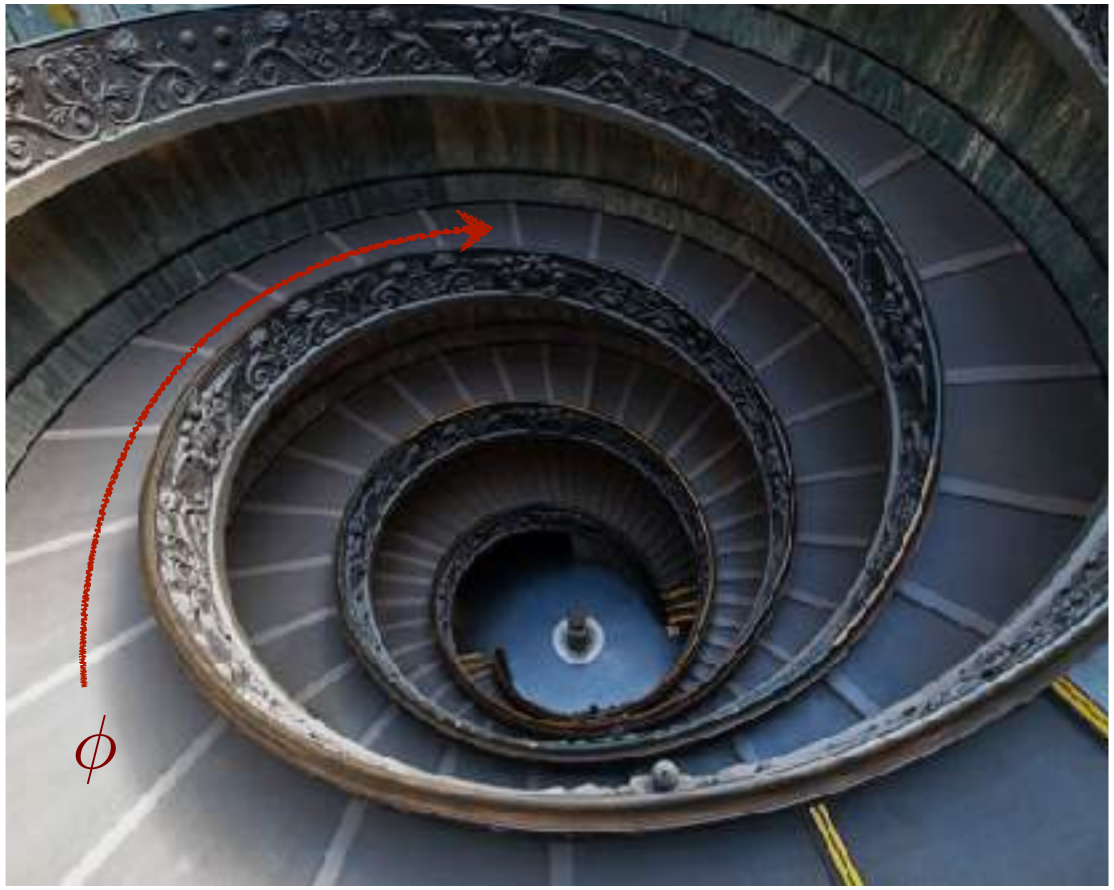
Axion Monodromy Inflation

Axion Monodromy

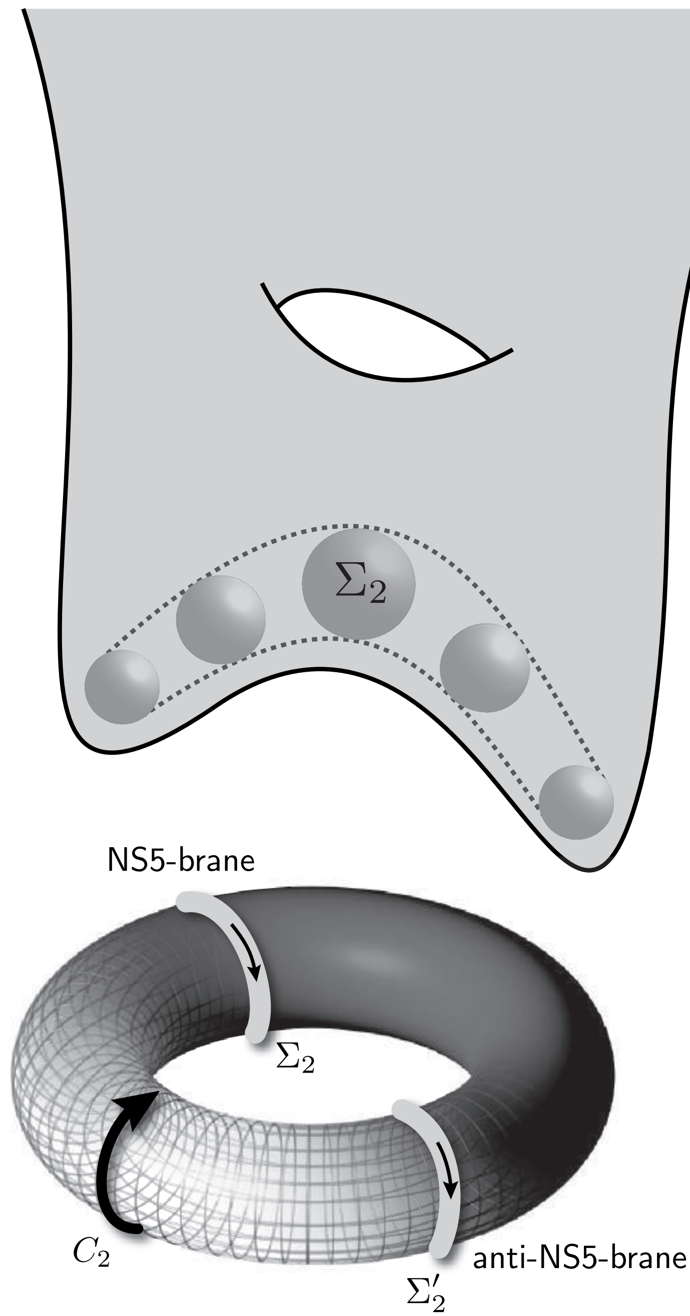
- Non-periodic axions whose shift symmetry is broken before non-perturbative effects set in → **non-compact field space**.



- Monodromy axions are mapped to ***massive*** U(1) vector fields in the particle picture, and do not lead to a long range force.
- WGC arguments not immediately apply, but what about the SDC?

Axion Monodromy

- When axion monodromy was first formulated, the shift symmetry was assumed to be broken by coupling to **branes**:



[Silverstein, Westphal, '08]; [McAllister, Silverstein, Westphal, 08]

Explicit constructions involve NS5-branes and **anti-NS5-branes** wrapping **homologous cycles** in **different warped throats**.

Harder to quantify corrections to the EFT upon super-Planckian field displacements.

F-term Axion Monodromy

- **Idea:** Flux compactifications stabilize moduli by giving them flux-induced masses. Same mechanism can generate **axion potential.**

[Marchesano, GS, Uranga, ’14] (see also [Blumenhagen, Plauschinn, ’14];[Hebecker; Kraus, Witkowski, ’14]).

- For example, consider dim reduction of kinetic terms of RR forms:

