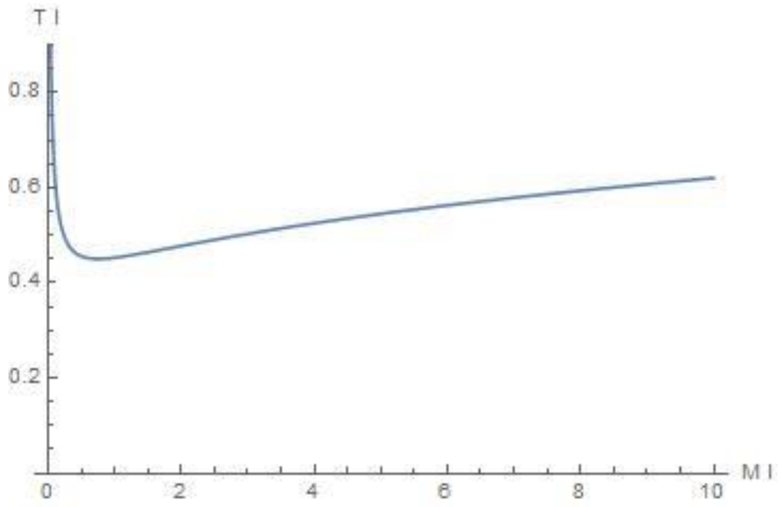
- Anyway, generally expect a zoo of competing BH's in a given system.
- Often ad hoc studies: Demands by hand the BH's one wishes to study. Single-center BH, multi-BH constituents, black ring, … Often employs different effective descriptions, …
- A setting in which one can study thermodynamics of all BH's systematically?

Black holes in AdS$_D$ $(D \geq 4)$

- Schwarzschild black holes in AdS (e.g. in AdS_5):
- Has S^3 horizon & boundary. Embedded in global AdS_5.
- Small BH: Similar to BH's in flat space. Negative specific heat.
- Large BH: important in AdS thermodynamics

$$T = \frac{r_+}{\pi \ell^2} + \frac{1}{2\pi r_+}$$

$$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



- Hawking-Page transition [Hawking, Page] (1983):

transition between two phases, at $T = \frac{3}{2\,\pi\ell}$ (order 1 in the unit of AdS radius ℓ)

- Low T phase: gas of gravitons in AdS. Doesn't see $1/\mathrm{G_N} \sim N^2$ so that $F \sim O(N^0)$
- High T phase: large AdS black holes ($F_{BH} = -T \log Z_{BH} < 0$). Sees N^2.

Black holes from gauge theory

- CFT dual (on $S^3 \times R$): confinement-deconfinement transition [Witten] (1998)
- Confined phase: $F \sim O(N^0)$, glue-balls (& mesons, etc.)
- Deconfined phase: $F \sim O(N^2)$ of gluons (& quarks) ~ matrices
- **deconfined (quark-)gluon plasma = black holes**

(Despite w/ finite spatial volume, large N makes sharp phase transitions possible…)

- Thermodynamics of YM on $S^3 \times R$ studied at weak coupling. Some qualitative AdS physics visible. [Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk] (2003)
- However, some qualitative & all quantitative aspects are different.

- BPS black holes in $AdS_5 \times S^5$ from 4d $N = 4$ SYM on $S^3 \times R$.
- First found in [Gutowski, Reall] (2004), and generalized thereof.
- Surprisingly, this basic question wasn't answered well till very recently.
- I'll show you today/tomorrow that, the basic solution is extremely simple.

Supersymmetric black holes

- BPS black hole solutions

[Gutowski, Reall] (2004) [Chong, Cvetic, Lu, Pope] (2005) [Kunduri, Lucietti, Reall] (2006)

- BPS energy (=mass): determined by

$U(1)^3 \subset SO(6)$ electric charges (momenta on S^5) & $U(1)^2 \subset SO(4)$ momenta on AdS_5

$$E\ell = Q_1 + Q_2 + Q_3 + J_1 + J_2$$

- Preserves only 2 of the 32 supercharges (1/16-BPS states)
- Very complicated solutions (many nonzero charges, little symmetry)

- Needs both electric charges Q_I & spins J_i to form BPS black holes
- $J_i = 0$: Chiral rings. Trace of commuting fields, e.g. $tr(XY^2Z) \sim tr(YZYX)$
- Not enough entropy $\sim \mathrm{O}(N^2)$: basic fields are effectively commutative, $\sim O(N^1)$

- $Q_I = 0$: Most operators are anomalous without $SO(6)$ R-charges. E.g. almost void graviton-like BPS operators in this sector (only "singletons")

SUSY black holes *(continued)*

- Charge relations: May naively expect 5 parameter solution, since they carry 5 charges.

