Sensitivities of Dark Matter direct detection experiments to effective WIMP-nucleus couplings

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The 3rd IBS-KIAS Joint Workshop

Based on arXiv:1805.06113 and arXiv:1810.00607

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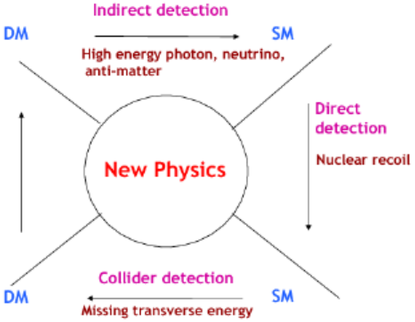
Overview

Introduction

Dark matter can be searched by many ways:



Status of Dark Matter Detection: 1707.06277

Direct Detection

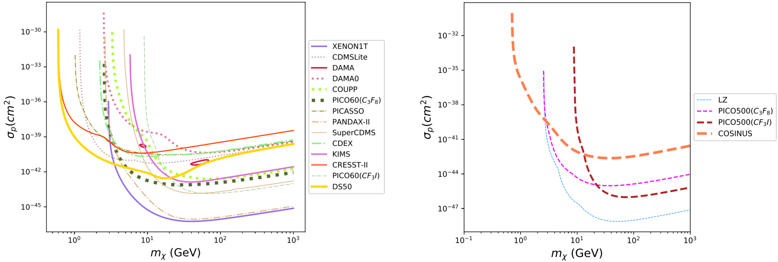
Elastic recoil of non relativistic halo WIMPs off the nuclei of an underground detector.

- Recoil energy of the nucleus lies in the keV range.
- Expected signal is very low.

- large exposures and extremely low background is required.

- Spin Independent interaction,

$$\sigma_{\chi N} \propto [c^p Z + (A - Z)c^n]^2 \,,$$



- Cross-section is enhanced for heavy nuclei (e.g. Xenon) and non-zero for all targets.
- Is it the case always?

- ◦ Isospin-violating models (1102.4331, 1205.2695),

$$\frac{c^n}{c^p} \simeq \frac{Z}{Z-A} \simeq -0.7$$

  ◦ WIMP-Xenon interaction is suppressed which reduces the sensitivity of Xenon detector.

- ◦ A Spin–Dependent WIMP–nucleon interaction,

$$\mathcal{L}_{int} \ni c^p \vec{S}_\chi \cdot \vec{S}_p + c^n \vec{S}_\chi \cdot \vec{S}_n,$$

  ◦ Only two isotopes with 47% of target number contribute reducing the sensitivity of Xenon detector.

- What about other non-standard interactions?

Non-relativistic EFT

- Hamiltonian density of WIMP-nucleus interaction,

$$\mathcal{H}(\mathbf{r}) = \sum_{j=1}^{15} (c_j^0 + c_j^1 \tau_3) \mathcal{O}_j(\mathbf{r})$$

$c_j^p = (c_j^0 + c_j^1)/2$ (proton) and $c_j^n = (c_j^0 - c_j^1)/2$ (neutron)

- All operators is guaranteed to be Hermitian if built out of the following four 3-vectors,

$$i\frac{\vec{q}}{m_N}, \ \vec{v}^\perp, \ \vec{S}_\chi, \ \vec{S}_N$$

with $\vec{v}^\perp = \vec{v} + \vec{q}/2\mu_N \ \Rightarrow \vec{v}^\perp \cdot \vec{q} = 0.$

A.L.Fitzpatrick, W.Haxton, E.Katz, N.Lubbers and Y.Xu, JCAP1302, 004 (2013),1203.3542.

