Black holes in AdS/CFT

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Plan

Lecture 1 (today):

• Introduction
• AdS black holes & gauge theories
• Supersymmetric AdS$^5$ black holes
  - Brief summary of analytic/numerical solutions
  - Index of SCFT & its Cardy limit
  - Black hole thermodynamics from QFT

Lecture 2 (tomorrow):

• Evidence of new BH saddle points
• Deconfinement & Hawking-Page transitions
• AdS black holes in diverse dimensions
  - AdS$^5$ and AdS$^7$: ‘t Hooft anomaly & background field method
  - AdS$^6$ from CFT$^5$ & AdS$^4$ from CFT$^3$
Black hole thermodynamics

- Thermodynamic “analogs” of classical black hole physics
  - 1\textsuperscript{st} law: perturb stationary BH
    \[ dE = \frac{\kappa}{8\pi} dA + \Omega\, dJ + \Phi\, dQ \]
  - 2\textsuperscript{nd} law: “area theorem”
    \[ \frac{dA}{dt} \geq 0 \]
    (Hawking 1971)

- Area \sim entropy carried by BH (cloaked behind event horizon): Bekenstein (1973)
- Hawking radiation (1974): Strengthens the statistical interpretation, also fixing
  \[ S_{\text{BH}} = \frac{kA}{4\ell_p^2} \]
  \[ \ell_p = \sqrt{\frac{G\hbar}{c^3}} \]

- A striking emergent aspect of gravity, encoding BH’s quantum & thermal info on its geometry. May be a good clue for QG itself. Holography, entropic force, …
Statistical approaches to BH

• Want to engineer BH’s w/ microscopic descriptions, e.g. D-branes.
  - It turns out that the emergent gravitational description of BH thermodynamics (large charge, etc.) demands strong coupling QM/QFT analysis, making quantitative studies difficult.
  - BPS black holes: protected physics, e.g. entropy ~ spectrum

• 5d BPS BHs from D1-D5-P: entropy from Cardy formula of 2d CFT [Strominger, Vafa] (1996)

\[
Z(\tau) \sim \text{Tr} \left[ e^{2\pi i \tau L_0} \right] \sim \exp \left[ \frac{\pi i c}{12\tau} \right] \quad \text{at } \tau \rightarrow i0^+
\]

\[
e^{S(P,c)} = \int d\tau Z(\tau) e^{-2\pi i \tau P} \sim \exp \left[ 2\pi \sqrt{\frac{cP}{6}} \right] \quad \text{at } P \gg 1 \quad \text{at } c = 6Q_1Q_5
\]

- Accounts for the Bekenstein-Hawking entropy \( S_{\text{BH}} = 2\pi \sqrt{Q_1Q_5P} \) of BH’s.

• Various studies of similar sort: 4d & 5d BH’s in asymptotically flat spacetime
  - BH’s in: heterotic strings, M-theory on CY_3, F-theory on CY_3, toroidal compactifications, orbifold compactifications, … ; small BH’s ; some limited studies on non-BPS, ……
Many black holes. Too many.

- Further studies: other types of BH’s than those first encountered & studied. e.g.
  - 4d N=2 compactifications: single-center, multi-centered [Denef et.al, …], wall-crossings, …
  - 5d: single-center, multi-center & threshold bounds [Vafa,Vandoren,Haghighat,Klemm,Murthy, …],
    black rings [Elvang-Empanan-Mateos-Reall, Bena-Warner, Gauntlett-Gutowski], hairs & “microstate
    geometry” [Bena, Warner, Shigemori, de Boer, …], …

- Certain black holes are simple (e.g. admit simple Cardy formula counting)
- Certain black holes are very tricky: e.g.
  - Become dominant saddle points only in non-Cardy regimes
  - Some BH’s are even argued to be invisible in the index, …

• Anyway, generally expect a zoo of competing BH’s in a given system.
• Often ad hoc studies: Demands by hand the BH’s one wishes to study. Single-center
  BH, multi-BH constituents, black ring, … Often employs different effective descriptions, …
• A setting in which one can study thermodynamics of all BH’s systematically?
Black holes in $\text{AdS}_D$ ($D \geq 4$)

