Sensitivities of Dark Matter direct detection experiments to effective WIMP-nucleus couplings

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Based on arXiv:1805.06113 and arXiv:1810.00607

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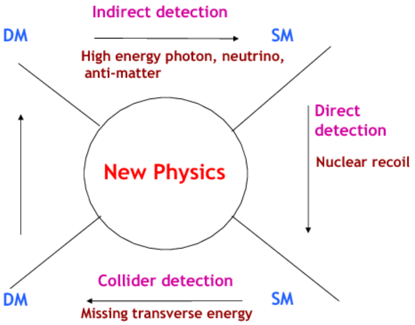
Overview

Introduction

Dark matter can be searched by many ways:



Status of Dark Matter Detection: 1707.06277

Direct Detection

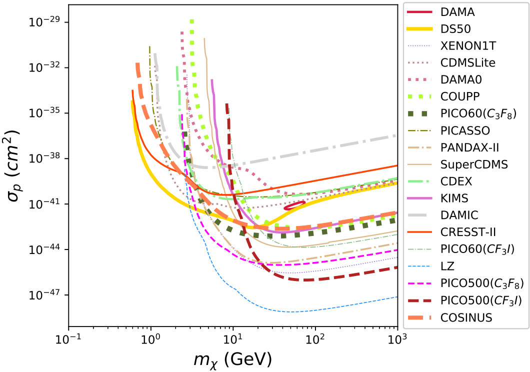
Elastic recoil of non relativistic halo WIMPs off the nuclei of an underground detector.

- Recoil energy of the nucleus lies in the keV range.
- Expected signal is very low.

- large exposures and extremely low background is required.

- Spin Independent interaction,

$$\sigma_{\chi N} \propto [c^p Z + (A - Z) c^n]^2 \,,$$



- Cross-section is enhanced for heavy nuclei (e.g. Xenon) and non-zero for all targets.
- Is it the case always?

- ◦ Isospin-violating models (1102.4331, 1205.2695),

$$\frac{c^n}{c^p} \simeq \frac{Z}{Z-A} \simeq -0.7$$

  ◦ WIMP-Xenon interaction is suppressed which reduces the sensitivity of Xenon detector.

- ◦ A Spin–Dependent WIMP–nucleon interaction,

$$\mathcal{L}_{int} \ni c^p \vec{S}_\chi \cdot \vec{S}_p + c^n \vec{S}_\chi \cdot \vec{S}_n,$$

  ◦ Only two isotopes with 47% of target number contribute reducing the sensitivity of Xenon detector.

- What about other non-standard interactions?

Non-relativistic EFT

- Hamiltonian density of WIMP-nucleus interaction,

$$\mathcal{H}(\mathbf{r}) \;=\; \sum_{j=1}^{15}(c_j^0 + c_j^1\tau_3)\mathcal{O}_j(\mathbf{r})$$

$c_j^p = (c_j^0 + c_j^1)/2$ (proton) and $c_j^n = (c_j^0 - c_j^1)/2$ (neutron)

- All operators is guaranteed to be Hermitian if built out of the following four 3-vectors,

$$i\frac{\vec{q}}{m_N},\ \vec{v}^{\perp},\ \vec{S}_\chi,\ \vec{S}_N$$

with $\vec{v}^{\perp} = \vec{v} + \vec{q}/2\mu_N \;\Rightarrow \vec{v}^{\perp}\cdot\vec{q} = 0.$

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu,
JCAP1302, 004 (2013),1203.3542.

N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

$$
\begin{aligned}
\mathcal{O}_1 &= 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp) \\
\mathcal{O}_4 &= \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp); \quad \mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N}) \\
\mathcal{O}_7 &= \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N}) \\
\mathcal{O}_{10} &= i\vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp) \\
\mathcal{O}_{13} &= i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}); \quad \mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp) \\
\mathcal{O}_{15} &= -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}).
\end{aligned}
$$

- The expected rate,

$$\frac{dR_{\chi T}}{dE_R}(t) = \sum_T N_T \frac{\rho_{\text{WIMP}}}{m_{\text{WIMP}}} \int_{v_{min}} d^3 v_T f(\vec{v}_T, t) v_T \frac{d\sigma_T}{dE_R},$$

with,

$$\frac{d\sigma_T}{dE_R} = \frac{2m_T}{4\pi v_T^2} \left[\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}_T|^2 \right],$$

$$\frac{1}{2j_\chi+1}\frac{1}{2j_T+1}|\mathcal{M}|^2 = \frac{4\pi}{2j_T+1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \sum_k \underbrace{R_k^{\tau\tau'}}_{\text{WIMP}} \left[c_j^\tau, (v_T^\perp)^2, \frac{q^2}{m_N^2} \right] \underbrace{W_{Tk}^{\tau\tau'}}_{\text{NUCLEUS}}(y)$$

$k = M,\ \Phi'',\ \Phi''M,\ \tilde{\Phi}',\ \Sigma'',\ \Sigma',\ \Delta, \Delta\Sigma'$

$y \equiv (qb/2)^2$
b=nuclear size
q=momentum transfer

- In general form,

$$R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{(v_T^\perp)^2}{c^2} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{v_T^2 - v_{min}^2}{c^2},$$

- Besides usual spin-dependent and spin-independent interactions, new contributions arise with explicit dependence on $\vec{q}$ and WIMP incoming velocity.