Possible operators

N.Anand, A.L.Fitzpatrick and W.C.Haxton, Phys.Rev.C89, 065501 (2014),1308.6288.

$$
\begin{aligned}
\mathcal{O}_1 &= 1_\chi 1_N; \quad \mathcal{O}_2 = (v^\perp)^2; \quad \mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp) \\
\mathcal{O}_4 &= \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp); \quad \mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N}) \\
\mathcal{O}_7 &= \vec{S}_N \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N}) \\
\mathcal{O}_{10} &= i\vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp) \\
\mathcal{O}_{13} &= i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N}); \quad \mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp) \\
\mathcal{O}_{15} &= -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N}).
\end{aligned}
$$

- The expected rate,

$$\frac{dR_{\chi T}}{dE_R}(t) = \sum_T N_T \frac{\rho_{\text{WIMP}}}{m_{\text{WIMP}}} \int_{v_{min}} d^3 v_T f(\vec{v}_T, t) v_T \frac{d\sigma_T}{dE_R},$$

with,

$$\frac{d\sigma_T}{dE_R} = \frac{2m_T}{4\pi v_T^2} \left[\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}_T|^2 \right],$$

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}|^2 = \frac{4\pi}{2j_T + 1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \sum_k \overset{\text{WIMP}}{R_k^{\tau\tau'}} \left[c_j^\tau, (v_T^\perp)^2, \frac{q^2}{m_N^2} \right] \overset{\text{NUCLEUS}}{W_{Tk}^{\tau\tau'}}(y)$$

$$k = M,\ \Phi'',\ \Phi''M,\ \tilde{\Phi}',\ \Sigma'',\ \Sigma',\ \Delta, \Delta\Sigma'$$

$y \equiv (qb/2)^2$
b=nuclear size
q=momentum transfer

- In general form,

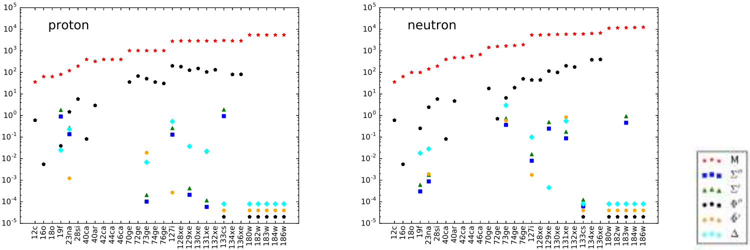
$$R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{(v_T^\perp)^2}{c^2} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \frac{v_T^2 - v_{min}^2}{c^2},$$

- Besides usual spin-dependent and spin-independent interactions, new contributions arise with explicit dependence on $\vec{q}$ and WIMP incoming velocity.

- Correspondence between WIMP and non-relativistic EFT nuclear response function

coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	coupling	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

- Nuclear response functions at vanishing momentum transfer



- Nuclear response function W's is normalized such as

$$\frac{16\pi}{(2j_T+1)} \times W_{TM}^{p}(y=0) = Z_T^2 \quad , \frac{16\pi}{(2j_T+1)} \times W_{TM}^{n}(y=0) = (A_T - Z_T)^2$$

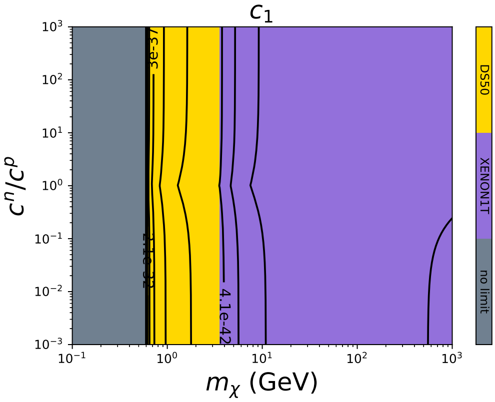
- We assume a Maxwellian velocity distribution.
- We assume that one coupling is dominant at a time.
- In our analysis, we included 14 existing experiments:

  XENON1T, PandaX-II, KIMS, CDMSLite, SuperCDMS, COUPP, PICASSO, PICO-60 (CF_3I and C_3F_8 targets), CRESST-II, DAMA (modulation data), DAMA0 (average count rate), CDEX, and DarkSide-50
- We have also included projections from LZ, COSINUS, PICO500 (CF_3I and C_3F_8 targets)
- Sensitivity is expressed in terms of 90% C.L. bounds on effective cross-section,

