2019 q-DAY MEETING: TITLES AND ABSTRACTS

$(n, n+1, \ldots, n+p)$-core partitions and Motzkin paths

Hyunsoo Cho (Yonsei University)

In this talk, we are concerned with $(n, n+1, \ldots, n+p)$-core partitions. Amderberhan conjectured that the number of $(n, n+1, n+2)$-core partitions is same as the nth Motzkin number. This conjecture was proved by several authors. We give an alternative proof of this result by establishing a bijection between the set of $(n, n+1, n+2)$-core partitions and that of Motzkin paths of length n. Similarly, we give a bijection between the set of $(n, n+1, \ldots, n+p)$-core partitions and that of Motzkin paths of length n with some restrictions. Also, we show the recurrence relation for the number of $(n, n+1, \ldots, n+p)$-core partitions with certain restrictions. Moreover, we enumerate the number of $(n, n+1, \ldots, n+p)$-core partitions with a specified number of corners.

A combinatorial approach to partition identities from representation theory

Jehanne Dousse (CNRS, Lyon)

Rogers-Ramanujan type identities are theorems stating that for all n, the number of partitions of n satisfying some difference conditions equals the number of partitions of n satisfying some congruence conditions. Lepowsky and Wilson were the first to exhibit a connection between Rogers-Ramanujan type partition identities and representation theory in the 1980's, followed by Capparelli, Primc, and others. This gave rise to many interesting new identities previously unknown to combinatorialists. In this talk, I will present a combinatorial approach to refine and generalise such partition identities.

Identities for overpartitions with even smallest parts

Min-Joo Jang (The University of Hong Kong)

We prove several combinatorial identities involving overpartitions whose smallest parts are even. These follow from an infinite product generating function for certain four-colored overpartitions. This is joint work with Jeremy Lovejoy.

Inequalities on Ranks and Cranks of Partitions

Kathy Q. Ji (Tianjin University)

Dyson's rank and the Andrews-Dyson-Garvan crank are two fundamental statistics in the theory of partitions. They are proved to give combinatorial explanations for Ramanujan's famous congruences of the partition function $p(n)$, where $p(n)$ counts the number of partitions of n. In this talk, I wish to report some recent work on the inequalities on ranks and cranks of partitions which includes the following three parts:

Let $N(\leq m,n)$ denote the number of partitions of n with rank not greater than m and let $M(\leq m,n)$ denote the number of partitions of n with crank not greater than m. Bringmann and Mahlburg observed that $N(\leq m,n) \leq M(\leq m,n) \leq N(\leq m+1,n)$ for $m<0$ and $1 \leq n \leq 100$ and conjectured that these two inequalities may also be restated in terms of ordered lists of partitions.

Andrews, Dyson, and Rhoades showed that the conjectured inequality $N(\leq m,n) \leq M(\leq m,n)$ of Bringmann and Mahlburg is equivalent to their conjecture on the unimodal of spt-crank. We have proved the conjecture of Andrews, Dyson, and Rhoades by a purely combinatorial argument. Recently, we also proved that the inequality $M(\leq m,n) \leq N(\leq m+1,n)$ holds for $m<0$ and $n \geq 1$. Based on these two inequalities, we are led to a bijection τ_n between the set of partitions of n and the set of partitions of n such that $|crank(\lambda)| - |rank(\tau_n(\lambda))| = 0,$ or 1. We then use this bijection to show that $spt(n) \leq \sqrt{2n}p(n)$, where $spt(n)$ counts the total number of smallest parts in all partitions of n.

Let $M(m,n)$ denote the number of partitions of n with the Andrews-Garvan-Dyson crank m, we show that $M(m,n) \geq M(m,n-1)$ for $n \geq 14$ and $0 \leq m \leq n-2$ and $M(m,n) \geq M(m+1,n)$ for $n \geq 44$ and $0 \leq m \leq n-2$. By means of the symmetry $M(m,n) = M(-m,n)$, we find that $M(m,n) \geq M(m+1,n)$ for $n \geq 44$ and $0 \leq m \leq n-2$ implies that the sequence $\{M(m,n)\}_{|m| \leq n-1}$ is unimodal for $n \geq 44$. We also give a proof of an upper bound for $ospt(n)$ conjectured by Chan and Mao in light of the inequality $M(m,n) \geq M(m+1,n)$ for $n \geq 44$ and $0 \leq m \leq n-2$. This is a joint work with William Y. C. Chen and Wenston J. T. Zang

Partial theta series and dissections of q-series

Byungchan Kim (SeoulTech)

In a study of congruences for the Fishburn numbers, Andrews and Sellers observed empirically that certain polynomials appearing in the dissections of the partial sums of the Kontsevich-Zagier series

$$\sum_{n\geq 0} (q;q)_n$$

are divisible by a certain q-factorial. In this talk, we discuss the proof of Andrews-Sellers conjecture and extension of this strong divisibility property to two generic families of q-hypergeometric series

which, like the Kontsevich-Zagier series, agree asymptotically with partial theta functions. This is a joint work with S. Ahlgren and J. Lovejoy.

Lecture hall tableaux

Jang Soo Kim (Sungkyunkwan University)

We introduce lecture hall tableaux, which are fillings of a skew Young diagram satisfying certain conditions. Lecture hall tableaux generalize both lecture hall partitions and anti-lecture hall compositions, and also contain reverse semistandard Young tableaux as a limit case. We show that the coefficients in the Schur expansion of multivariate little q-Jacobi polynomials are generating functions for lecture hall tableaux. Using a Selberg-type integral we show that the moment of multivariate little q-Jacobi polynomials, which is equal to a generating function for lecture hall tableaux of a Young diagram, has a product formula. We also explore various combinatorial properties of lecture hall tableaux. This is joint work with Sylvie Corteel.