Quantum Gravity and the Swampland

Lecture 2

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The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa ‘06

- The conjecture:

“Gravity is the Weakest Force”

- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{“1”}$$

- This is often known as the ***mild form***, as it only requires a state satisfying the bound.

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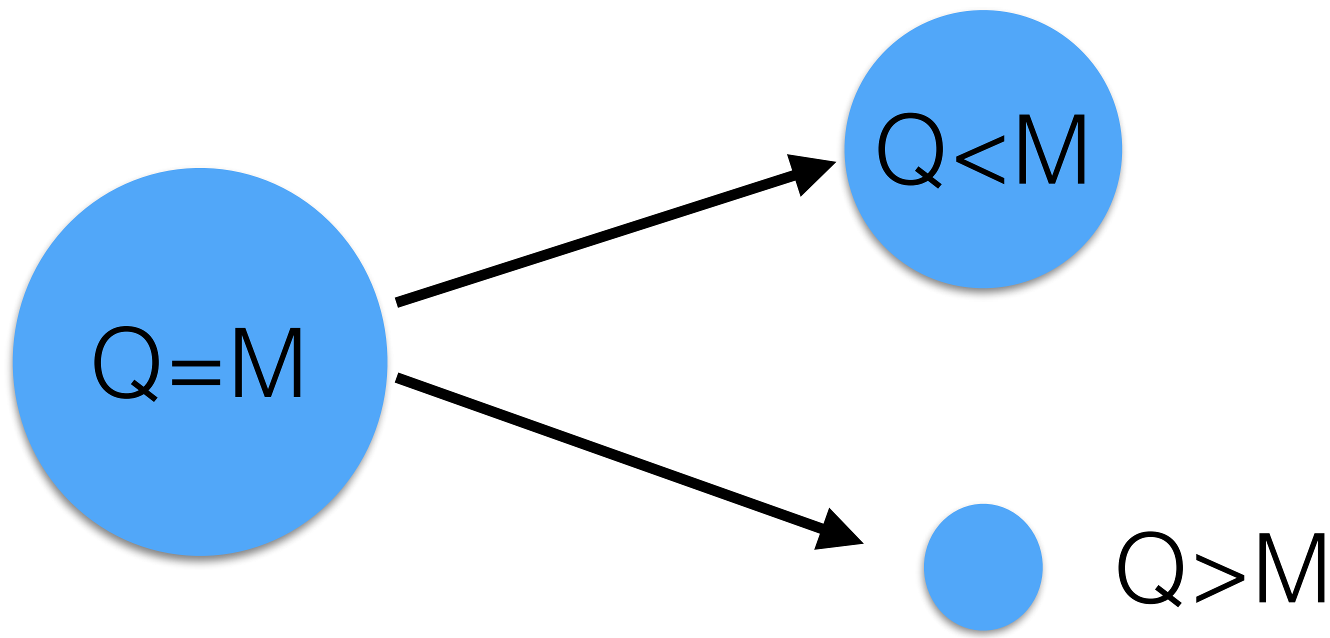
- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{``}1\text{''} \equiv \frac{Q_{Ext}}{M_{Ext}} M_P$$

- This is often known as the ***mild form***, as it only requires a state satisfying the bound.

The Weak Gravity Conjecture

- The motivation of the (mild) WGC is for an extremal BH to decay:



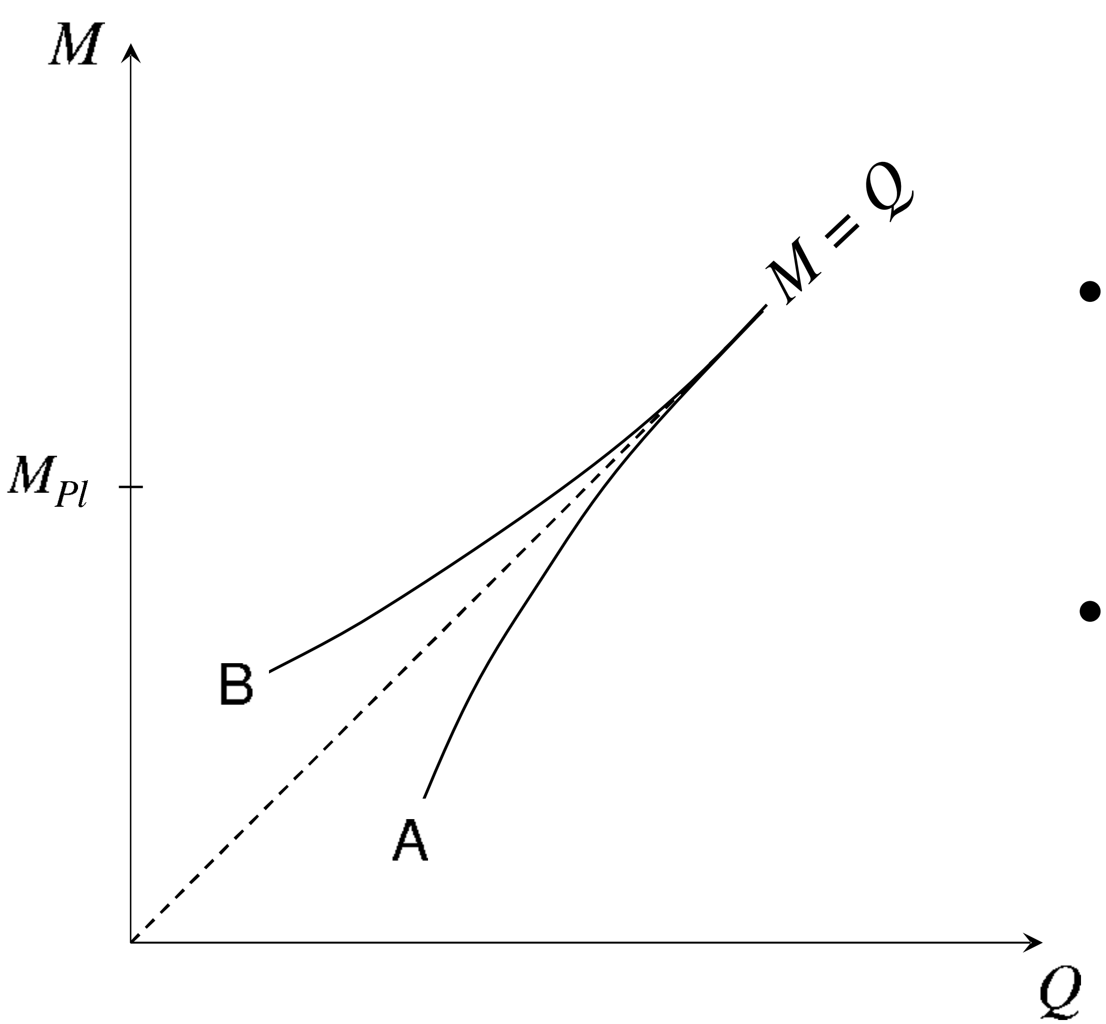
- ***Strong forms*** of the WGC have been proposed, e.g., sLWGC **[Montero, GS, Soler, '16],[Heidenreich, Reece, Rudelius, '16]**, tower WGC **[Andriolo, Junghans, Noumi, GS, '18].**

- We first prove for a wide class of theories the mild form using unitarity and causality, then present evidence for the tower WGC.

