

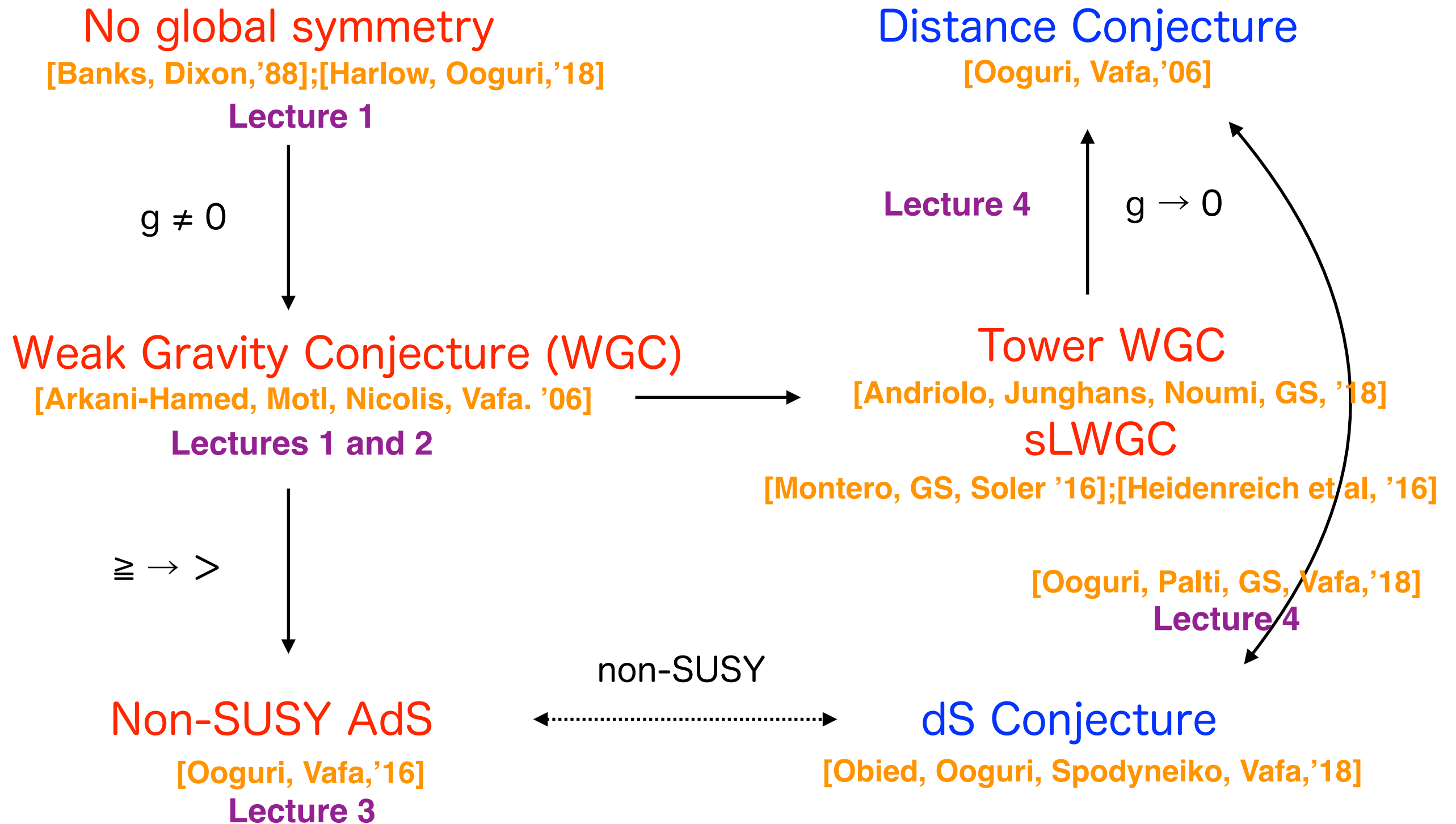
The background of the slide is a painting of a swampy landscape. In the foreground, several tall, thin, light-colored tree trunks (likely cypresses) stand in dark, reflective water. The water shows ripples and reflections of the trees and the sky. In the middle ground, more trees are visible, some with sparse foliage. In the far background, a hazy city skyline with various buildings and structures is visible under a pale, overcast sky. The overall color palette is muted, with greens, browns, and greys, giving it a somber and atmospheric feel.

Quantum Gravity and the Swampland

Lecture 3

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Web of Conjectures



Swampland Conjectures and Cosmology

- The WGC constrains the mass of particles charged under gauge forces.
- The SDC constrains the field range within the validity of an EFT coupled consistently to gravity.
- **What does this web of conjectures have to do with cosmology?**
- While not all inflation models predict a detectable level of **gravitational waves**, some do (known as **large-field inflation**).
- Much of the efforts in the Swampland has been in constraining large-field inflation.
- We will see in this and the next lecture that:
 - Large field Inflation is constrained by the WGC & the SDC
 - SDC is connected to the dS conjecture at weak coupling points
 - The AdS instability conjecture relates neutrino masses/type with Λ .

WGC for Axions

Axions and ALPs

The QCD axion [Wilczek, '78]; [Weinberg, '78] was introduced in the context of the Pecci-Quinn mechanism and the strong CP problem.

An axion enjoys a **perturbative shift symmetry**.

String theory has many **higher-dimensional form-fields**:

e.g.

$$F = dA$$

3-form flux $\xrightarrow{\quad}$ $\xrightarrow{\quad}$ 2-form gauge potential:

gauge symmetry: $A \rightarrow A + d\Lambda$

Integrating the 2-form over a 2-cycle gives an **axion-like particle (ALP)**:

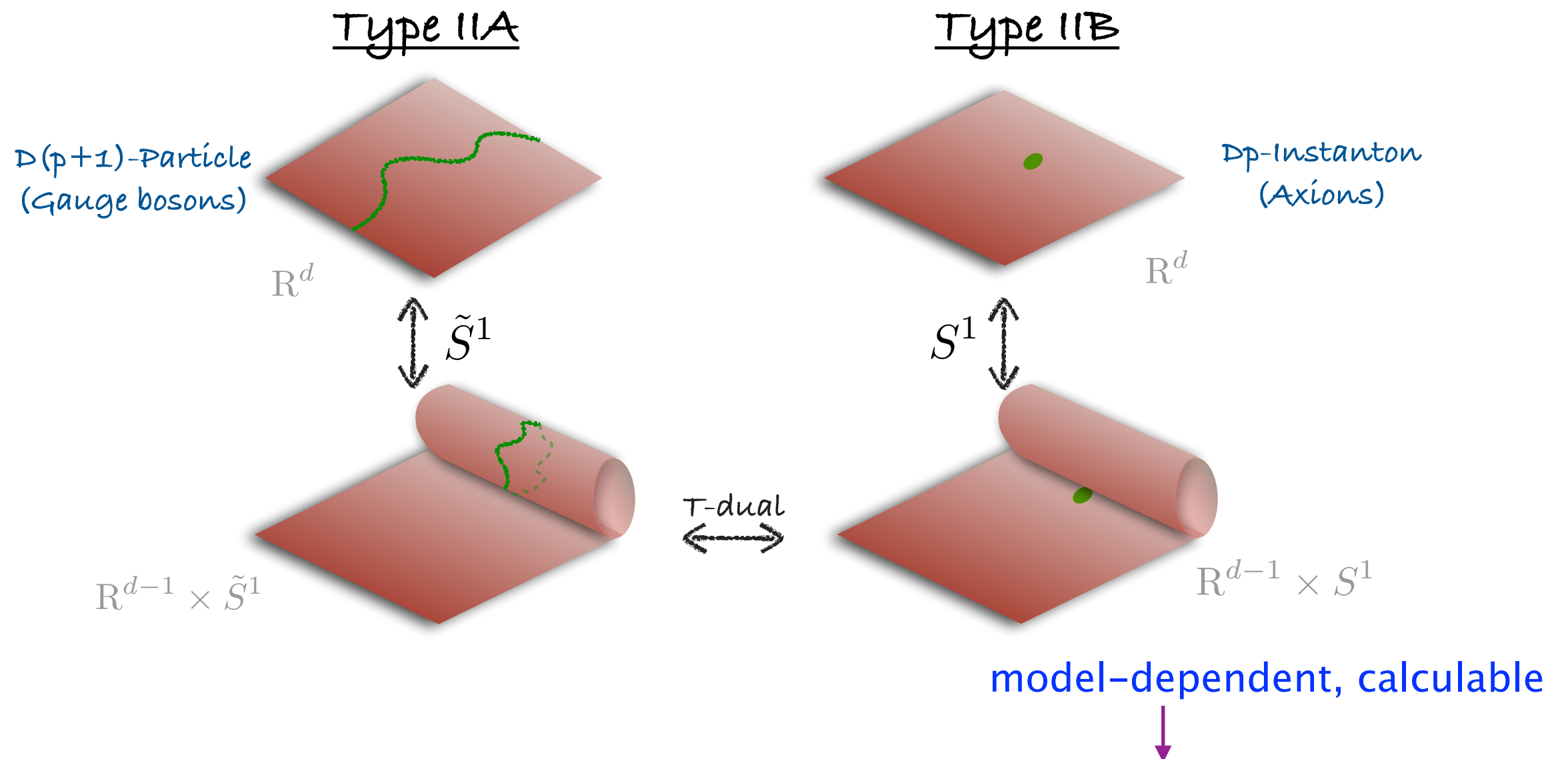
$$a(x) \equiv \int_{\Sigma_2} A$$

The gauge symmetry becomes a **shift symmetry**, that is broken by non-perturbative (instanton) effects.

WGC and Axions

Brown, Cottrell, GS, Soler

- Formulate the WGC in a duality frame where the axions and instantons turn into gauge fields and particles, e.g.