- Only finds solutions w/ 4 independent parameters [Kunduri, Lucietti, Reall] (2006)
- Has to do w/ demanding smooth event horizon w/ SUSY (E.g. see [Gutowski, Reall] 1)

- Similar properties hold for many known BPS BH's in flat spacetime:

  5d BPS BH's w/ rotations [Breckenridge, Myers, Peet, Vafa] :

  microscopic charges: electric charges Q_I and spins J_1, J_2.

  BH charge relation: $J_1 = J_2$ (equal spins)

  4d BPS BH's [Ferrara, Kallosh, Strominger] :

  microscopic charges: dyonic charges P^I, Q_I and spin J.

  BH charge relation: $J = 0$ (no rotation)

- Recently, some numerical studies on more general BH's in $AdS_5 \times S^5$.
- Hairy BPS BH's [Markeviciute,Santos] [Markeviciute] [Bhattacharyya,Minwalla,Papadodimas]

- Charge condensates outside BH horizon. No charge relations.
- Mild horizon singularity (finite metric, horizon area, … but certain tidal forces diverge.)

SUSY black holes from QFT

- Study the degeneracy of BPS local operators = BPS states on $S^3 \times R$.
- More easily, study a "Witten index" type partition function on $S^3 \times R$,
- … at the risk of losing certain states by boson/fermion cancelation from $(-1)^F$.

- This index was defined in [Romelsberger] [Kinney,Maldacena,Minwalla,Raju] (2005):

$$Z(\Delta_I, \omega_i) = \mathrm{Tr}\left[(-1)^F e^{-\sum_{I=1}^{3}\Delta_I Q_I - \sum_{i=1}^{2}\omega_i J_i}\right]$$

$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 0$$

- Matrix integral expression for $\mathrm{U(N)}, \mathcal{N} = 4$ SYM (QFT on $S^3 \times S^1$) [KMMR] (2005) :

$$Z = \frac{1}{N!}\int \prod_{a=1}^{N}\frac{d\alpha_a}{2\pi}\prod_{a<b}\left(2\sin\frac{\alpha_{ab}}{2}\right)^2 \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(1 - \frac{\prod_{I=1}^{3} 2\sinh\frac{n\Delta_I}{2}}{2\sinh\frac{n\omega_1}{2}\cdot 2\sinh\frac{n\omega_2}{2}}\right)\sum_{a,b=1}^{N} e^{in\alpha_{ab}}\right]$$

- Questions:
- Does the low "temperature" index agree w/ that of BPS gravitons in $AdS_5 \times S^5$?
- Does this index undergo a deconfinement transition at certain $O(1)$ temperature?

Large N index

- Large N matrix integral → eigenvalue distribution:
- Integral variables α_a are $U(1)^N \subset U(N)$ holonomies on temporal circle.
- Identical particles on S^1, $\theta \sim \theta + 2\pi$: densely distributed at large N.
- Replaced at large N by a functional integral over their distribution,

$$\rho(\theta) = \frac{1}{2\pi} + \frac{1}{2\pi}\sum_{n=1}^{\infty}\left[\rho_n e^{in\theta} + \rho_{-n}e^{-in\theta}\right] \quad , \quad \rho_{-n} = \rho_n^*$$

$$\int_0^{2\pi} d\theta \rho(\theta) = 1$$
$$\rho(\theta) \geq 0$$

- From fine-grained picture:

$$\rho(\theta) = \frac{1}{N}\sum_{a=1}^{N}\delta(\theta - \alpha_a) = \frac{1}{2\pi N}\sum_{n=-\infty}^{\infty}\sum_{a=1}^{N} e^{in(\theta-\alpha_a)}$$

- Large N index: [Aharony,Marsano,Minwalla,Papadodimas,Raamsdonk] (2003) [KMMR] (2005)

$$Z = \int \prod_{n=1}^{\infty}[d\rho_n d\rho_{-n}] \exp\left[-N^2 \sum_{n=1}^{\infty}\frac{1}{n}\rho_n\rho_{-n}\frac{\prod_I(1-e^{-n\Delta_I})}{\prod_i(1-e^{-n\omega_i})}\right]$$

- Low / high "temperature" ~ large/small Δ_I, ω_i (all being positive).
- "Low T": Saddle point at uniform distribution $\rho(\theta) = 1/2\pi$. "Confining phase"
- The confining index agrees w/ BPS graviton index on $AdS_5 \times S^5$ [KMMR] (2005).