WGC and Blackholes

Extremality of Blackholes

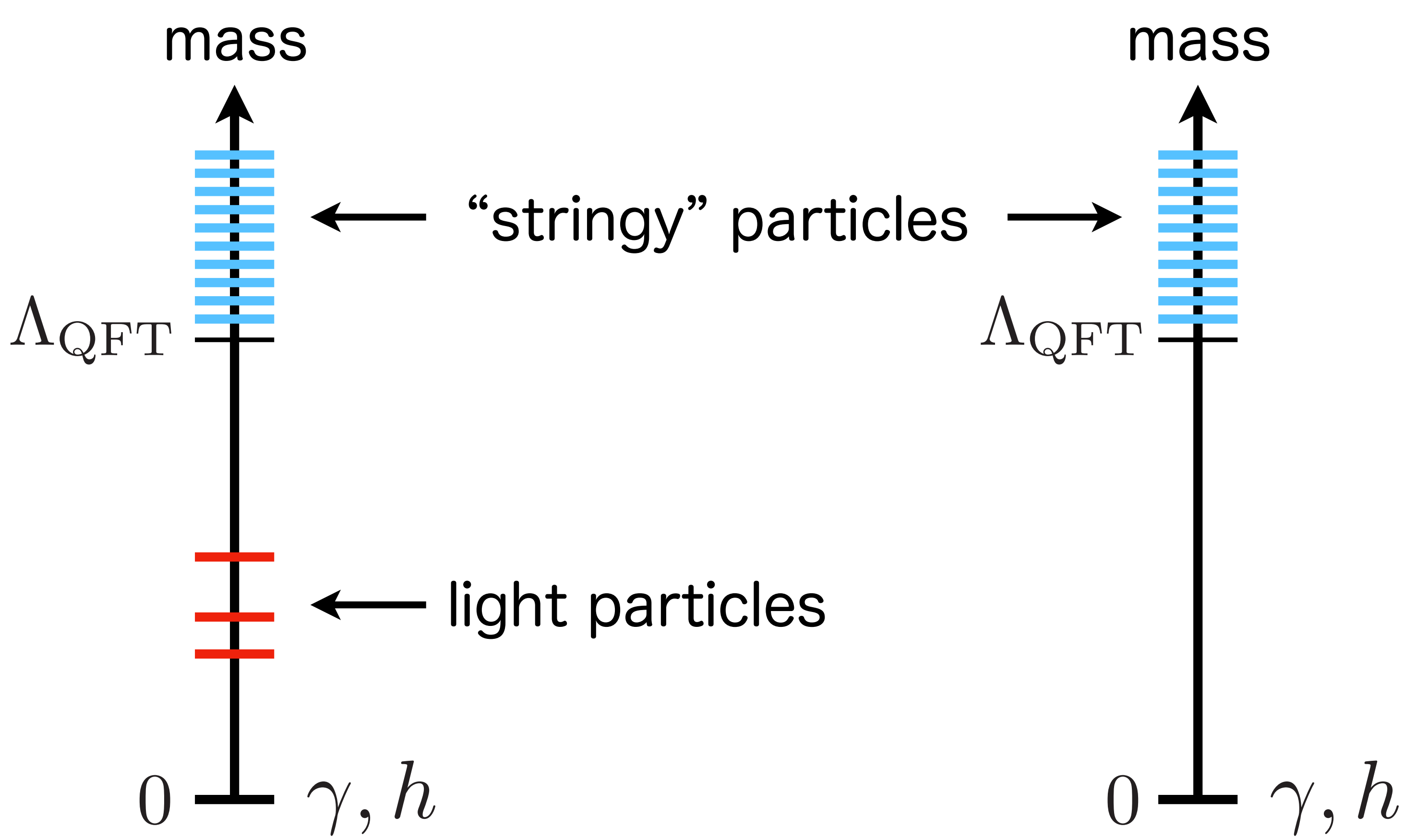
- The mild form of the WGC requires only ***some*** state for an extremal BH to decay to.
- **Can an extremal BH satisfy the WGC?**



- Higher derivative corrections can make extremal BHs lighter than the **classical bound** Q=M
- Demonstrated to be the case for 4D heterotic extremal BHs. **[Kats, Motl, Padi, '06]**
- We showed that this behavior (A) follows from unitarity (at least for some classes of theories). **[Hamada, Noumi, GS]**

WGC from Unitarity and Causality

- We assume a ***weakly coupled UV completion*** at scale Λ_{QFT}. Our proof for the strict WGC bound applies to at least two classes of theories:



- Theories with ***light*** (compared with Λ_{QFT})***, neutral i) parity-even scalars*** (e.g., dilaton, moduli), or **ii) spin ≥ 2 particles**
- ***UV completion*** where the photon & the graviton are accompanied by different sets of Regge states (as in open string theory).

Higher Derivative Corrections

- In the IR, the BH dynamics is described by an EFT of the photon and the graviton.
- In D=4, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$S = \int d^4x \sqrt{-g} \left[\frac{2M_{\rm Pl}^2}{4} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \Delta\mathcal{L} \right]$$

where $$\begin{aligned} \Delta\mathcal{L} = \quad & c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \\ & + c_4 R F_{\mu\nu} F^{\mu\nu} + c_5 R_{\mu\nu} F^{\mu\rho} F^{\nu}{}_{\rho} + c_6 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \\ & + c_7 F_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + c_8 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu}. \end{aligned}$$

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- In D=4, the general effective action up to 4-derivative operators (assume parity invariance for simplicity):

$$S = \int d^4x\sqrt{-g}\Big[\frac{2M_{\rm Pl}^2}{4}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_1}{4M_{\rm Pl}^4}(F_{\mu\nu}F^{\mu\nu})^2 + \frac{\alpha_2}{4M_{\rm Pl}^4}(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \frac{\alpha_3}{2M_{\rm Pl}^2}F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}\Big]$$

by field redefinition. Here, $W^{\mu\nu\rho\sigma}$ is the **Weyl tensor:**

$$R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma} + \frac{1}{2}\left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) - \frac{1}{3}Rg_{\mu[\rho}g_{\sigma]\nu}$$

Extremality Condition

- The higher derivative operators modify the BH solutions, so the charge-to-mass ratio of an extremal BH is corrected:

$$z = \frac{\sqrt{2} M_{\rm Pl} |Q|}{M} = 1 + \frac{2}{5} \frac{(4\pi)^2}{Q^2} (2\alpha_1 - \alpha_3)$$

[Kats, Motl, Padi, ’06]

applicable when the BH is sufficiently heavy: $M^2 \sim Q^2 M_{\rm Pl}^2 \gg \alpha_i M_{\rm Pl}^2$

because extremal BHs in Einstein-Maxwell theory satisfy:

$$R \sim M_{\rm Pl}^4 / M^2 \text{ and } F^2 \sim M_{\rm Pl}^6 / M^2$$

- Proving the WGC (mild form) amounts to showing:

$$2\alpha_1 - \alpha_3 \geq 0 \, .$$

so large extremal BHs can decay into smaller extremal BHs.

Sketch of the Proof

[Hamada, Noumi, GS]

- We first show that for the aforementioned theories, **causality** implies

$$|\alpha_1| \gg |\alpha_3|$$

- The helicity amplitudes $\mathcal{M}\,(1^+, 2^+, 3^{+2})$ & $\mathcal{M}\,(1^-, 2^-, 3^{-2})$ induced by α_3 lead to causality violation at the energy scale: $E \sim M_{\rm Pl}/\sqrt{\alpha_3}$

- Moreover, an infinite tower of massive higher spin particles with

$$m \gtrsim M_{Pl}/\sqrt{\alpha_3}$$

(just like string theory!) is required to UV complete the EFT at tree-level **[Camanho, Edelstein, Maldacena, Zhibodev].**

- This infinite tower is also confirmed by a holographic derivation using the conformal bootstrap approach **[Li, Melzer, and Poland].**

- If there are light fields or different Regge towers, α_3 is **subdominant** compared with the causality preserving terms α_1 and α_2.

Sketch of the Proof

[Hamada, Noumi, GS]

- The forward limit t→0 of γγ scattering for the aforementioned theories:

$$\mathcal{M}^{1234}(s) = \sum_n \left[\frac{g_{h_1 h_2 n} g_{\bar{h}_3 \bar{h}_4 n}}{m_n^2 - s} P^{1234}_{\mathbf{s}_n}(1) + \frac{g_{h_1 h_4 n} g_{\bar{h}_3 \bar{h}_2 n}}{m_n^2 + s} P^{1432}_{\mathbf{s}_n}(1) \right] + \text{analytic}$$

Spinning polynomials

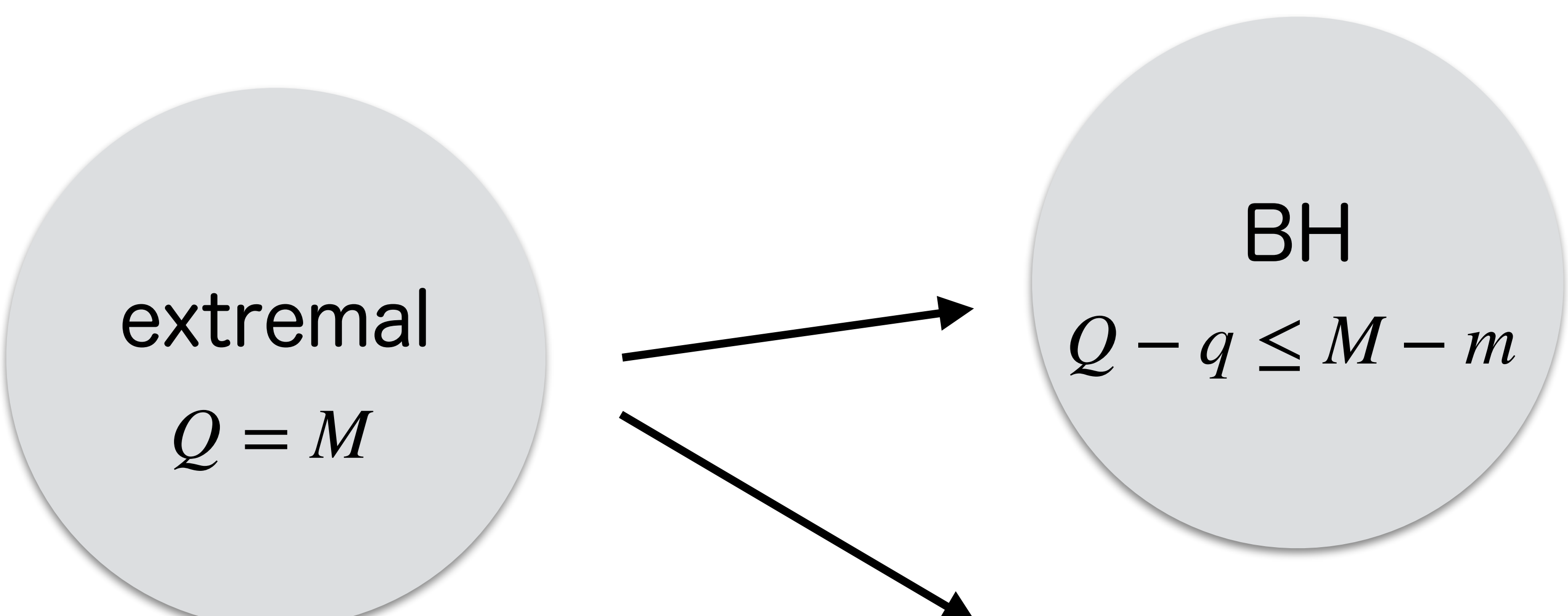
[Arkani-Hamed, Huang, Huang, ’17]

