Black holes in AdS/CFT 2

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Kavli Asian winter school

Jan 15, 2019

Plan

Lecture 1 (yesterday):

- Introduction
- AdS black holes & gauge theories
- Supersymmetric AdS_5 black holes

- Brief summary of analytic/numerical solutions
- Index of SCFT & its Cardy limit
- Black hole thermodynamics from QFT

Lecture 2 (today):

- Evidence of new BH saddle points
- Deconfinement & Hawking-Page transitions
- AdS black holes in diverse dimensions

- AdS_5 and AdS_7: ‘t Hooft anomaly & background field method
- AdS_6 from CFT_5 & AdS_4 from CFT_3

New black holes?

- Are the known analytic BH's all saddle pts? Dominant away from Cardy limit?
- They may NOT be true. We shall discuss two evidences:

- Index in the 1/8-BPS Macdonald sector. (~ Schur operators)
- Compute an upper bounds on deconfinement transition point

- 1/8-BPS sectors: On top of BPS condition $E = Q_1 + Q_2 + Q_3 + J_1 + J_2$, further impose

- $J_1 + J_2 = 0$: "chiral ring" sector. Completely solved. Doesn't host BH's.
- $Q_1 + Q_2 = 0$: Completely solved. No BH's. [Mandal,Suryanarayana] [Grant,Grassi,SK,Minwalla]

- $Q_3 + J_2 = 0$: No complete solution known. Called "Schur operators"
- Limit of the index, projecting states to $Q_3 + J_2 = 0$: $\Delta_3, \omega_2 \to \infty$, $\Delta_3/\omega_2 \to 1$

"Macdonald index" [Gadde, Rastelli, Razamat, Yan] (2011)

$$Z = \mathrm{Tr}\left[(-1)^F e^{-\sum_{I=1}^{2}\Delta_I(Q_I+J_2)-\omega_1(J_1-J_2)}\right]$$

$$Z = \frac{1}{N!}\int\prod_{a=1}^{N}\frac{d\alpha_a}{2\pi}\prod_{a<b}\left(2\sin\frac{\alpha_{ab}}{2}\right)^2\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(1-\frac{(1-e^{-n\Delta_1})(1-e^{-n\Delta_2})}{1-e^{-n\omega_1}}\right)\sum_{a,b=1}^{N}e^{in\alpha_{ab}}\right]$$

Analytic BH's & beyond in 1/8-BPS sector

- BH charge relation at $Q_3 + J_2 = 0$:

$$0 = \left(Q_1 + Q_2 + \frac{N^2}{2}\right)\left(\underbrace{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_1+J_2)}_{(S/2\pi)^2 \geq 0} + Q_3^2\right)$$

$$\longrightarrow \quad Q_3 \to 0\ , \quad Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_1+J_2) = \left(\frac{S}{2\pi}\right)^2 \to 0$$

- So in the Macdonald limit, the known BH's shrink to "small black holes"
- Therefore, had the known BPS BH's been the unique solutions, one would expect no regular BH's in this 1/8-BPS sector. And thus no deconfined saddle points.

- However, QFT predicts a different structure.

- Consider the Macdonald-Cardy limit $|\omega_1| \to 0$.
- again at maximally deconfining saddle point $\alpha_1 = \cdots = \alpha_N$.

$$Z = \frac{1}{N!}\int \prod_{a=1}^{N} \frac{d\alpha_a}{2\pi} \prod_{a<b}\left(2\sin\frac{\alpha_{ab}}{2}\right)^2 \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(1 - \frac{(1-e^{-n\Delta_1})(1-e^{-n\Delta_2})}{1-e^{-n\omega_1}}\right)\sum_{a,b=1}^{N} e^{in\alpha_{ab}}\right]$$

$$\longrightarrow \quad \log Z \sim -\frac{N^2}{\omega_1}\left[\mathrm{Li}_2(1) - \mathrm{Li}_2(e^{-\Delta_1}) - \mathrm{Li}_2(e^{-\Delta_2}) + \mathrm{Li}_2(e^{-\Delta_1-\Delta_2})\right]$$

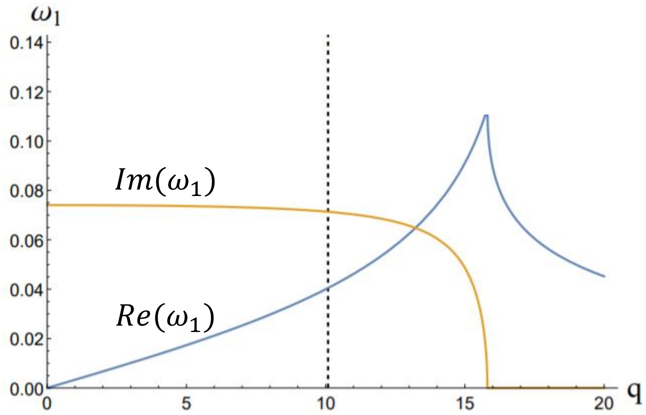
needs numerical analysis to study its Legendre transform

New deconfining saddle points

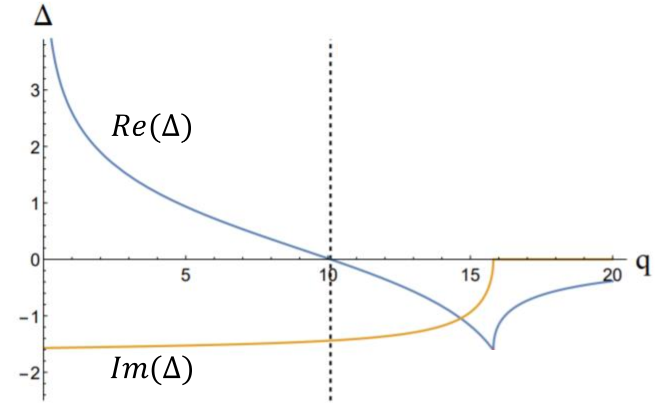
- We study the case w/ $\Delta \equiv \Delta_1 = \Delta_2$: charges $Q \equiv Q_1 = Q_2$ & J_1