- The WGC takes the form $f \cdot S_{\text{instanton}} \leq \mathcal{O}(1) M_P$

WGC and Axions: An Example

Brown, Cottrell, GS, Soler

Type IIA

Gauge fields: $A_i \sim \int_{\Sigma_2^{(i)}} C_3$

Particles: D2 on $\Sigma_2^{(i)}$

WGC

$$\tilde{m}_k = m_k \frac{\sqrt{g_{33}}}{2\pi l_s}$$

$$\tilde{q}_k^i = (f_k^i)^{-1} \frac{\sqrt{2}}{4\pi l_s}$$

“Couplings”:

$$\tilde{g}_s = \frac{g_s}{\sqrt{g_{33}}}$$

$$\tilde{M}_P = M_P \sqrt{g_{33}}$$

Type IIB

Axions: $\phi_i \sim \int_{\Sigma_2^{(i)}} C_2$

Instantons: D1 on $\Sigma_2^{(i)}$

$$S_{inst_k} \sim -m_k + i(f_k^i)^{-1} \phi_i$$

“Couplings”:

$$g_s$$

$$M_P$$

WGC and Axions: An Example

Brown, Cottrell, GS, Soler

**4d Type IIB
D1-instantons**

$$m_i$$
$$f_i$$
$$g_s \ll 1$$

**4d Type IIA
D2-particles**

$$\tilde{m}_i \sim m_i$$
$$\tilde{q}_i \sim f_i^{-1}$$
$$\tilde{g}_s \gg 1$$

**5d M-theory
M2-particles**

$$M_i^{(5d)} \sim m_i$$
$$Q_i^{(5d)} \sim f_i^{-1}$$
$$R_M \rightarrow \infty$$

- Apply the WGC to 5d particles:

$$\frac{Q^{(5d)}}{M_i^{(5d)}} M_P^{(5d)} = \frac{M_P^{(IIB)}}{\sqrt{2} f_i m_i} \geq \text{“1”} \equiv \left(\frac{Q}{M} M_P \right)_{\text{Ext}_{5d}} = \sqrt{\frac{2}{3}}$$

WGC and Axions: An Example

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WGC and axions

- Consider a U(1) gauge theory in 5d, and compactify on S to 4d. Upon dimensional reduction: $A_M(x, x_4) \rightarrow (A_\mu(x), \phi(x))$

$$S = \int d^5x \frac{-1}{4g_5^2} F_{MN} F^{MN} \longrightarrow \int d^4x \left(\frac{-1}{4g_4^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \right)$$

The gauge symmetry leads to an axion shift symmetry $\phi = \phi + c$

- Topologically non-trivial Euclidean configurations (instantons) with charged fields wrapping the 5d circle generate a potential

$$V(\phi) = e^{-S_{inst}} \cos\left(\frac{\phi}{f}\right) \quad \begin{aligned} S_{inst} &= 2\pi R m_5 \\ f &= q_5 \sqrt{2\pi R} \end{aligned}$$

- The 5d WGC for charged particles $m_5 < q_5 M_{p,5d}^3$ translates into:

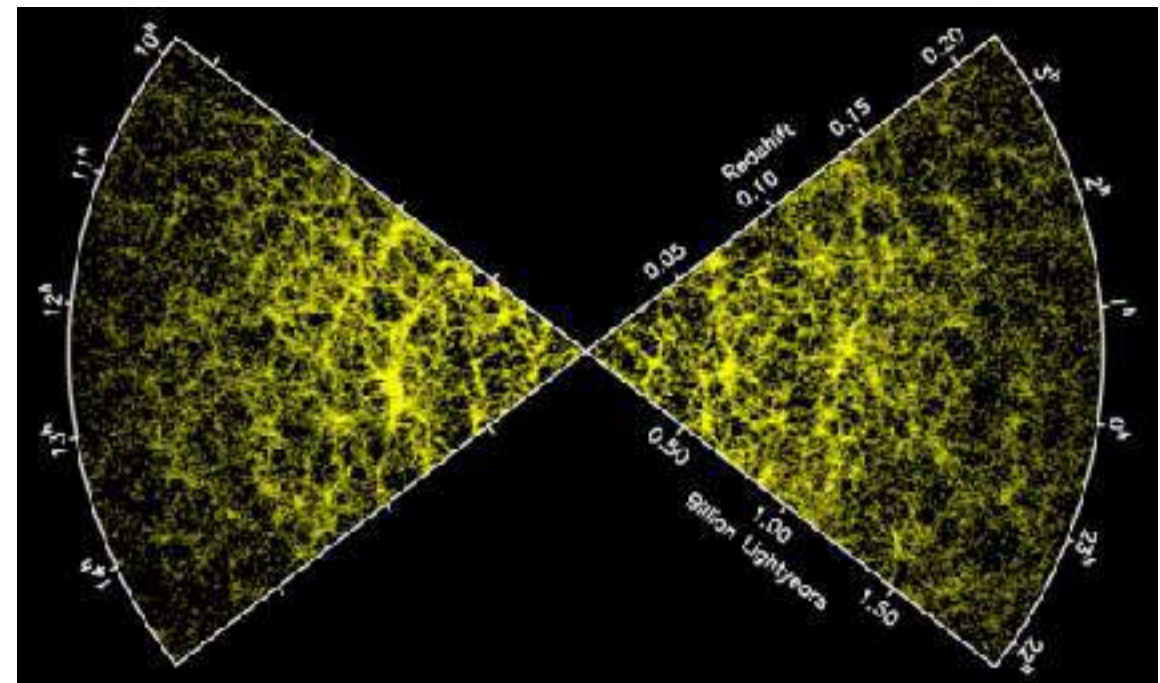
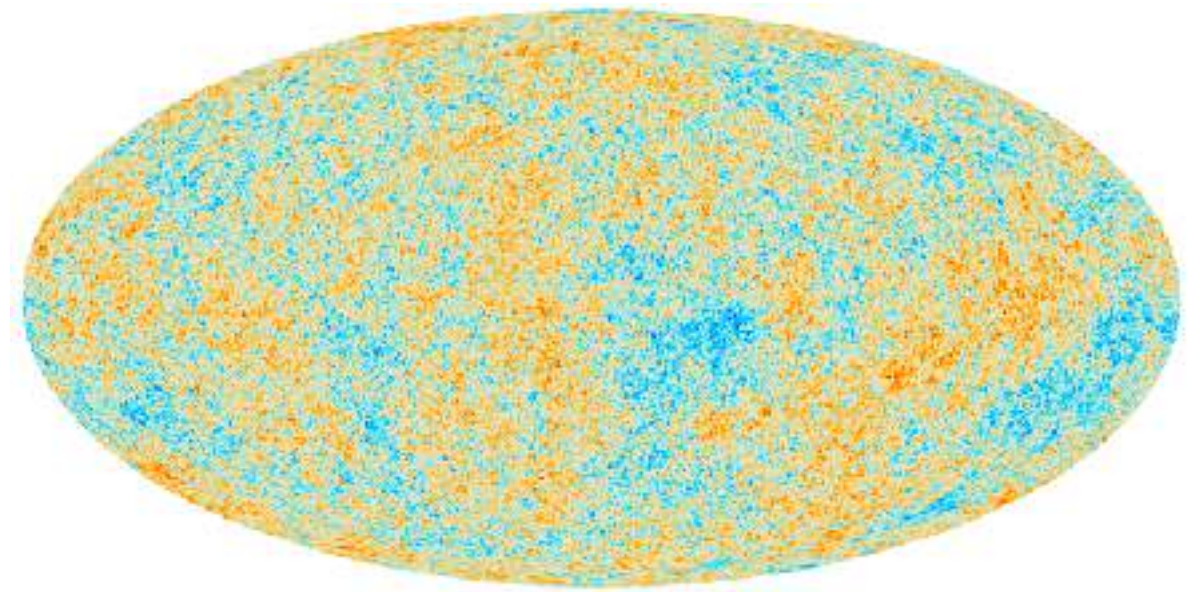
$$f \cdot S_{inst} \leq \mathcal{O}(1) M_P$$

WGC and Inflation

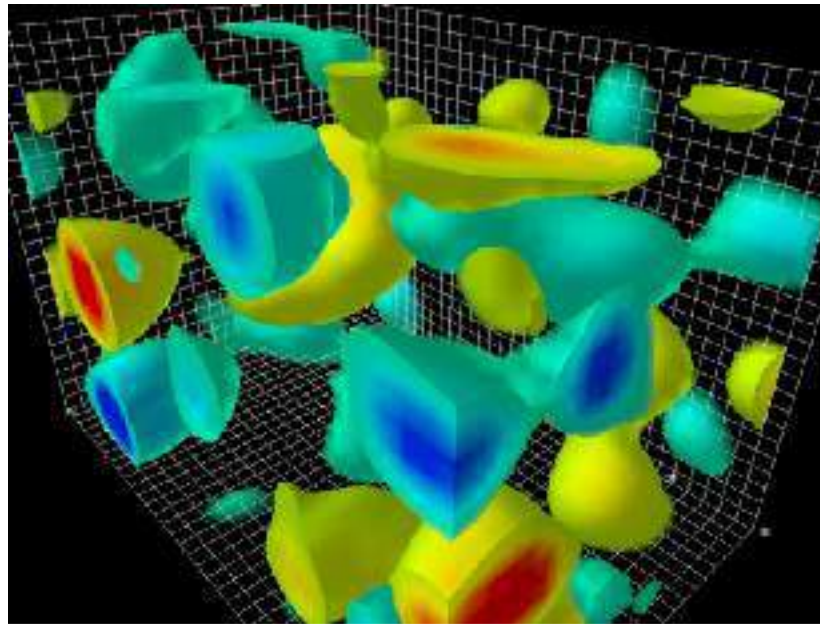
Inflation

[Starobinsky];[Guth];[Linde];[Albrecht, Steinhardt];...

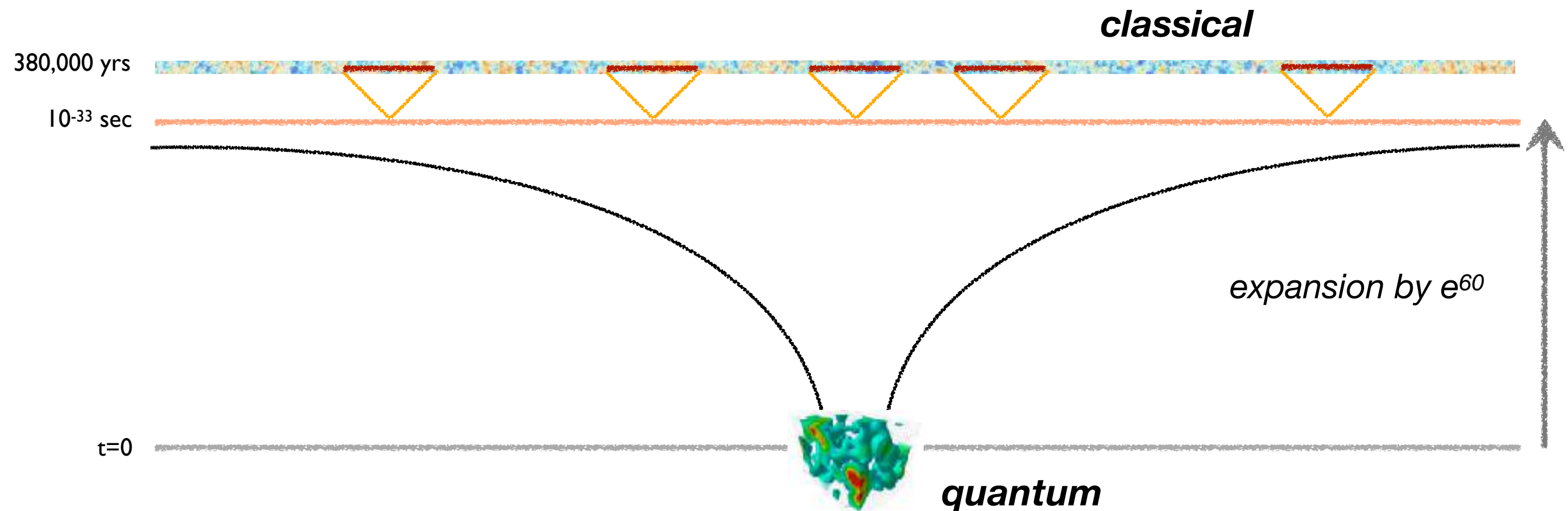
- Period of **accelerated expansion** in early universe
 - Solves flatness, horizon, and monopole problems
 - Predicts **nearly scale-invariant, Gaussian** curvature fluctuations
 - Source anisotropies in CMB, inhomogeneities in LSS
- A myriad of models. Taxonomy done mostly through their observables (n_s , r)