Deconfinement from index?

- Does this index deconfine at high enough T?

$$Z = \int \prod_{n=1}^{\infty} [d\rho_n d\rho_{-n}] \exp\left[-N^2 \sum_{n=1}^{\infty} \frac{1}{n} \rho_n \rho_{-n} \frac{\prod_I (1 - e^{-n\Delta_I})}{\prod_i (1 - e^{-n\omega_i})} \right]$$

- Apparently, no, as the coefficients of Gaussian integrals are always positive,

$$f(\Delta_I, \omega_i) \equiv \frac{\prod_{I=1}^{3} (1 - e^{-\Delta_I})}{\prod_{i=1}^{2} (1 - e^{-\omega_i})} \qquad Z_{N\to\infty} = \prod_{n=1}^{\infty} f(n\Delta_I, n\omega_i)^{-1} = Z_{\text{gravitons}}$$

- … at real fugacities.
- So the index never seems to deconfine: Never sees a free energy at order $\log Z \sim N^2$.
- $\rho_1 = \langle tr_f [e^{i\int d\tau\, A_\tau}] \rangle$: Polyakov loop. Probes deconfinement at finite T : $\sim e^{-F_{quark}=-\infty}$

- Apparent reason for this may be a severe B/F cancelation due to $(-1)^F$.
- By this, I think most people imagined "microscopic cancelations" so far,

$$Z(x) = \sum_j \Omega_j x^j \qquad \log \Omega_j \sim \mathcal{O}(N^0) \text{ at } j \sim \mathcal{O}(N^2)$$

- However, there seems to be a weaker and subtler "macroscopic cancelation"

Macroscopic B/F cancelation

- Consider the following unrefined index, for simplicity. $e^{-\omega} = x^3,\ e^{-\Delta} = x^2$

  $\Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta,\ \omega_1 = \omega_2 \equiv \omega$

- E.g. fugacity expansion of U(2) index is given by

$$
\begin{aligned}
&1 + 3x^2 - 2x^3 + 9x^4 - 6x^5 + 11x^6 - 6x^7 + 9x^8 + 14x^9 - 21x^{10} + 36x^{11} - 17x^{12} - 18x^{13} \\
&+114x^{14} - 194x^{15} + 258x^{16} - 168x^{17} - 112x^{18} + 630x^{19} - 1089x^{20} + 1130x^{21} - 273x^{22} \\
&-1632x^{23} + 4104x^{24} - 5364x^{25} + 3426x^{26} + 3152x^{27} - 13233x^{28} + 21336x^{29} - 18319x^{30} \\
&-2994x^{31} + 40752x^{32} - 76884x^{33} + 78012x^{34} - 11808x^{35} + \cdots \ .
\end{aligned}
$$

- Degeneracy grows at large charges, but with randomly alternating signs.

- Consider a large charge $j \sim O(N^2)$ approximation of $\Omega_j = \frac{1}{2\pi i}\oint \frac{dx}{x^{j+1}} Z(x)$
- Saddle pt. calculus (Legendre transform) is insensitive to changing j by a quantum
- Unable to capture single Ω_j w/ wild oscillation. “Smears out” nearby Ω_j’s.

- If this is the case, one can try to obstruct cancelations of nearby Ω_j’s.
- Tune the fugacity phases to maximally obstruct cancelations of nearby B/F

Cardy limit

- Tomorrow, I'll explain how index sees deconfinement transition, w/ this idea.

- Today, we study something simpler: "high temperature limit"
  - large spin limit ~ high T limit, $|\omega_i| \ll 1$.
  - higher D analogue of 2d Cardy formula at $|\tau| \ll 1$

$$Z(\Delta_I, \omega_i) = \mathrm{Tr}\left[(-1)^F e^{-\sum_{I=1}^{3}\Delta_I Q_I - \sum_{i=1}^{2}\omega_i J_i}\right]$$

- There are studies on the "Cardy limit" of 4d index [Di Pietro, Komargodski] [Ardehali] ……
  - They all stick to real fugacities. Don't see macroscopic free energy for BH's.
  - E.g. in a class of models discussed in [Di Pietro, Komargodski], one finds

$$-\log Z \sim \frac{a-c}{\beta} \qquad \beta \sim \omega_{1,2} \ll 1$$

  - For 4d CFTs w/ AdS5 duals, $a \approx c$, so this formula only captures subleading corrections.

