Feynman Rules of massive gauge theory in physical gauges

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coming out soon

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Motivations:

1. power counting — longitudinal

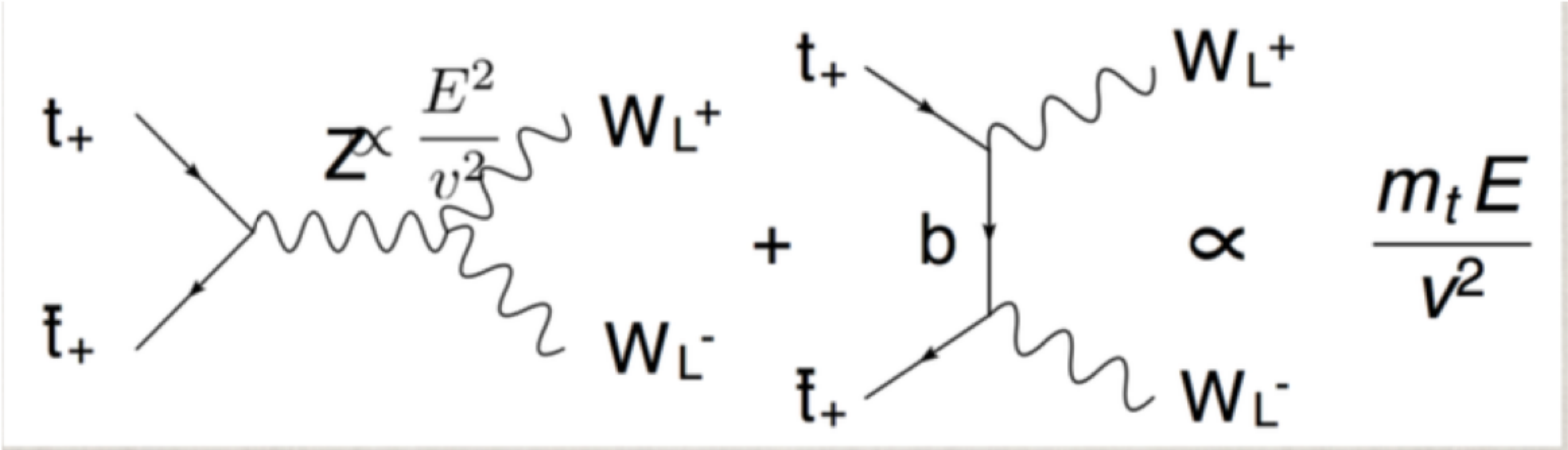
2. goldstone equivalence

3. (longitudinal) vector = goldstone + gauge

4. Better understanding Feynman rules from it

Motivations:

Longitudinal vector boson $\epsilon_L \sim \frac{k^\mu}{m_W}$, bad energy behavior and huge interference!