- Numerical results:

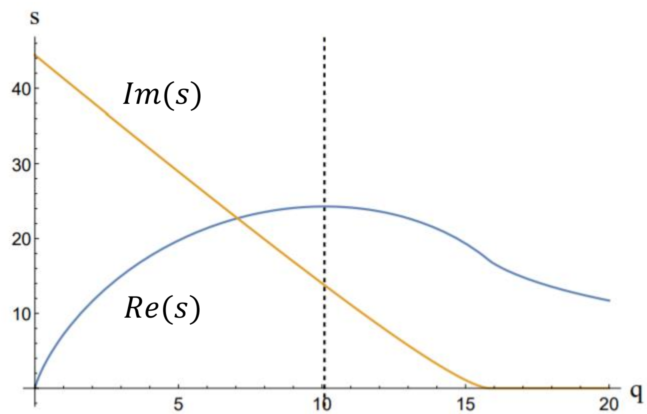
$$q \equiv \frac{Q+J_1}{N^2}\ ,\ j \equiv \frac{J_1 - J_2}{N^2}\ ,\ s \equiv \frac{S}{N^2}$$



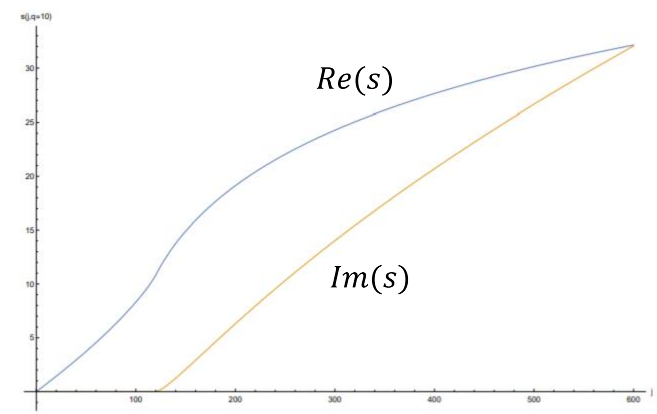
(a) ω_1 as a function of q at $j = 300$



(b) Δ as a function of q at $j = 300$



(a) $s(q)$ at $j = 300$

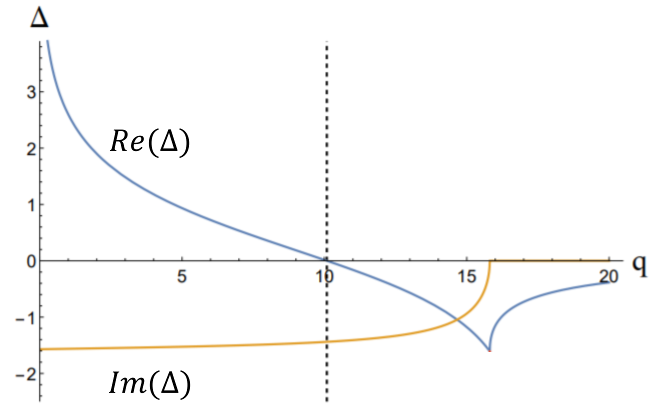


(b) $s(j)$ at $q = 10$

- We find saddle points with macroscopic entropy ($Re(S) \propto N^2$). Predicts new BH's…!?

Hairy black holes…?

- The resulting BH's are somewhat queer.
- At given angular momentum $j \equiv J/N^2$, the `temperature' $Re(\Delta)^{-1}$ reaches infinity at finite electric charge $q(j)$.
- Then "negative temperature" at super-critical charge.



(b) Δ as a function of q at $j = 300$

- These properties are very reminiscent of non-BPS and BPS hairy black holes in $AdS_5 \times S^5$, studied in the literature mostly numerically (in the sector w/ $Q \equiv Q_1 = Q_2 = Q_3, J \equiv J_1 = J_2$) [Bhattacharyya, Minwalla, Papadodimas] (2010) [Santos, Markeviciute] (2016), (2018) [Markevicuite] (2018)

- In the non-BPS sector, hairy black holes have been explored as more general BH's with charge/spin condensates outside the event horizon.
- BPS limits of these BH's exist without satisfying any charge relations.
- Implies rich thermodynamics in AdS/CFT. Our QFT saddles could be such BH's.

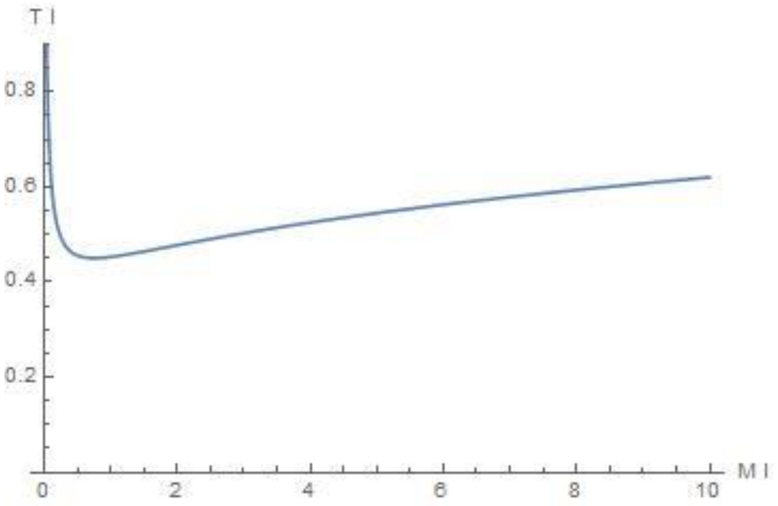
AdS$_5$ Schwarzschild black hole again

- Schwarzschild black holes in AdS$_5$:

- Small BH: Negative specific heat.
- Large BH: important in AdS thermodynamics

$$T = \frac{r_+}{\pi \ell^2} + \frac{1}{2\pi r_+}$$

$$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



- Hawking-Page transition [Hawking, Page] (1983):

transition between two phases, at $T = \frac{3}{2\,\pi\ell}$ (order 1 in the unit of AdS radius ℓ)

- Low T phase: gas of gravitons in AdS. Doesn't see $1/G_N \sim N^2$
- High T phase: large AdS black holes ($F_{BH} = -T \log Z_{BH} < 0$). Sees N^2.

- Yesterday, we saw similar structures w/ known analytic solutions for BPS BH's.

Large N index again

- The index from free 4d N=4 SYM [Kinney, Maldacena, Minwalla, Raju] (2005)

$$Z = \frac{1}{N!}\int \prod_{a=1}^{N}\frac{d\alpha_a}{2\pi}\prod_{a<b}\left(2\sin\frac{\alpha_{ab}}{2}\right)^2 \exp\left[\sum_{n=1}^{\infty}\frac{1}{n}\left(1-\frac{\prod_{I=1}^{3}2\sinh\frac{n\Delta_I}{2}}{2\sinh\frac{n\omega_1}{2}\cdot 2\sinh\frac{n\omega_2}{2}}\right)\sum_{a,b=1}^{N}e^{in\alpha_{ab}}\right]$$

$$\Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2=0$$

- Large N matrix integral → eigenvalue distribution:

$$\rho(\theta)=\frac{1}{N}\sum_{a=1}^{N}\delta(\theta-\alpha_a)=\frac{1}{2\pi N}\sum_{n=-\infty}^{\infty}\sum_{a=1}^{N}e^{in(\theta-\alpha_a)}$$

$$\rho(\theta)=\frac{1}{2\pi}+\frac{1}{2\pi}\sum_{n=1}^{\infty}\left[\rho_n e^{in\theta}+\rho_{-n}e^{-in\theta}\right] \quad , \quad \rho_{-n}=\rho_n^*$$

$$\int_0^{2\pi}d\theta\rho(\theta)=1$$

$$\rho(\theta)\geq 0$$

- Large N index:

$$Z=\int\prod_{n=1}^{\infty}[d\rho_n d\rho_{-n}]\exp\left[-N^2\sum_{n=1}^{\infty}\frac{1}{n}\rho_n\rho_{-n}\frac{\prod_I(1-e^{-n\Delta_I})}{\prod_i(1-e^{-n\omega_i})}\right]$$

- If the coefficients of Gaussian integrals are positive, ρ_n's don't condense: $F\sim N^0$
- "Low T": Saddle point w/ $\rho_n=0$ at uniform distribution $\rho(\theta)=1/2\pi$. "Confining phase"

No deconfinement & caveat

- Does this index deconfine at high enough T?

$$Z = \int \prod_{n=1}^{\infty} [d\rho_n d\rho_{-n}] \exp\left[-N^2 \sum_{n=1}^{\infty} \frac{1}{n} \rho_n \rho_{-n} \frac{\prod_I (1-e^{-n\Delta_I})}{\prod_i (1-e^{-n\omega_i})}\right]$$

- Not in this setting, as the coefficients of Gaussian integrals are always positive,

$$f(\Delta_I, \omega_i) \equiv \frac{\prod_{I=1}^{3}(1-e^{-\Delta_I})}{\prod_{i=1}^{2}(1-e^{-\omega_i})} \qquad Z_{N\to\infty} = \prod_{n=1}^{\infty} f(n\Delta_I, n\omega_i)^{-1} = Z_{\text{gravitons}}$$

- At real fugacities, as emphasized yesterday.
- So the index doesn't deconfine in this setting: $\log Z \sim N^0$.

- Again: introduce fugacity phases to obstruct B/F cancelation due to $(-1)^F$.
- As emphasized yesterday, we want to obstruct "macroscopic cancelation" between nearby B/F states at macroscopic charges.
- For simplicity, we shall set $\Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta$, $\omega_1 = \omega_2 \equiv \omega$: only one T-like fugacity

$$e^{-\omega} = x^3,\ e^{-\Delta} = x^2$$

Instability of the confining saddle point

- The index w/ $x \to x\, e^{i\phi}$ $$Z = \int \prod_{n=1}^{\infty} [d\rho_n d\rho_{-n}] \exp\left[-N^2 \sum_{n=1}^{\infty} \frac{f(x^n)}{n} \rho_n \rho_{-n}\right] \qquad f(x) = \frac{(1-x^2)^3}{(1-x^3)^2}$$