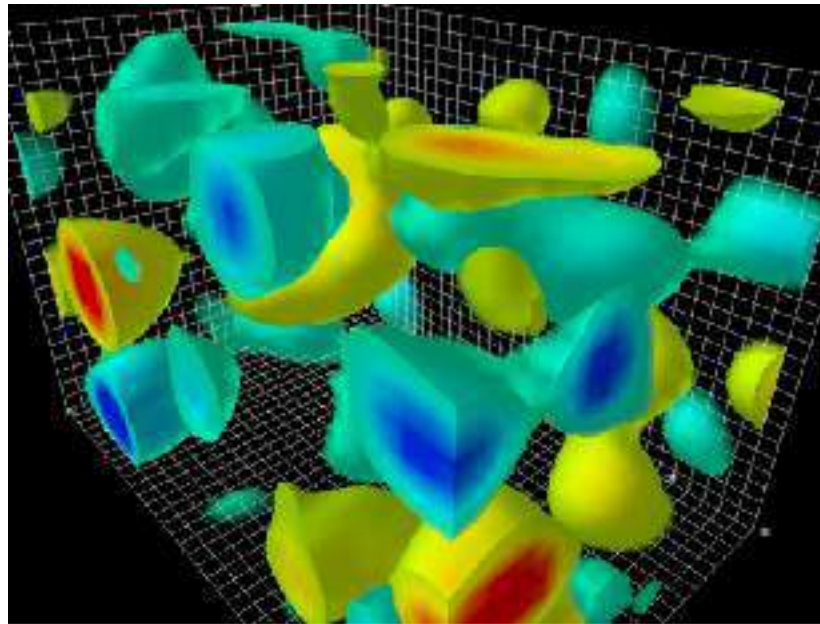
Any massless field experiences quantum fluctuations during inflation:



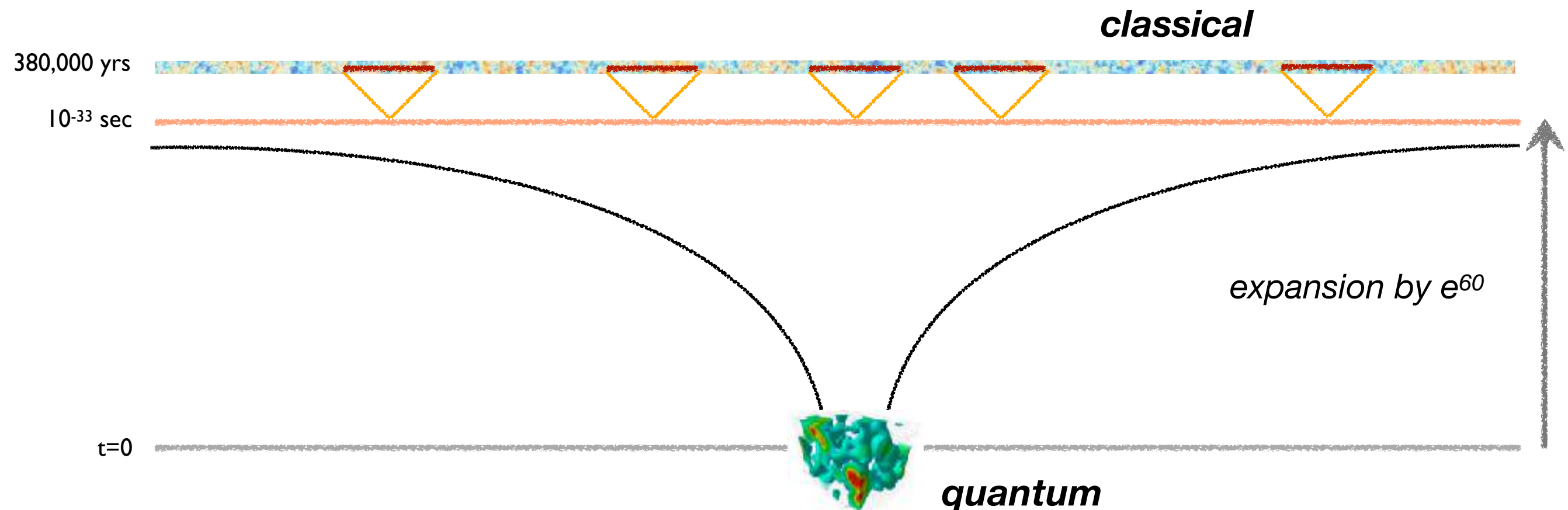
Inflation stretches these to macroscopic scales:



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Inflation stretches these to macroscopic scales:



Two massless fields that are guaranteed to exist are:

ζ

Goldstone boson

of broken time translations

h_{ij}

graviton

Two massless fields that are guaranteed to exist are:

ζ

Goldstone boson

of broken time translations

$$\Delta_s^2 = \frac{1}{4\pi^2} \frac{H^4}{f_\pi^4}$$

symmetry breaking scale

($= \dot{\phi}^2$ for slow-roll inflation)

h_{ij}

graviton

expansion rate

$$\Delta_t^2 \equiv \frac{2}{\pi^2} \frac{H^2}{M_{\text{pl}}^2}$$

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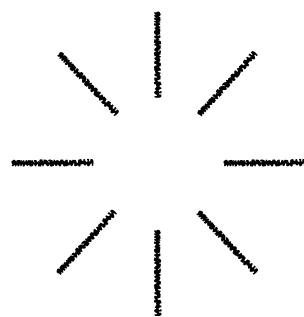
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E-modes:



B-modes:

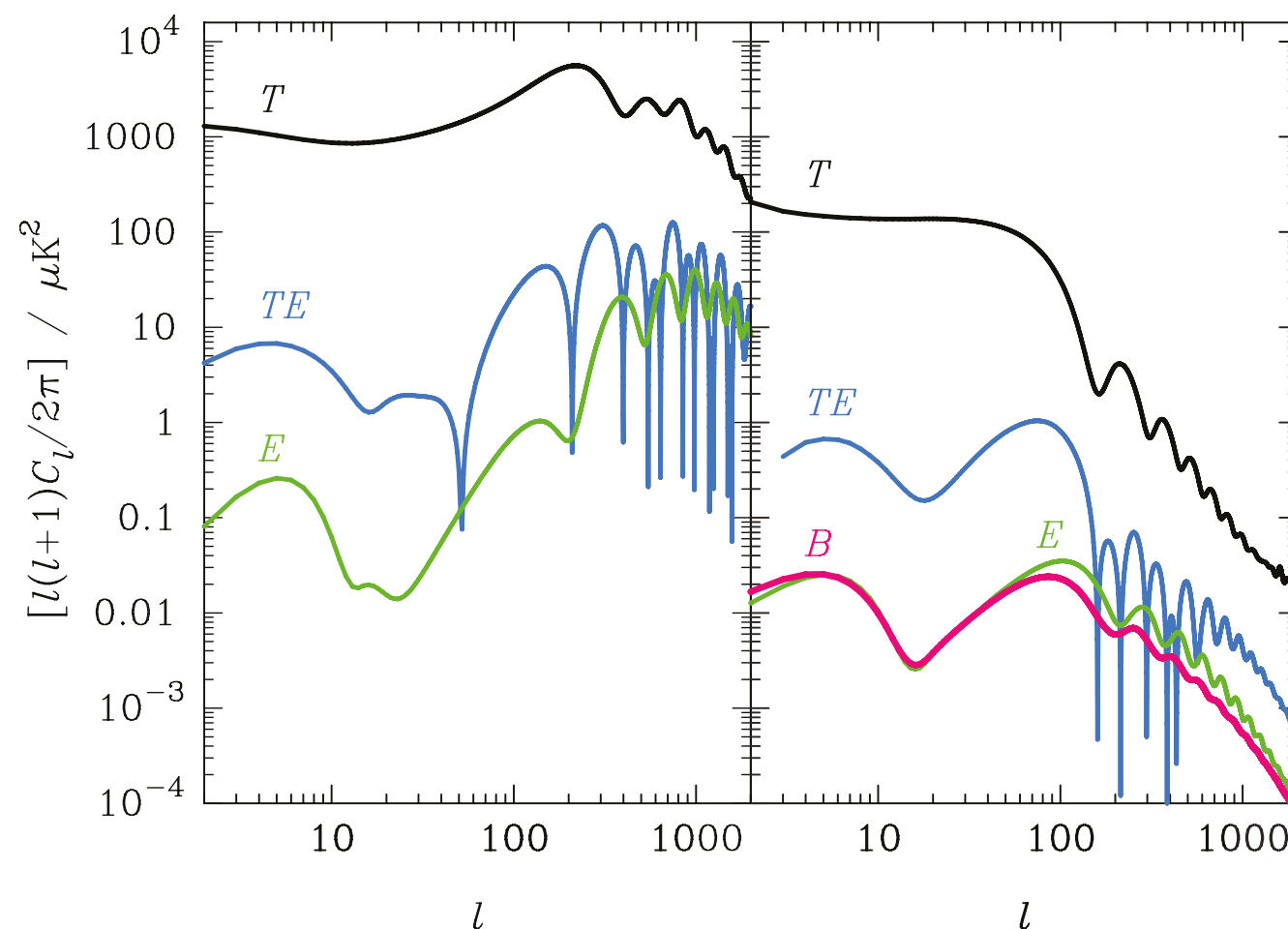


h_{ij}

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expansion rate

$$\Delta_t^2 \equiv \frac{2}{\pi^2} \frac{H^2}{M_{\text{pl}}^2}$$



Inflation and Gravity Waves

- Assuming i) single field slow-roll inflation, ii) the observed fluctuations are generated by the vacuum fluctuation of the inflaton:

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon \quad \text{where} \quad \epsilon = \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2$$

where we have used the fact that for slow-roll:

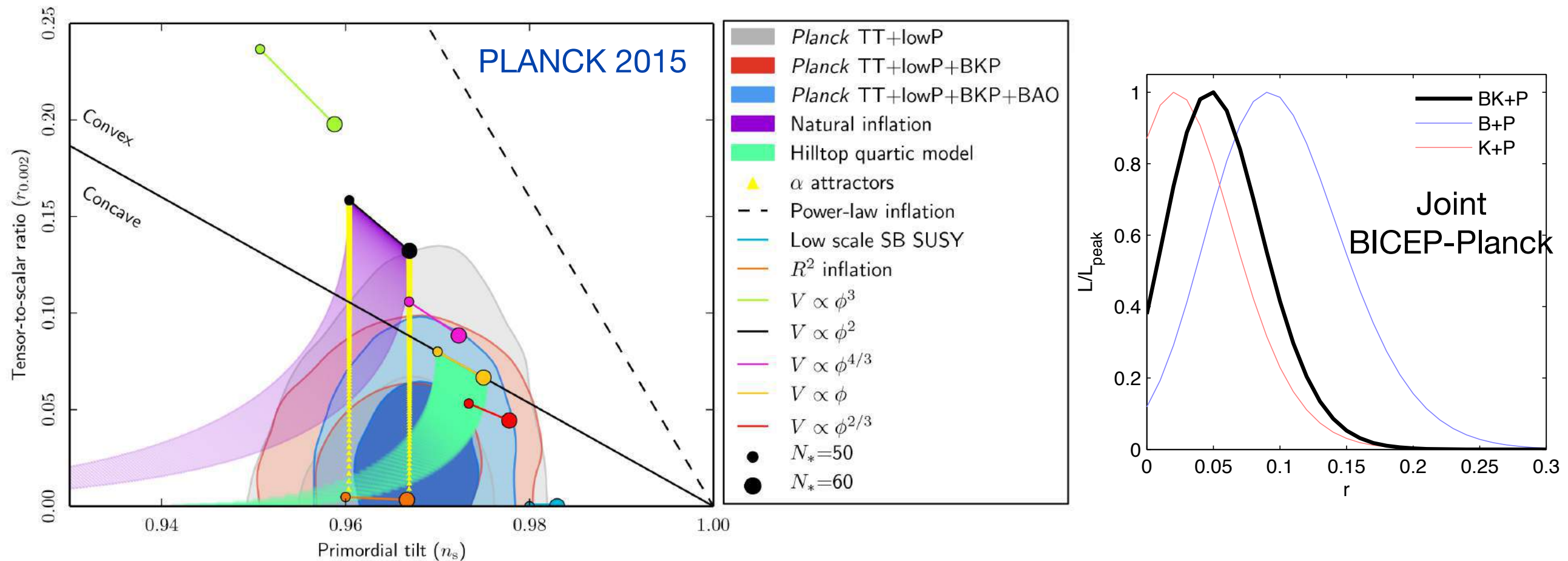
$$\dot{\phi}^2 = \frac{V'^2}{9H^2} = \frac{M_P^2}{3} \frac{V'^2}{V} = \frac{2}{3} \epsilon V ; \quad \frac{d\phi}{dN} = \frac{\dot{\phi}}{H} = M_P \sqrt{2\epsilon} = M_P \sqrt{\frac{r}{8}}$$

$$\frac{\Delta\phi}{M_P} = \int \sqrt{\frac{r(N)}{8}} dN \sim \sqrt{\frac{r_{CMB}}{2.5 \times 10^{-3}}}$$

Lyth bound

for 50 – 60 e – folds and $\frac{dr}{d\log N} \sim \mathcal{O}(\epsilon)$

Primordial Gravitational Waves



Many experiments including BICEP/KECK, PLANCK, ACT, PolarBeaR, SPT, SPIDER, QUEIT, Clover, EBEX, QUaD, ... can potentially detect primordial B-mode at the sensitivity $r \sim 10^{-2}$.

Further experiments, such as CMB-S4, PIXIE, LiteBIRD, DECIGO, Ali, .. may improve further the sensitivity to eventually reach $r \sim 10^{-3}$.

B-mode and Inflation

If primordial B-mode is detected, natural interpretations:

- ◆ Inflation took place at an energy scale around the GUT scale

$$E_{\text{inf}} \simeq 0.75 \times \left(\frac{r}{0.1} \right)^{1/4} \times 10^{-2} M_{\text{Pl}}$$

- ◆ The inflaton field excursion was super-Planckian

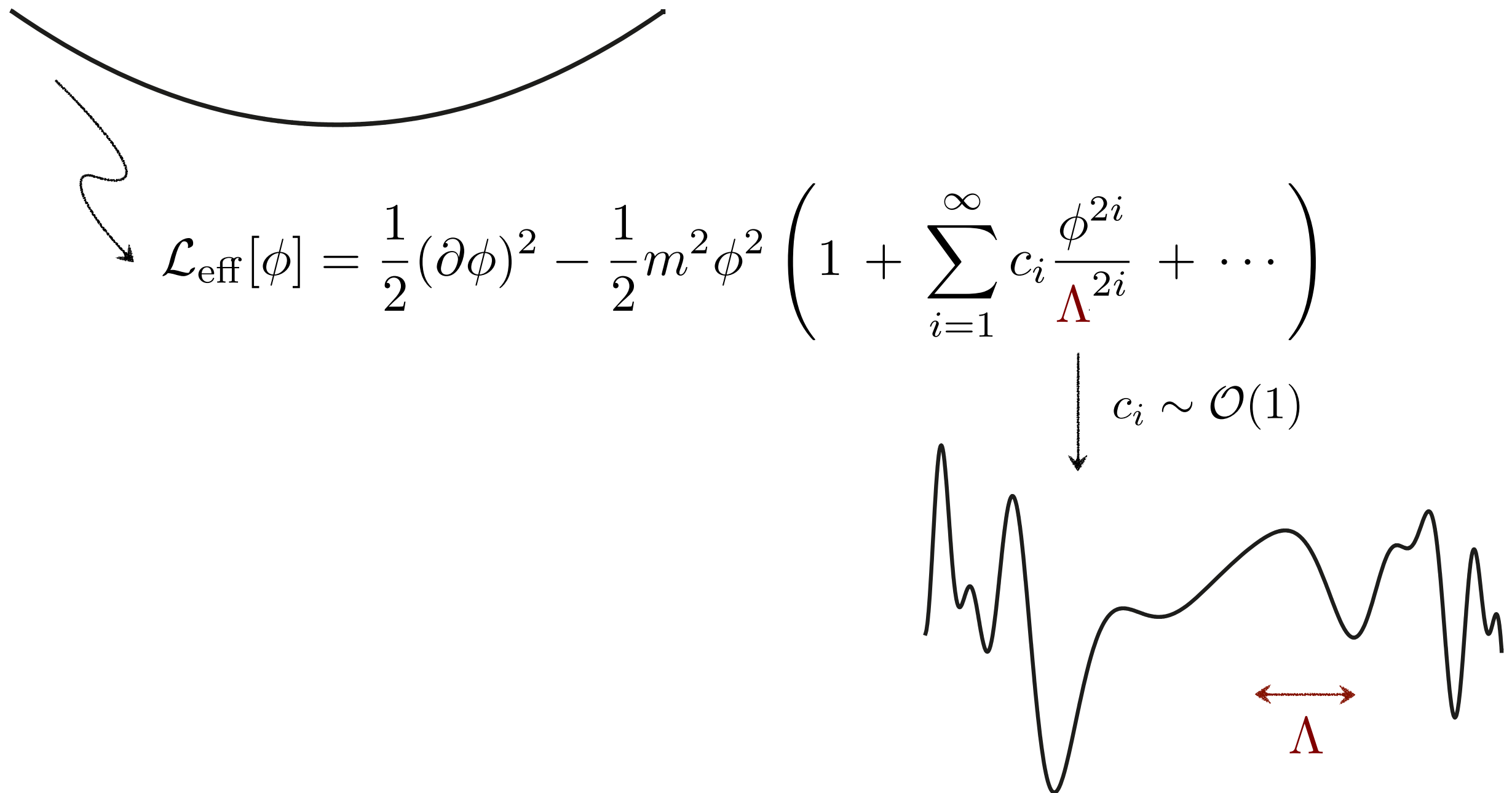
$$\Delta\phi \gtrsim \left(\frac{r}{0.01} \right)^{1/2} M_{\text{Pl}}$$

Lyth '96

- ◆ Great news for string theory due to strong UV sensitivity!

Large field inflation and UV Sensitivity

UV sensitivity of large field inflation:



The diagram illustrates the UV sensitivity of large field inflation. At the top, a smooth, parabolic curve represents the potential $V(\phi)$. A curved arrow points from this curve down to the effective Lagrangian equation. The equation is:

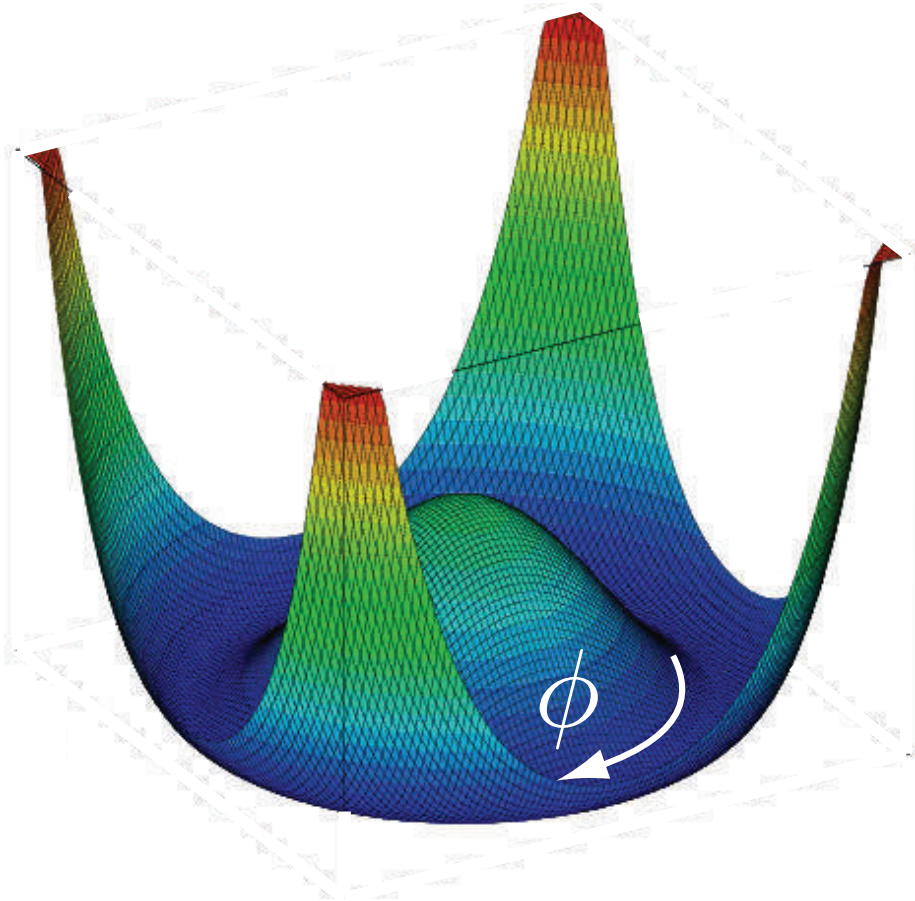
$$\mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 \left(1 + \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}} + \dots \right)$$

Below the equation, a vertical arrow points down to a highly oscillatory, noisy curve, indicating the breakdown of the smooth potential at high energies. To the right of this arrow is the text $c_i \sim \mathcal{O}(1)$. At the bottom right, a horizontal double-headed arrow labeled Λ indicates the energy scale associated with the UV sensitivity.