$$S_{10} = \int d^{10}x \; G_p \wedge *G_p \qquad\qquad G_p = \underset{\substack{\uparrow \\ dC_{p-1}}}{F_p} - \underset{\substack{\uparrow \\ dB_2}}{H_3} \wedge C_{p-3} + \mathscr{F} \wedge e^{B_2}$$

$$\int_{\Sigma_p} C_p \; , \int_{\Sigma_2} B_2 \rightarrow \text{ 4D axions} \quad \text{e.g., Kahler moduli } T^i = \int_{\Sigma_2^i} B + i \int_{\Sigma_2^i} J$$

F-term Axion Monodromy

- Dimensionally reduce kinetic terms → **scalar potential** for axions

$$V = Z_{ab}(s)\ F_4^a \wedge *F_4^b - 2F_4^a \rho_a(\theta) + \ldots$$

- Note the structure that Z (s) depends only on the **saxions s** and all the **axion** θ dependence appears in $\rho(\theta)$.
- $\rho(\theta)$ also depends on quantized internal fluxes.
- Integrating out the 3-form C_3:

$$* F_4^a = [Z^{-1}(s)]^{ab} \rho_b(\theta) \rightarrow V = [Z^{-1}(s)]^{ab} \rho_a(\theta) \rho_b(\theta)$$

saxion and axion dependence nicely separated

A Simple Model

- Consider a single axion and a single 4-form:

$$\mathscr{L} = -f^2 d\theta \wedge *d\theta - F_4 \wedge *F_4 + 2F_4\left(m\theta + f_0\right)$$

quantized fluxes

- The eom for C_3:

$$d * F_4 = d(m\theta + f_0) \quad \Rightarrow \quad * F_4 = f_0 + m\theta$$

- This gives a **multi-branched** effective potential:

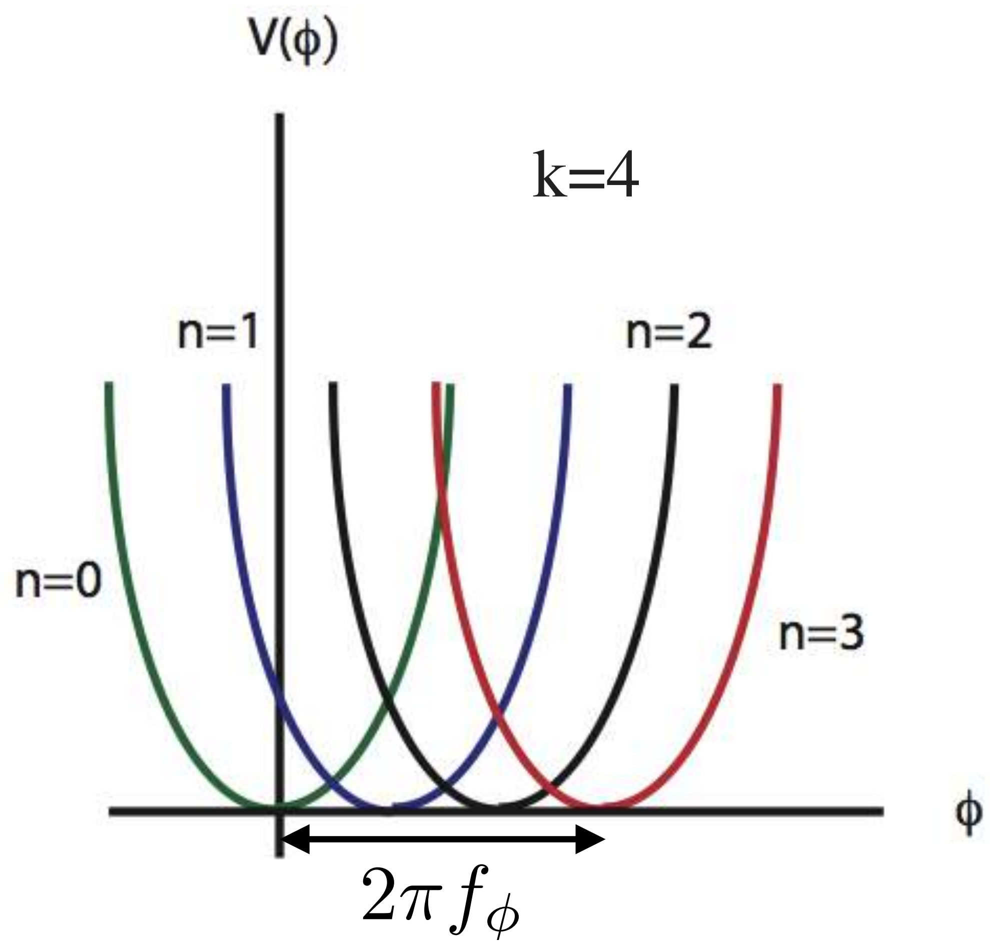
$$V = (f_0 + m\theta)^2$$

- Moreover, V is invariant under the **combined discrete shift**:

$$\theta \to \theta - c/m, \quad f_0 \to f_0 + c$$

Multi-branched Potential

- This identifies gauge equivalent branches when $c/m = 2\pi f$



Z_n discrete symmetry

- The combined discrete shift symmetry and gauge symmetry of C_3 constrain the higher order corrections:

$$\delta V \sim \sum_n c_n F_4^{2n} \sim \sum_n c_n V_0^n$$

higher order corrections are under control

Dual Formulation

- This model can be described in terms of the dual 2-form:

$$d\phi = *_4 db_2$$

- The axion 4-form Lagrangian can be rewritten in terms of the Stuckelberg coupling:

$$|db_2 + nc_3|^2 + |F_4|^2$$ **2-form eaten by 3-form**

- The **gauge invariance**:

$$C_3 \to C_3 + d\Lambda_2, \quad b_2 \to b_2 - n\Lambda_2$$

- This gauge invariance protects the axion potential from dangerous UV corrections.