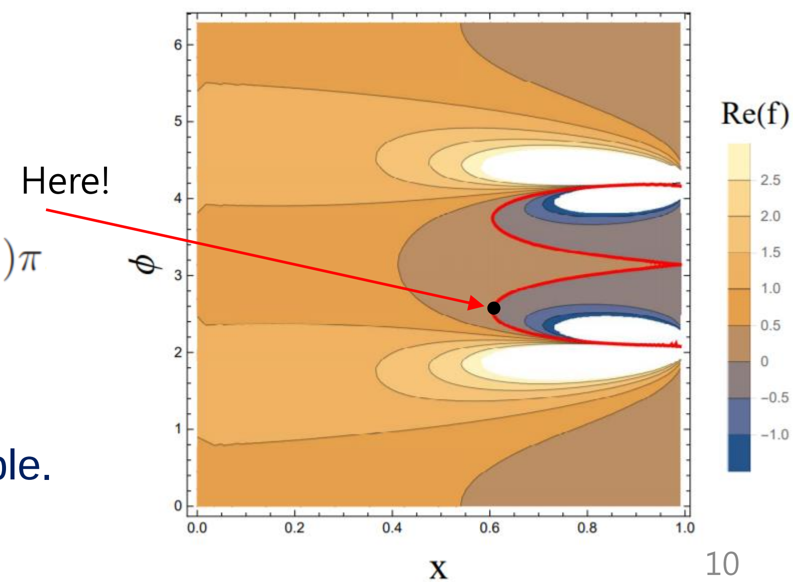
- Search for condensation of ρ_1, dialing ϕ : $\rho_1 = 0$ is unstable if $Re\left[f\left(xe^{i\phi}\right)\right] < 0$.
- Depending on ϕ, macroscopic cancelation may be partly obstructed, so that phase transition may be less delayed. Seek for the least delayed transition at optimized ϕ.

- $Re[f(xe^{i\phi})]$: $$\frac{(1-x^2)(1+x^2-2x\cos\phi)^2\,(2x(2+5x^2+2x^4)\cos\phi + (1+x^2)(1+4x^2+x^4+3x^2\cos(2\phi)))}{(1+x^6-2x^3\cos(3\phi))^2}$$

- red curve: $Re\left[f\left(xe^{i\phi}\right)\right] = 0$.
- least delayed transition at…

$$x_H = \sqrt{\frac{\sqrt{3}-1}{2}} \approx 0.605 \qquad \cos\phi = -\frac{1}{2x_H}$$

$$\phi \approx 0.81\pi \ \text{or} \approx (2-0.81)\pi$$



- Sets an upper bound of deconfinement, because the confining saddle point becomes locally unstable.

Deconfinement scenarios

- Scenario 1: condensation of ρ_1 triggers deconfinement precisely at $x_H \equiv e^{-1/T_H}$.

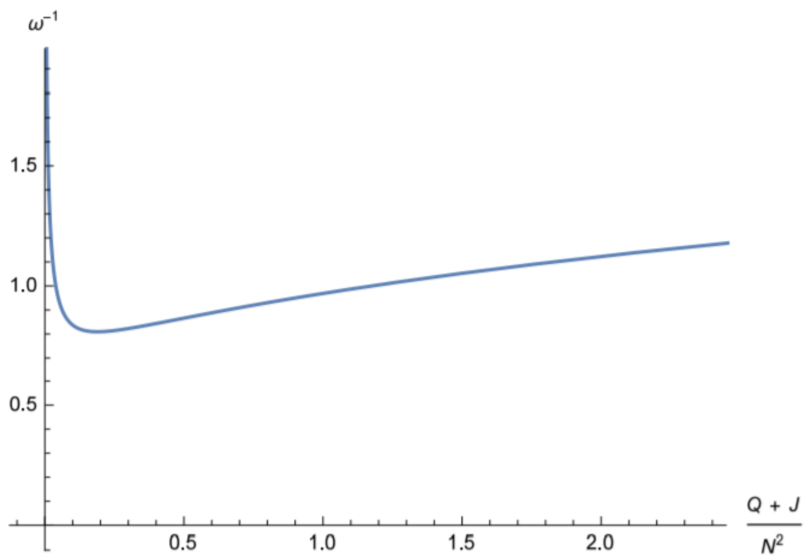
- Scenario 2: New saddle point appears at $\mathrm{T_0} < \mathrm{T_H}$, and dominates at T_c (where $T_0 < T_c < T_H$) before the confining saddle point is locally destabilized.

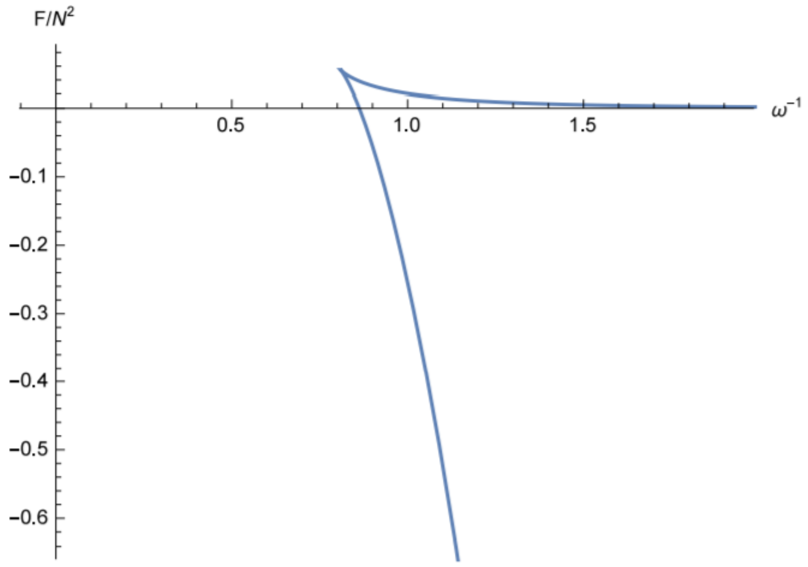
- Here, we expect a first order phase transition
- This seems more natural to explain Hawking-Page transition in AdS.
- For this scenario, one should find new complexified large N saddle points for $e^{i\alpha_a}$'s deviating from the unit circle, due to complex $f(xe^{i\phi})$ in the effective action.
- Looks challenging…

- In any case, T_H is the destabilizing point of the confining saddle point.
- Which sets an upper bound for deconfinement ~ HP transition temperature

Implications: new black holes?