Froissart bound $a_n + b_n s$

- The higher derivative operator parametrized by α_1 leads to:

$$\alpha_1 (F_{\mu\nu} F^{\mu\mu})^2 \quad \Rightarrow \quad \mathcal{M} \sim \alpha_1 s^2$$

Unitarity ⇒ $\alpha_1 > 0$



- a state $q \geq m$ **can be an extremal BH!**

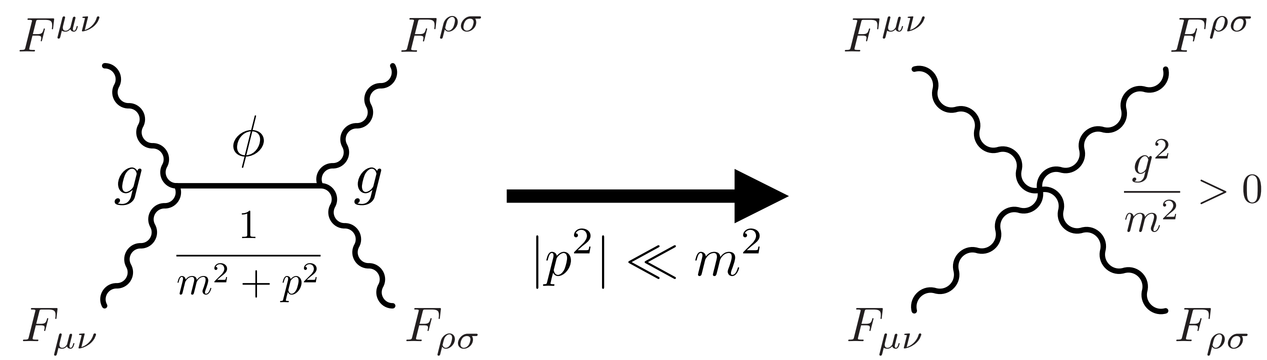
Proof in more details

Sources of Higher Dimensional Operators

- There are 3 sources of higher dimensional operators, which we refer to as (a), (b), (c):

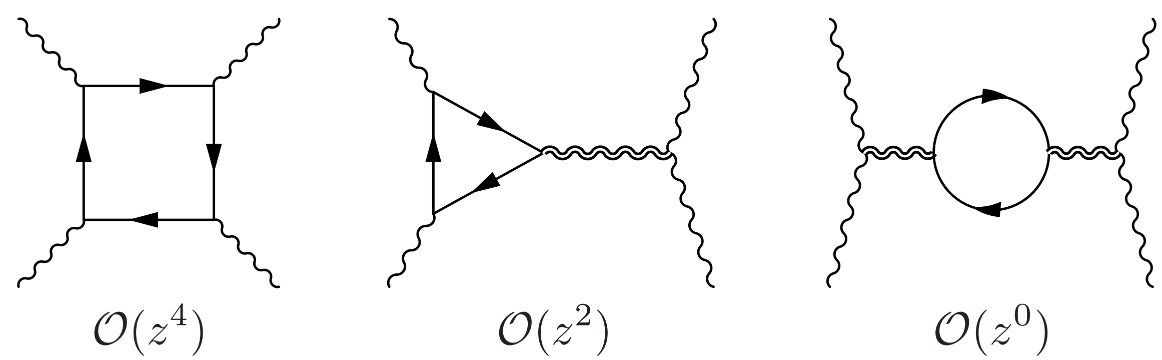
(a) Neutral Bosons

(dilaton, axion, moduli)



(b) Loop Effects

(charged particles)



(c) UV Effects

(string states)

- We now discuss in turn their unitarity constraints.

(a) Light Neutral Bosons

- Consider a scalar (dilaton) and a pseudoscalar (axion):

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_\phi^2}{2}\phi^2 + \frac{\phi}{f_\phi} F_{\mu\nu}F^{\mu\nu} \,,$$

$$\mathcal{L}_a = -\frac{1}{2}(\partial_\mu a)^2 - \frac{m_a^2}{2}a^2 + \frac{a}{f_a} F_{\mu\nu}\widetilde{F}^{\mu\nu} \,,$$

- Integrating them out leads to tree-level effective couplings:

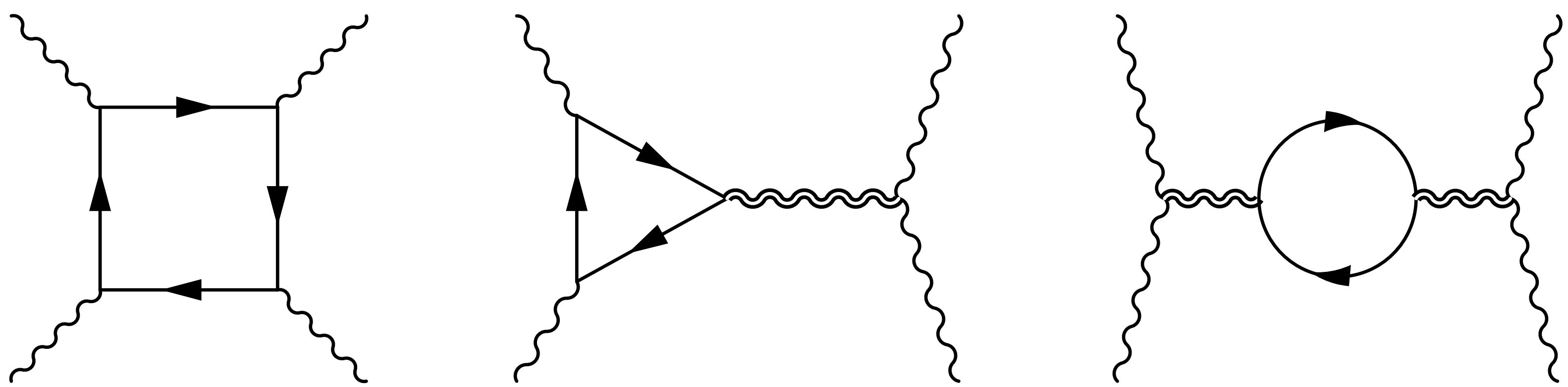
$$\alpha_1 = \frac{2M_{\rm Pl}^4}{m_\phi^2 f_\phi^2} \,, \quad \alpha_2 = \frac{2M_{\rm Pl}^4}{m_a^2 f_a^2}$$

- More generally, the positivity of $\alpha_{1,2}$ is consequence of unitarity ⇒ $\alpha_1>0$ ($\alpha_2>0$) for parity-even (odd) neutral scalar or spin ≥ 2 particle

- The proof is a bit technical (see **[Hamada, Noumi, GS]**) but it follows from expressing scattering amplitudes in terms of the spinning polynomials basis **[Arkani-Hamed, Huang, Huang]** and the fact that the forward limit amplitude < s^2.