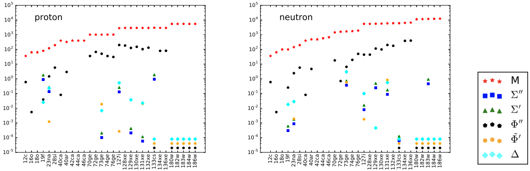
Six distinct nuclear response functions are defined as,

- M : vector-charge (**spin-independent part**, non-zero for all nuclei)
- Φ'' : vector-longitudinal, related to spin-orbit coupling $\sigma \cdot l$ (also spin-independent, non-zero for all nuclei)
- Σ', Σ'' : longitudinal and transverse components of nuclear spin, **their sum is the usual spin-dependent interaction**, require $j > 0$
- Δ : associated to orbital angular momentum operator l, requires $j > 0$
- $\tilde{\Phi}'$: related to the vector-longitudinal operator, transforms as a tensor under rotation, require $j > 1/2$

- Correspondence between WIMP and non-relativistic EFT nuclear response function,

coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

- Nuclear response functions at vanishing momentum transfer ,



- Nuclear response function W's is normalized such as,

$$\frac{16\pi}{(j_T+1)} \times W_{TM}^{p}(y=0) = Z_T^2 \quad , \frac{16\pi}{(j_T+1)} \times W_{TM}^{n}(y=0) = (A_T - Z_T)^2$$

- We assume a Maxwellian velocity distribution.
- We assume that one coupling is dominant at a time.
- In our analysis, we included 15 existing experiments:

  XENON1T, PandaX-II, KIMS, CDMSLite, SuperCDMS, COUPP, PICASSO, PICO-60 (CF_3I and C_3F_8 targets), CRESST-II, DAMA (modulation data), DAMA0 (average count rate), CDEX, DAMIC, and DarkSide-50
- We have also included projections from LZ, COSINUS, PICO500 (CF_3I and C_3F_8 targets)
- Sensitivity is expressed in terms of 90% C.L. bounds on effective cross-section,

$$\sigma_{\mathcal{N},lim} = \max(\sigma_p, \sigma_n)$$

$$\sigma_p = (c_j^p)^2 \frac{\mu_{\chi\mathcal{N}}^2}{\pi} \quad , \quad \sigma_n = (c_j^n)^2 \frac{\mu_{\chi\mathcal{N}}^2}{\pi}$$

- Categorize the couplings,

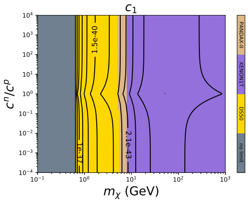
$$\begin{aligned} M &: \quad c_1,\ c_{11},\ c_5,\ c_8, \\ \Sigma'/\Sigma'' &: \quad c_4,\ c_6,\ c_9,\ c_{10},\ c_3,\ c_7,\ c_{12},\ c_{13},\ c_{14},\ c_{15} \\ \Phi'' &: \quad c_3,\ c_{12},\ c_{15} \\ \tilde{\Phi}' &: \quad c_{12},\ c_{13} \\ \Delta &: \quad c_5,\ c_8 \end{aligned}$$

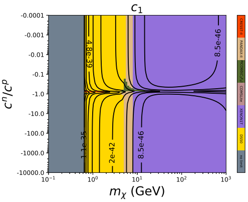
- Velocity dependent contribution is important in 5 cases,

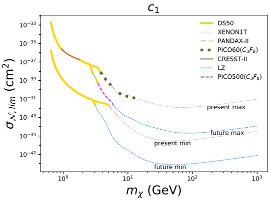
$$c_7,\ c_{14},\ c_5,\ c_8, c_{13}$$

- N.B. $W_{\Sigma'}^{\tau\tau'} \sim 2W_{\Sigma''}^{\tau\tau'}$

- Two free parameters viz. WIMP mass m_χ and $r = c^n/c^p$.
- Spin-independent coupling, no velocity dependence in the cross-section, M response function :)







- Similar results exist for $c_{11}(q^2)$.