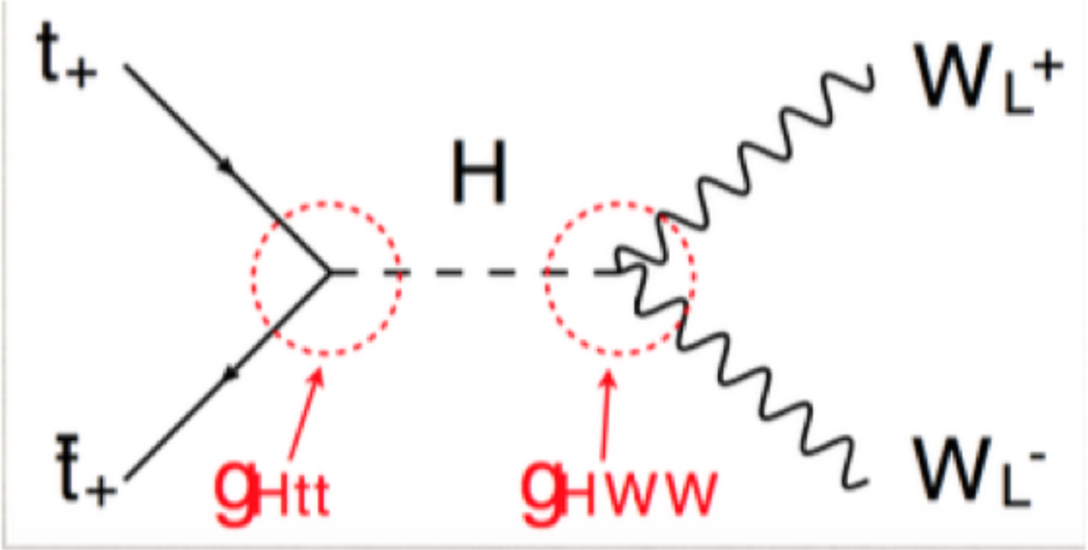


Figure: higgs contribution $\propto \frac{m_t m_h}{v^2}$ fixes the problem.

MOTIVATION2: on-shell scattering amplitudes

MHV diagrams, Park-Tayler formula

$$\mathcal{A}(1^+ \cdots i^- \cdots j^- \cdots n^+) = i(-g)^{n-2} \frac{\langle i\, j\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle \cdots \langle (n-1)\, n\rangle\langle n\, 1\rangle}$$

1. Huge simplifications in calculations

2. Underlying structures uncovered.

Literature

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11042050, Nathaniel Craig[a,b], Henriette Elvang[c], Michael Kiermaier[d] and Tracy Slatyer[a]

11042280, Rutger H. Boels, Christian Schwinn

150606134, Stephen G. Naculich

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on-shell approach for massive

1. "Miracle" don't extend to massive case

2. Little group

massless particles : U(1)

massive particles : SU(2)

3. Jump of Degrees of freedom

4. It might be beneficial to understand the theory in QFT formulation better

Our goals:

1. Rederive the Feynman rules by treating gauge field W and goldstone field

2. Understand the possible underlying structure (not noticed before)

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Gauge Choice:

Covariant gauge v.s. Physical gauges

$$\mathcal{L}_\xi = -\frac{1}{2\xi}(\partial^\mu A^a_\mu)^2$$

$$\mathcal{L}_\xi = -\frac{1}{2\xi}(n \cdot A^a)^2$$

Covariant gauge:

1. Manifest Lorentz symmetry
2. Gauge redundancies
3. Ghosts — unitarity is not straightforward

Physical gauges:

1. No manifest Lorentz symmetry
2. Only physical degrees of freedom remain
3. Ghosts decouple

Physical gauges (axial gauge)

Propagator:

$$< A_a^\mu A_b^\nu > = \frac{-i\delta_{ab}(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k} + \frac{n^2}{(n\cdot k)^2} k^\mu k^\nu)}{k^2 + i\epsilon}$$

$n^2 < 0,$ axial gauge

$n^2 = 0$ light-cone gauge

$n^2 > 0,$ temporal gauge

Physical gauges (axial gauge)

Propagator:

$$< A_a^\mu A_b^\nu > = \frac{-i\delta_{ab}(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k} + \frac{n^2}{(n\cdot k)^2} k^\mu k^\nu)}{k^2 + i\epsilon}$$

$$k^2 k_\mu < A^\mu A^\nu > = ik^2 (\frac{n^\nu}{n \cdot k})$$

goes to 0 when $k^2 \to 0$,
only transverse (physical)
polarizations remain

Impose on massive gauge theory

$$\mathcal{L}_{W_a^2} = -\frac{1}{2}\partial^\mu W_a^\nu \partial_\mu W_{a\nu} + \frac{1}{2}\partial^\mu W_{a\mu}\partial^\nu W_{a\nu} + \frac{1}{2}m_W^2 W_{a\mu}W^{a\mu} + \frac{1}{2\xi}(n\cdot\partial\, n\cdot W_a)(n\cdot\partial\, n\cdot W_a)^*$$

$$\mathcal{L}_{\phi_a W^a} = -m_W W^{a\mu}\partial_\mu\phi_a$$

$$\mathcal{L}_{\phi_a^2} = \frac{1}{2}(\partial^\mu\phi_a)^2$$

Not easy to work out the propagator.