- A slight generalization reveals amazing structures, as we shall see.

[Choi, Joonho Kim, SK, Nahmgoong] 1810.nnnnn, 1811.nnnnn (2018)

Generalized Cardy limit

- Large charge (~ BPS energy) means small chemical potentials: "real parts"

- Keep O(1) imaginary parts of chemical potentials for internal symmetries.
- Convenient to change convention for the chemical potentials:

From:

$$Z(\Delta_I,\omega_i) = \mathrm{Tr}\left[(-1)^F e^{-\sum_{I=1}^3 \Delta_I Q_I - \sum_{i=1}^2 \omega_i J_i}\right]$$

$$\Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2=0$$

To:

$$\mathrm{Tr}\left[e^{-\sum_{I=1}^3 \Delta_I Q_I - \sum_{i=1}^2 \omega_i J_i}\right]$$

$$\Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2=2\pi i \pmod{4\pi i}$$

Should optimally distribute $2\pi i$ (caused by $(-1)^F$) to chemical potentials, minimizing B/F cancelations

- Matrix integral expression in this basis:

From:

$$Z=\frac{1}{N!}\int\prod_{a=1}^N\frac{d\alpha_a}{2\pi}\prod_{a<b}\left(2\sin\frac{\alpha_{ab}}{2}\right)^2\exp\left[\sum_{n=1}^\infty\frac{1}{n}\left(1-\frac{\prod_{I=1}^3 2\sinh\frac{n\Delta_I}{2}}{2\sinh\frac{n\omega_1}{2}\cdot 2\sinh\frac{n\omega_2}{2}}\right)\sum_{a,b=1}^N e^{in\alpha_{ab}}\right]$$

To:

$$\frac{1}{N!}\oint\prod_{a=1}^N\frac{d\alpha_a}{2\pi}\cdot\prod_{a<b}\left(2\sin\frac{\alpha_{ab}}{2}\right)^2\exp\left[\sum_{a,b=1}^N\sum_{n=1}^\infty\frac{1}{n}\left(1+\sum_{s_1,s_2,s_3=\pm1}\frac{s_1s_2s_3(-1)^{n-1}e^{\frac{ns_I\Delta_I}{2}}}{2\sinh\frac{n\omega_1}{2}\cdot 2\sinh\frac{n\omega_2}{2}}\right)e^{in\alpha_{ab}}\right]$$

- Approximate at $|\omega_i| \ll 1$. (chose to give O(1) phases to all Δ_I's)

$$Z\sim\frac{1}{N!}\oint\prod_{a=1}^N\frac{d\alpha_a}{2\pi}\exp\left[-\frac{1}{\omega_1\omega_2}\sum_{a\neq b}\sum_{s_1,s_2,s_3=\pm1}s_1s_2s_3\mathrm{Li}_3\left(-e^{\frac{s_I\Delta_I}{2}}e^{i\alpha_{ab}}\right)\right]$$

$$\mathrm{Li}_3(x)\equiv\sum_{n=1}^\infty\frac{x^n}{n^3}$$

Free energy in the Cardy limit

- Cardy saddle point for holonomies α_a's:

- "Maximally deconfining" saddle points: $\alpha_1 = \alpha_2 = \cdots = \alpha_N$. Clearly a local saddle point.
- Likely to be the dominant one for 4d U(N) MSYM [Aharony, et.al.] (2003) [Di Pietro, Komargodski]
- However, for some 4d N=1 SCFTs, expect caveats [Ardehali] (2015)

- The resulting free energy: [Choi, Joonho Kim, SK, Nahmgoong] (2018)

Use: $\mathrm{Li}_3(-e^{x}) - \mathrm{Li}_3(-e^{-x}) = -\frac{x^3}{6} - \frac{\pi^2 x}{6}$