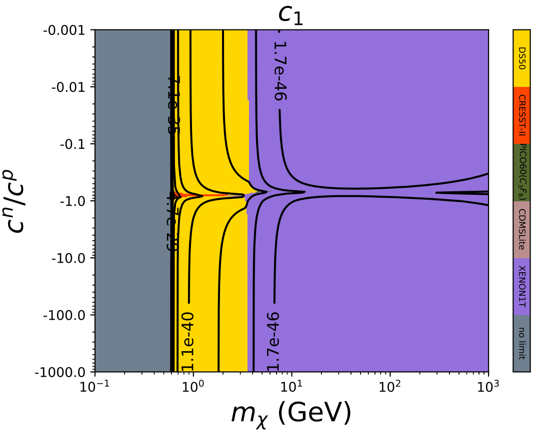
$$\sigma_{\mathcal{N},lim} = \max(\sigma_p, \sigma_n)$$

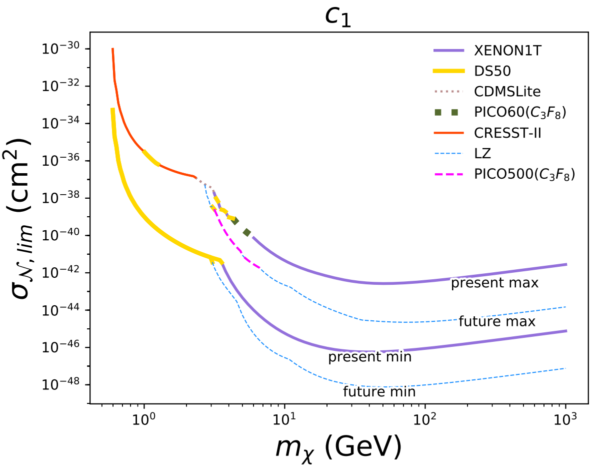
$$\sigma_p = (c_j^p)^2 \frac{\mu_{\chi\mathcal{N}}^2}{\pi} \quad , \quad \sigma_n = (c_j^n)^2 \frac{\mu_{\chi\mathcal{N}}^2}{\pi}$$

- Two free parameters viz. WIMP mass m_χ and $r = c^n/c^p$.
- Spin-independent coupling, no velocity dependence in the cross-section, M response function



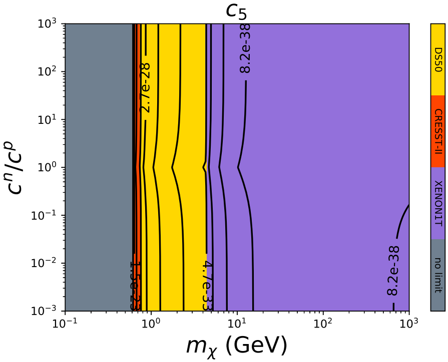
- Similar results exist for $c_{11}(q^2)$.


c_1
1.7e-46
1.1e-40
1.7e-46
-0.001
-0.01
-0.1
-1.0
-10.0
-100.0
-1000.0
c^n/c^p
10^{-1}
10^0
10^1
10^2
10^3
m_χ (GeV)
DS50
CRESST-II
PICO60(C_3F_8)
CDMSLite
XENON1T
no limit