- Compare w/ known BPS BH's: $Q_1 = Q_2 = Q_3 \equiv Q$, $J_1 = J_2 \equiv J$ [Gutowski,Reall] $\omega^{-1} \equiv T/3$





- Its Hawking-Page transition would happen at $\omega_{\rm HP}^{\rm known} \equiv \frac{\pi}{16}\sqrt{414 - 66\sqrt{33}} \approx 1.16$
- $\left(\omega_{HP}^{known}\right)^{-1}$ is higher than our upper bound $\omega_{\rm H}^{-1} \approx 1.508^{-1}$

- This appears to predict new BPS black holes, since the known one cannot trigger the transition below our upper bound. They might also be hairy BPS black holes, similar to [Santos, Markeviciute] (2018)

Other AdS black holes

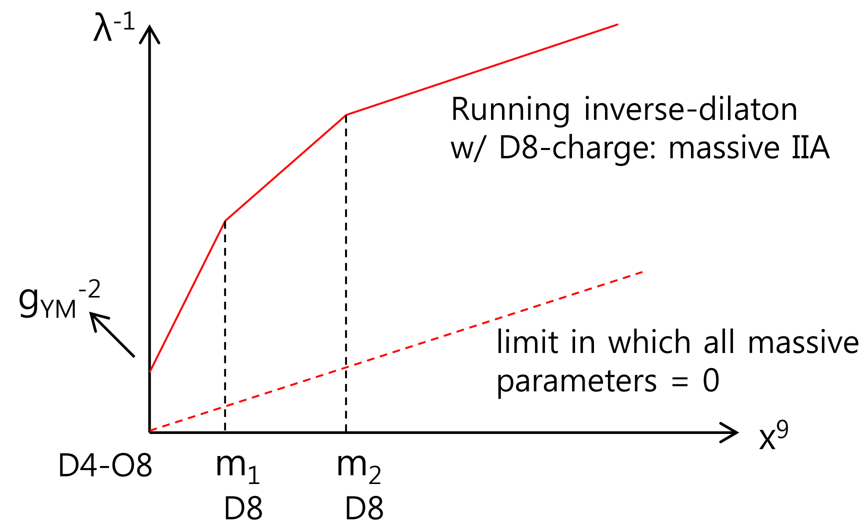
- SCFT_D in $D = 3,5,6$. Major examples are:

- 3d SCFT w/ maximal SUSY: lives on N M2's. $AdS_4 \times S^7$ dual. # of d.o.f. at large N ~ $N^{3/2}$

- 6d (2,0) SCFTs: lives on N M5's. $AdS_7 \times S^4$ dual. # of d.o.f. ~ N^3.

- 5d SCFT: [Seiberg] (1996) live on N D4's probing D8-O8
- Strong coupling limit of 5d Sp(N) gauge theories (w/ suitable matters)
- Warped $AdS_6 \times S^4/Z_2$ dual. # of d.o.f. ~ $N^{5/2}$.

	0	1	2	3	4	5	6	7	8	9
D4	•	•	•	•	•					v
D8/O8	•	•	•	•	•	•	•	•	•	m_i



- What does deconfinement mean?

- Relations between strong coupling CFTs & gauge theories are subtle/indirect.
- #'s of d.o.f. are either enhanced or reduced than ~ N^2.

Cardy free energy from ‘t Hooft anomalies

- 4d maximal SYM: chemical potentials on $S^3 \times S^1$ as background fields

$$ds^2 = r^2\left[d\theta^2 + \sum_{i=1}^{2} n_i^2\left(d\phi_i - \frac{i\omega_i}{\beta}d\tau\right)^2\right] + d\tau^2 \qquad A^I = -\frac{i\Delta_I}{\beta}d\tau$$

for $U(1)^3 \subset SO(6)_R$ symmetry

$$(n_1, n_2) = (\cos\theta, \sin\theta) \qquad Z(\beta, \Delta_I, \omega_i) = \mathrm{Tr}\left[e^{-\beta E} e^{-\sum_{I=1}^{3}\Delta_I Q_I} e^{-\sum_{i=1}^{2}\omega_i J_i}\right]$$

- $\beta \to 0^+$ is a “regulating parameter” at $\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = 2\pi i \pmod{4\pi i}$
- Can use the formal high T limit (small temporal circle) at $\beta \ll \omega_{1,2} \ll 1$.

- High T effective description of $\log Z$: effective action of 3d background fields on S^3

$$ds_4^2 = r^2\left[d\theta^2 + \sum_i n_i^2 d\phi_i^2 + \frac{r^2(\sum_i \omega_i n_i^2 d\phi_i)^2}{\beta^2(1 - r^2\sum_i \frac{n_i^2\omega_i^2}{\beta^2})}\right] + e^{-2\Phi}(d\tau + a)^2 \equiv ds_3^2 + e^{-2\Phi}(d\tau + a)^2$$

$$e^{-2\Phi} = 1 - r^2\sum_i \frac{n_i^2\omega_i^2}{\beta^2} \qquad a = -i\frac{r^2\sum_i \omega_i n_i^2 d\phi_i}{\beta(1 - r^2\sum_i \frac{n_i^2\omega_i^2}{\beta^2})}$$

dilaton — gravi-photon

$$A^I = A_4^I(d\tau + a) + \mathcal{A}^I \qquad A_4^I = -\frac{i\Delta^I}{\beta} \equiv \frac{\alpha^I}{\beta}, \quad \mathcal{A}^I = -A_4^I a$$

- ∞ tower of terms in derivative expansion: generated by integrating out KK fields on S^1.
- Cardy limit: All but finite # of terms in $\log Z$ are suppressed by $\beta \to 0^+$ or $\omega_{1,2} \ll 1$.
- Leading terms are all given by: 3d Chern-Simons terms.