$-\pi < \mathrm{Im}(x) < \pi$

$$\log Z \sim -\frac{N^2}{\omega_1\omega_2} \sum_{s_1 s_2 s_3 = +1} \left[\mathrm{Li}_3\left(-e^{\frac{s_I \Delta_I}{2}}\right) - \mathrm{Li}_3\left(-e^{-\frac{s_I\Delta_I}{2}}\right)\right] \longrightarrow \log Z \sim \frac{N^2 \Delta_1\Delta_2\Delta_3}{2\omega_1\omega_2}$$

in the "canonical chamber" (octahedron):

$$-2\pi < \mathrm{Im}(\Delta_1 + \Delta_2 + \Delta_3) < 2\pi\ , \quad -2\pi < \mathrm{Im}(\Delta_1 - \Delta_2 - \Delta_3) < 2\pi\ ,$$
$$-2\pi < \mathrm{Im}(-\Delta_1 + \Delta_2 - \Delta_3) < 2\pi\ , \quad -2\pi < \mathrm{Im}(-\Delta_1 - \Delta_2 + \Delta_3) < 2\pi$$

- This is our Cardy free energy, valid at $|\omega_i| \ll 1$ and $\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i$
- Macroscopic in large N, $\propto N^2$, since Δ_I's have O(1) imaginary parts
- Explained later [Benini, Milan] that it holds beyond Cardy limit, at "certain" local saddle pt.
- Tomorrow, I'll give alternative derivation in which 4d 't Hooft anomaly determines it.

Macroscopic entropy

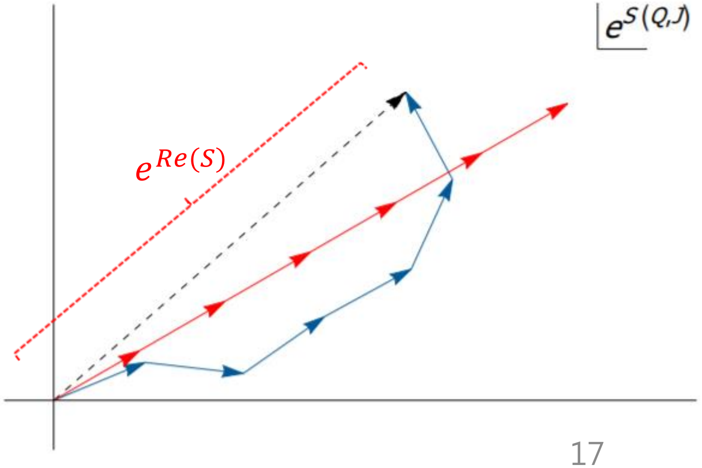
- So far, we established the Cardy formula for the index of 4d U(N) MSYM
- In the grand canonical ensemble.
- Macroscopic free energy, $\propto N^2$ as we further take a large N limit.

- To compute the macroscopic entropy, go to the microcanonical ensemble.
- inverse Laplace transform: large charge approx. by Legendre transform

$$S(\Delta_I, \omega_i; Q_I, J_i) = \frac{N^2}{2}\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} + \sum_{I=1}^{3} Q_I \Delta_I + \sum_{i=1}^{2} J_i \omega_i \qquad \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i$$

- This extremization problem was already discussed [Hosseini, Hristov, Zaffaroni] (2017), as a tool to neatly express properties of known BPS BH's in $AdS_5 \times S^5$.

- $S(Q_I, J_i)$ at the saddle pt. is in general complex.
- $Im(S)$: remnant of phase rotations at counting
- $Re(S)$: a lower bound to the true BPS entropy
- We successfully count BH's if $Re(S)$ saturates S_{BH}.



Counting large black holes

- Here, recall that known BPS BH's satisfy a charge relation.
- Technically from gravity, comes from demanding smooth event horizon. (See later, contrasting w/ hairy BPS BH's.) No intrinsic QFT explanation, at least so far.
- Anyway, to see if our index counts these BH's, impose it by hand.
- Charge relation of known BH's is extremely simple, $Im\big(S(Q_I, J_i)\big) = 0$: an explanation from near-horizon AdS_2 & superconformal QM [Benini, Milan]

- Obtains the following eqn for complex $S(Q_I, J_i)$: [Choi, Joonho Kim, SK, Nahmgoong]

$$(S - 2\pi i Q_1)(S - 2\pi i Q_2)(S - 2\pi i Q_3) - \pi i N^2 (S + 2\pi i J_1)(S + 2\pi i J_2) = 0$$