(b) Charged Particles

- Do not contribute at tree-level, leading contribution is 1-loop:



$\mathcal{O}(z^4)$ $\mathcal{O}(z^2)$ $\mathcal{O}(z^0)$

- For example, 1-loop effective couplings generated by minimally coupled charged particles

$$\alpha_{1,2} = \max\left\{\mathcal{O}(z^4), \mathcal{O}(1)\right\} \ , \ \alpha_3 = \mathcal{O}(z^2)$$

- If $z \gg 1$, $|\alpha_1|$, $|\alpha_2| \gg |\alpha_3| \gg 1$. In this limit, gravity is negligible and unitarity for QFT implies

$$\alpha_1 > 0 \text{ and } \alpha_2 > 0$$

[Adams et al, '06]; [Hamada, Noumi, GS]

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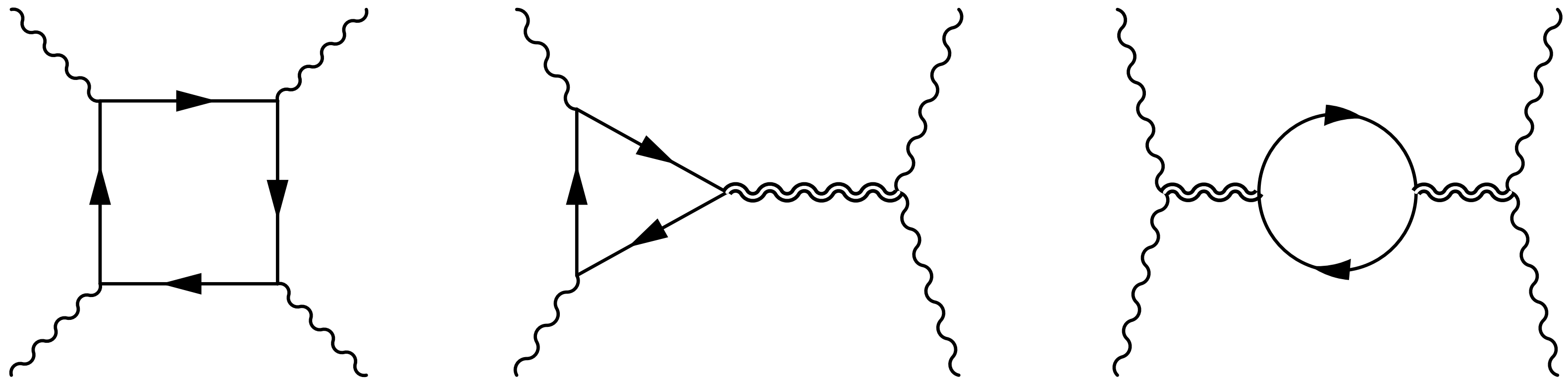

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- If z ≫ 1, $|\alpha_1|$, $|\alpha_2|$ ≫ $|\alpha_3|$ ≫1. Not only do we have superextremal particles, there are extremal BHs with z>1:

$$2\alpha_1 - \alpha_3 \geq 0\,.$$

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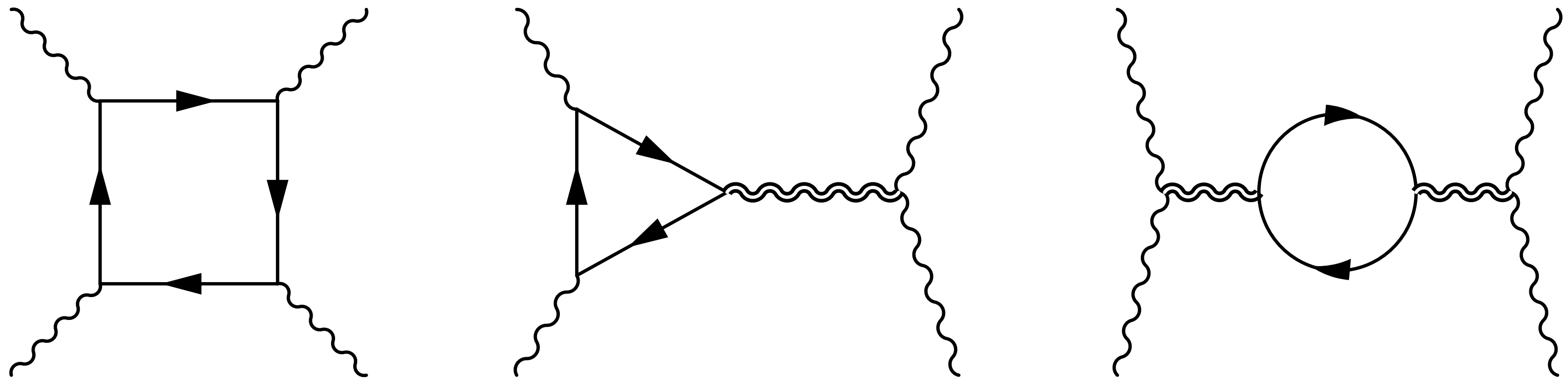

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$$\alpha_{1,2} = \max\left\{\mathcal{O}(z^4), \mathcal{O}(1)\right\}\ ,\ \ \alpha_3 = \mathcal{O}(z^2)$$

- If $z \lesssim 1$, $\alpha_i \sim O\,(1)$, no rigorous unitarity bound is known, but other effects (A) and (C) dominate.

(c) UV Effects

- Higher derivative operators can also be generated by integrating out UV effects:

$$\alpha_{1,2} = \mathcal{O}\Big(\frac{M_{\rm Pl}^4}{\Lambda_{\rm QFT}^4}\Big)\,, \quad \alpha_3 = \mathcal{O}\Big(\frac{M_{\rm Pl}^2}{\Lambda_{\rm QFT}^2}\Big)$$

  where $\Lambda_{\rm QFT}$ is the scale above which ordinary QFT breaks down. In string theory, these are α' effects.

- Crucial obstruction in deriving a unitarity bound for gravitational theories is that the t-channel graviton exchange in the forward limit t→0 dominates and diverges quadratically in s:

$$\mathcal{M}(s,t) \sim -\frac{1}{M_{\rm Pl}^2}\frac{s^2}{t}$$

- The UV behavior is made mild if the graviton is accompanied by a Regge tower of higher spin states.

(c) UV Effects

- In the Regge limit s→∞ (with t<0 fixed), the amplitude:

$$\mathcal{M}(s,t) \sim -\frac{1}{M_{\rm Pl}^2}\frac{s^{2+\gamma t+\mathcal{O}(t^2)}}{t}$$

  is bounded by < s^2 for t<0 as long as γ>0.

- Expanding the amplitude in powers of t:

$$\mathcal{M}(s,t) \sim -\frac{1}{M_{\rm Pl}^2}\frac{s^2}{t} - \frac{\gamma}{M_{\rm Pl}^2}s^2\log s + \mathcal{O}(t)$$

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bounded by s^3, but less bounded than non-gravitational case

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bounded by s^3, but less bounded than non-gravitational case

Analytic part of M may contain $\mathcal{O}(s^2)$ terms with a negative coefficient, thus cannot derive positivity bound on the $\mathcal{O}(s^2)$ term.

(c) UV Effects

- In string theory, γ is α', the Regge states contributions to

$$[\alpha_{1,2}]_{\text{Regge}} \sim \frac{M_{\text{Pl}}^2}{M_s^2}$$

- The unitarity bound we found earlier is applicable if ∃ other more dominant contribution from, e.g., a light neutral boson with $m \ll M_s$ or a different Regge tower.
- For example, the open string coupling $g_0 \sim g_s^{1/2} >> g_s$

$$[\alpha_{1,2}]_{\text{open}} \sim \frac{M_{\text{Pl}}^2}{g_s M_s^2}$$

- In UV completions where the higher spin states Reggeizing the graviton exchange are subdominant in the photon scattering:

$$\text{unitarity} \Rightarrow \alpha_1 > 0 \text{ and } \alpha_2 > 0$$

WGC from Unitarity

	magnitude	unitarity
(a) neutral bosons	$\alpha_i \gtrsim \mathcal{O}\left(\frac{M_{\mathrm{Pl}}^2}{m^2}\right)$	$\alpha_1, \alpha_2 > 0$
(b) loop effects		
(b-1) $z \gg 1$	$\lvert\alpha_1\rvert, \lvert\alpha_2\rvert \gg \lvert\alpha_3\rvert \gg 1$	$\alpha_1, \alpha_2 > 0$
(b-2) $z = \mathcal{O}(1)$	$\alpha_i = \mathcal{O}(1)$	N.A.
(c) UV effects	$\alpha_{1,2} = \mathcal{O}\left(\frac{M_{\mathrm{Pl}}^4}{\Lambda_{\mathrm{QFT}}^4}\right)$ $\alpha_3 = \mathcal{O}\left(\frac{M_{\mathrm{Pl}}^2}{\Lambda_{\mathrm{QFT}}^2}\right)$	$\alpha_1, \alpha_2 > 0\,(\star)$

- When (b-1) dominates, $2\alpha_1 - \alpha_3 > 0$

⇒ large extremal BHs can decay but then we already have a superextremal particle satisfying the WGC.

- We are interested in whether extremal BHs may play the role of the WGC state when there are no particles with z ≥ 1

⇒ Effects (a) or (c) (which are tree-effects) dominate.