General Properties of F-term Axion Monodromy

In general, with ***more fluxes and more axions***, we expect:

- **Non-linear couplings**: shift invariant function $\rho(\theta)$ can include mixings and higher order terms for axions ⇒ more general Vθ).

- **Multiple 3-forms:** higher dim. operators cannot always be written as V^n but rather as $\rho^n(\theta)$. However still under control if $\rho(\theta) \ll 1$.

- **Axion-Saxion Mixings:** saxion-dependent axion kinetic term ⇒

$$V = [Z^{-1}(s)]^{ab}\rho_a(\theta)\rho_b(\theta)$$

- as θ traverses along the inflationary trajectory, s changes, thus the backreaction of s modifies the field space metric for axions.

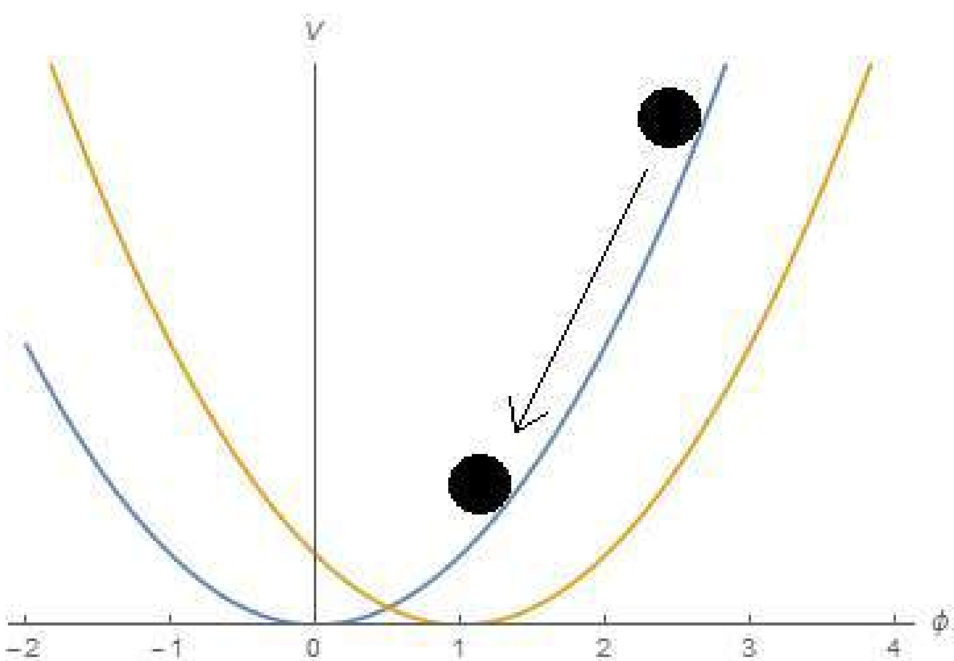
Symmetry and Constraints of F-term Axion Monodromy