- Imposing $Im(S) = 0$, this complex eqn. w/ real S yields $S^3 + \alpha S = 0,\ \beta S^2 + \gamma = 0$

$$S\ (= \sqrt{-\alpha}) = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_3 Q_1 - \frac{N^2}{2}(J_1 + J_2)}$$

$$S\ (= \sqrt{-\gamma/\beta}) = 2\pi \sqrt{\frac{Q_1 Q_2 Q_3 + \frac{N^2}{2} J_1 J_2}{\frac{N^2}{2} + Q_1 + Q_2 + Q_3}}\ .$$

known expression for S_{BH} [K.Lee, SK] (2006)

Compatibility of two expressions: charge relation of known BH's

- Accounts for large BPS BH's in $AdS_5 \times S^5$ from dual QFT, as the dominant saddle point.

Away from large BH's

- Even away from large BH's, $\log Z \sim \frac{N^2\Delta_1\Delta_2\Delta_3}{2\omega_1\omega_2}$ describe known BH's.
- Study $F = -Re(\log Z)$ to anticipate the fate of these BH's at generic size.
- For simplicity, set $Q \equiv Q_1 = Q_2 = Q_3$, $J \equiv J_1 = J_2$. Or, $\Delta \equiv \Delta_1 = \Delta_2 = \Delta_3$, $\omega \equiv \omega_1 = \omega_2$.
- Index: $3\Delta = 2\omega + 2\pi i$ admits one chemical potential ω, conjugate to $2(Q + J)$.

$$\omega = -\xi\sqrt{\frac{3\pi+3\xi}{\pi-3\xi}}\ , \quad -\pi < \xi < 0$$

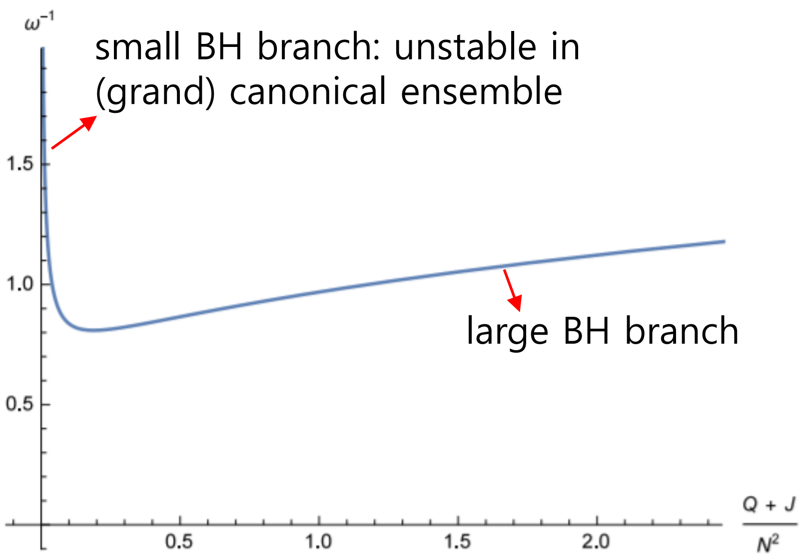
$$F = -\mathrm{Re}(\log Z) = -\frac{N^2}{18}\frac{\pi^3 - 9\pi\xi^2 - 8\xi^3}{\xi^2}\sqrt{\frac{\pi+\xi}{3\pi-9\xi}}$$

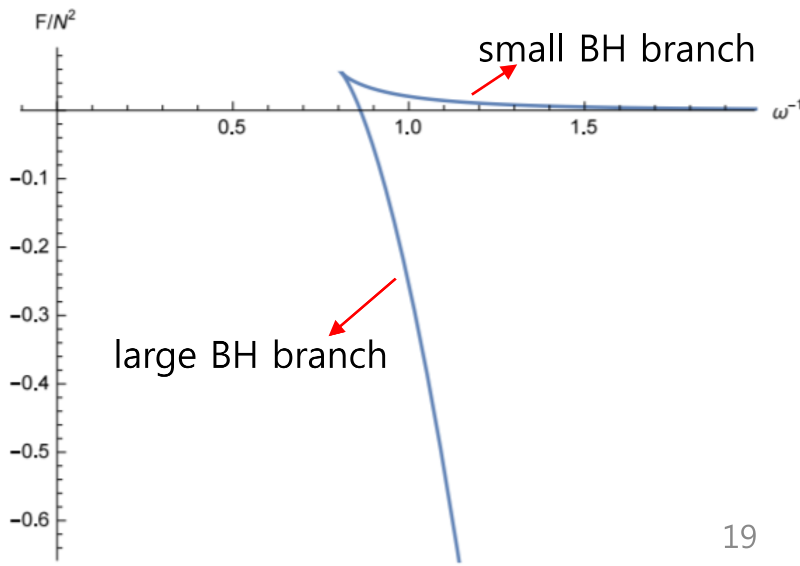
$$Q + J = -\frac{N^2}{54}\frac{(\pi-2\xi)^2(\pi+\xi)}{\xi^3}\ .$$