- Schwarzschild black holes in AdS (e.g. in $\text{AdS}_5$):
  - Has $S^3$ horizon & boundary. Embedded in global $\text{AdS}_5$.
  - Small BH: Similar to BH’s in flat space. Negative specific heat.
  - Large BH: important in AdS thermodynamics

\[
T = \frac{r_+}{\pi \ell^2} + \frac{1}{2 \pi r_+}
\]

\[
r_+^2 = -\frac{\ell^2}{2} + \ell \sqrt{\frac{\ell^2}{4} + \omega M}
\]

\[
\omega \equiv \frac{16 \pi G_N}{3 \text{vol}(S^3)}
\]

  transition between two phases, at $T = \frac{3}{2 \pi \ell}$ (order 1 in the unit of AdS radius $\ell$)
  - Low T phase: gas of gravitons in AdS. Doesn’t see $1/G_N \sim N^2$ so that $F \sim O(N^0)$
  - High T phase: large AdS black holes ($F_{BH} = -T \log Z_{BH} < 0$). Sees $N^2$. 

Black holes from gauge theory

• CFT dual (on $S^3 \times R$): confinement-deconfinement transition [Witten] (1998)
  - Confined phase: $F \sim O(N^0)$, glue-balls (& mesons, etc.)
  - Deconfined phase: $F \sim O(N^2)$ of gluons (& quarks) ~ matrices
• deconfined (quark-)gluon plasma = black holes

(Despite w/ finite spatial volume, large N makes sharp phase transitions possible…)

• Thermodynamics of YM on $S^3 \times R$ studied at weak coupling. Some qualitative AdS physics visible. [Aharony, Marsano, Minwalla, Papadodimas, van Raamsdonk] (2003)
• However, some qualitative & all quantitative aspects are different.

• BPS black holes in $AdS_5 \times S^5$ from 4d $N = 4$ SYM on $S^3 \times R$.
  - First found in [Gutowski, Reall] (2004), and generalized thereof.
  - Surprisingly, this basic question wasn’t answered well till very recently.
  - I’ll show you today/tomorrow that, the basic solution is extremely simple.
Supersymmetric black holes

- **BPS black hole solutions**
  

- BPS energy (=mass): determined by

  \[ U(1)^3 \subset SO(6) \text{ electric charges (momenta on } S^5) \& U(1)^2 \subset SO(4) \text{ momenta on } AdS_5 \]

  \[ E\ell = Q_1 + Q_2 + Q_3 + J_1 + J_2 \]

- Preserves only 2 of the 32 supercharges (1/16-BPS states)
- Very complicated solutions (many nonzero charges, little symmetry)

- Needs both electric charges \( Q_I \) & spins \( J_i \) to form BPS black holes

- \( J_i = 0 \): Chiral rings. Trace of commuting fields, e.g. \( tr(XY^2Z) \sim tr(YZYX) \)
- Not enough entropy \( \sim O(N^2) \): basic fields are effectively commutative, \( \sim O(N^1) \)
- \( Q_I = 0 \): Most operators are anomalous without \( SO(6) \) R-charges. E.g. almost void graviton-like BPS operators in this sector (only “singletons”)
SUSY black holes (continued)

- Charge relations: May naively expect 5 parameter solution, since they carry 5 charges.
  - Has to do w/ demanding smooth event horizon w/ SUSY (E.g. see [Gutowski, Reall] 1)

- Similar properties hold for many known BPS BH’s in flat spacetime:
  5d BPS BH’s w/ rotations [Breckenridge, Myers, Peet, Vafa] :
    microscopic charges: electric charges $Q_I$ and spins $J_1, J_2$.
    BH charge relation: $J_1 = J_2$ (equal spins)
  4d BPS BH’s [Ferrara, Kallosh, Strominger] :
    microscopic charges: dyonic charges $P^I, Q_I$ and spin $J$.
    BH charge relation: $J = 0$ (no rotation)

- Recently, some numerical studies on more general BH’s in $AdS_5 \times S^5$.
- Hairy BPS BH’s [Markeviciute,Santos] [Markeviciute] [Bhattacharyya,Minwalla,Papadodimas]
  - Charge condensates outside BH horizon. No charge relations.
  - Mild horizon singularity (finite metric, horizon area, … but certain tidal forces diverge.)
SUSY black holes from QFT

- Study the degeneracy of BPS local operators = BPS states on $S^3 \times R$.
  - More easily, study a “Witten index” type partition function on $S^3 \times R$,
  - … at the risk of losing certain states by boson/fermion cancelation from $(-1)^F$.