Causality

- The helicity amplitudes $\mathcal{M}\,(1^+, 2^+, 3^{+2})$ & $\mathcal{M}\,(1^-, 2^-, 3^{-2})$ induced by α_3 lead to causality violation at the energy scale: $E \sim M_{\rm Pl}/\sqrt{\alpha_3}$
- Moreover, an **infinite tower of massive higher spin particles** with m $\gtrsim M_{\rm Pl}/\sqrt{\alpha_3}$ is required to UV complete the EFT at tree-level **[Camanho, Edelstein, Maldacena, Zhibodev]**
- This infinite tower is also confirmed by a holographic derivation using the conformal bootstrap approach **[Li, Melzer, and Poland]**
- The scale at which QFT breaks down: $\Lambda_{\rm QFT} \sim M_{\rm Pl}/\sqrt{\alpha_3}$

$$\Rightarrow \qquad \alpha_3 \sim \frac{M_{\rm Pl}^2}{\Lambda_{\rm QFT}^2} \qquad \Rightarrow$$ **effect (c)**

time delay/advancement on shock waves

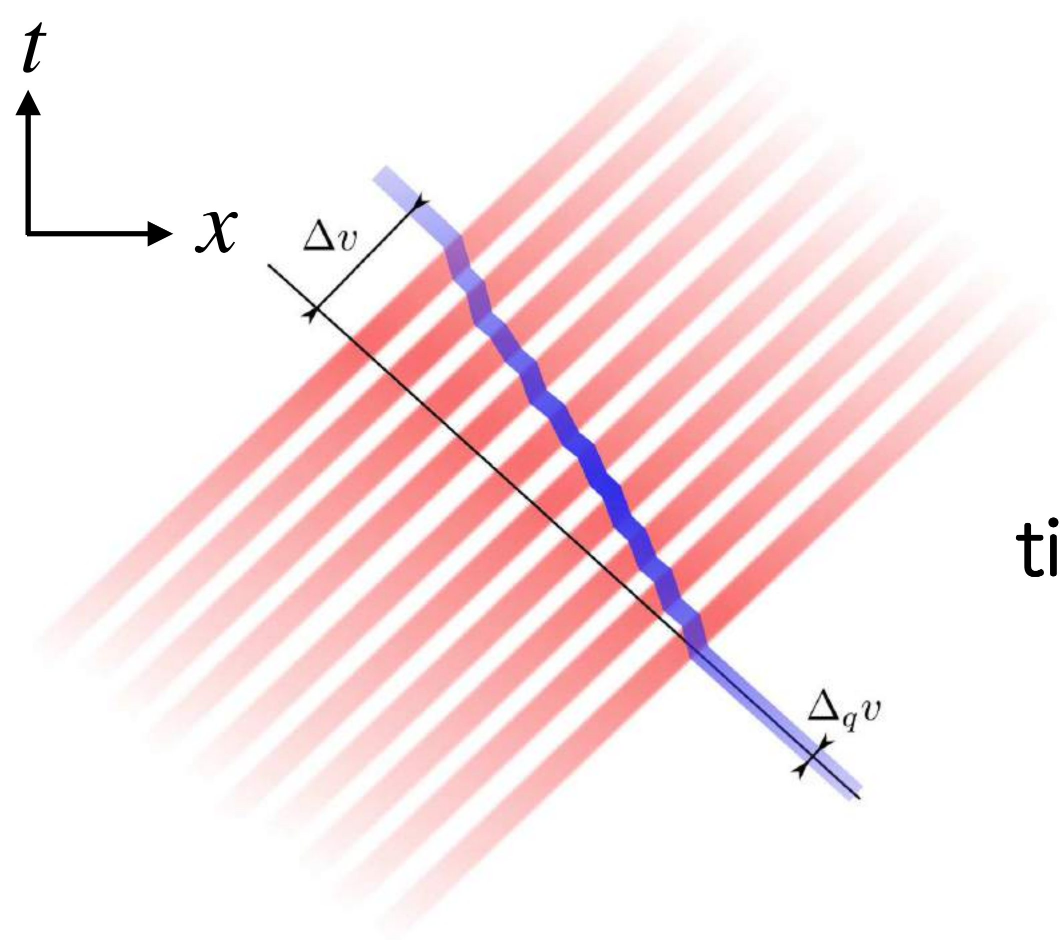


fig: Camanho et al '14

phase shift of photon propagation

$$\delta \sim s \left(\ln(L_{\mathrm{IR}}/b) \pm \frac{|\alpha_3|}{b^2} + \dots \right)$$

time delay in GR

helicity dependent phase shit

b : impact parameter L_{IR} : IR cutoff

※ time advancement for $b^2 \ln(L/b) \ll |\alpha_3|$

→ causality violation

unless this scale is beyond UV cutoff

※ similar argument shows

massive particle w/spin $J \leq 2$ does not help

an infinite tower of higher spins!

phase shift generated by spin J is $\delta \sim s^{J-1}$

consistency requirements

[Camanho-Edelstein-Maldacena-Zhiboedov '14, Maldacena-Shenker-Stanford '15]

- causality requires that $e^{i\delta(s)}$ is analytic on the complex s UHP
- unitarity requires $|e^{i\delta(s)}| \leq 1$ on the UHP

suppose that $\delta \sim s^{p}$ for large |s|, then $p \leq 1$

$\rightarrow$ finite spinning particles do not help

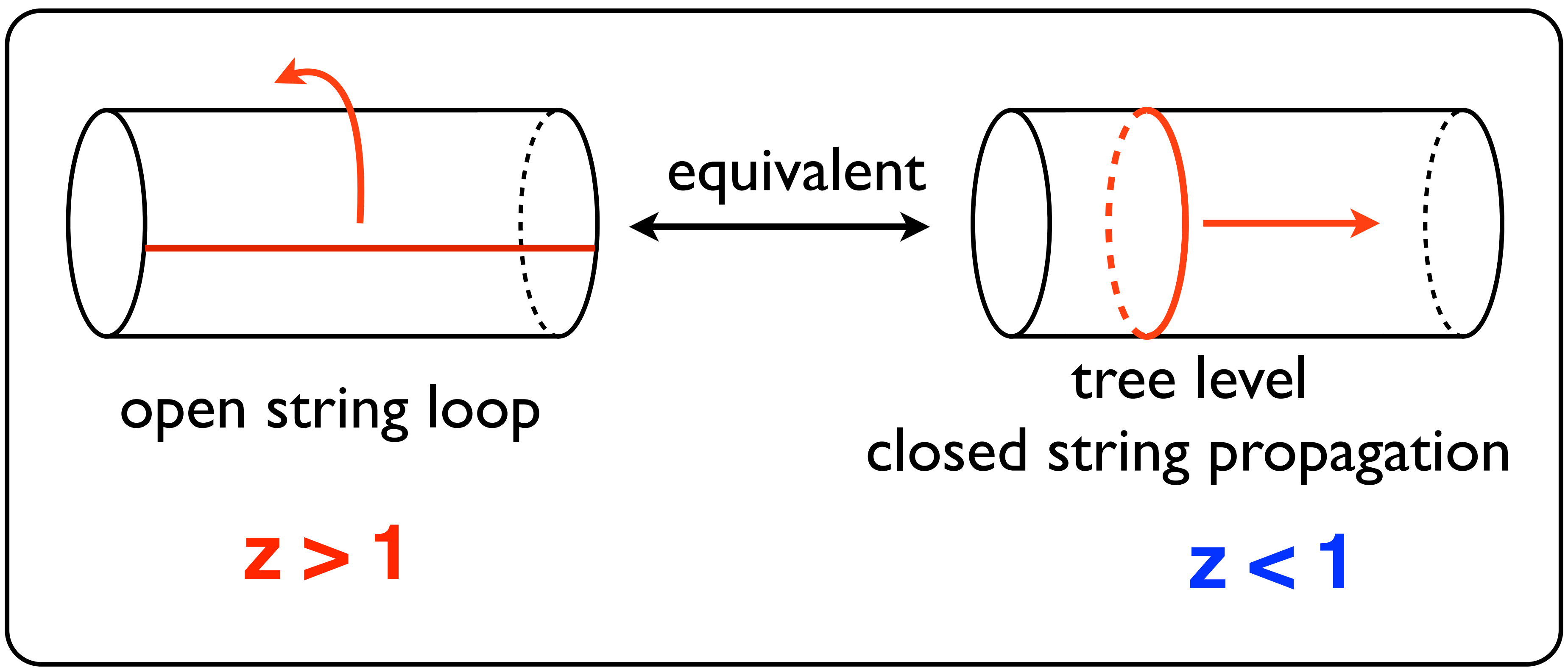
$\rightarrow$ at least requires an infinite higher spin particles (Regge tower)

Class 1: Theories with Light Neutral Bosons

- If tree-level **effect (a)** dominates, casualty implies

$$|\alpha_1|\,,|\alpha_2| \gg |\alpha_3|$$

- The WGC can be satisfied by extremal BHs if ∃ a **parity-even neutral scalar** or a **spin ≥ 2 neutral particle** with $m \ll \Lambda_{QFT.}$
- **Open-closed string duality** interpretation:



Class 2: Open-String Type UV Completion

- The photon and the graviton can be accompanied by different sets of Regge states, e.g., in theories with open strings:

$$[\alpha_{1,2}]_{\rm open} \sim \frac{M_{\rm Pl}^2}{g_s M_s^2}\,, \quad [\alpha_{1,2}]_{\rm closed} \sim \frac{M_{\rm Pl}^2}{M_s^2}\,.$$

- Unitarity implies: $\alpha_{1,2} \simeq [\alpha_{1,2}]_{\rm open} > 0$ which can be explicitly seen in the photon scattering of Type I string theory.