Solution: treat gauge fields and goldstone fields as a whole.

Impose on massive gauge theory

$$n^M = (n^\mu, 0),\ W_M^a = (W_\mu^a, \phi^a),\ \partial^M = (\partial^\mu, -m_W),\ \eta^{MN} = \mathrm{diag}(1,-1,-1,-1,-1).$$

Kinetic Lagrangian:

$$\mathcal{L}_{W_M^2} = -\frac{1}{2}\partial_M W_N^a \partial^M W_a^N + \frac{1}{2}(\partial_M W_a^M)^2 + \frac{1}{2\xi}(n\cdot\partial\, n_M W_a^M)(n\cdot\partial\, n_M W_a^M)^*$$

$\partial^M = (\partial^\mu, -m_W)$ gives $k^M = (k^\mu, -im_W)$ for incoming momentum

$k^{M*} = (k^\mu, im_W)$ for outgoing momentum

$k\cdot k^* = \eta^{MN} k_M k_N^* = k^2 - m_W^2$ equals to 0 when on shell

$$\mathcal{L}_{W_M^2} = -\frac{1}{2}\partial_M W_N^a \partial^M W_a^N + \frac{1}{2}(\partial_M W_a^M)^2 + \frac{1}{2\xi}(n\cdot\partial\, n_M W_a^M)(n\cdot\partial\, n_M W_a^M)^*$$

$$n^M = (n^\mu, 0),\ W_M^a = (W_\mu^a, \phi^a),\ \partial^M = (\partial^\mu, -m_W),\ \eta^{MN} = \mathrm{diag}(1,-1,-1,-1,-1).$$

Algebra is completely analogue to the massless case.

massless:

$$< A_a^\mu A_b^\nu > = \frac{-i\delta_{ab}(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k} + \xi\frac{k^2}{(n\cdot k)^4}k^\mu k^\nu)}{k^2 + i\epsilon}$$

massive:

$$< W_a^M W_b^N > = \frac{-i\delta_{ab}(g^{MN} - \frac{n^M k^{*N} + k^M n^{*N}}{n\cdot k} + \xi\frac{k\cdot k^*}{(n\cdot k)^4}k^M k^{*N})}{k\cdot k^* + i\epsilon}$$

Propagator: write gauge components and goldstone components separately

$$< (W_a^\mu, \phi_a), (W_b^\nu, \phi_b) >= \frac{i\delta_{ab}}{k^2 - m_W^2 + i\epsilon} \begin{pmatrix} -(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n\cdot k}) & i\frac{m_W}{n\cdot k} n^\mu \\ -i\frac{m_W}{n\cdot k} n^\nu & 1 \end{pmatrix}$$

same mass for gauge and goldstone modes

when on-shell

$$< W_a^M W_b^{*N} >= \frac{i\delta_{ab}\sum_{s=\pm,L} \epsilon_s^M \epsilon_s^{N*}}{k\cdot k^* + i\epsilon}$$

with

longitudinal pol. is interpolated by gauge fields and goldstone fiel

$$\epsilon_\pm^{M(*)} = (\epsilon_\pm^{\mu(*)}, 0)$$

$$\epsilon_L^M = (-\frac{m_W}{n\cdot k} n^\mu, i)$$

$$\epsilon_L^{*M} = (-\frac{m_W}{n\cdot k} n^\mu, -i)$$

Five-component treatment for vertices

Bottom line: Higgs multiplet is a representation of the gauge group

Question: Arrange interactions with goldstone modes in a way similar to gauge ?

Good if there is custodial symmetry with all the goldstone bosons.