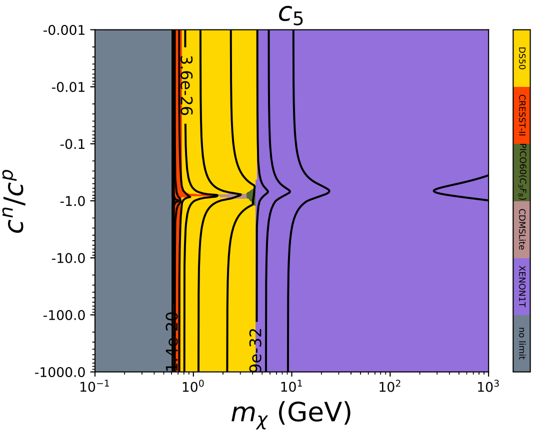

c_1
XENON1T
DS50
CDMSLite
PICO60(C_3F_8)
CRESST-II
LZ
PICO500(C_3F_8)
present max
future max
present min
future min
$\sigma_{\mathcal{N},lim}$ (cm^2)
10^{-30}
10^{-32}
10^{-34}
10^{-36}
10^{-38}
10^{-40}
10^{-42}
10^{-44}
10^{-46}
10^{-48}
10^0
10^1
10^2
10^3
m_χ (GeV)

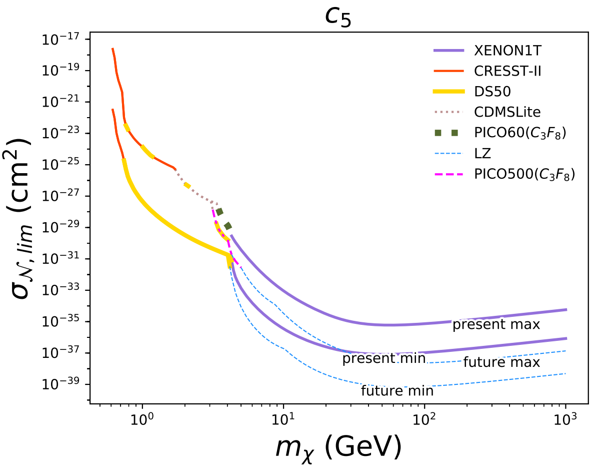
- $\Delta(q^4)$ and velocity-dependent $M(q^2)$ response functions.

- The velocity-dependent contribution is of the same order of the velocity independent one.

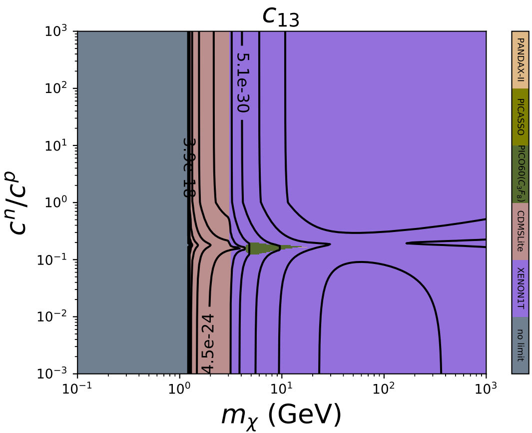


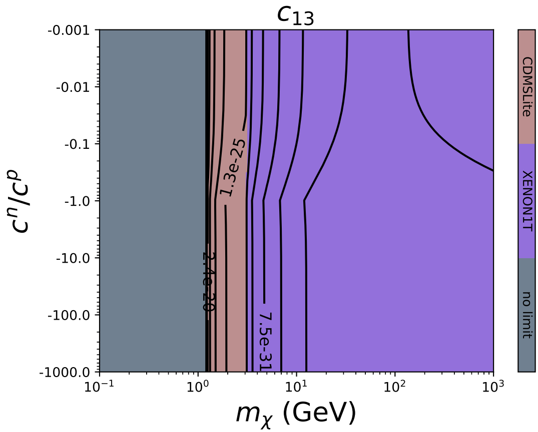
- Similar results exist for couplings c_8 with $\Delta(q^2)$ and M.