- The graviton comes with a Regge tower with a mass scale:

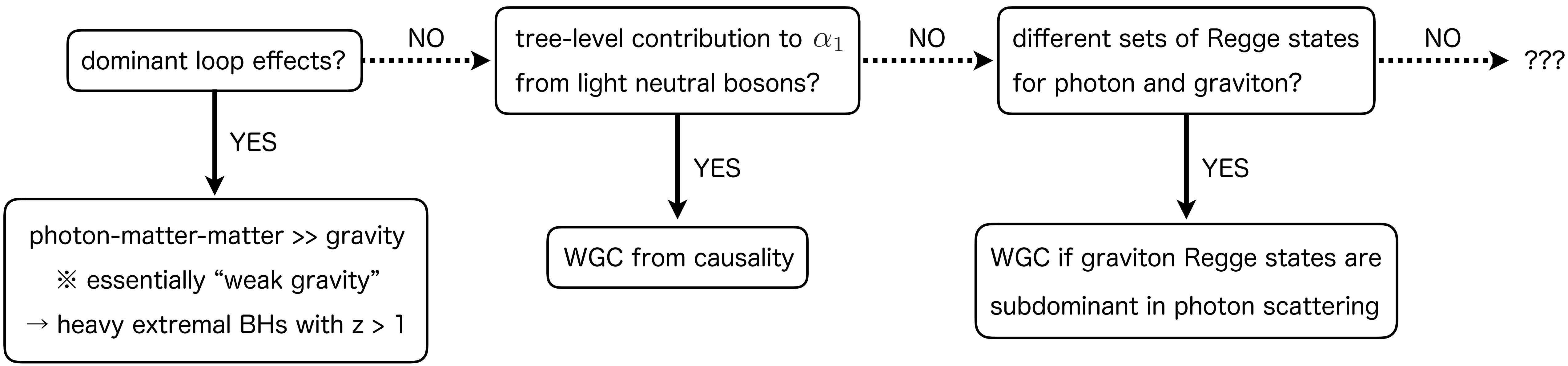
$$m \sim M_{Pl}/\alpha_3^{1/2}$$

e.g., bosonic string: $\alpha_3 \sim \frac{M_{\rm Pl}^2}{M_s^2}$

- α_3=0 for SUSY theories since $\mathcal{M}$ $(1^+, 2^+, 3^{+2})$ and $\mathcal{M}$ $(1^-, 2^-, 3^{-2})$ are incompatible with the SUSY Wald-Takahashi identity.

- Regardless of SUSY: $\alpha_1 + \frac{1}{2}\alpha_3 \simeq [\alpha_1]_{\rm open} > 0$

Summarizing the Unitarity Constraints



- Theories not covered by our proof are those with one type of Regge states (e.g., heterotic string) & no light bosons below M_S.
- If the WGC follows from field theoretical consistencies alone, it won’t be a swampland condition!
- Nonetheless, explicit calculations of scattering amplitudes give $\alpha_1>0$ and $\alpha_2>0$. Moreover, $\alpha_3=0$ because of SUSY.
- Argument is applicable as long as the tree-level scattering inherits the same structure, e.g., the O(16)xO(16) string **[Alvarez-Gaume, Ginsparg, Moore, Vafa].**

WGC and Blackhole Entropy

WGC from Blackhole Entropy?

- There has been a recent claim of a proof of the WGC from blackhole entropy **[Cheung, Liu, Remmen]**
- The argument basically boils down to:
  - If there exist a light field with $m \ll \Lambda_{QFT}$, integrating out this field leads to $\Delta S>0$ (more dof, more entropy).
  - Explicit calculations show that $\Delta S>0 \Leftrightarrow z_{ext} > 1$
- Other than the limited applicability, this begs the questions:
  - Does more dof means $\Delta S>0$?
  - Is there a physical reason for $\Delta S>0 \Leftrightarrow z_{ext} > 1$

A Counterexample

- Consider a massive spin 2 field $h_{\mu\nu}$ coupled with the F^2 term:

$$\mathcal{L} = \mathcal{L}_{\rm EM} + \Delta\mathcal{L}\,,$$
$$\Delta\mathcal{L} = -\frac{1}{4}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{m^2}{8}(h_{\mu\nu}^2 - h^2) + \frac{1}{M}hF_{\rho\sigma}F^{\rho\sigma}\,,$$

where $h := h_\mu{}^\mu$ and $\mathcal{E}^{\alpha\beta}{}_{\mu\nu}$ is the kinetic operator.

- Since the trace part h has no propagating mode, we can remove it from the interaction term by a field redefinition:

$$h_{\mu\nu} \to h_{\mu\nu} - \frac{4}{3m^2M}\left(\eta_{\mu\nu} + \frac{2}{m^2}\partial_\mu\partial_\nu\right)F_{\rho\sigma}F^{\rho\sigma}$$

and the Lagrangian becomes:

$$\Delta\mathcal{L} = -\frac{1}{4}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} - \frac{m^2}{8}(h_{\mu\nu}^2 - h^2)$$
$$- \frac{4}{3m^4M^2}F_{\rho\sigma}F^{\rho\sigma}(2m^2 + \Box)F_{\alpha\beta}F^{\alpha\beta}\,.$$

A Counterexample

- The BH entropy is smaller than that of Einstein-Maxwell theory due to the negative coefficient of the F^4 term.
- **[Cheung, Liu, Remmen]** assumed that the Euclidean action with a vanishing UV field χ=0 is equivalent to that of the Einstein-Maxwell theory for any configuration of the metric & gauge field.
- This assumption is not invariant under field redefinition.
- Our argument based on scattering amplitudes (rather than Lagrangians) is invariant under field redefinition.
- This counterexample is excluded by our analysis as we demand a mild UV behavior of scattering amplitudes at large s.

Blackhole Entropy Corrections

- We provided a physical explanation for $\Delta S>0 \Leftrightarrow z_{ext}>1$ and thus our unitarity bounds imply that $\Delta S>0$ for extremal BHs.
- The leading correction to the BH entropy:

$$\Delta S = \Delta S_{\text{int}} + \Delta S_{\text{horizon}}$$

higher derivative correction to Wald entropy → ΔS_{int}: $\mathcal{O}(\alpha_i)$

horizon shift Δr_H by higher derivative correction → $\Delta S_{\text{horizon}}$: $\mathcal{O}(\alpha_i^{1/2})$

- The dominant contribution to ΔS is positive if $\Delta r_H > 0$:

$$\frac{\Delta S_{\text{horizon}}}{S_{EM}} = \frac{(r_H + \Delta r_H)^{D-2}}{r_H^{D-2}} - 1 \simeq (D-2)\frac{\Delta r_H}{r_H}$$

Blackhole Entropy Corrections

- $\Delta S>0$ If the higher derivative corrections resolve the degeneracy of the two horizons without introducing a naked singularity.

$$r_H^{\pm} = r_H \pm \Delta r_H$$

$r_H^+ - r_H > 0$ for the **outer horizon**, so is the entropy correction.

- The absence of naked singularity is nothing but $z_{\mathrm{ext}} > 1$!
- **BH solution:** $ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega^2_{D-2}$
- Horizon**:** $0 = g(r_H + \Delta r_H) = g_{EM}(r_H) + \Delta g(r_H) + \Delta r_H g'_{EM}(r_H) + \frac{1}{2}\Delta r^2_{EM} g''_{EM}(r_H)$

$$\Rightarrow \quad \frac{\Delta r_H^2}{r_H^2} = -\frac{2\Delta g(r_H)}{r_H^2 g''_{EM}(r_H)} = \frac{4\mathcal{F}(\alpha_i)}{(D-3)^2 m^{\frac{2}{D-3}}} > 0$$

from unitarity

- We have shown that $\Delta S>0$ for any charged BHs in any D (some additional assumption for the Gauss-Bonnet term is needed for D=4).

Stronger forms of the WGC

Einstein-Maxwell + massive charged particles

↓ integrate out matters

IR effective theory of photon & graviton

Positivity of EFT coefficients follow from unitary, causality, and analyticity of scattering amplitudes.

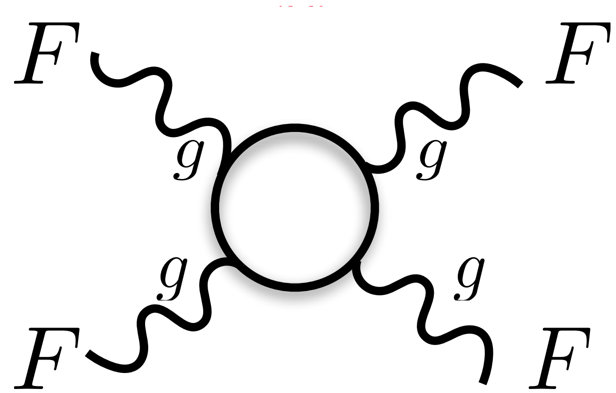
Q. What does the positivity of this EFT imply?