Axions & Large Field Inflation

Natural Inflation [Freese, Frieman, Olinto]

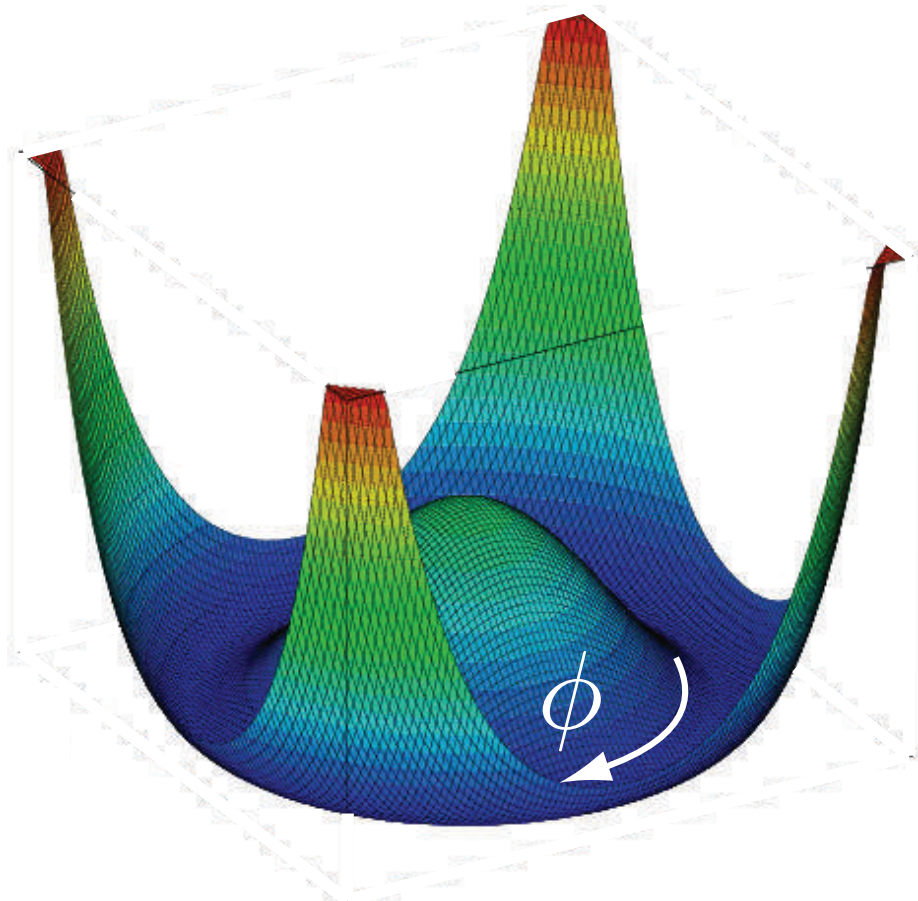
Pseudo-Nambu-Goldstone bosons are natural inflaton candidates.



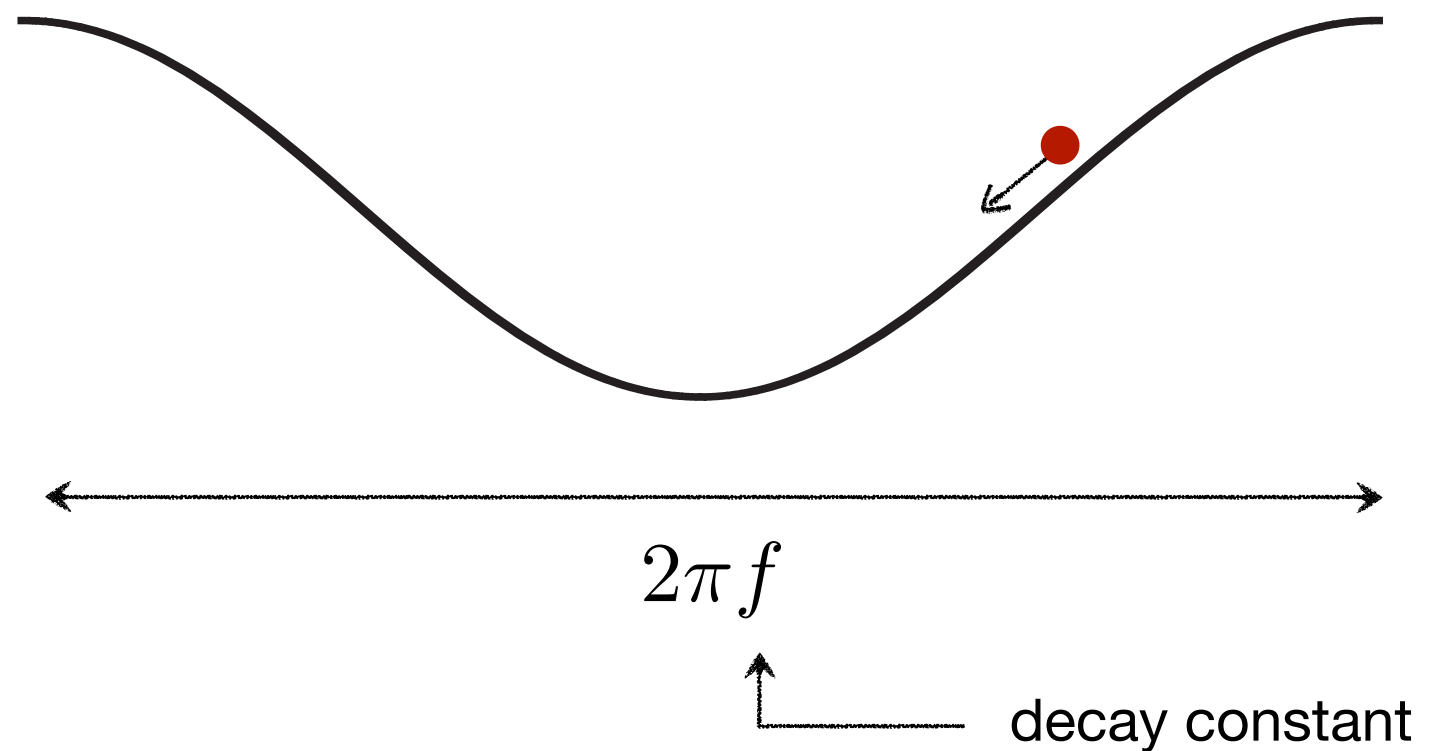
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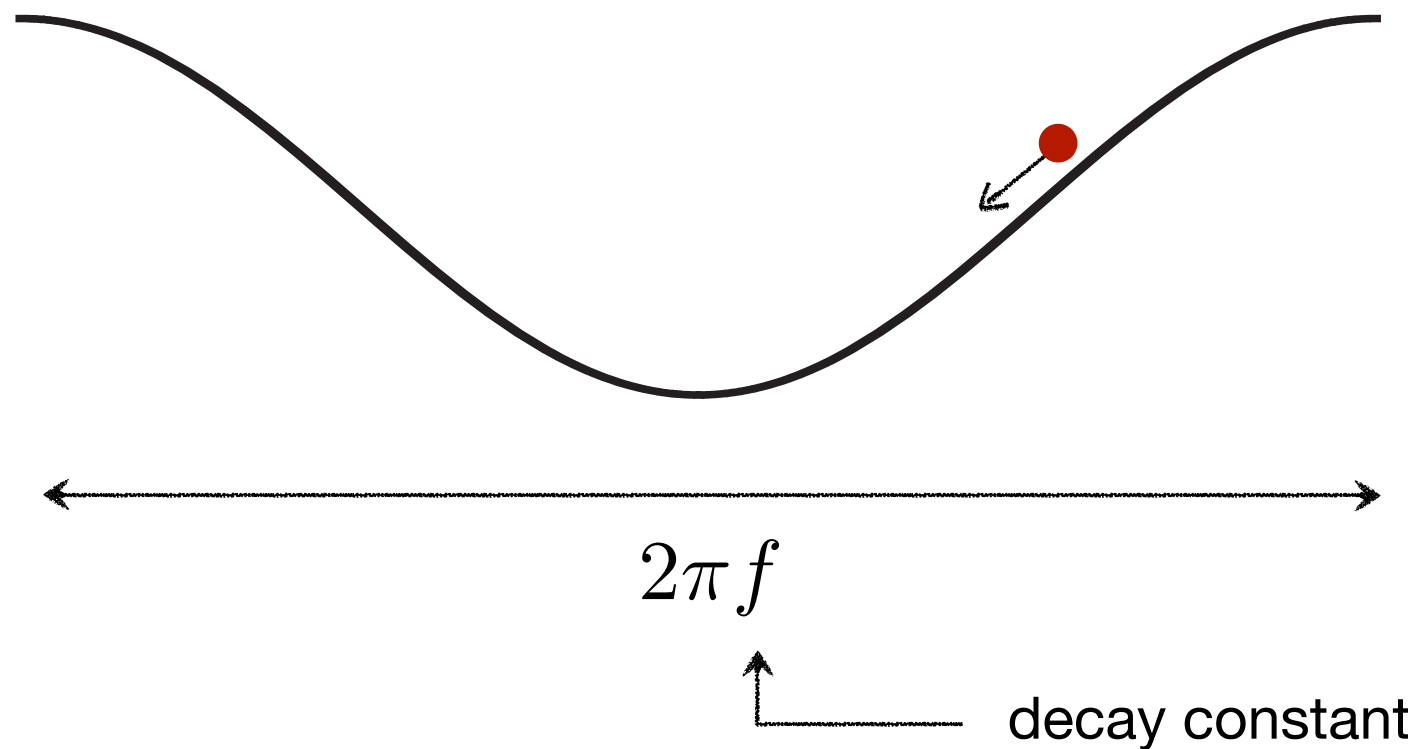
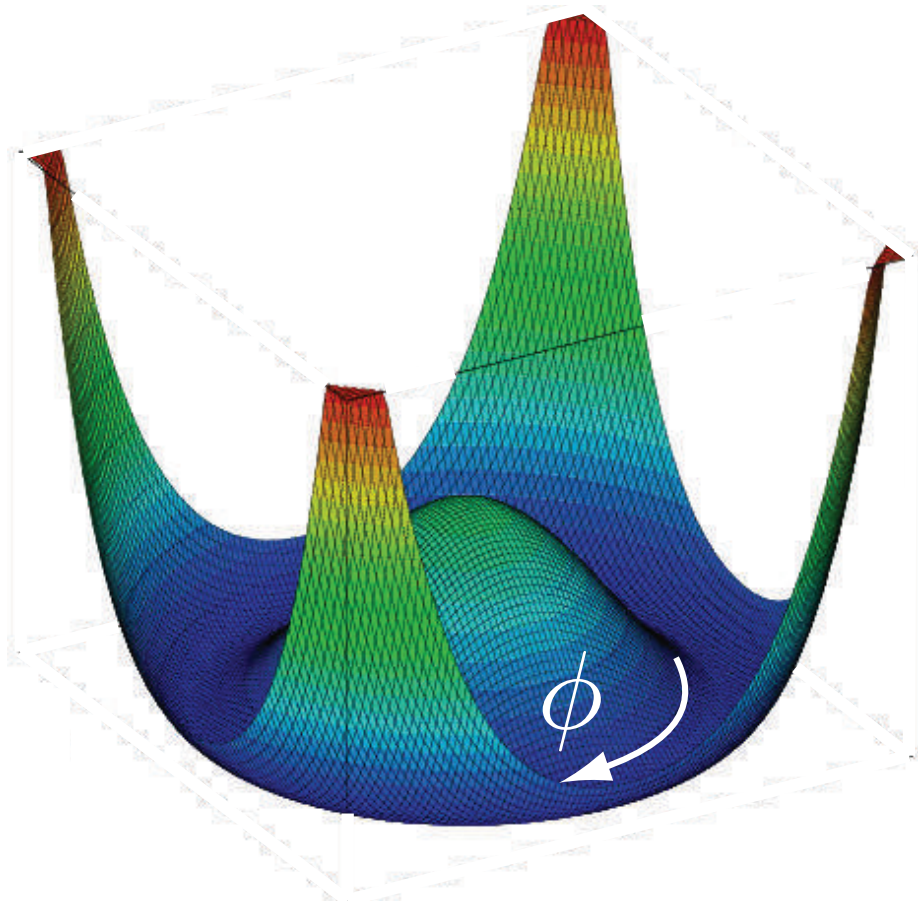