SU(2) GAUGE THEORY: CUSTODIAL SYMMETRY

$$\mathcal{H} = \frac{1}{\sqrt{2}}(i\sigma_2\Phi^*, \Phi) = \frac{1}{2}(h - i\sigma^a\phi_a)$$

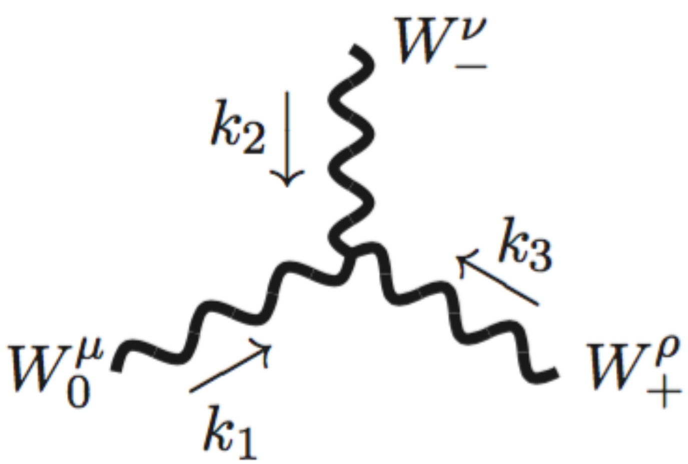
$$D_\mu\mathcal{H} = (\partial_\mu + igW_{a\mu}\frac{\sigma^a}{2})(h\frac{\mathbf{1}}{2} - i\frac{\sigma^b}{2}\phi_b)$$
$$= (\partial_\mu + igW_{a\mu}\frac{\sigma^a}{2}) \cdot h\frac{\mathbf{1}}{2} - \frac{\sigma^a}{2}(\partial_\mu\delta^{ac} - \frac{g}{2}\epsilon^{abc}W^b_\mu)\phi^c + \frac{1}{4}gW^a_\mu\phi_a\mathbf{1}$$

Resulting in, for example the Lagrangian term

$$\mathcal{L}_{W^3_M} = g\epsilon^{abc}\partial_\mu W_N W^{\mu b}W^c_K\eta'^{NK}$$

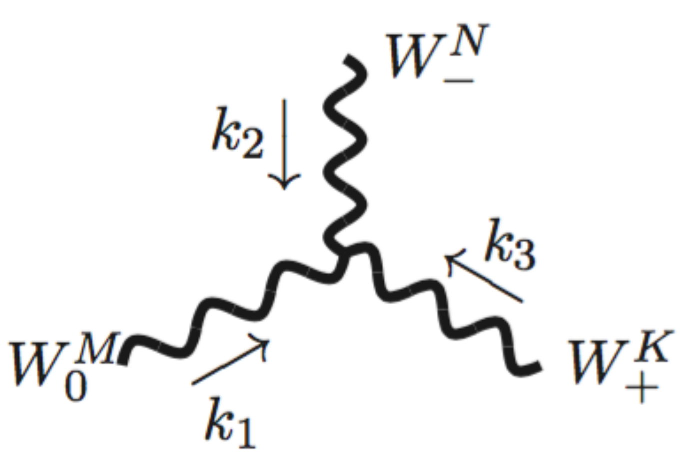
Higgs-Kibble model with SU(2) gauge group

Triple gauge vertices for massless theory



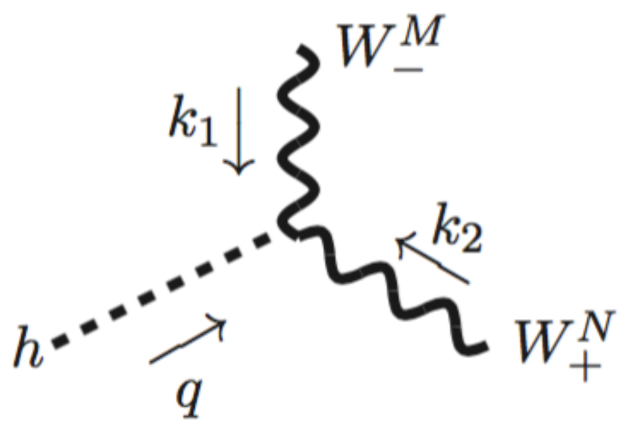
$$= \quad -ig(\eta^{\mu\nu}(k_1 - k_2)^\rho + \eta^{\nu\rho}(k_2 - k_3)^\mu + \eta^{\rho\mu}(k_3 - k_1)^\nu)$$