3d CS from 4d anomalies

- CS terms of background fields: Quantized coefficients.

- Gauge invariant: For N=4 SYM, generalized parity & SO(6) Weyl symmetry forbid them.

$$\beta^{-2}\int a\wedge da\ ,\quad \beta^{-1}\int \mathcal{A}^I\wedge da\ ,\quad \int \mathcal{A}^I\wedge d\mathcal{A}^J$$

- Gauge non-invariant: required by 't Hooft anomaly matching for SO(6) [Banerjee, et.al.] (2012)

$$S_{\rm CS}=-\frac{iN^2}{8\pi}\cdot\frac{\beta}{2\pi}\int_{S^3}C_{IJK}\left(A_4^I\mathcal{A}^J\wedge d\mathcal{A}^K+A_4^IA_4^J\mathcal{A}^K\wedge da+\frac{1}{3}A_4^IA_4^JA_4^Ka\wedge da\right)$$

C_{IJK} is symmetric in I,J,K, $C_{123}=1$, and $C_{IJK}=0$ if any two of I,J,K are same

- For Lagrangian QFT's, one can compute all CS terms at weak coupling, integrating out KK fermions of S^1. However, abstract arguments extend to non-Lagrangian QFTs.

- Other ∞-ly many terms: subdominant in $\beta\to 0^+$ and/or $\omega_{1,2}\ll 1$. [CKKN] (2018)

- Leading CS term: plug in background fields $$S_{\rm CS}\to-\frac{N^2C_{IJK}\Delta^I\Delta^I\Delta^K}{12\omega_1\omega_2}=-\frac{N^2\Delta_1\Delta_2\Delta_3}{2\omega_1\omega_2}$$

- Same Cardy free energy: $F=-\log Z\sim -S_{CS}$. Determined by 't Hooft anomaly.

6d anomalies & AdS_7 BH's

- Similar studies for 6d $(2,0)$ SCFT on $S^5 \times S^1$, for N M5-branes [CKKN] (2018)
- Gauge non-invariant CS terms: from $U(1)^2 \subset SO(5)_R$ 't Hooft anomaly

$$S_{CS} = \frac{i(N^3 - \frac{N}{4})\beta}{192\pi^3} \int_{S^5} \Big[2\Big(A_6^1 \mathcal{A}^1 \wedge d\mathcal{A}^2 \wedge d\mathcal{A}^2 + A_6^2 \mathcal{A}^2 \wedge d\mathcal{A}^1 \wedge d\mathcal{A}^1\Big)$$
$$+\Big(4A_6^1 A_6^2 \mathcal{A}^1 \wedge d\mathcal{A}^2 \wedge da + (A_6^1)^2 \mathcal{A}^2 \wedge d\mathcal{A}^2 \wedge da + (A_6^2)^2 \mathcal{A}^1 \wedge d\mathcal{A}^1 \wedge da\Big)$$
$$+2\Big((A_6^2)^2 A_6^1 \mathcal{A}^1 \wedge da \wedge da + (A_6^1)^2 A_6^2 \mathcal{A}^2 \wedge da \wedge da\Big) + (A_6^1)^2 (A_6^2)^2 a \wedge da \wedge da\Big]$$
$$+ \frac{iN\beta}{1536\pi^3} \sum_{I=1}^{2} \int_{S^5} \Big[4A_6^I \mathcal{A}^I \wedge d\mathcal{A}^I \wedge d\mathcal{A}^I + 6(A_6^I)^2 \mathcal{A}^I \wedge d\mathcal{A}^I \wedge da$$
$$+ 4(A_6^I)^3 \mathcal{A}^I \wedge da \wedge da + (A_6^I)^4 a \wedge da \wedge da\Big] .$$

our background → $S_{CS} = \frac{N^3}{24} \frac{\Delta_1^2 \Delta_2^2}{\omega_1 \omega_2 \omega_3} + \mathcal{O}(N^1)$

- Again, other ∞-ly many terms suppressed, except gauge invariant CS terms:

$$\beta^{-3} a \wedge da \wedge da, \quad \beta^{-2} \mathcal{A}^I \wedge da \wedge da, \quad \beta^{-1} \mathcal{A}^I \wedge d\mathcal{A}^J \wedge da, \quad \mathcal{A}^I \wedge d\mathcal{A}^J \wedge d\mathcal{A}^K$$

- Unlike 4d N=4 SYM, cannot argue w/ discrete symmetries that they are forbidden.
- Assume $1/N$ suppressions. (Some support in literature, but requires better justification)

- Then, the Cardy free energy at large N: $\log Z \sim -S_{\rm CS} = -\frac{N^3}{24} \frac{\Delta_1^2 \Delta_2^2}{\omega_1 \omega_2 \omega_3}$

- Correctly counts large BPS black holes in $AdS_7 \times S^4$. [CKKN] [Hosseini, Hristov, Zaffaroni]

Large N index of SCFT$_5$ on N D4's

- The index formula: uses intuitions from 5d SYM theories,

$$Z_{S^4\times S^1} = \oint [da] Z_{\mathbb{R}^4\times S^1}(ia, \epsilon_{1,2}, m_a, q) Z_{\mathbb{R}^4\times S^1}(-ia, \epsilon_{1,2}, -m_a, q^{-1})$$

$$(t, u) = e^{-\epsilon_\pm} = e^{-\frac{\epsilon_1 \pm \epsilon_2}{2}}$$

- Obtained from 5d SYM intuitions & SUSY [Hee-Cheol Kim, Sung-Soo Kim, Kimyeong Lee] (2012)