- **Results:** (quite similar to AdS Schwarzschild…!)

  Here, ω denotes $Re(\omega)$, ξ denotes $Im(\omega)$.

  plays the role of T[-1], in the BPS sector



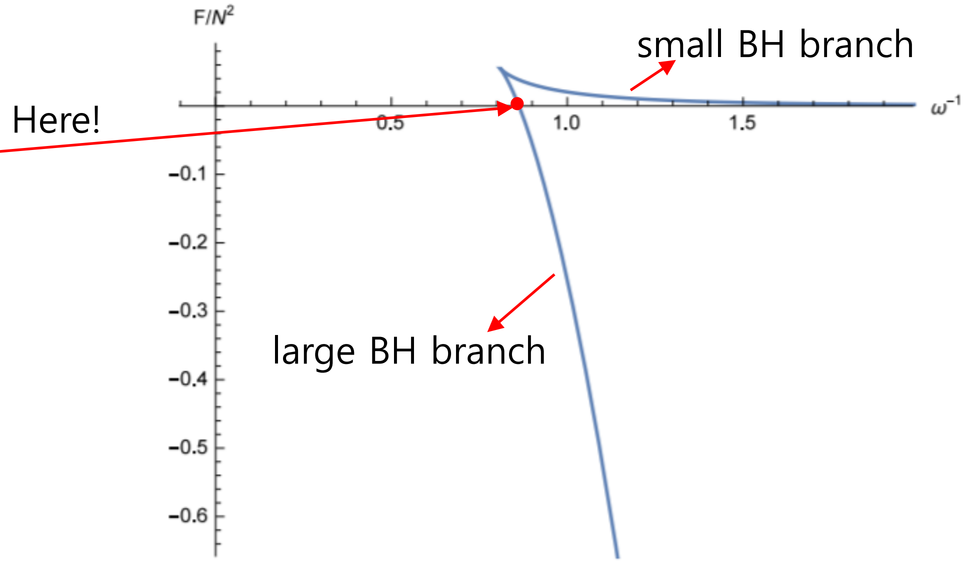


Hawking-Page transition of known BH's

- In canonical ensemble, many large N saddle points compete at given $T \sim \omega^{-1}$.
- One saddle point is the thermal graviton phase. $F \sim O(N^0)$
- There is one black hole phase, given by known analytic solutions. $F \sim O(N^2)$
- There could possibly be more. (We shall argue later/tomorrow that they SHOULD exist.)

- The competition of graviton phase & black hole phase decides the Hawking-Page transition temperature.
- Transition "temperature" for known BH saddle point:

$$\omega_{\rm HP}^{\rm known} \equiv \frac{\pi}{16}\sqrt{414 - 66\sqrt{33}} \approx 1.16$$



- I'll explain tomorrow that the index should deconfine below this T :
  This probably predicts new BH's.

Concluding remarks

- Unlike a long (& wrong) belief, index of SCFT_4 sees BPS AdS_5 black holes.
- Counts known BH's in large BH limit, as the dominant saddle point
- Tomorrow: Away from this limit, new BH's should exist. Likely to be "hairy black holes"

- Some questions:
- Explicit construction of BPS operators at weak coupling? [Grant,Grassi,SK,Minwalla] [Chang,Yin]
- Exists some trials to construct BPS operators at large spin: "Fermi liquid operator"

- Some topics to be discussed tomorrow:
- New BH saddle points?
- Deconfinement transition from the index: Again predicts new BPS BH's.
- Alternative anomaly-based approach to the Cardy free energy in $\text{SCFT}_{D=\text{even}}$

- Exotic deconfinements in SCFTs in low & high dimensions (AdS_4 & AdS_6 BH's)