Triple gauge-goldstone vertices for massive theory



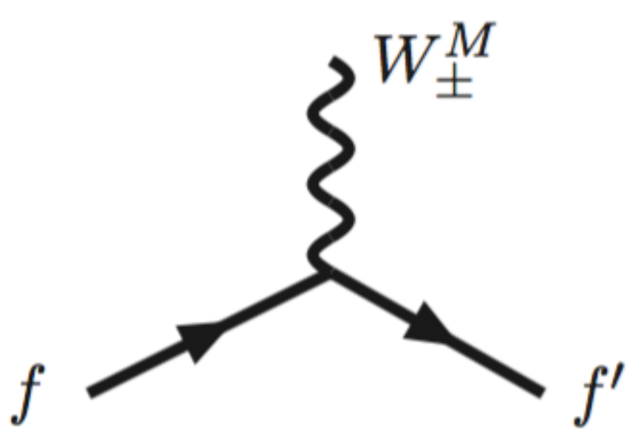
$$= \quad -ig(\eta'^{MN}(k_1 - k_2)^\rho + \eta'^{NK}(k_2 - k_3)^\mu + \eta'^{KM}(k_3 - k_1)^\nu)$$

hWW



$$= \; -\frac{g}{2}\left((k_1-q)^\mu g^{N4} + g^{M4}(k_2-q)^\nu\right) + i g m_W g^{\mu\nu} - i\frac{\lambda_h v}{2} g^{M4} g^{N4}$$

ffW



$$= \; -i\frac{g}{\sqrt{2}}\gamma^\mu P_L - (y_f P_R + y_{f'} P_L) g^{M4}$$

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Onshel gauge symmetry

two forms of longitudinal polarization

$$\epsilon_{1L}^{M} = \begin{pmatrix} -\epsilon_n^{\mu} \\ i \end{pmatrix}$$

$$\epsilon_{2L}^{M} = \begin{pmatrix} -\epsilon_n^{\mu} + \frac{k^{\mu}}{m_W} \\ 0 \end{pmatrix} = \epsilon_{1L}^{M} - \frac{k^M}{m_W}$$

—>two forms gives the same results

$$\epsilon_{2L}^{M_1} ... \epsilon_{2L}^{M_i} S_{M_1 ... M_i}(k_1 ... k_i ...) = \epsilon_{1L}^{M_1} ... \epsilon_{1L}^{M_i} S_{M_1 ... M_i}(k_1 ... k_i ...)$$

$$k_1^{M_1} ... k_i^{M_i} S_{M_1 ... M_i}(k_1 ... k_i ...) = 0.$$

On-shell gauge symmetry to GET

$$\epsilon_{2L}^{M_1}...\epsilon_{2L}^{M_i} S_{M_1...M_i}(k_1...k_i...) = \epsilon_{1L}^{M_1}...\epsilon_{1L}^{M_i} S_{M_1...M_i}(k_1...k_i...)$$

keep the goldstone components

$$\epsilon_{2L}^{M_1}...\epsilon_{2L}^{M_i} S_{M_1...M_i}(k_1...k_i...) = S(\phi_1...\phi_i...) + O(\frac{m_W}{n \cdot k})$$

this is simply goldstone equivalence theorem

On-shell gauge symmetry for 3-point amplitudes

$$i\mathcal{M}(1^{s_1}2^{s_2}3^{s_3})|_{\epsilon^M_{s_i}\to\frac{k_i^M}{m_W}} = 0$$