- This index was defined in [Romelsberger] [Kinney,Maldacena,Minwalla,Raju] (2005):

\[
Z(\Delta_I, \omega_i) = \text{Tr} \left[ (-1)^F e^{-\sum_{I=1}^3 \Delta_I Q_I - \sum_{i=1}^2 \omega_i J_i} \right] \\
\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 0
\]

- Matrix integral expression for $U(N), N = 4$ SYM (QFT on $S^3 \times S^1$) [KMMR] (2005):

\[
Z = \frac{1}{N!} \int \prod_{a=1}^N \frac{d\alpha_a}{2\pi} \prod_{a<b} \left( 2 \sin \frac{\alpha_{ab}}{2} \right)^2 \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} \left( 1 - \frac{\prod_{I=1}^3 2 \sinh \frac{n\Delta_I}{2}}{2 \sinh \frac{n\omega_1}{2} \cdot 2 \sinh \frac{n\omega_2}{2}} \right) \sum_{a,b=1}^N e^{in\alpha_{ab}} \right]
\]

- Questions:
  - Does the low “temperature” index agree w/ that of BPS gravitons in $AdS_5 \times S^5$?
  - Does this index undergo a deconfinement transition at certain $O(1)$ temperature?
Large N index

- Large N matrix integral → eigenvalue distribution:
  - Integral variables $\alpha_a$ are $U(1)^N \subset U(N)$ holonomies on temporal circle.
  - Identical particles on $S^1$, $\theta \sim \theta + 2\pi$: densely distributed at large N.
  - Replaced at large N by a functional integral over their distribution,
    \[
    \rho(\theta) = \frac{1}{2\pi} + \frac{1}{2\pi} \sum_{n=1}^{\infty} \left[ \rho_n e^{in\theta} + \rho_{-n} e^{-in\theta} \right], \quad \rho_{-n} = \rho_n^* \quad \int_0^{2\pi} d\theta \rho(\theta) = 1 \quad \rho(\theta) \geq 0
    \]
  - From fine-grained picture:
    \[
    \rho(\theta) = \frac{1}{N} \sum_{a=1}^{N} \delta(\theta - \alpha_a) = \frac{1}{2\pi N} \sum_{n=-\infty}^{\infty} \sum_{a=1}^{N} e^{in(\theta - \alpha_a)}
    \]

  \[
  Z = \int \prod_{n=1}^{\infty} [d\rho_n d\rho_{-n}] \exp \left[ -N^2 \sum_{n=1}^{\infty} \frac{1}{n} \rho_n \rho_{-n} \prod_i \left( \frac{1 - e^{-n\Delta_i}}{1 - e^{-n\omega_i}} \right) \right]
  \]
  - Low / high “temperature” ~ large/small $\Delta_i, \omega_i$ (all being positive).
  - “Low T”: Saddle point at uniform distribution $\rho(\theta) = 1/2\pi$. “Confining phase”
  - The confining index agrees w/ BPS graviton index on $AdS_5 \times S^5$ [KMMR] (2005).
Deconfinement from index?

- Does this index deconfine at high enough $T$?

$$Z = \int \prod_{n=1}^{\infty} [d\rho_n d\rho_{-n}] \exp \left[ -N^2 \sum_{n=1}^{\infty} \frac{1}{n} \rho_n \rho_{-n} \prod_{i=1}^{n} \left( 1 - e^{-n\Delta_i} \right) \prod_{i=1}^{n} \left( 1 - e^{-\omega_i} \right) \right]$$

- Apparently, no, as the coefficients of Gaussian integrals are always positive,

$$f(\Delta_I, \omega_i) \equiv \frac{\prod_{l=1}^{3} (1 - e^{-\Delta_I})}{\prod_{l=1}^{2} (1 - e^{-\omega_i})}$$

$$Z_{N \to \infty} = \prod_{n=1}^{\infty} f(n\Delta_I, n\omega_i)^{-1} = Z_{\text{gravitons}}$$