- **Symmetry-based protection mechanism:** higher order corrections appear as

$$\rho^n(\theta) \text{ rather than } \theta^n$$

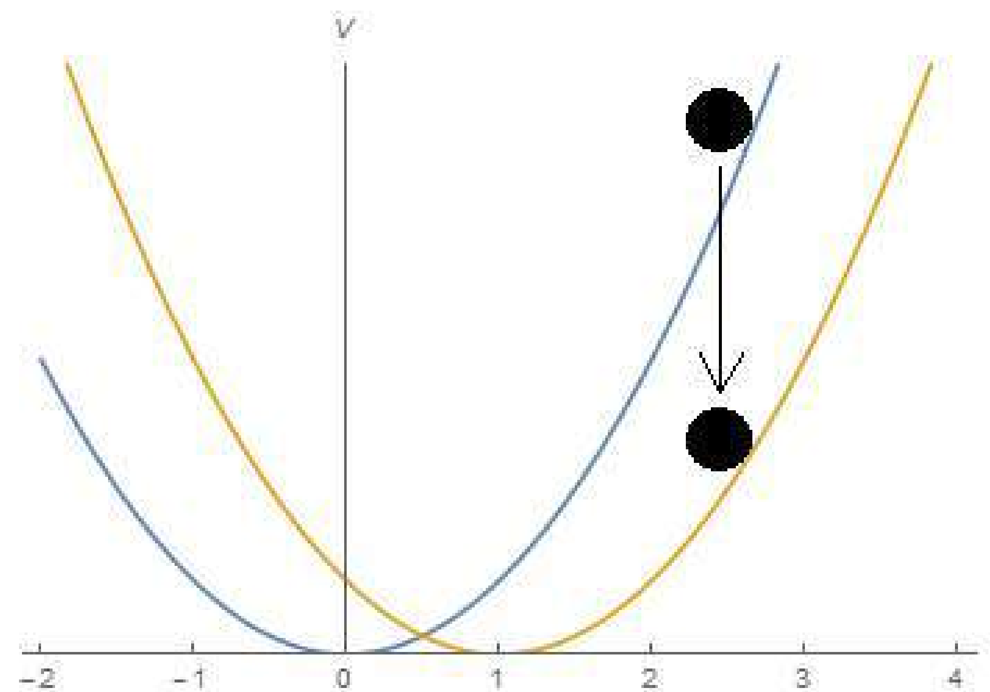
- **Tunneling between branches:** not strong constraints though $\Delta\phi$ *cannot be parametrically larger than* M_P. **[Brown, Cottrell, GS, Soler, '16]**

$$V(\phi) = \frac{1}{2}(ne + \mu\phi)^2$$



Slow Roll Inflation

$$V(\phi) = \frac{1}{2}(ne + \mu\phi)^2$$



Tunneling Event: $n \to n-1$

Swampland Distance Conjecture

- Can SDC, a more general statement about field ranges, limit the field range of axion monodromy?
- Naively, the argument we used earlier for KK reduction:

$$\mathcal{L}_{eff} = \int \left(\frac{dR}{R}\right)^2 + \ldots$$

does not apply to **axions** because of their **shift symmetries**.

- Consider the **axio-dilaton:**

$$K = -\log(\Phi + \overline{\Phi}) \quad \text{where} \quad \Phi = s + i\theta$$

$$\rightarrow \mathscr{L}_{kin} = \frac{1}{4s^2}(\partial s)^2 + \frac{1}{4s^2}(\partial \theta)^2 + \ldots$$

Backreaction and SDC

[Palti, Klawer, '16]

- The saxions are displaced as the axions traverses in its field space:

$$V = [Z^{-1}(s)]^{ab}\rho_a(\theta)\rho_b(\theta)$$

- This leads to an axion-dependent kinetic term:

$$\mathscr{L}_{kin} \to \frac{1}{4s^2(\theta)}(\partial\theta)^2 + \ldots$$

- One way to estimate this backreaction is to consider the trajectory traced by the minimum of V with respect to s as θ varies:

$$\langle s \rangle = s_0 + \delta s(\theta) \approx s_0 + \lambda\theta \to \lambda\theta$$

- If we evaluate the field range using:

$$\Delta\theta = \int \sqrt{G_{\theta\theta}(s)}\, d\theta \sim \int \frac{d\theta}{s_0 + \lambda\theta} \sim \frac{1}{\lambda}\log\theta$$

Backreaction and SDC

- The critical field range is set by λ, let θ_c be the value of θ when:

$$s_0 = \delta s(\theta_c) = \lambda\theta_c \quad \rightarrow \quad \Delta\theta_c \sim \frac{1}{\lambda}$$

- In string examples, one finds that λ is of order 1 if the saxion and the axion have about the same mass, but in general:

$$\lambda \sim \frac{m_\theta}{m_s}$$

- The critical field range is **enhanced**. For large field inflation, only a mild hierarchy $\lambda \sim 10^{-1}$ is sufficient.