hWW

$$\begin{aligned} i\mathcal{M}(1^h 2^{s_2=L}3^{s_3})|_{\epsilon^M_{s_3}\to\frac{k_3^M}{m_W}} &= igm_W\epsilon_2^n\cdot\frac{k_3}{m_W} + \frac{g}{2}((k_1-k_3)\cdot\epsilon_2)(i) + \frac{g}{2}(k_1-k_2)\cdot\frac{k_3}{m_W}(-i) - ig\frac{m_h^2}{2m_W}i\cdot(-i) \\ &= ig(\epsilon_2^n\cdot k_3 - \epsilon_2^n\cdot k_3 - \frac{1}{2}k_2\cdot\epsilon_2^n - \frac{(k_1-k_2)(k_1+k_2)}{2m_W} - \frac{m_h^2}{2m_W}) \end{aligned}$$

$$\begin{aligned} i\mathcal{M}(1^h 2^{s_2=L}3^{s_3})|_{\epsilon^M_{s_3}\to\frac{k_3^M}{m_W}} &= ig(\frac{m_W}{2} + \frac{m_h^2-m_W^2}{2m_W} - \frac{m_h^2}{2m_W}) \\ &= 0, \end{aligned}$$

confirmed!

1.Similarly, for ffW

$$i\mathcal{M}(1^{s_1}2^{s_2}3^{s_3})\Big|_{\epsilon^M_{s_3}\to\frac{k_3^M}{m_W}} = \mathcal{M}(1^{s_1}2^{s_2}3^{s_3})\Big|_{\epsilon^\mu_{s_3}\to\frac{k_3^\mu}{m_W},\epsilon^4_{s_3}\to 0} + \mathcal{M}(1^{s_1}2^{s_2}3^{s_3})\Big|_{\epsilon^\mu_{s_3}\to 0,\epsilon^4_{s_3}\to -i} = 0 \quad (42)$$

making use of E.O.Ms. confirmed!

2.for WWW

making use of transverse condition $k^{*M}\epsilon_{LM} = 0$

and on-shell condition $k^{*M}k_M = 0.$

confirmed!

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COLLINEAR SPLITTING AMPLITUDES

$$i\mathcal{M} = \sum_s i\mathcal{M}^s_{split} \cdot \frac{i}{k_3^2 - m_3^2} \cdot i\mathcal{M}_0^s + \text{power suppressed}$$

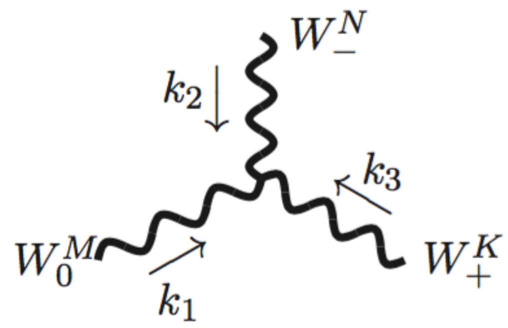
k3 approach the pole: on-shell amplitudes

COLLINEAR SPLITTING FUNCTIONS

$$\frac{d\mathcal{P}}{dz dk_T^2} \propto |\mathcal{M}_{split}|^2$$

Example: 1—> 2 splitting amplitudes

$$W_L^+ \to W_L^+ W_L^0$$



$$= \quad -ig(\eta'^{MN}(k_1-k_2)^\rho + \eta'^{NK}(k_2-k_3)^\mu + \eta'^{KM}(k_3-k_1)^\nu)$$

$$\begin{aligned} i\mathcal{M}(W_L^+ \to W_L^0 W_L^+) = -ig\ \{ & [\epsilon_{n_1}(k_1)\cdot\epsilon_{n_2}(k_2) - \frac{i^2}{2}](-k_1+k_2)\cdot\epsilon_{n_3}(k_3) \\ & + [\epsilon_{n_1}(k_2)\cdot\epsilon_{n_3}(k_3) - \frac{i(-i)}{2}](-k_2-k_3)\cdot\epsilon_{n_1}(k_1) \\ & + [\epsilon_{n_3}(k_3)\cdot\epsilon_{n_1}(k_1) - \frac{i(-i)}{2}](k_3+k_1)\cdot\epsilon_{n_2}(k_2)\} \end{aligned}$$