- ... at real fugacities.
- So the index never seems to deconfine: Never sees a free energy at order $\log Z \sim N^2$.
- $\rho_1 = \langle tr_f [e^{i\int \tau A_\tau}] \rangle$ : Polyakov loop. Probes deconfinement at finite $T$ : $\sim e^{-F_{\text{quark}}=-\infty}$

- Apparent reason for this may be a severe B/F cancelation due to $(-1)^F$.
- By this, I think most people imagined “microscopic cancelations” so far,

$$Z(x) = \sum_j \Omega_j x^j \quad \log \Omega_j \sim \mathcal{O}(N^0) \quad \text{at} \quad j \sim \mathcal{O}(N^2)$$

- However, there seems to be a weaker and subtler “macroscopic cancelation”
**Macroscopic B/F cancelation**

- Consider the following unrefined index, for simplicity.
  - E.g. fugacity expansion of U(2) index is given by
    \[
    e^{-\omega} = x^3, \quad e^{-\Delta} = x^2
    \]
    \[
    \Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta, \quad \omega_1 = \omega_2 = \omega
    \]
    \[
    1 + 3x^2 - 2x^3 + 9x^4 - 6x^5 + 11x^6 - 6x^7 + 9x^8 + 14x^9 - 21x^{10} + 36x^{11} - 17x^{12} - 18x^{13} \\
    +114x^{14} - 194x^{15} + 258x^{16} - 168x^{17} - 112x^{18} + 630x^{19} - 1089x^{20} + 1130x^{21} - 273x^{22} \\
    -1632x^{23} + 4104x^{24} - 5364x^{25} + 3426x^{26} + 3152x^{27} - 13233x^{28} + 21336x^{29} - 18319x^{30} \\
    -2994x^{31} + 40752x^{32} - 76884x^{33} + 78012x^{34} - 11808x^{35} + \cdots
    \]
  - Degeneracy grows at large charges, but with randomly alternating signs.

- Consider a large charge \( j \sim O(N^2) \) approximation of \( \Omega_j = \frac{1}{2\pi i} \int \frac{dx}{x^{j+1}} Z(x) \)
  - Saddle pt. calculus (Legendre transform) is insensitive to changing \( j \) by a quantum
  - Unable to capture single \( \Omega_j \) w/ wild oscillation. “Smears out” nearby \( \Omega_j \)’s.

- If this is the case, one can try to obstruct cancelations of nearby \( \Omega_j \)’s.
- Tune the fugacity phases to maximally obstruct cancelations of nearby B/F
Cardy limit

• Tomorrow, I’ll explain how index sees deconfinement transition, w/ this idea.

• Today, we study something simpler: “high temperature limit”
  - large spin limit ~ high T limit, $|\omega_i| \ll 1$.
  - higher D analogue of 2d Cardy formula at $|\tau| \ll 1$

\[
Z(\Delta_i, \omega_i) = \text{Tr} \left[ (-1)^F e^{-\sum_{i=1}^{3} \Delta_i Q_i - \sum_{i=1}^{2} \omega_i J_i} \right]
\]

• There are studies on the “Cardy limit” of 4d index [Di Pietro, Komargodski] [Ardehali] ……
  - They all stick to real fugacities. Don’t see macroscopic free energy for BH’s.
  - E.g. in a class of models discussed in [Di Pietro, Komargodski], one finds

\[
- \log Z \sim \frac{a - c}{\beta} \quad \beta \sim \omega_{1,2} \ll 1
\]

  - For 4d CFTs w/ AdS5 duals, $a \approx c$, so this formula only captures subleading corrections.