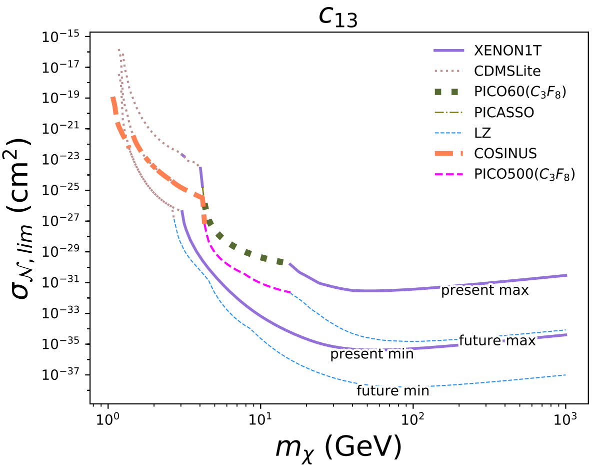

c_5
-0.001
-0.01
-0.1
-1.0
-10.0
-100.0
-1000.0
c^n/c^p
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
m_χ (GeV)
3.6e-26
1.4e-20
9e-32
DS50
CRESST-II
PICO60(C_3F_8)
CDMSLite
XENON1T
no limit


c_5
XENON1T
CRESST-II
DS50
CDMSLite
PICO60(C_3F_8)
LZ
PICO500(C_3F_8)
present max
present min
future max
future min
$\sigma_{\mathcal{N},lim}$ (cm^2)
m_χ (GeV)
10^{-17}
10^{-19}
10^{-21}
10^{-23}
10^{-25}
10^{-27}
10^{-29}
10^{-31}
10^{-33}
10^{-35}
10^{-37}
10^{-39}
10^0
10^1
10^2
10^3

- $\tilde{\Phi}'(q^4)$ and velocity dependent $\Sigma''(q^2)$ response functions, require nuclear spin $j > 1/2$, non-zero for Na^{23}, Ge^{73}, I^{121}, Xe^{131}.



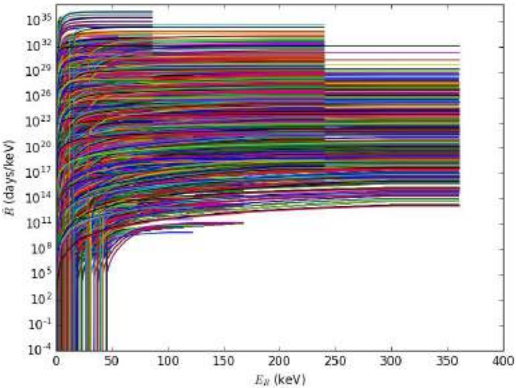

c_{13}
-0.001
-0.01
-0.1
-1.0
-10.0
-100.0
-1000.0
c^n/c^p
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
m_χ (GeV)
1.3e-25
2.4e-20
7.5e-31
CDMSLite
XENON1T
no limit



c_{13}
XENON1T
CDMSLite
PICO60(C_3F_8)
PICASSO
LZ
COSINUS
PICO500(C_3F_8)
present max
future max
present min
future min
$\sigma_{\mathcal{N},lim}$ (cm^2)
m_χ (GeV)
10^{-15}
10^{-17}
10^{-19}
10^{-21}
10^{-23}
10^{-25}
10^{-27}
10^{-29}
10^{-31}
10^{-33}
10^{-35}
10^{-37}
10^0
10^1
10^2
10^3

Efforts at a glance

- We have calculated 75768 response functions for 19 experiments and 14 couplings.
- If include interferences then 37884 more response functions.



Relativistic EFT

Motivations

1-We practically analyze all existing experiments, while usually people only include few of them.

2-We discuss the relation between the NR limits in the $m_\chi - r$ planes and the relativistic bounds, explaining when and how the former can be used to obtain the latter.

- We extended our analysis to relativistic operators.
- Set of operators upto dim-7 is considered.
- Lagrangian describing DM interactions with quarks and gluons,

$$\mathcal{L}_\chi = \sum_q \sum_{a,d} \mathcal{C}_{a,q}^{(d)} \mathcal{Q}_{a,q}^{(d)} + \sum_{b,d} \mathcal{C}_b^{(d)} \mathcal{Q}_b^{(d)},$$

  $\mathcal{C}_{a,q}^{(d)}, \mathcal{C}_b^{(d)} \equiv \frac{1}{\tilde{\Lambda}^{d-4}}$ are dimensional Wilson coefficients, fixed at EW scale.
- Once NR Wilson coefficients c_j^τ is obtained from $\mathcal{C}_{a,q}^{(d)}, \ \mathcal{C}_b^{(d)}$ our previous analysis holds.