1-loop effective action for photon & graviton

$$\mathcal{L}_{\rm eff} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}^2$$
$$+\alpha_1 (F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \alpha_3 F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma} + \dots$$

- positivity implies $\alpha_1 + \alpha_2 \geq 0$
- α_i depends on mass and charge of particles integrated out

$\alpha_i =$



$+\underbrace{\mathcal{O}(g^2) + \mathcal{O}(g^0)}_{\text{gravitational effects}}$

- Cheung-Remmen found positivity implies $z^4 - z^2 + \gamma \geq 0$

※ $z = \dfrac{qg}{m/M_{\rm Pl}}$, γ is a UV sensitive $\mathcal{O}(z^0)$ coefficient

(free parameter in the EFT framework)

Positivity of photon-graviton EFT implies $z^4 - z^2 + \gamma \geq 0$

$\rightarrow$ at lest one of the following two should be satisfied

1) WGC type lower bound on charge-to-mass ratio

in particular when $\gamma = 0$, WGC $z^2 \geq 1$ is reproduced!

2) not so small value of UV sensitive parameter $\gamma > 0$

[Cheung, Remmen]

In [Andriolo, Junghans, Noumi, GS], we discussed

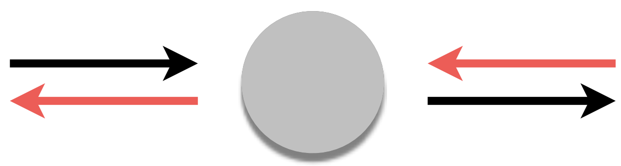
- multiple U(1)'s
- implications for KK reduction

and found qualitatively new features.

Multiple U(1)'s

for example, let us consider $U(1)_1 \times U(1)_2$

a new ingredient is positivity of $\gamma_1 + \gamma_2 \to \gamma_1 + \gamma_2$

Im  ≥ 0 implies $z_1^2 z_2^2 - z_1^2 - z_2^2 \geq 0$

- $z_i = q_i/m$ is the charge-to-mass ratio for each U(1)

- we set $\mathcal{O}(z^0) = 0$ for illustration (same as γ = 0 before)

the punchline here:

positivity bound cannot be satisfied unless $z_1^2 z_2^2 \neq 0$

$\to$ requires existence of a bifundamental particle!

Implications for KK reduction

\# S^1 compactify d+1 dim Einstein-Maxwell with single U(1) into d dim Einstein-Maxwell with $U(1) \times U(1)_{\mathrm{KK}}$

d+1 dim charged particle (q,m)

$\rightarrow$ KK tower with the charged-to-mass ratios

$$(z, z_{\mathrm{KK}}) = \left(\frac{q}{\sqrt{m^2 + n^2 m_{\mathrm{KK}}^2}}, \frac{n}{\sqrt{(m/m_{\mathrm{KK}})^2 + n^2}} \right)$$

in the small radius limit $m_{\mathrm{KK}} \rightarrow \infty$,

the lowest mode (n = 0): $(z, z_{\mathrm{KK}}) = (q/m, 0)$

KK modes (n ≠ 0): $(z, z_{\mathrm{KK}}) \simeq (0, 1)$

※ no bifundamentals → ~~positivity bound~~ generically

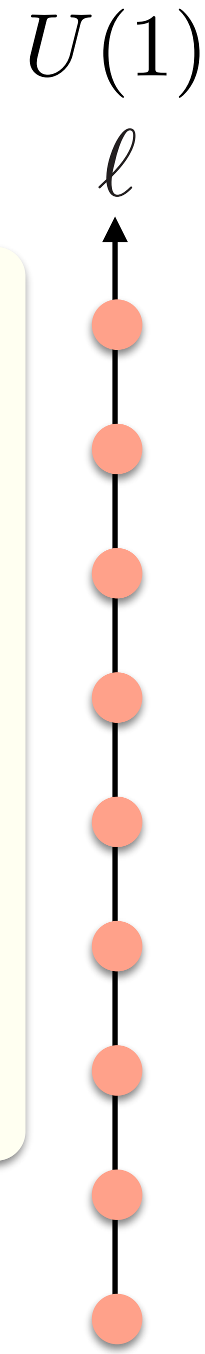
d+1 dim

charged particles

labeled by $\ell = 1, 2, \ldots$

$$(q, m) = (\ell\, q_*, \ell\, m_*)$$

s.t. $z_* = \frac{q_*}{m_*} = \mathcal{O}(1)$



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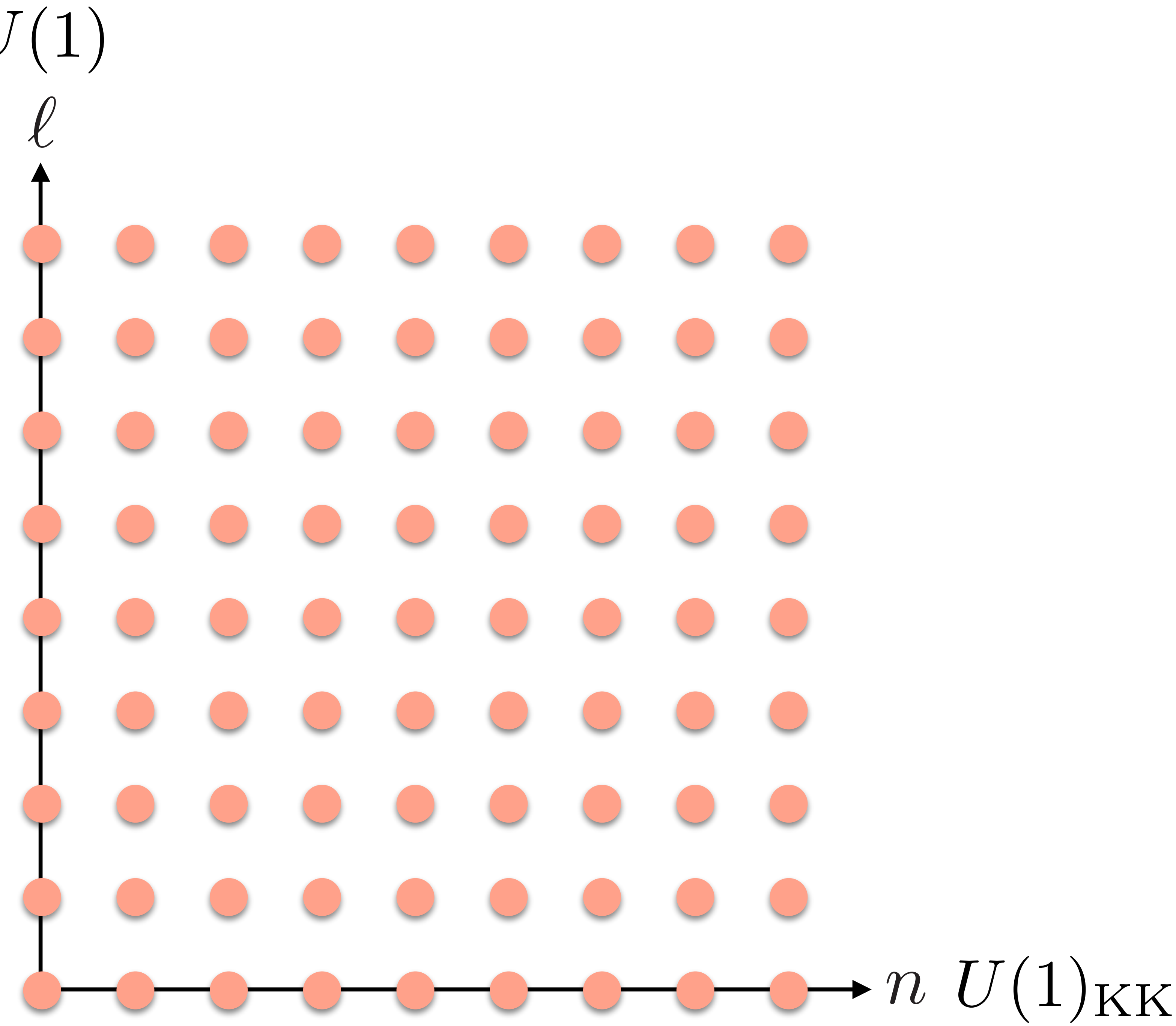

d dim charged particles

$$(z, z_{\mathrm{KK}}) = \left(\frac{\ell\, z_*}{\sqrt{\ell^2 (m_*/m_{\mathrm{KK}})^2 + n^2}}, \; \frac{n}{\sqrt{\ell^2 (m_*/m_{\mathrm{KK}})^2 + n^2}} \right)$$