- The Cardy limit:
- "perturbative part"

$$\exp\left[-\frac{1}{\omega_1\omega_2}\sum_{\pm}\left(\sum_{a=1}^{N}\left(\mathrm{Li}_3(-e^{\pm\frac{\Delta}{2}-2i\alpha_a}) + \mathrm{Li}_3(-e^{\pm\frac{\Delta}{2}+2i\alpha_a})\right)\right.\right.$$
$$\left.\left.+\sum_{a<b}^{N}\left(\mathrm{Li}_3(-e^{\pm\frac{\Delta}{2}-i\alpha_a-i\alpha_b}) + \mathrm{Li}_3(-e^{\pm\frac{\Delta}{2}-i\alpha_a+i\alpha_b}) + \mathrm{Li}_3(-e^{\pm\frac{\Delta}{2}+i\alpha_a-i\alpha_b}) + \mathrm{Li}_3(-e^{\pm\frac{\Delta}{2}+i\alpha_a+i\alpha_b})\right)\right)\right]$$
$$\exp\left[\frac{1}{\omega_1\omega_2}\sum_{a<b}^{N}\sum_{\pm}\left(\mathrm{Li}_3(e^{\pm\frac{m}{2}-i\alpha_a-i\alpha_b}) + \mathrm{Li}_3(e^{\pm\frac{m}{2}-i\alpha_a+i\alpha_b}) + \mathrm{Li}_3(e^{\pm\frac{m}{2}+i\alpha_a-i\alpha_b}) + \mathrm{Li}_3(e^{\pm\frac{m}{2}+i\alpha_a+i\alpha_b})\right)\right]$$
$$\exp\left[\frac{1}{\omega_1\omega_2}\sum_{l=1}^{N_f}\sum_{a=1}^{N}\sum_{\pm}\left(\mathrm{Li}_3(e^{\pm\frac{M_l}{2}+i\alpha_a}) + \mathrm{Li}_3(e^{\pm\frac{M_l}{2}-i\alpha_a})\right)\right]$$

- The instanton correction: very very complicated (later)
- Instantons of SCFTs in Coulomb phase: quantum particles related to the stationary soliton solving $F_{\mu\nu} = \star_4 F_{\mu\nu}$. ∞ tower of BPS bound states. Like mesons of QCD.

Instanton solitons, Cardy limit & AdS_6 BH's

- Large N saddle point: maximally deconfining one, on the unit circle for $e^{i\alpha_a}$'s?
- This saddle doesn't exist: Instanton part of effective action divergent at this point.

- E.g. SU(N) single instanton sector: $$\log Z_{\text{inst}} \propto \frac{1}{\omega_1\omega_2}\sum_{a=1}^{N}\prod_{b(\neq a)}\frac{1}{\sin^2\frac{\alpha_b-\alpha_a}{2}}$$
- Technically, from instanton moduli space dynamics: non-compact scale 0-modes
- More conceptually, responsible for ∞-tower of "mesonic" spectrum in Coulomb branch.
- Presumably, key to a novel "meson → quark-gluon" like deconfinement in 5d

- Large N eigenvalues pushed into complex plane (cylinder).

- Eigenvalue spreading: $\alpha_a \sim iN^{1/2}$ (~ fundamental Wilson/Polyakov loops in AdS_6)

- Cardy limit: Obtains a free energy which counts the AdS6 BH's [Chow], [Choi, Hwangk SK, Nahmgoong] (2018), [Sunjin Choi, SK] to appear

$$\log Z \sim -i\,\boxed{\frac{\pi\ell^4}{81G}}\,\frac{\Delta^3}{\omega_1\omega_2} \qquad \Delta-\omega_1-\omega_2=2\pi i$$

$\propto N^{5/2}$

M2-brane QFTs

- Unlike $D > 4$, we have too many QFT descriptions by now.

- ABJM: $U(N)_1 \times U(N)_{-1}$ Chern-Simons matter theory.
- Complicated brane engineering. D2's probing D6 & Taub-NUT [ABJM] (2008)
- Its index on $S^2 \times R$ takes the following schematic form: [SK] (2009)

$$Z_{S^2\times S^1} \sim \sum_{\{n\}\in \text{GNO monopole charge}} \oint [d\alpha] Z_{\text{1-loop}}(\alpha, n, \cdots)$$

Large N calculus is difficult…! Not just α_a integrals, but also monopole sum

- 3d maximal SYM: $U(N)$ Yang-Mills w/ $N = 8$ SUSY
- Stack of D2-branes probing flat space R^7. Hard to use it to make strong coupling studies

- "Mirror dual" of maximal SYM
- N D2-branes probing 1 D6-brane in flat space. $N = 4$ SUSY in UV.
- $U(N)$ gauge theory w/ 1 fundamental & 1 adjoint hypers. Believed to flow to same SCFT.
- has Higgs branch & vortices: factorization of $Z[S^2 \times S^1]$ into vortex partition functions
- Use this to study large N Cardy free energy & large AdS_4 BH's. [Choi, Hwang, SK] to appear.