- The dimension-five operators:

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}i\gamma_5\chi)F_{\mu\nu}$$

- The dimension-six operators are:

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu q)\,,$$
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{q}\gamma^\mu\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5 q)\,,$$

- The dimension-seven operators are:

$$\mathcal{Q}_1^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}\chi)G^{a\mu\nu}G^a_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}G^a_{\mu\nu}\,,$$
$$\mathcal{Q}_3^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}\chi)G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}\,, \qquad \mathcal{Q}_4^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}\widetilde{G}^a_{\mu\nu}\,,$$
$$\mathcal{Q}_{5,q}^{(7)} = m_q(\bar{\chi}\chi)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(7)} = m_q(\bar{\chi}i\gamma_5\chi)(\bar{q}q)\,,$$
$$\mathcal{Q}_{7,q}^{(7)} = m_q(\bar{\chi}\chi)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(7)} = m_q(\bar{\chi}i\gamma_5\chi)(\bar{q}i\gamma_5 q)\,,$$
$$\mathcal{Q}_{9,q}^{(7)} = m_q(\bar{\chi}\sigma^{\mu\nu}\chi)(\bar{q}\sigma_{\mu\nu}q)\,, \qquad \mathcal{Q}_{10,q}^{(7)} = m_q(\bar{\chi}i\sigma^{\mu\nu}\gamma_5\chi)(\bar{q}\sigma_{\mu\nu}q)$$

Assumption:

We assume single coupling, $\mathcal{C}_{a,q}^{(d)} = 1$ to all quarks and gluon $\mathcal{C}_b^{(d)} = 1$.

- A sizable mixing between vector and axial-vector currents is very important

$$\mathcal{Q}_{1,q}^{(6)} \to F_1^{q/N} \mathcal{O}_1^N \,, \mathcal{Q}_{2,q}^{(6)} \to 2F_1^{q/N} \mathcal{O}_8^N + 2(F_1^{q/N} + F_2^{q/N}) \mathcal{O}_9^N \,,$$

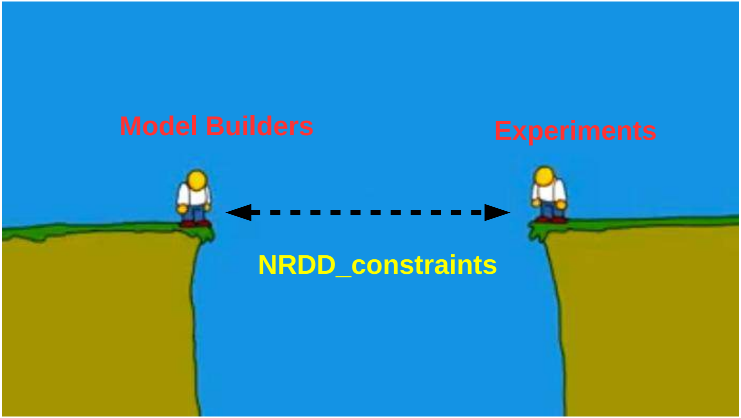
$$\mathcal{Q}_{3,q}^{(6)} \to -2F_A^{q/N} \mathcal{O}_7^N - \frac{m_N}{m_\chi} \mathcal{O}_9^N \,, \mathcal{Q}_{4,q}^{(6)} \to -4F_A^{q/N} \mathcal{O}_4^N + F_{p'}^{q/N} \mathcal{O}_6^N$$

- Specifically the pion form factors are:

$$F_{P,P'}^{q/N}(q^2) = \frac{m_N^2}{m_\pi^2 - q^2} a_\pi^{q/N} + \frac{m_N^2}{m_\eta^2 - q^2} a_\eta^{q/N} + b^{q/N},$$
$$F_{\tilde{G}}^{N}(q^2) = \frac{q^2}{m_\pi^2 - q^2} a_{\tilde{G},\pi}^{N} + \frac{q^2}{m_\eta^2 - q^2} a_{\tilde{G},\eta}^{N} + b_{\tilde{G}}^{N}.$$

- We study the mixing effect by fixing the coupling at scale $\tilde{\Lambda} = 2$ TeV and run it to EW scale through runDM (1605.04917).
- Third family of quarks gives the dominant contribution.
- We further run the coupling to nucleon scale using directDM (1708.02678) and obtain $c_j^{\mathcal{T}}$.