• A slight generalization reveals amazing structures, as we shall see.
  [Choi, Joonho Kim, SK, Nahmgoong] 1810.nnnnn, 1811.nnnnn (2018)
Generalized Cardy limit

- Large charge (~ BPS energy) means small chemical potentials: “real parts”
  - Keep O(1) imaginary parts of chemical potentials for internal symmetries.
  - Convenient to change convention for the chemical potentials:

\[
Z(\Delta_I, \omega_i) = \text{Tr} \left[ (-1)^F e^{-\sum_{i=1}^{3} \Delta_I Q_i - \sum_{i=1}^{2} \omega_i J_i} \right]
\]
\[\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 0\]

From:
\[
\frac{1}{N!} \int \prod_{a=1}^{N} \frac{d\alpha_a}{2\pi} \prod_{a<b} \left( 2 \sin \frac{\alpha_{ab}}{2} \right)^2 \exp \left[ \sum_{n=1}^{\infty} \frac{1}{n} \left( 1 - \frac{\prod_{I=1}^{3} 2 \sinh \frac{n\Delta_I}{2}}{2 \sinh \frac{n\omega_1}{2} \cdot 2 \sinh \frac{n\omega_2}{2}} \right) \sum_{a,b=1}^{N} e^{in\alpha_{ab}} \right]
\]

To:
\[
\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i \pmod{4\pi i}
\]

Should optimally distribute \(2\pi i\) (caused by \((-1)^F\)) to chemical potentials, minimizing B/F cancelations

- Matrix integral expression in this basis:

\[
Z \sim \frac{1}{N!} \int \prod_{a=1}^{N} \frac{d\alpha_a}{2\pi} \exp \left[ -\frac{1}{\omega_1 \omega_2} \sum_{a \neq b, s_1, s_2, s_3 = \pm 1} s_1 s_2 s_3 \text{Li}_3 \left( -e^{\frac{s_1 \Delta_I}{2} + i\alpha_{ab}} \right) \right] \text{Li}_3(x) \equiv \sum_{n=1}^{\infty} \frac{x^n}{n^3}
\]
Free energy in the Cardy limit

- Cardy saddle point for holonomies $\alpha_a$'s:
  - “Maximally deconfining” saddle points: $\alpha_1 = \alpha_2 = \cdots = \alpha_N$. Clearly a local saddle point.
  - Likely to be the dominant one for 4d U(N) MSYM [Aharony, et.al.] (2003) [Di Pietro, Komargodski]
  - However, for some 4d N=1 SCFTs, expect caveats [Ardehali] (2015)

- The resulting free energy: [Choi, Joonho Kim, SK, Nahmgoong] (2018)

$$\log Z \sim -\frac{N^2}{\omega_1 \omega_2} \sum_{s_1 s_2 s_3 = +1} \left[ \text{Li}_3 \left( -e^{\frac{s_1 \Delta I}{2}} \right) - \text{Li}_3 \left( -e^{-\frac{s_1 \Delta I}{2}} \right) \right]$$

in the “canonical chamber” (octahedron):

- This is our Cardy free energy, valid at $|\omega_i| \ll 1$ and $\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i$
- Macroscopic in large $N$, $\propto N^2$, since $\Delta_i$'s have $O(1)$ imaginary parts
- Explained later [Benini, Milan] that it holds beyond Cardy limit, at “certain” local saddle pt.
- Tomorrow, I’ll give alternative derivation in which 4d ‘t Hooft anomaly determines it.
Macroscopic entropy

• So far, we established the Cardy formula for the index of 4d U(N) MSYM
  - In the grand canonical ensemble.
  - Macroscopic free energy, $\propto N^2$ as we further take a large N limit.

• To compute the macroscopic entropy, go to the microcanonical ensemble.
  - inverse Laplace transform: large charge approx. by Legendre transform
    \[
    S(\Delta_I, \omega_i; Q_I, J_i) = \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_1 \omega_2} + \sum_{I=1}^{3} Q_I \Delta_I + \sum_{i=1}^{2} J_i \omega_i \\
    \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i
    \]

  - This extremization problem was already discussed [Hosseini, Hristov, Zaffaroni] (2017), as a tool to neatly express properties of known BPS BH’s in $AdS_5 \times S^5$.