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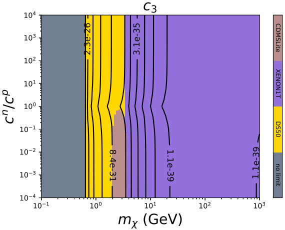
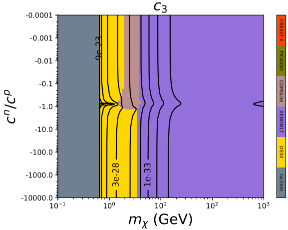
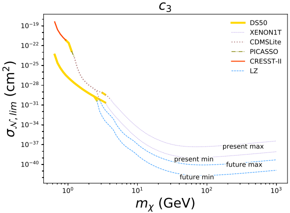
- Φ'' response function, favors heavy nuclei with partially filled orbitals :)







- Similar results exist for couplings $c_{12}(q^2)$ and $c_{15}(q^6)$

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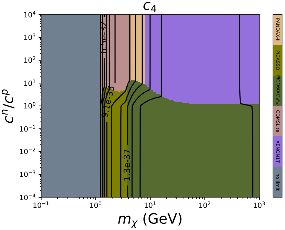
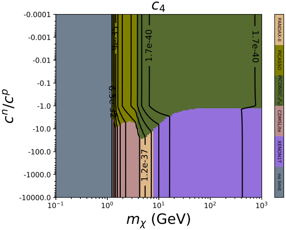
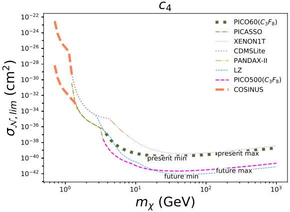
- Standard spin-dependent coupling with no velocity dependent term in the cross-section, Σ', Σ'' response functions :)



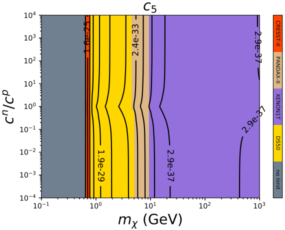


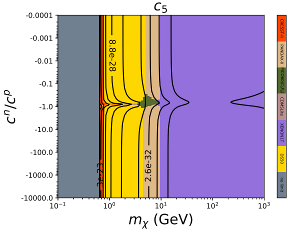


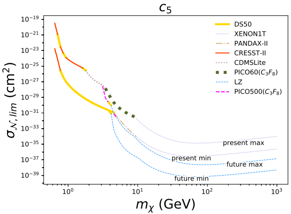
- Similar results exist for couplings $c_6(q^4)$, $c_9(q^2)$, and $c_{10}(q^2)$

Results

- $\Delta(q^4)$ and velocity dependent $M(q^2)$ response functions. :)







- Similar results exist for couplings c_8 with $\Delta(q^2)$ and M.

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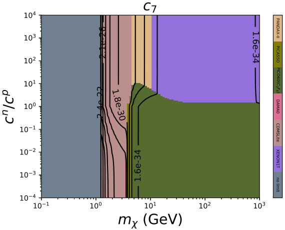
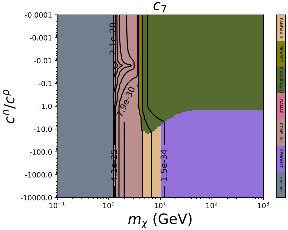
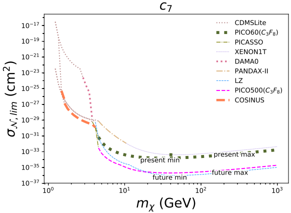
- Σ' response functions, only velocity dependent term in the cross-section. :)







- Similar results exist for couplings $c_{14}(q^2)$.

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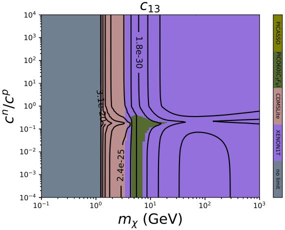
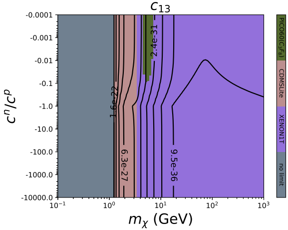
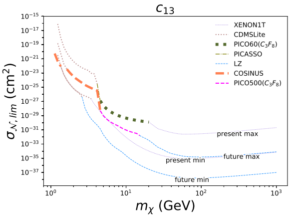
Results

- $\tilde{\Phi}'(q^4)$ and velocity dependent $\Sigma''(q^2)$ response functions, require nuclear spin $j > 1/2$, non-zero for Na^{23}, Ge^{73}, I^{121}, Xe^{131}. :)







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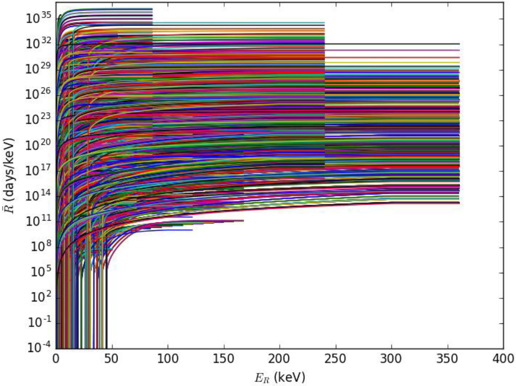
- We have calculated 75768 response functions for 19 experiments and 14 couplings.
- If include interferences then 37884 more response functions.