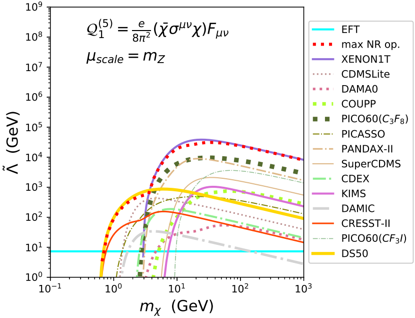
Bridging the gap!

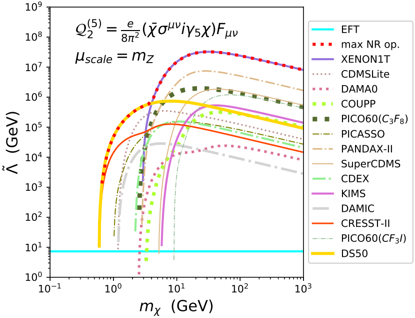


- Lower bound on the effective scale $\tilde{\Lambda} = (c\mu_{\chi\mathcal{N}}/\sqrt{\sigma_{NR}\pi})^{1/d-4}$, $\mu_{scale} = m_Z$,

$$\mathcal{Q}_1^{(5)} \rightarrow -\frac{\alpha}{2\pi}F_1^N\left(\frac{1}{m_\chi}\mathcal{O}_1^N - 4\frac{m_N}{\vec{q}^{\,2}}\mathcal{O}_5^N\right) - \frac{2\alpha}{\pi}\frac{\mu_N}{m_N}\left(\mathcal{O}_4^N - \frac{m_N^2}{\vec{q}^{\,2}}\mathcal{O}_6^N\right),$$

$$\mathcal{Q}_2^{(5)} \rightarrow \frac{2\alpha}{\pi}\frac{m_N}{\vec{q}^2}F_1^N\mathcal{O}_{11}^N$$