• $S(Q_I, J_i)$ at the saddle pt. is in general complex.
  - $Im(S)$: remnant of phase rotations at counting
  - $Re(S)$: a lower bound to the true BPS entropy
  - We successfully count BH’s if $Re(S)$ saturates $S_{BH}$. 
Counting large black holes

- Here, recall that known BPS BH’s satisfy a charge relation.
  - Technically from gravity, comes from demanding smooth event horizon. (See later, contrasting w/ hairy BPS BH’s.) No intrinsic QFT explanation, at least so far.
  - Anyway, to see if our index counts these BH’s, impose it by hand.
  - Charge relation of known BH’s is extremely simple, $Im(S(Q_I,J_i)) = 0$: an explanation from near-horizon AdS$_2$ & superconformal QM [Benini, Milan]

- Obtains the following eqn for complex $S(Q_I,J_i)$: [Choi, Joonho Kim, SK, Nahmgoong]
  \[(S - 2\pi i Q_1)(S - 2\pi i Q_2)(S - 2\pi i Q_3) - \pi i N^2(S + 2\pi i J_1)(S + 2\pi i J_2) = 0\]
  - Imposing $Im(S) = 0$, this complex eqn. w/ real $S$ yields $S^3 + \alpha S = 0, \beta S^2 + \gamma = 0$

- Accounts for large BPS BH’s in $AdS_5 \times S^5$ from dual QFT, as the dominant saddle point.
Away from large BH’s

- Even away from large BH’s, \( \log Z \sim \frac{N^2 \Delta_1 \Delta_2 \Delta_3}{2\omega_1 \omega_2} \) describe known BH’s.
  - Study \( F = -Re(\log Z) \) to anticipate the fate of these BH’s at generic size.
  - For simplicity, set \( Q \equiv Q_1 = Q_2 = Q_3, J \equiv J_1 = J_2 \). Or, \( \Delta \equiv \Delta_1 = \Delta_2 = \Delta_3, \omega \equiv \omega_1 = \omega_2 \).
  - Index: \( 3\Delta = 2\omega + 2\pi i \) admits one chemical potential \( \omega \), conjugate to \( 2(Q + J) \).

- Results: (quite similar to AdS Schwarzschild…!)

Here, \( \omega \) denotes \( Re(\omega) \), \( \xi \) denotes \( Im(\omega) \).

\[
\omega = -\xi \sqrt{\frac{3\pi + 3\xi}{\pi - 3\xi}}, \quad -\pi < \xi < 0
\]

\[
F = -Re(\log Z) = -\frac{N^2 \pi^3 - 9\pi \xi^2 - 8\xi^3}{18 \xi^2 \sqrt{\frac{\pi + \xi}{3\pi - 9\xi}}}
\]

\[
Q + J = -\frac{N^2}{54} \frac{(\pi - 2\xi)^2(\pi + \xi)}{\xi^3}.
\]

small BH branch: unstable in (grand) canonical ensemble

large BH branch

small BH branch

large BH branch
Hawking-Page transition of known BH’s

• In canonical ensemble, many large N saddle points compete at given $T \sim \omega^{-1}$.
  - One saddle point is the thermal graviton phase. $F \sim O(N^0)$
  - There is one black hole phase, given by known analytic solutions. $F \sim O(N^2)$
  - There could possibly be more. (We shall argue later/tomorrow that they SHOULD exist.)

• The competition of graviton phase & black hole phase decides the Hawking-Page transition temperature.

• Transition “temperature” for known BH saddle point:

\[ \omega_{\text{HP}}^{\text{known}} = \frac{\pi}{16} \sqrt{414 - 66\sqrt{33}} \approx 1.16 \]

- I’ll explain tomorrow that the index should deconfine below this $T$:
  This probably predicts new BH’s.
Concluding remarks

• Unlike a long (& wrong) belief, index of SCFT$_4$ sees BPS $AdS_5$ black holes.
  - Counts known BH’s in large BH limit, as the dominant saddle point
  - Tomorrow: Away from this limit, new BH’s should exist. Likely to be “hairy black holes”

• Some questions:
  - Explicit construction of BPS operators at weak coupling? [Grant, Grassi, SK, Minwalla] [Chang, Yin]
  - Exists some trials to construct BPS operators at large spin: “Fermi liquid operator”

• Some topics to be discussed tomorrow:
  - New BH saddle points?
  - Deconfinement transition from the index: Again predicts new BPS BH’s.
  - Alternative anomaly-based approach to the Cardy free energy in SCFT$_{D={\text{even}}}$
  - Exotic deconfinements in SCFTs in low & high dimensions (AdS$_4$ & AdS$_6$ BH’s)