Relativistic EFT

- We extended our analysis to relativistic operators.
- Set of operators upto dim-7 is considered.
- Lagrangian describing DM interactions with quarks and gluons,

$$\mathcal{L}_\chi = \sum_q \sum_{a,d} \mathcal{C}_{a,q}^{(d)} \mathcal{Q}_{a,q}^{(d)} + \sum_{b,d} \mathcal{C}_b^{(d)} \mathcal{Q}_b^{(d)},$$

  $\mathcal{C}_{a,q}^{(d)}, \mathcal{C}_b^{(d)} \equiv \frac{1}{\tilde{\Lambda}^{d-4}}$ are dimensional Wilson coefficients, fixed at EW scale.
- Once NR Wilson coefficients c_j^τ is obtained from $\mathcal{C}_{a,q}^{(d)}, \ \mathcal{C}_b^{(d)}$ our previous analysis holds.

Operators considered

- The dimension-five operators:

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}i\gamma_5\chi)F_{\mu\nu}$$

- The dimension-six operators are:

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu q)\,,$$
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q)\,,$$

- The dimension-seven operators are:

$$\mathcal{Q}_1^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}\chi)G^{a\mu\nu}G^a_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}G^a_{\mu\nu}\,,$$
$$\mathcal{Q}_3^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}\chi)G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}\,, \qquad \mathcal{Q}_4^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}\,,$$
$$\mathcal{Q}_{5,q}^{(7)} = m_q(\bar{\chi}\chi)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(7)} = m_q(\bar{\chi}i\gamma_5\chi)(\bar{q}q)\,,$$
$$\mathcal{Q}_{7,q}^{(7)} = m_q(\bar{\chi}\chi)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(7)} = m_q(\bar{\chi}i\gamma_5\chi)(\bar{q}i\gamma_5 q)\,,$$
$$\mathcal{Q}_{9,q}^{(7)} = m_q(\bar{\chi}\sigma^{\mu\nu}\chi)(\bar{q}\sigma_{\mu\nu}q)\,, \qquad \mathcal{Q}_{10,q}^{(7)} = m_q(\bar{\chi}i\sigma^{\mu\nu}\gamma_5\chi)(\bar{q}\sigma_{\mu\nu}q)$$

Assumptions:

1-We assume single coupling, $\mathcal{C}^{(d)}_{a,q} = 1$ to all quarks and gluon $\mathcal{C}^{(d)}_{b} = 1$.
2-Single NR coupling is dominant at a time.

- A sizable mixing between vector and axial-vector currents is very important, :)

$$\mathcal{Q}^{(6)}_{1,q} \to F_1^{q/N}\mathcal{O}_1^N\,,\ \mathcal{Q}^{(6)}_{2,q} \to 2F_1^{q/N}\mathcal{O}_8^N + 2(F_1^{q/N} + F_2^{q/N})\mathcal{O}_9^N\,,$$
$$\mathcal{Q}^{(6)}_{3,q} \to -2F_A^{q/N}\mathcal{O}_7^N - \frac{m_N}{m_\chi}\mathcal{O}_9^N\,,\ \mathcal{Q}^{(6)}_{4,q} \to -4F_A^{q/N}\mathcal{O}_4^N + F_{p'}^{q/N}\mathcal{O}_6^N$$

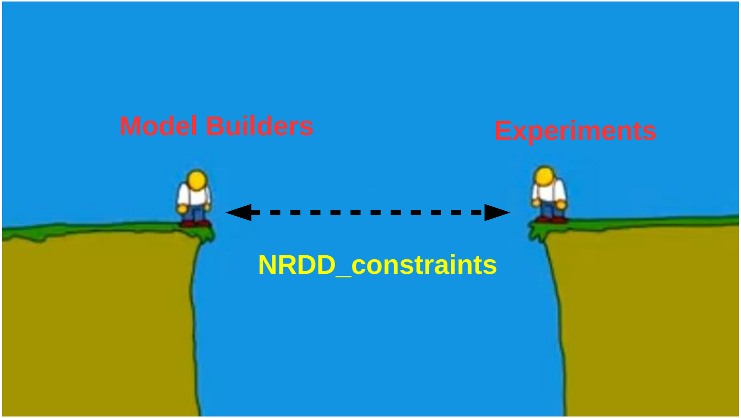
- Specifically the pion form factors are: :)

$$F_{P,P'}^{q/N}(q^2) = \frac{m_N^2}{m_\pi^2 - q^2} a_\pi^{q/N} + \frac{m_N^2}{m_\eta^2 - q^2} a_\eta^{q/N} + b^{q/N},$$

$$F_{\tilde{G}}^{N}(q^2) = \frac{q^2}{m_\pi^2 - q^2} a_{\tilde{G},\pi}^{N} + \frac{q^2}{m_\eta^2 - q^2} a_{\tilde{G},\eta}^{N} + b_{\tilde{G}}^{N}.$$

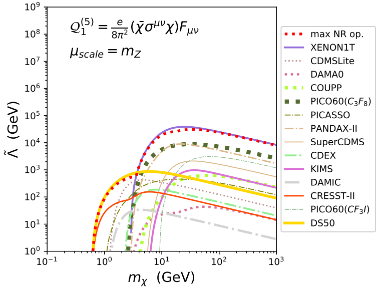
- We study the mixing effect by fixing the coupling at scale $\tilde{\Lambda} = 2$ TeV and run it to EW scale through runDM (1605.04917).
- Third family of quarks gives the dominant contribution.
- We further run the coupling to nucleon scale using directDM (1708.02678) and obtain $c_j^{\mathcal{T}}$.