Example: 1—> 2 splitting amplitudes

$$W_L^+ \to W_L^+ W_L^0$$

$$n_1 = n_2 = n_3 = n,$$

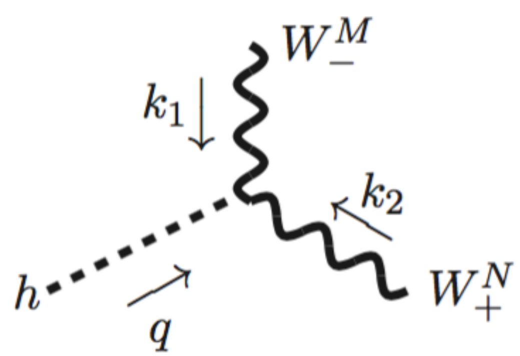
$$i\mathcal{M}(W_L^+ \to W_L^0 W_L^+) = \frac{ig}{2} m_W \left[-\frac{(k_1 - k_2)\cdot n}{n \cdot k_3} + \frac{(k_2 + k_3)\cdot n}{n \cdot k_1} + \frac{-(k_3 + k_1)\cdot n}{n \cdot k_2} \right]$$

$$z = \frac{n \cdot k_1}{n \cdot k_3} \qquad \bar{z} = \frac{n \cdot k_2}{n \cdot k_3}$$

$$i\mathcal{M}_{W_L^+ \to W_L^+ W_L^0} = \frac{ig^2 v}{2} \frac{z - \bar{z}}{z\bar{z}} (1 + \frac{z\bar{z}}{2})$$

Example: 1—> 2 splitting amplitudes

$$h \rightarrow W_L^+ W_L^-$$



$$= -\frac{g}{2}\left((k_1-q)^\mu g^{N4} + g^{M4}(k_2-q)^\nu\right) + igm_W g^{\mu\nu} - i\frac{\lambda_h v}{2} g^{M4} g^{N4}$$

Multiple parameters, not so simple

Alternatively, choose $\epsilon_L^\mu = \frac{k^\mu}{m_W} - \frac{m_W}{n\cdot k} n^\mu$,

and evaluate amplitude on-shell.

Example: 1—> 2 splitting amplitudes

$$h \to \boldsymbol{W_L^+ W_L^-}$$

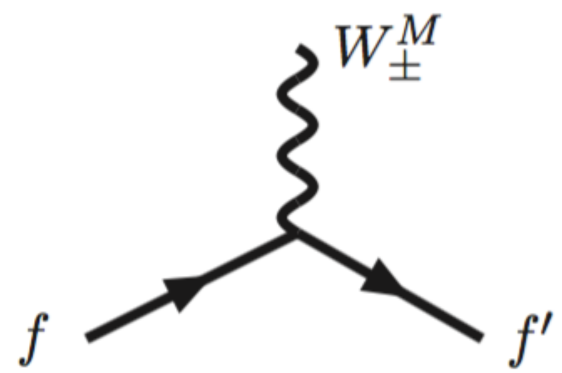
$$\begin{aligned}
i\mathcal{M}(h \to W_L^+ W_L^-) &= igm_W \left(\frac{k_2^\mu}{m_W} + \epsilon_{2n}^\mu\right)\left(\frac{k_{1\mu}}{m_W} + \epsilon_{n_1\mu}\right) \\
&= igm_W \left(\frac{k_3^2 - k_2^2 - k_1^2}{2m_W^2} + \frac{k_2 \cdot \epsilon_{n_1} + k_1 \cdot \epsilon_{n_2}}{m_W} + \epsilon_{n_2}\epsilon_{n_1}\right) \\
&\overset{\text{onshell}}{=} igm_W \left(\frac{m_h^2 - 2m_W^2}{2m_W^2} - \frac{k_2 \cdot n_1}{k_1 \cdot n_2} - \frac{k_1 \cdot n_2}{k_2 \cdot n_1} + m_W^2 \frac{n_2 \cdot n_1}{(n_2 \cdot k_2)(n_1 \cdot k_1)}\right) \\
&= igm_W \left(\frac{m_h^2 - 2m_W^2}{2m_W^2} - \frac{\bar{z}}{z} - \frac{z}{\bar{z}}\right)
\end{aligned}$$