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Slow roll: $f > M_P$

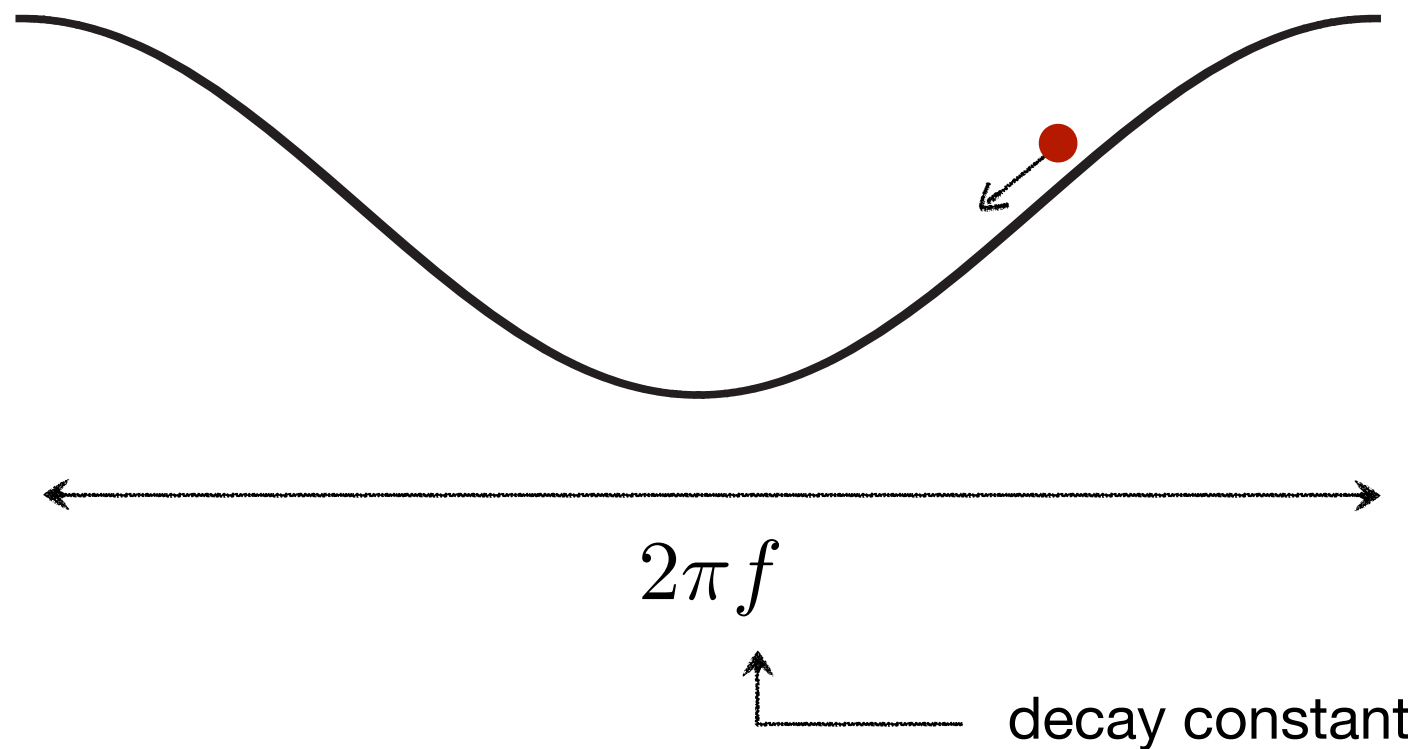
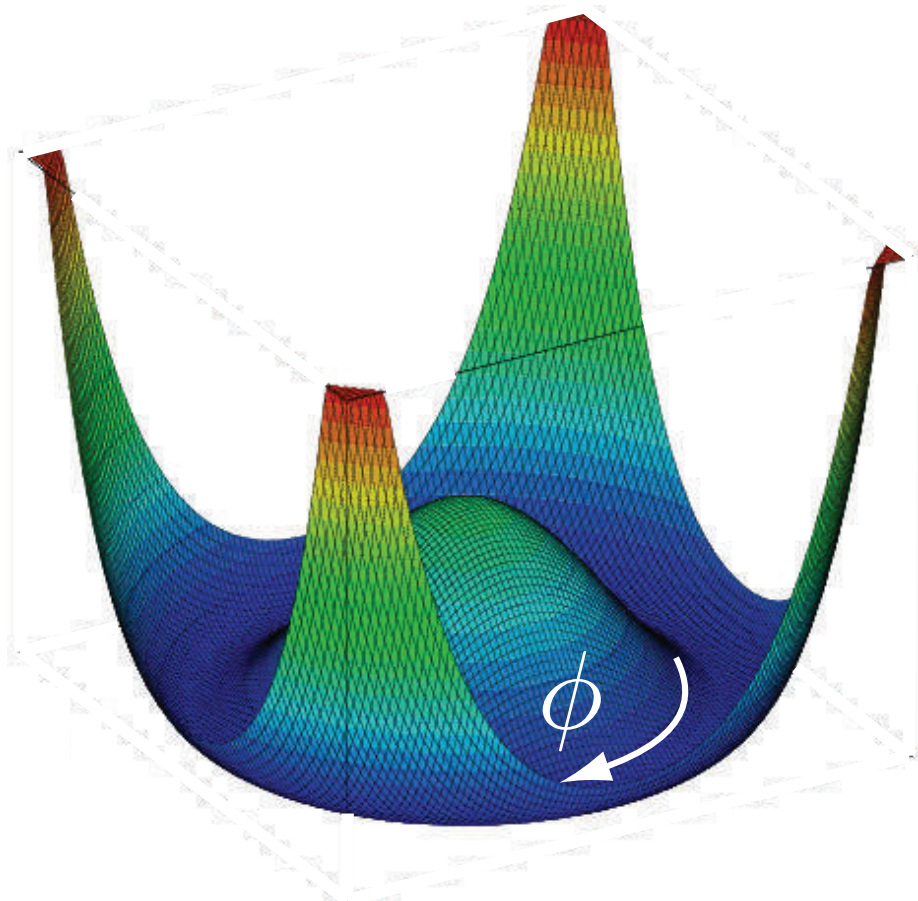
$$V(\phi) = 1 - \Lambda^{(1)} \cos\left(\frac{\phi}{f}\right) + \sum_{k>1} \Lambda^{(k)} \left[1 - \cos\left(\frac{k\phi}{f}\right) \right] \quad \text{if} \quad \frac{\Lambda^{(n+1)}}{\Lambda^{(n)}} \sim e^{-S_{\text{inst}}} \ll 1$$