- In fact, we need to maintain this hierarchy for single field inflation:

$$m_\theta < H < m_s$$

Dynamical Field Range

- **Questions:**
  - Does minimizing V (s,θ) at each θ and substituting s(θ) into $\Delta\theta$ give a proper estimate of the **dynamical field range**?
  - If there is no (or weak) mass hierarchy, is $\Delta\theta$ limited to O(1) M_P ?
- **We found the answers to both questions are no!** **[Landete, GS, '18]**
- In evaluating the field range:

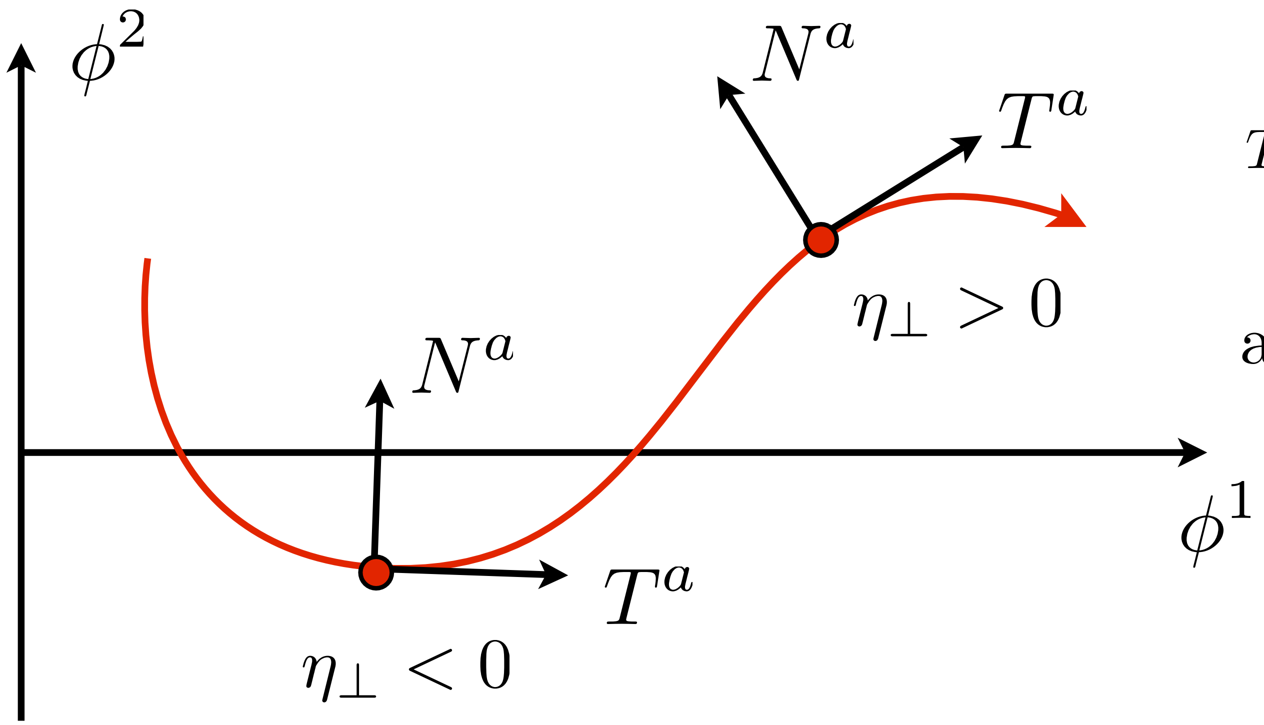
$$\Delta\theta = \int \sqrt{G_{\theta\theta}} d\theta \quad \text{instead of} \quad \int \sqrt{G_{ab}\dot{\phi}^a \dot{\phi}^b} dt$$

this assumed the kinetic terms of the other fields can be ignored. This assumption, by default, implies that the trajectory is a **geodesic** in field space.

Dynamical Field Range

- Minimizing V w.r.t s does not give a trajectory that solves the eom!

- It is known in the study of inflation with multiple fields that the light field trajectory does not always follow a geodesic.



orthogonal basis in field space:

$$T^a = \frac{\dot{\phi}_0^a}{\dot{\phi}_0}. \qquad N^a = s_N(t)\,\frac{D_t T^a}{\sqrt{G_{cd} D_t T^c D_t T^d}},$$