In third step, made us of $k_3^2 = m_h^2,\ k_1^2 = m_W^2 \text{ and } k_2^2 = m_W^2$

$$i\mathcal{M}(h \to W_L^+ W_L^-) = igm_W \frac{1}{z\bar{z}}\left(\frac{m_h^2}{2m_W^2} z\bar{z} - (1 - z\bar{z})\right)$$

Example: 1—> 2 splitting amplitudes

$$f \to f' W_L^+$$



$$= \quad -i\frac{g}{\sqrt{2}}\gamma^\mu P_L - (y_f P_R + y_{f'} P_L) g^{M4}$$

Multiple parameters, the best way is still choose $\epsilon_L^\mu = \frac{k^\mu}{m_W} - \frac{m_W}{n\cdot k} n^\mu$

and evaluate amplitude on-shell.

Example: 1—> 2 splitting amplitudes

$f \to f' W_L^+$

$$
\begin{aligned}
i\mathcal{M}(f^{s_3} \to f'^{s_2} W_L^+) &= i\frac{g}{\sqrt{2}}\bar{u}_L^{s_2}(k_2)\gamma^\mu u_L^{s_3}(k_3) \cdot \left(\frac{k_{1\mu}}{m_W} + \epsilon_{n_1\mu}\right) \\
&= i\frac{g}{\sqrt{2}m_W}\bar{u}_L^{s_2}(k_2)(\not{k}_3 - \not{k}_2)u_L^{s_1}(k_3) - i\frac{g m_W}{\sqrt{2} n_1 \cdot k_1}\bar{u}_L(k_2)\not{n}_1 u_L(k_3) \\
&\overset{\text{onshell}}{=} i\frac{g}{\sqrt{2}m_W}(m_2\bar{u}_R^{s_2}(k_2)u_L^{s_3}(k_3) - m_3\bar{u}_L^{s_2}(k_2)u_R^{s_3}(k_3)) \\
&\quad -i\frac{g}{\sqrt{2}}\frac{m_W}{n_1 \cdot k_1}\bar{u}_L^{s_2}(k_2)\not{n}_1 u_L^{s_3}(k_3)
\end{aligned}
$$

In third step, made us of the equations of motion for on-shell particles

$$
\begin{aligned}
i\mathcal{M}(f^{-\frac{1}{2}} \to f'^{-\frac{1}{2}} W_L^+) &= \frac{ig}{\sqrt{2}}\frac{1}{\sqrt{\bar{z}z}}\left(\frac{m_2^2}{m_W}z - \frac{m_3^2}{m_W}z\bar{z} - 2m_W\bar{z}\right) \\
&= i\left(y_{f_2} m_2 \frac{1}{\sqrt{\bar{z}}} - y_{f_1} m_1 \sqrt{\bar{z}} - \frac{g}{\sqrt{2}} 2m_W \frac{\sqrt{\bar{z}}}{z}\right)
\end{aligned}
$$

Conclusions

1. In a Physical Gauge: longitudinal polarization vector: gauge components + goldstone components

2. 5-component treatment $W_M^a = (W_\mu^a, \phi^a)$, simplifies the Feynman rules: propagators, vertices.

3. Two equivalent forms of longitudinal polarizations: on-shell gauge symmetry, on-shell match of 3-point amplitudes

4. Calculations examples for collinear splitting amplitudes.