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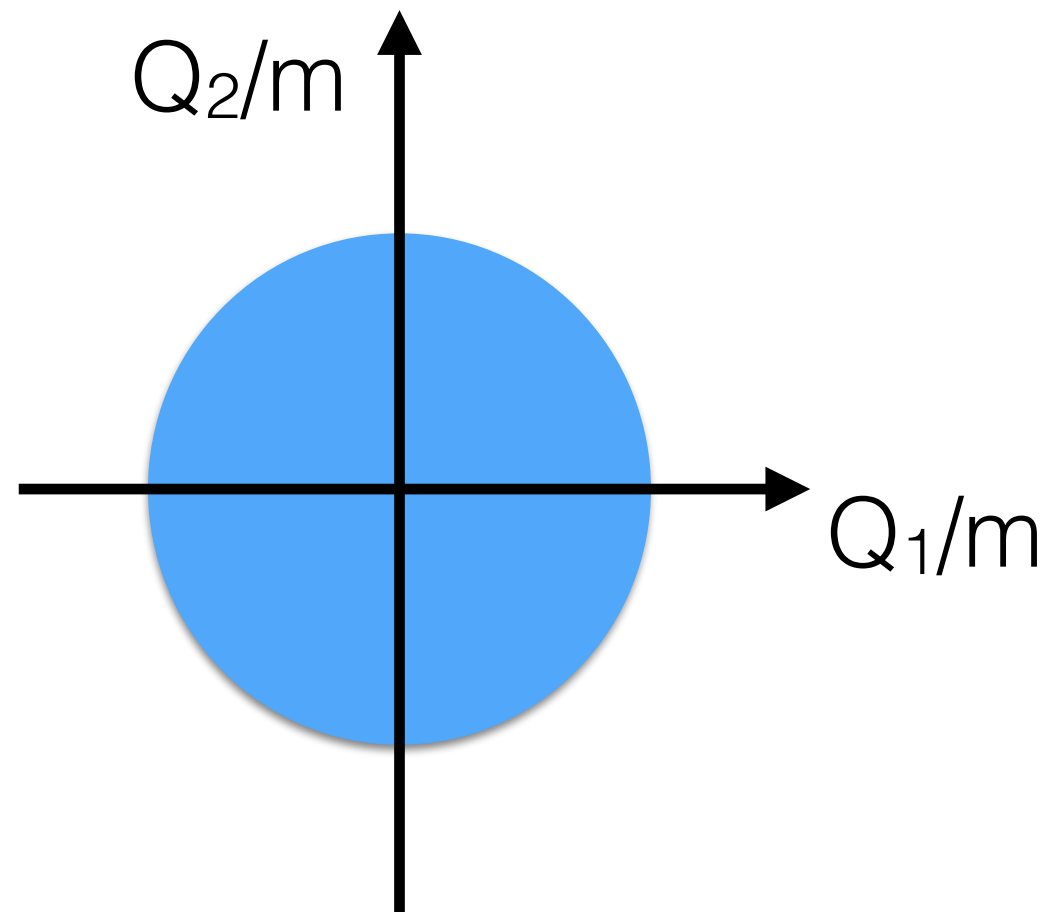
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The WGC implies that these conditions cannot be *simultaneously* satisfied.

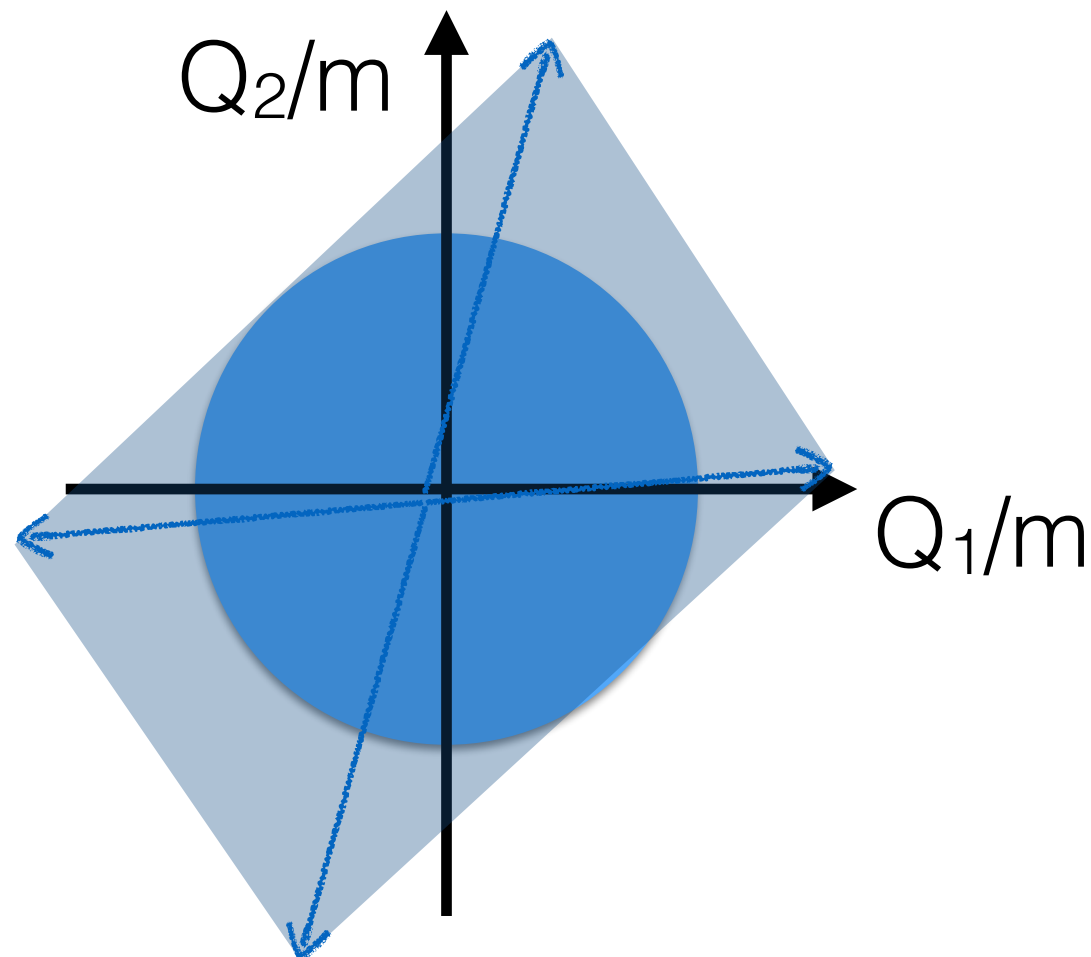
WGC and Axion Inflation

- Effective models of natural inflation in direct conflict with WGC.
- Thorough searches for transplanckian axions in the string landscape have not been successful. Banks et al. '03 ...
- Models with multiple axions have been proposed but they violate the **convex hull condition**. Recall the WGC implies:



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WGC and Axions

Multiple axions/U(1)s

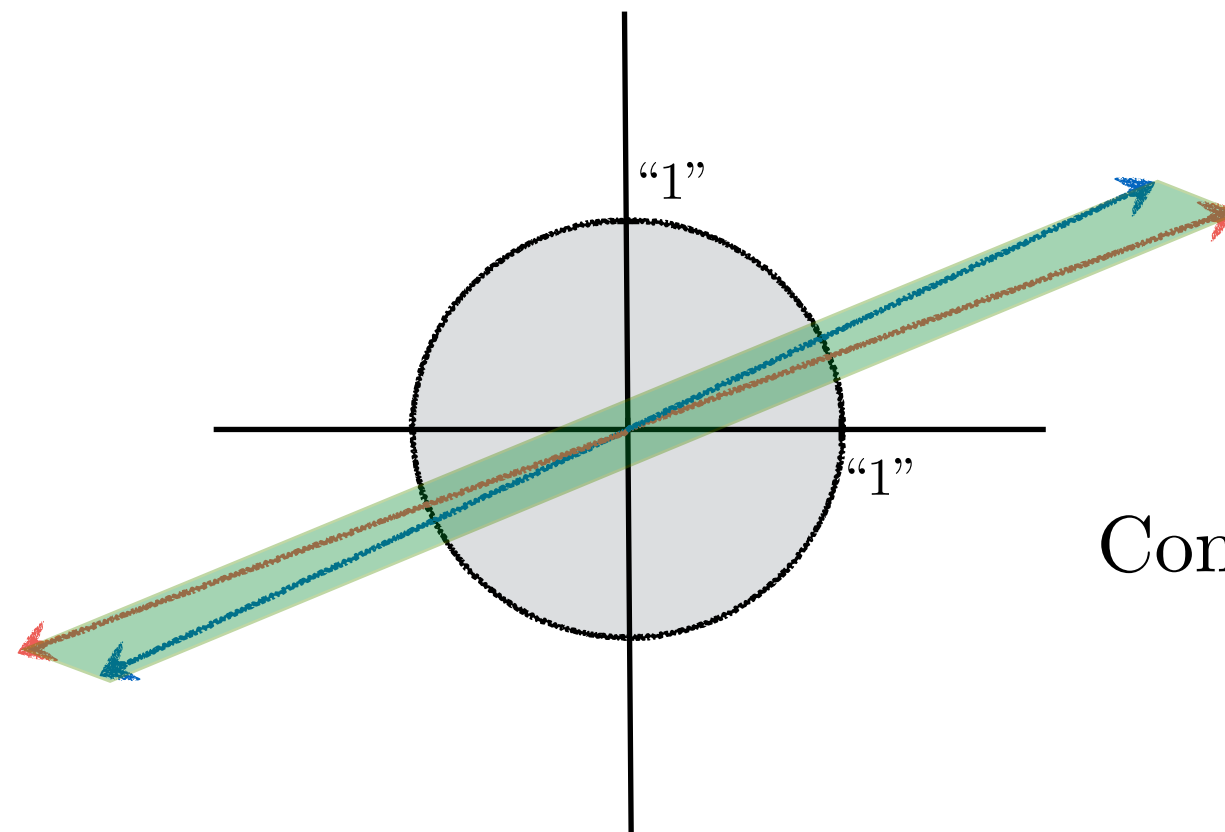
Brown, Cottrell, GS, Soler '15
Rudelius '15

- Consider two U(1) bosons (axions): there must be 2 particles (instantons) $i=1,2$, so that BH's can decay.

$$\vec{z}_i \equiv \frac{M_P}{M_i} \begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix} \quad \left(= \frac{M_P}{\sqrt{2} m_i} \begin{pmatrix} 1/f_i^1 & 1/f_i^2 \end{pmatrix} \right)$$