and $D_t^2 T^a, D_t^3 T^a, \ldots$

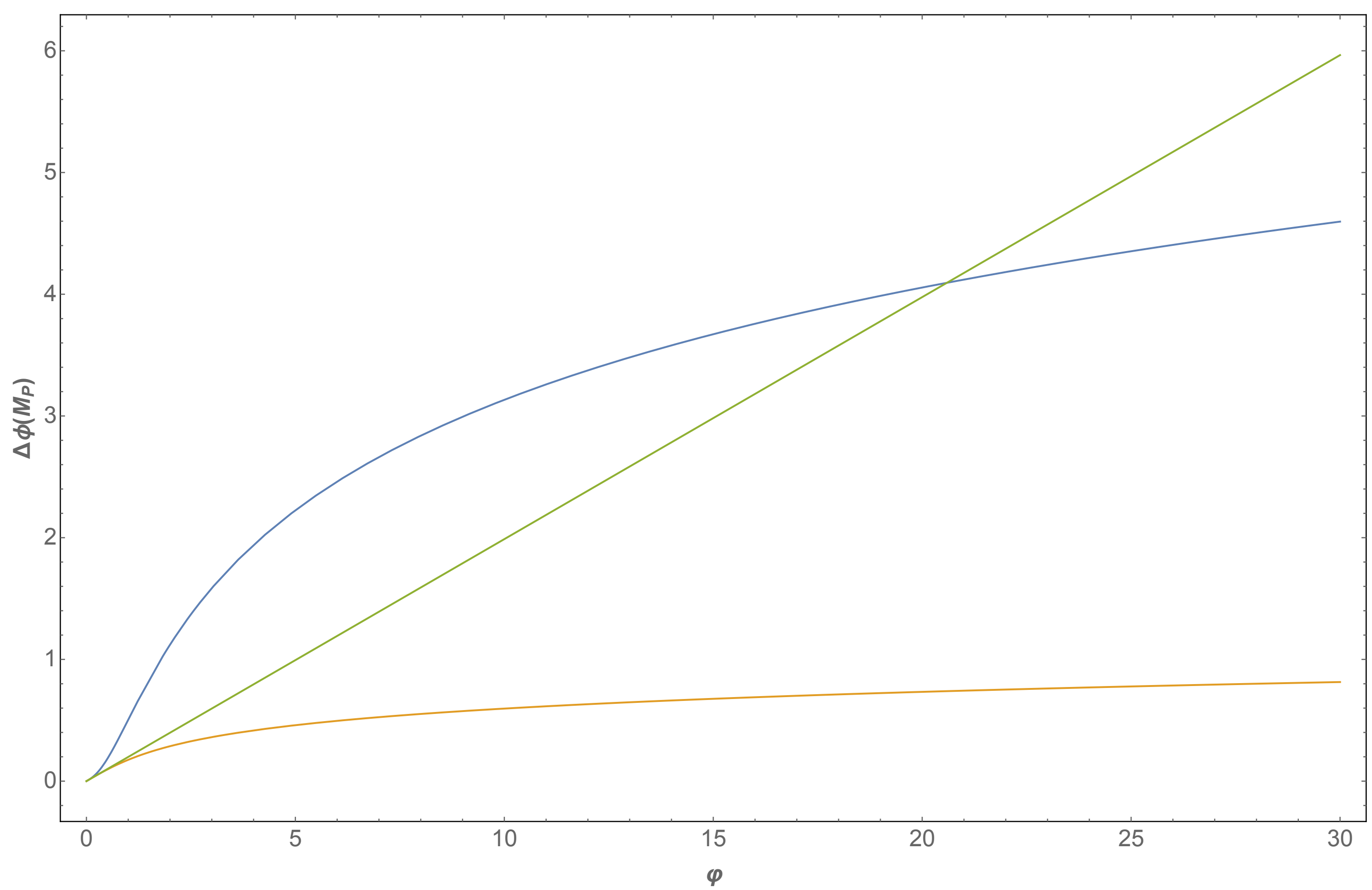
$$D_t X^a \equiv \partial_t X^a + \Gamma^a_{bc} \dot{\phi}^b_o X^c$$

- When is the trajectory a geodesic? i.e., $D_t T^a = 0$

Mass Hierarchy and Dynamical Field Range

- If there is a mass hierarchy, the kinetic terms of the heavy fields can be ignored, but the critical field range is **enhanced by mass ratio.**

- If there is no mass hierarchy, the dynamical field range deviates significantly from the earlier RSDC estimate:



no backreaction

physical field range

RSDC field range

[Landete, GS, '18]