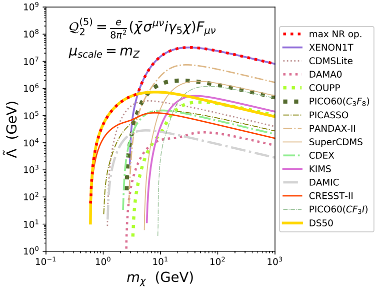
Bridging the gap!



- Lower bound on the effective scale $\tilde{\Lambda} = (c\mu_{\chi\mathcal{N}}/\sqrt{\sigma_{NR}\pi})^{1/d-4}$, $\mu_{scale} = m_Z$, :)

$$\mathcal{Q}_1^{(5)} \to -\frac{\alpha}{2\pi} F_1^N \left(\frac{1}{m_\chi} \mathcal{O}_1^N - 4\frac{m_N}{\vec{q}^{\,2}} \mathcal{O}_5^N \right) - \frac{2\alpha}{\pi}\frac{\mu_N}{m_N}\left(\mathcal{O}_4^N - \frac{m_N^2}{\vec{q}^{\,2}} \mathcal{O}_6^N \right),$$





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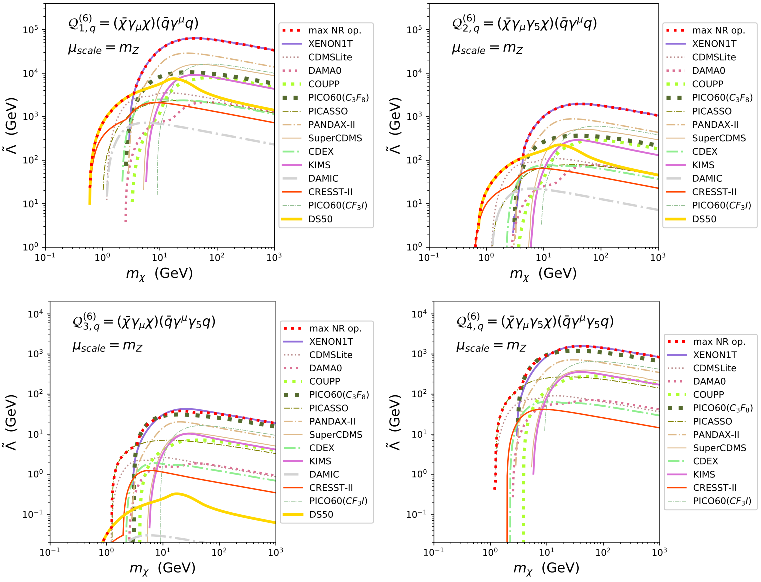
Results



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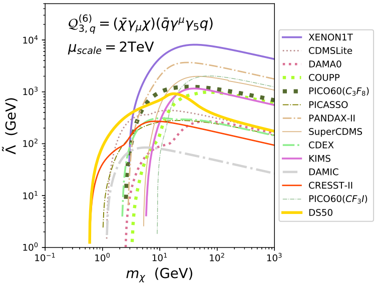
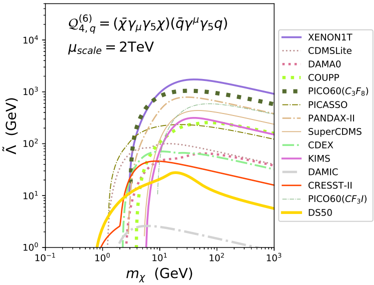
- Lower bound on the effective scale $\tilde{\Lambda}$, $\mu_{scale} = 2\,TeV$, :)





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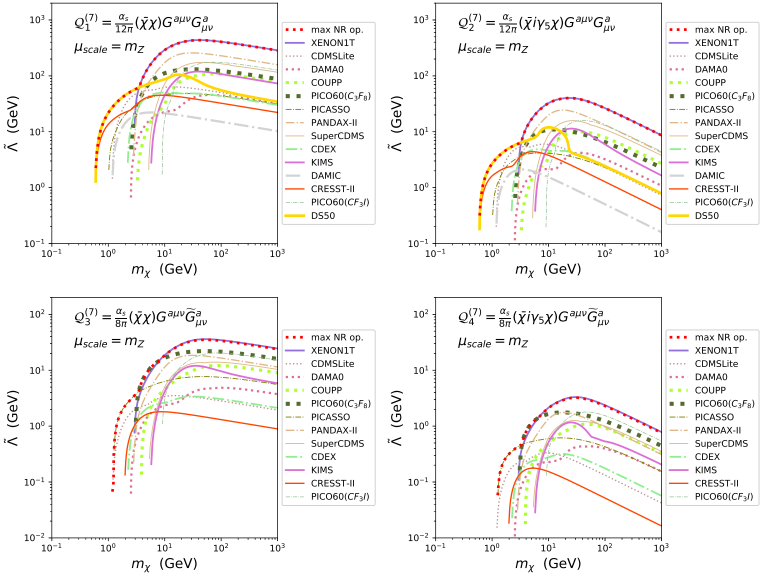
Results: dimension 7 operators