Higgs branch & vortices

- Higgs vacuum w/ FI $\xi > 0$. Consider the vacuum at $\tilde{q} = 0$, $\tilde{\phi} = 0$.

$$qq^\dagger - \tilde{q}^\dagger\tilde{q} + [\phi, \phi^\dagger] + [\tilde{\phi}, \tilde{\phi}^\dagger] = \xi \ , \quad q\tilde{q} + [\phi, \tilde{\phi}] = 0$$

$$q^\dagger = (\sqrt{N\xi}, 0, \cdots, 0)$$

$$\phi = \sqrt{\xi} \begin{pmatrix} 0 & \cdots & & & & \\ \sqrt{N-1} & 0 & \cdots & & & \\ 0 & \sqrt{N-2} & 0 & \cdots & & \\ \vdots & & & \ddots & & \\ 0 & \cdots & & \sqrt{2} & 0 & 0 \\ 0 & \cdots & & 0 & 1 & 0 \end{pmatrix}$$

$e^{in_1\varphi}$, $e^{in_2\varphi}$, $e^{in_3\varphi}$, $e^{in_{N-1}\varphi}$, $e^{in_N\varphi}$

-
- BPS vortex: space-dependent VEV, windings at ∞.
- winding #'s $n_i \geq 0$ & vortex charge = $U(1)^N$ flux k_i

$$n_1 = k_1 \ , \quad n_2 = k_2 - k_1 \ , \quad \cdots \ , \quad n_N = k_N - k_{N-1}$$

$$M = \xi \sum_{i=1}^{N} k_i \quad , \quad k_1 \leq k_2 \leq \cdots \leq k_N$$

- Index $Z[R^2_\beta \times S^1]$ with suitable Higgs branch VEV at infinity

$$Z(q, t, z, \tilde{Q}) = \mathrm{Tr}\left[(-1)^F q^{R+r+2j} t^{R-r} z^{2L} \tilde{Q}^T\right]$$

- R, r: $SO(4)$ R-charges ($q = e^{-\beta}$: Ω-deformation)
- z: flavor symmetry for adjoint hyper. $\tilde{Q}$: vortex charge, topological $U(1)_T$
- Related to $Z[D_2 \times S^1]$ w/ suitable b.c. at $\partial D_2 = S^1$

$$D_n q = 0 \ , \quad \tilde{q} = 0 \ , \quad D_n\phi = 0 \ , \quad \tilde{\phi} = 0$$

(similar D/N b.c. given to fields in vector multiplet)

Vortices, Cardy free energy & AdS4 BH's

- Contour integral expression [Yoshida, Sugiyama]:

$$Z = \frac{1}{N!}\int \prod_{a=1}^{N}\left[\frac{ds_a}{2\pi i s_a} s_a^{-2\pi r\zeta}\right]\prod_{a=1}^{N}\frac{(s_a t^{-\frac{1}{2}} q^{\frac{3}{2}};q^2)_\infty}{(s_a t^{\frac{1}{2}} q^{\frac{1}{2}};q^2)_\infty}\cdot\frac{\prod_{a\neq b}(s_a s_b^{-1};q^2)_\infty}{\prod_{a,b=1}^{N}(s_a s_b^{-1} t^{-1} q;q^2)_\infty}\cdot\prod_{a,b=1}^{N}\frac{(s_a s_b^{-1} z t^{-\frac{1}{2}} q^{\frac{3}{2}};q^2)_\infty}{(s_a s_b^{-1} z t^{\frac{1}{2}} q^{\frac{1}{2}};q^2)_\infty}$$

$$\tilde{Q} \equiv q^{4\pi r\zeta}. \quad (a;q)_\infty \equiv \prod_{n=0}^{\infty}(1-aq^n)$$

- relatively simple large N analysis in the Cardy limit

- Factorization of $Z[S^2 \times S^1]$ to vortex partition functions [Hwang, H.-C.Kim, Park] [CHK]
- Many Higgs vacuum points contribute $qq^\dagger - \tilde{q}^\dagger\tilde{q} + [\phi,\phi^\dagger] + [\tilde{\phi},\tilde{\phi}^\dagger] = \xi\ ,\quad q\tilde{q} + [\phi,\tilde{\phi}] = 0$
- The factorization formula: $Z_{S^2\times S^1}(Q,t,z,q) = \sum_{p\in \text{Higgs}} Z^{(p)}_{\text{pert}}(t,z,q) Z^{(p)}_{\text{vortex}}(Q,t,z,q) Z^{(p)}_{\text{vortex}}(Q^{-1},t^{-1},z^{-1},q^{-1})$

- Large N vortex partition function: Similar to AdS_6/CFT_5 examples (repulsion into complex planes), but shows short-ranged eigenvalue interactions: $F \sim N^{\frac{3}{2}} \ll N^2$

$$\log Z_{S^2\times S^1} \sim i\frac{2\sqrt{2}N^{\frac{3}{2}}\sqrt{T_1T_2T_3T_4}}{3\beta} \qquad T_1+T_2+T_3+T_4-2\beta = 2\pi i$$

T_I: suitable linear combinations of f, T, ξ, β

- Its Legendre transform yields accounts for known BPS BH's in $AdS_4 \times S^7$. [CHK+Nahmgoong]

[Derived for certain complex fugacities, but still away from saddle pts of large AdS4 BH's.]

Conclusion

- Macdonald index of 4d MSYM: new large N saddle points. Predicts hairy BH's?
- Index sees deconfinement transition: upper bound of T_c. Predicted new BPS BH's.

- Cardy free energy of SCFT_D indices in various dimensions:

- D=even: strongly constrained by 't Hooft anomaly, discrete symmetry.
- Rederives it for 4d N=4 SYM. Almost derives it for 6d (2,0) CFT: counts large AdS_7 BH's

- Some 5d SCFTs: novel deconfinement, $N^{5/2}$ d.o.f. Accounts for large AdS_6 BH's.
- Hopefully, hints on the partonic structure of 5d instanton solitons

- 3d SCFT on M2's: novel vortex partition functions. Restricted deconfinement to $N^{3/2}$ d.o.f.
- We are getting close to accounting for large AdS_4 BH's.

- Grand summary of my two lectures:

Many interesting, fundamental (& **easy**) problems waiting to be solved…!

(E.g. more (hairy) BH's & QFT duals; Cardy limits in $D > 2$; SCFT d.o.f. in $D > 4$; …)