<u>d+1 dim</u>

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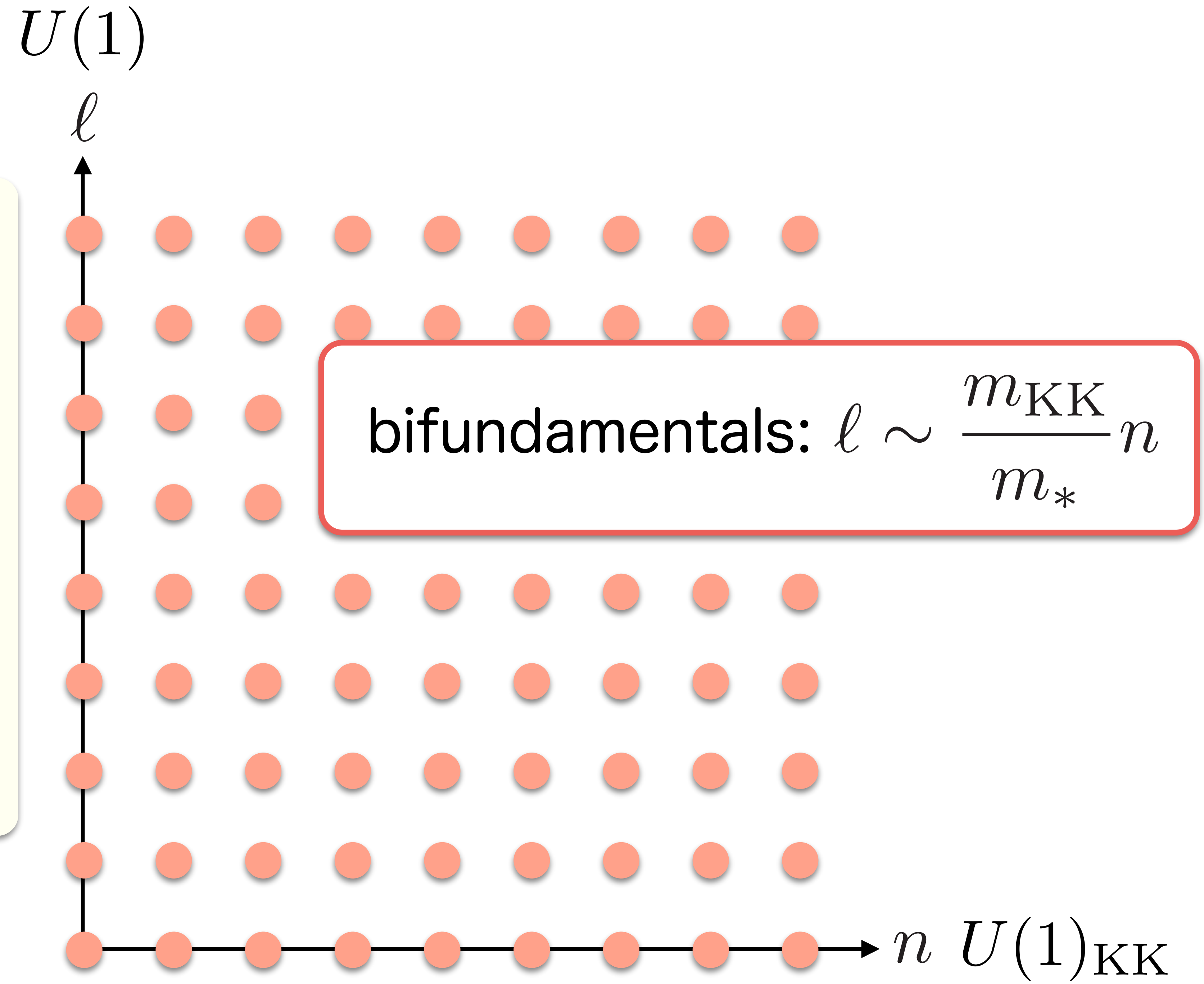

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d+1 dim

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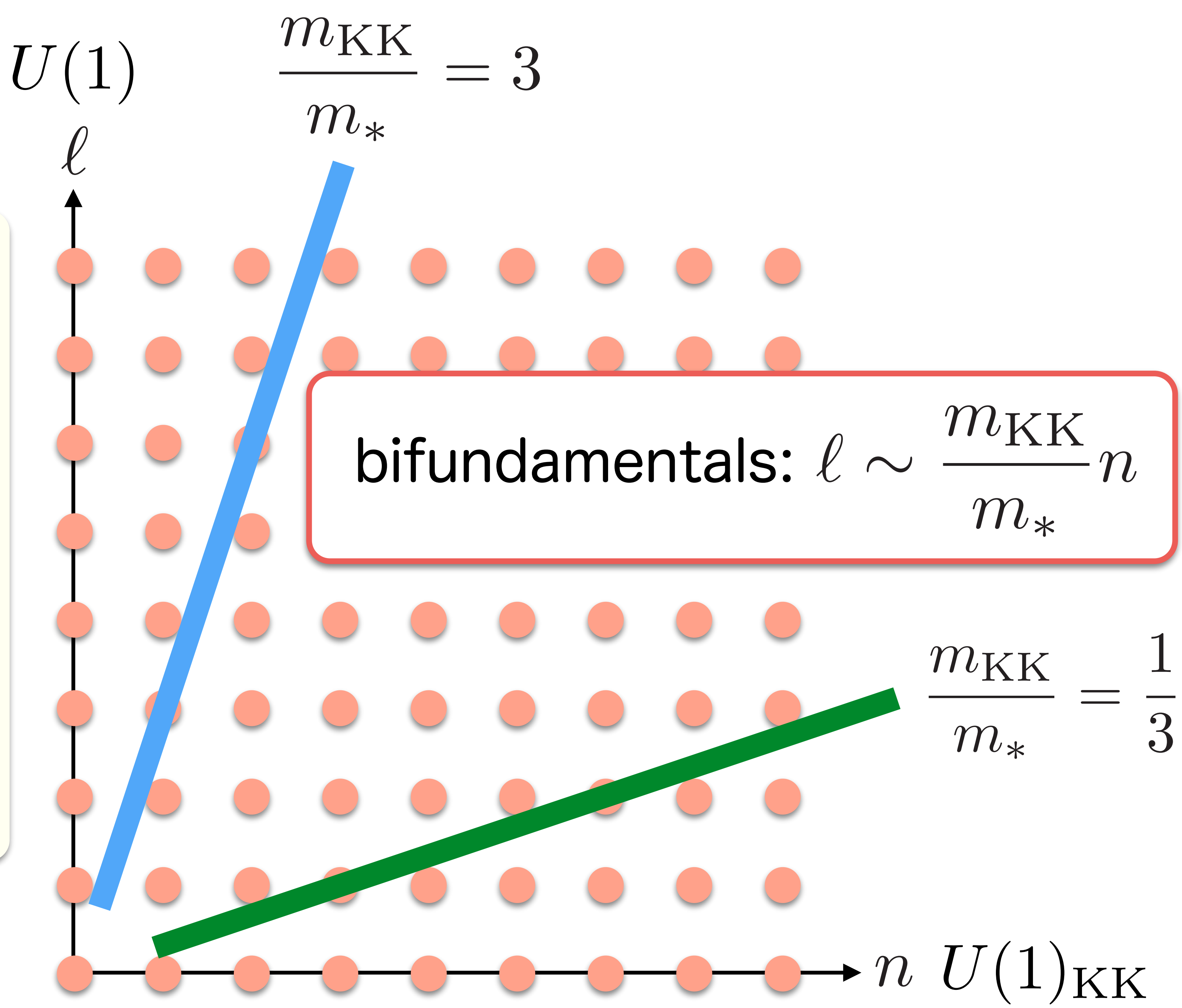

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Tower WGC

[Andriolo, Junghans, Noumi, GS]

Consistency with KK reduction seems to imply a tower of d+1 dim U(1) charged particles

→ Tower Weak Gravity Conjecture!

※ a similar “(sub)lattice WGC” was proposed based on modular invariance or holography

[Montero, GS, Soler, ’16];[Heidenreich, Reece, Rudelius, ’16]

Summary of Lecture 2

Summary of Lecture 2

- The mild form of the WGC follows from unitarity and causality for wide classes of theories that can be naturally realized in string theory:
    - Theories with light neutral scalars (e.g., dilaton, moduli)
    - Open string theory type UV completion
- We have shown that correction to the BH entropy from higher derivative operators is positive in these theories.
- We have extended our proof to higher dimensions and multiple U(1)'s.
- In Lecture 3, we will see that the WGC for branes implies that non-SUSY AdS vacua are unstable **[Ooguri, Vafa, '16].**

Summary of Lecture 2

- We showed that the decay of an extremal BH (whose near-horizon geometry is AdS) is kinematically allowed, giving support to the AdS instability conjecture.

- Plan for the next 2 lectures:

- ***Lecture 3:*** Applications to inflation and particle physics.

- ***Lecture 4:*** de Sitter vacua in string theory and the Swampland.

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