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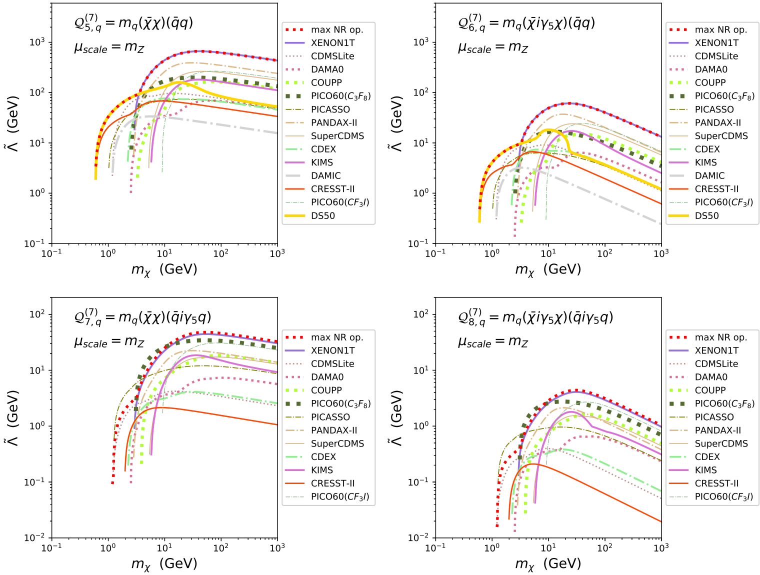
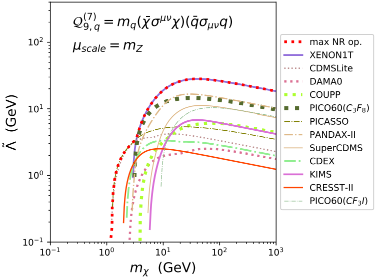
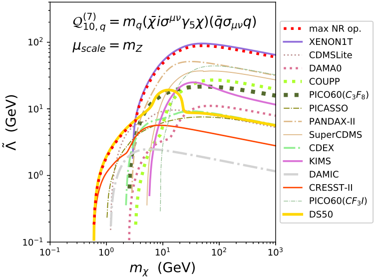


Sunghyun Kang, S. Scopel, G. Tomar, J.H. Yoon, arXiv: 1810.00607

Results: dimension 7 operators

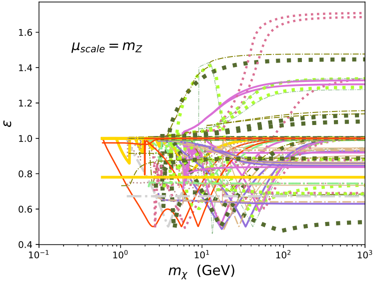




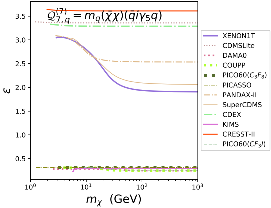
Sunghyun Kang, S. Scopel, G. Tomar, J.H. Yoon, arXiv: 1810.00607

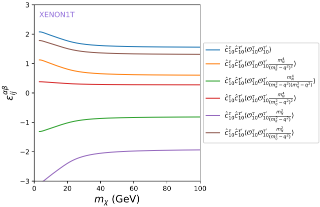
- With the exception of the operators $\mathcal{Q}_{7,q}^{(7)}$ and $\mathcal{Q}_{8,q}^{(7)}$, the single coupling assumption works very well.
- The interference parameter,

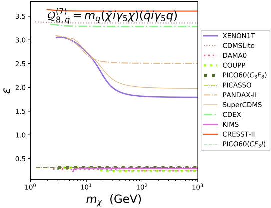
$$\epsilon_{ij}^{\alpha\beta} = \frac{\hat{c}_{i,\alpha}^{\tau}\hat{c}_{j,\beta}^{\tau'}\langle \mathcal{O}_i^{\tau}\mathcal{O}_j^{\tau'} F_i^{\alpha}(q^2)F_j^{\beta}(q^2)\rangle}{\sum_{lm}\sum_{\rho\sigma}\hat{c}_{l,\rho}^{\tau}\hat{c}_{m,\sigma}^{\tau'}\langle \mathcal{O}_l^{\tau}\mathcal{O}_m^{\tau'} F_l^{\rho}(q^2)F_m^{\sigma}(q^2)\rangle}, \quad \epsilon \equiv \max(|\epsilon_{ij}^{\alpha\beta}|),$$

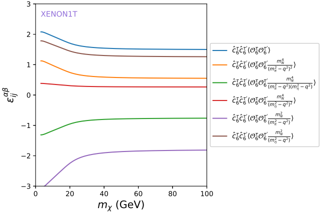


- The interference parameter for operators $\mathcal{Q}_{7,q}^{(7)}$ and $\mathcal{Q}_{8,q}^{(7)}$, :)









All results are based on our Direct Detection code.