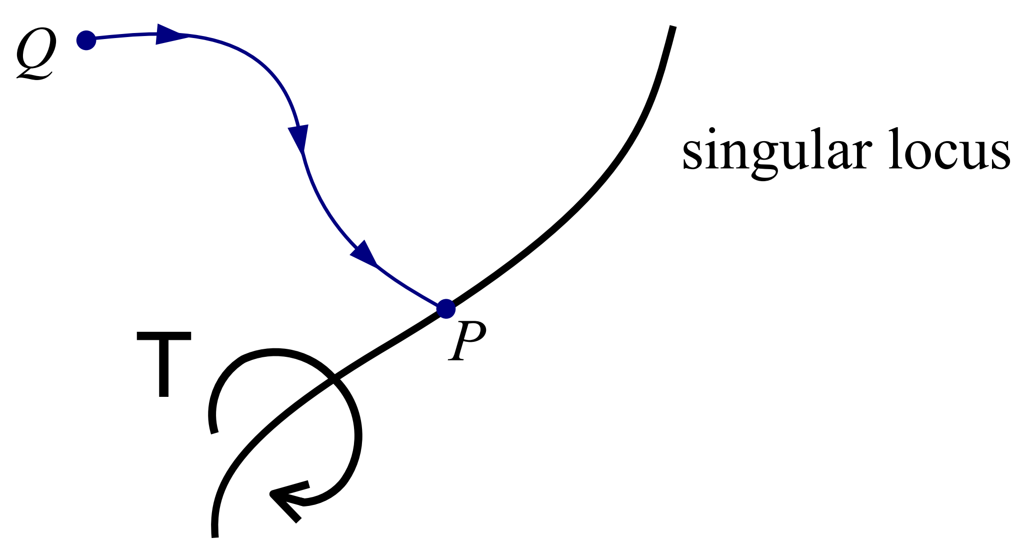
Axion Mondromy and Swampland Constraints

- **Upshot: WGC & SDC do not exclude axion monodromy inflation.**

- There may be other quantum gravity constraints of these models, but before we know what they are, we should keep an open mind.

- Recently, there has been further evidence for the SDC from studying the complex structure moduli space of Type IIB compactifications:

[Grimm, Palti, Valenzuela, '18]



Massless BPS states (wrapped D3-branes) arise at the singularities

- Infinite geodesic distance occur only if approaching a singularity.

Quantum Corrections and Emergence

- While this confirms the asymptotic behavior suggested by the SDC, the interesting question is the behavior at large but large distances.

- Quantum corrections to the field space metric from integrating out the infinite tower:

$$\delta g_{\phi\phi} \sim \frac{1}{\phi^2}$$

- This resembles the corrections from classical backreaction, though the origin is different (quantum vs classical).

- At a general point in the moduli space, both corrections should be included.

- Regardless of which contribution dominates, we still need to address similar issues in finding the dynamical field range.

WGC for the QCD Axion

WGC for the QCD Axion

- **The QCD instanton action**

$$S_{\rm QCD} = 4 \ln M_*/\Lambda_{\rm QCD} \approx 160\,, \quad \text{where } M_* = \text{UV scale, e.g.,} M_{GUT}$$

- **The WGC implies a bound:** $f_{\rm QCD} \lesssim 10^{16}\ \text{GeV}$

- While weaker than the commonly quoted **cosmological bound**:

$$f_{\rm QCD} < 10^{12}\ \text{GeV}$$

scenarios that allow larger $f_{\rm QCD}$ have been proposed, e.g. [Wilczek, '04]

- QCD axion with decay constants above the GUT scale can be tested:
  - **laboratory searches** e.g., ABRACADABRA
  - **gravitational wave observatory** e.g., LIGO (via black hole super-radiance, [Arvanitaki, Dimopoulos, Dubovsky, Kaloper, March-Russell, '09])

Summary of Lecture 3

Summary of Lecture 3

- Inflation models with detectable GWs are sensitive to UV physics.
- The WGC and the SDC can constrain the inflation field range, and hence the amplitude of gravitational waves generated by inflation.
- The WGC for the 0-forms is more subtle, but can be argued using duality or dimensional reduction.
- The WGC rules out simple models of axion inflation, though there are loopholes involving spectator instantons.
- Axion monodromy is not ruled out by the WGC or SDC
- The WGC when applied to the **QCD axion** implies $f_{\rm QCD} \lesssim 10^{16}~{\rm GeV}$ which can be tested by lab. axion searches or GW detectors.