KNP

WGC



$$|\vec{z}| = \text{"1"}$$

\cap

Convex Hull $\{\vec{z}_{p1}, \vec{z}_{p2}\}$

WGC and Axions

Multiple axions/U(1)s

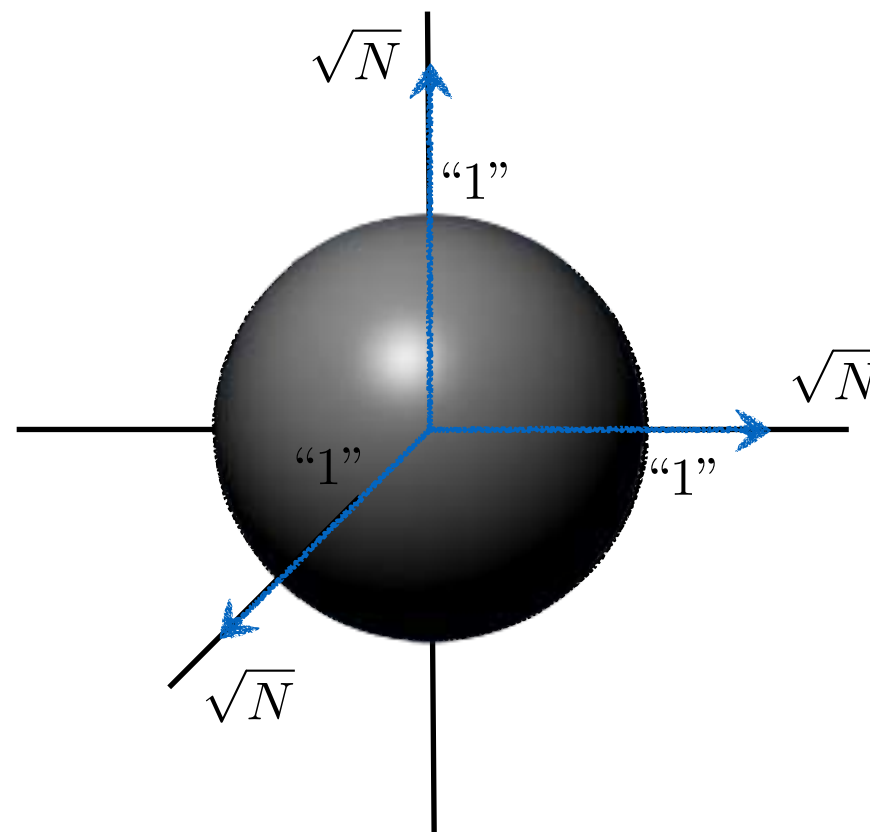
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N-flation

$$z_i^k \geq \sqrt{N} \delta_i^k$$



WGC

$$|\vec{z}| = \text{"1"}$$

\cap

Convex Hull $\{\vec{z}_{p1}, \vec{z}_{p2}\}$

A possible loophole

[Brown, Cottrell, GS, Soler, '15]

- The WGC requires $f \cdot m < 1$ for ONE instanton, but not ALL

$$V = e^{-m} \left[1 - \cos \left(\frac{\Phi}{F} \right) \right] + e^{-M} \left[1 - \cos \left(\frac{\Phi}{f} \right) \right]$$

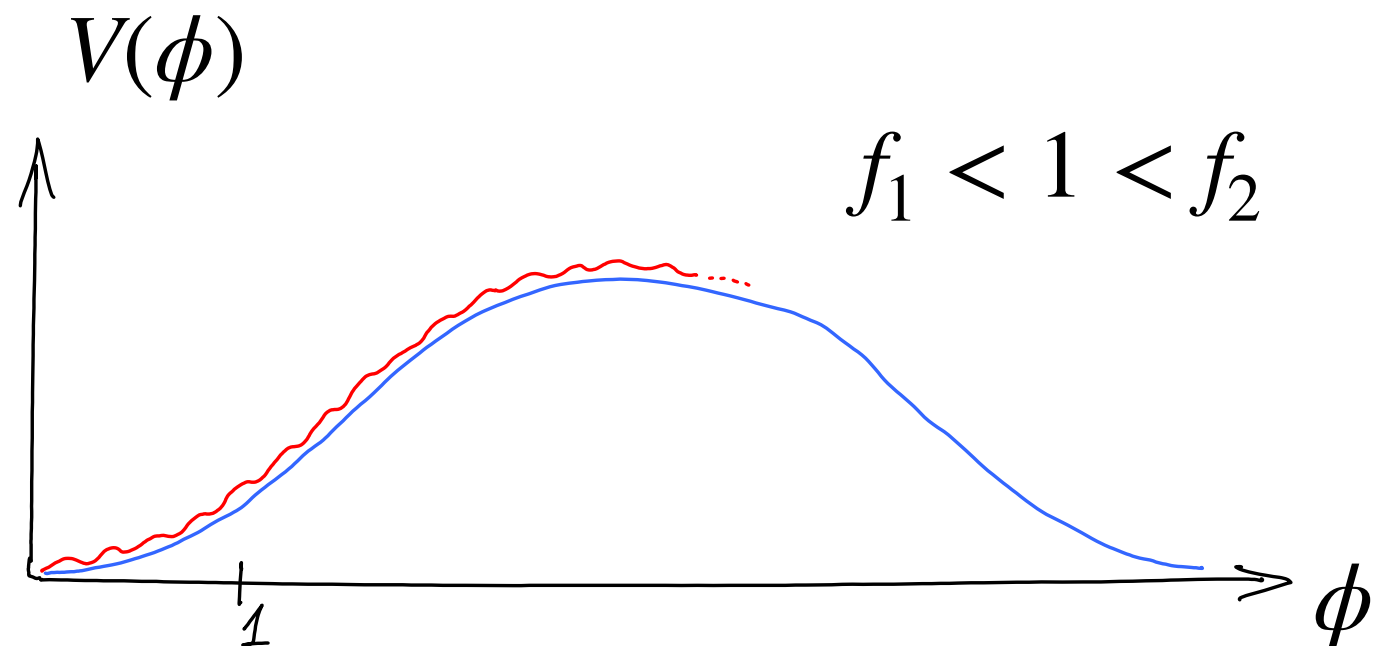
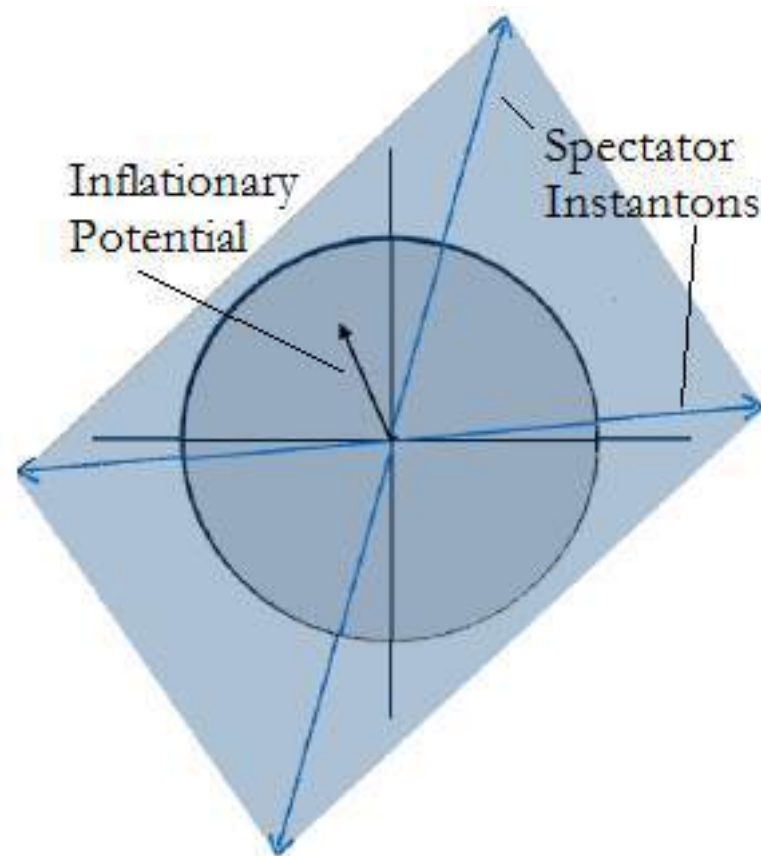
with $1 < m \ll M$, $F \gg M_P > f$, $M \times f \ll 1$

- The second instanton fulfills the WGC, but is negligible, an “spectator”. Inflation is governed by the first term.

A possible loophole

[Brown, Cottrell, GS, Soler, '15]

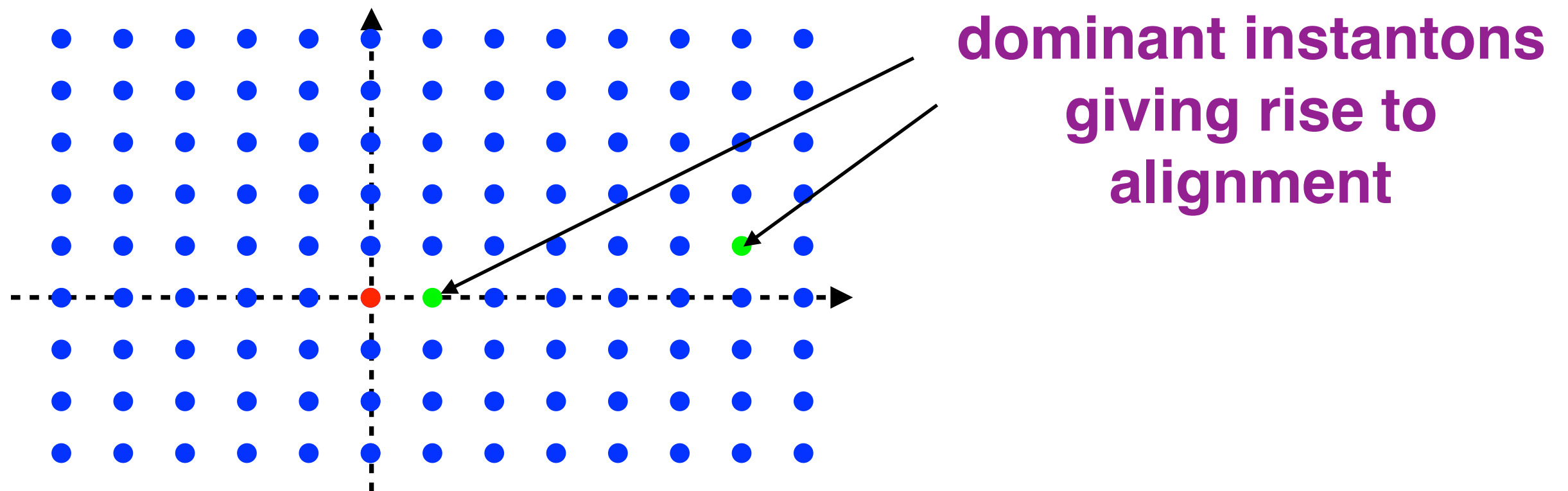
- “Spectator” instantons satisfy the WGC, while dominant instantons can generate an inflationary potential



- Tiny wiggles in the potential may lead to interesting signatures e.g, non-Gaussianity.

Stronger forms of the WGC

- Even stronger forms (e.g., sLWGC, tower WGC, lightest state,...) can be satisfied with **spectator instantons**.



- **Main message** is not that these loopholes are natural or can be realized easily, but that models satisfying the WGC come with **extra baggage** that may lead to new signatures.