- Object-oriented, based on Python.
- Flexible to easily implement any new experiment and/or update new information. Efficient to calculate and handle a large number of response functions.
- Valid for any velocity distribution of WIMPs.

- Development with rigorous testing is in progress. Plan to eventually make it publicly available.
- A python routine named **NRDD-constraints** has been released by our group.
  https://github.com/NRDD-constraints/NRDD

Summary

Summary

- Expected cross-section $\sigma_{\mathcal{N},lim}$ varies many orders of magnitude depending on effective couplings.
- In most cases, it is driven by,
  - **Xenon target:** C_1, C_3, C_5, C_6, C_8, C_{11}, C_{12}, C_{13}, and C_{15}
  - **Fluorine target:** C_4, C_7, C_9, C_{10}, C_{14}
- Out of 15 considered experiments, there are 9 experiments which provide the most stringent bounds on effective couplings:

  XENON1T, PandaX-II, CDMSLite, PICASSO, PICO-60, CRESST-II, DAMA0, DarkSide-50
- It is due to the complementarity between different targets, combinations of count rate and energy thresholds.
- For all the couplings the future experiments could improve the limits by two to three order of magnitudes.

Summary

- We extended our analysis to the relativistic operators.
- There are two cases,
  - The operators $\mathcal{Q}_{1,q}^{(5)}$, $\mathcal{Q}_{2,q}^{(5)}$, $\mathcal{Q}_{1,q}^{(6)}$, $\mathcal{Q}_{2,q}^{(6)}$, $\mathcal{Q}_{1}^{(7)}$, $\mathcal{Q}_{2}^{(7)}$, $\mathcal{Q}_{5,q}^{(7)}$, $\mathcal{Q}_{6,q}^{(7)}$ and $\mathcal{Q}_{10,q}^{(7)}$ follow the SI scaling of cross section and constrained by DS50 and Xenon1T.
  - The operators $\mathcal{Q}_{3,q}^{(6)}$, $\mathcal{Q}_{4,q}^{(6)}$, $\mathcal{Q}_{3}^{(7)}$, $\mathcal{Q}_{4}^{(7)}$, $\mathcal{Q}_{7,q}^{(7)}$, $\mathcal{Q}_{8,q}^{(7)}$, and $\mathcal{Q}_{9,q}^{(7)}$ follow the SD scaling of cross section and constrained by PICO60, PICASSO, and Xenon1T.
- In all models with exception of $\mathcal{Q}_{7,q}^{(7)}$ and $\mathcal{Q}_{8,q}^{(7)}$ the expected rate is driven by one of the NR operator with an accuracy of 60%.
- We released a Python tool called **NRDD-constraints** using which it is very easy to test any relativistic model against DM direct detection.

Namaste!

- Connection to relativistic effective theory: 1203.3542

j	$\mathcal{L}^j_{\text{int}}$	Nonrelativistic reduction	$\sum_i c_i \mathcal{O}_i$	P/T
1	$\bar\chi\chi\bar N N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
2	$i\bar\chi\chi\bar N\gamma^5 N$	$i\frac{\vec q}{m_N}\cdot\vec S_N$	$\mathcal{O}_{10}$	O/O
3	$i\bar\chi\gamma^5\chi\bar N N$	$-i\frac{\vec q}{m_\chi}\cdot\vec S_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$	O/O
4	$\bar\chi\gamma^5\chi\bar N\gamma^5 N$	$-\frac{\vec q}{m_\chi}\cdot\vec S_\chi\frac{\vec q}{m_N}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$	E/E
5	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
6	$\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q^{\,2}}{2m_N m_M}1_\chi 1_N + 2(\frac{\vec q}{m_\chi}\times\vec S_\chi + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$\frac{\vec q^{\,2}}{2m_N m_M}\mathcal{O}_1 - 2\frac{m_N}{m_M}\mathcal{O}_3$ $+2\frac{m_N^2}{m_M m_\chi}\left(\frac{q^2}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6\right)$	E/E
7	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu\gamma^5 N$	$-2\vec S_N\cdot\vec v^\perp + \frac{2}{m_\chi}i\vec S_\chi\cdot(\vec S_N\times\vec q)$	$-2\mathcal{O}_7 + 2\frac{m_N}{m_\chi}\mathcal{O}_9$	O/E
8	$i\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec q}{m_M}\cdot\vec S_N$	$2\frac{m_N}{m_M}\mathcal{O}_{10}$	O/O
9	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma_\mu N$	$-\frac{\vec q^{\,2}}{2m_\chi m_M}1_\chi 1_N - 2(\frac{\vec q}{m_N}\times\vec S_N + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)$	$-\frac{\vec q^{\,2}}{2m_\chi m_M}\mathcal{O}_1 + \frac{2m_N}{m_M}\mathcal{O}_5$ $-2\frac{m_N}{m_M}\left(\frac{\vec q^{\,2}}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6\right)$	E/E
10	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$4(\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_4 - \frac{m_N^2}{m_M^2}\mathcal{O}_6)$	E/E
11	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma^\mu\gamma^5 N$	$4i(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
12	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-[i\frac{\vec q^{\,2}}{m_\chi m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)]\frac{\vec q}{m_M}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_{10} - 4\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_{12} - 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
13	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma_\mu N$	$2\vec v^\perp\cdot\vec S_\chi + 2i\vec S_\chi\cdot(\vec S_N\times\frac{\vec q}{m_N})$	$2\mathcal{O}_8 + 2\mathcal{O}_9$	O/E
14	$\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4i\vec S_\chi\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
15	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma^\mu\gamma^5 N$	$-4\vec S_\chi\cdot\vec S_N$	$-4\mathcal{O}_4$	E/E
16	$i\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i\vec v^\perp\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_{13}$	E/O
17	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu N$	$2i\frac{\vec q}{m_M}\cdot\vec S_\chi$	$2\frac{m_N}{m_M}\mathcal{O}_{11}$	O/O
18	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q}{m_M}\cdot\vec S_\chi[i\frac{\vec q^{\,2}}{m_N m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_N)]$	$\frac{\vec q^{\,2}}{m_M^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
19	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu\gamma^5 N$	$-4i\frac{\vec q}{m_M}\cdot\vec S_\chi\vec v_\perp\cdot\vec S_N$	$-4\frac{m_N}{m_M}\mathcal{O}_{14}$	E/O
20	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4\frac{\vec q}{m_M}\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N^2}{m_M^2}\mathcal{O}_6$	E/E

- WIMPs response funtions: 1203.3542

$$
\begin{aligned}
R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right] \\
R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2} \\
R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2} \\
R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2} \\
R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\left. \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right] \\
R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\left. \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right] \\
R_{\Delta}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2} \\
R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}.
\end{aligned}
$$

- Maxwellian velocity distribution:

$$f(\vec{v}_T, t) = N\left(\frac{3}{2\pi v_{rms}^2}\right)^{3/2} e^{-\frac{3|\vec{v}_T+\vec{v}_E|^2}{2v_{rms}^2}}\Theta(u_{esc} - |\vec{v}_T + \vec{v}_E(t)|)$$

$$N = \left[erf(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2}\right]^{-1},$$

with $z = 3u_{esc}^2/(2v_{rms}^2)$. In the isothermal sphere model hydrothermal equilibrium between the WIMP gas pressure and gravity is assumed, leading to $v_{rms}=\sqrt{3/2}v_0$ with v_0 the galactic rotational velocity.

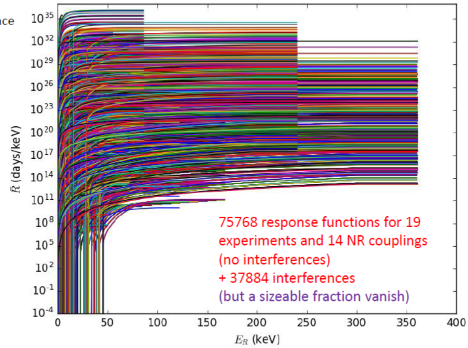
Back Up

Tabulate full calculation of R response function for each:

1) Experiment
2) Energy bin/energy threshold/energy value
3) Isospin value (c_n/c_p=-1,0,1)
4) Nuclear target (including all stable isotopes)
5) Effective coupling
6) 4 terms including explicit velocity dependence

Isospin rotation with $r=c^n/c^p$:

$$R(r) = \frac{r(r+1)}{2}R(r=1) + (1-r^2)R(r=0) + \frac{r(r-1)}{2}R(r=-1)$$



Back Up

The rate can be written as

$$R = \sum_{k=1}^{N} \delta\tilde{\eta}^k \times$$

$$\left\{ \bar{\mathcal{R}}_0 \left[E_R^{max}(v_k)\right] + (v_k^2 - \frac{\delta}{\mu_{\chi N}})\bar{\mathcal{R}}_1 \left[E_R^{max}(v_k)\right] \right.$$

$$\left. - \frac{m_N}{2\mu_{\chi N}^2}\bar{\mathcal{R}}_{1E} \left[E_R^{max}(v_k)\right] - \frac{\delta^2}{2m_N}\bar{\mathcal{R}}_{1E^{-1}} \left[E_R^{max}(v_k)\right] \right\}$$

In terms of four response functions that do not depend on the WIMP mass or mass splitting:

$$\bar{\mathcal{R}}_{0,1}(E_R) \equiv \int_0^{E_R} dE'_R \mathcal{R}_{0,1}(E'_R)$$

$$\bar{\mathcal{R}}_{1E}(E_R) \equiv \int_0^{E_R} dE'_R E'_R \mathcal{R}_1(E'_R)$$

$$\bar{\mathcal{R}}_{1E^{-1}}(E_R) \equiv \int_0^{E_R} dE'_R \frac{1}{E'_R} \mathcal{R}_1(E'_R)$$

that can be tabulated for later use.