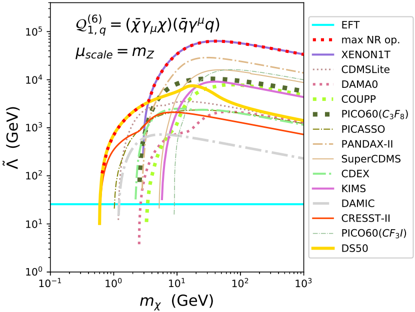


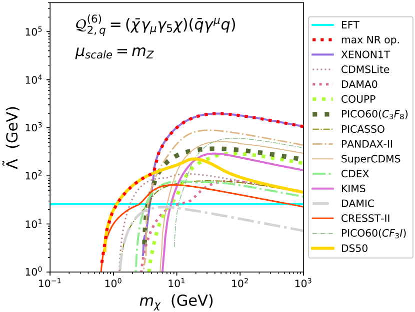


Sunghyun Kang, S. Scopel, G. Tomar, J.H. Yoon, arXiv: 1810.00607

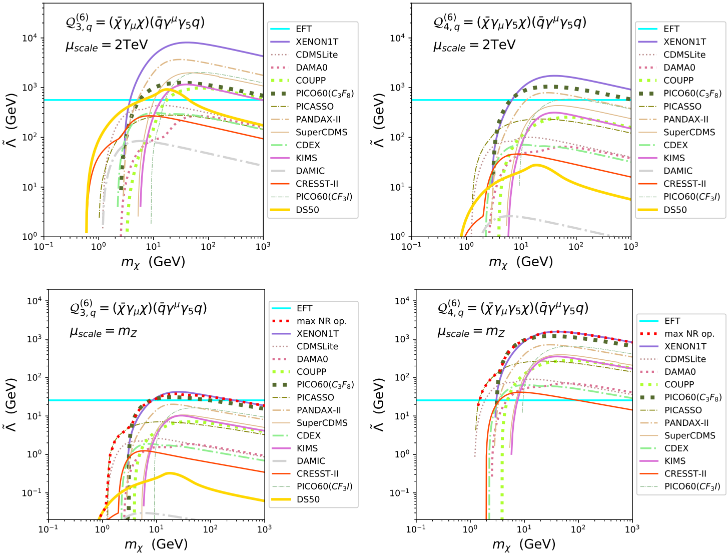
Validity of the effective theory:

$$\tilde{\Lambda} > \mu_{scale}/(4\pi)^{1/(d-4)}$$

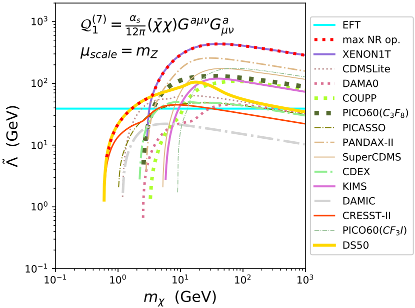


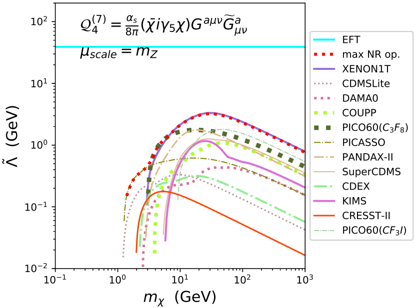


- Lower bound on the effective scale $\tilde{\Lambda}$, $\mu_{scale} = 2\,TeV$,

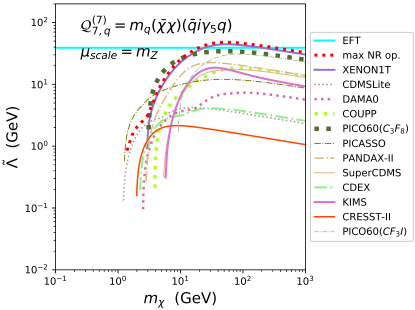


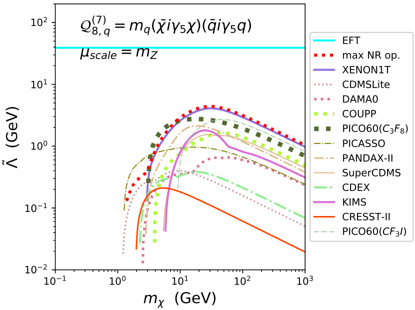
$$\mathcal{Q}_1^{(7)} \to F_G^N \mathcal{O}_1^N, \qquad \mathcal{Q}_4^{(7)} \to \frac{m_N}{m_\chi} F_{\tilde{G}}^N \mathcal{O}_6^N$$





$$\mathcal{Q}^{(7)}_{7,q} \to F_P^{q/N}\mathcal{O}^N_{10}, \qquad \mathcal{Q}^{(7)}_{8,q} \to \frac{m_N}{m_\chi}F_P^{q/N}\mathcal{O}^N_6$$





Sunghyun Kang, S. Scopel, G. Tomar, J.H. Yoon, arXiv: 1810.00607

All results are based on our Direct Detection code.

- Object-oriented, based on Python.
- Flexible to easily implement any new experiment and/or update new information. Efficient to calculate and handle a large number of response functions.
- Valid for any velocity distribution of WIMPs.

- Development with rigorous testing is in progress. Plan to eventually make it publicly available.
- A python routine named **NRDD-constraints** has been released by our group.
  https://github.com/NRDD-constraints/NRDD

Summary

Summary

- Expected cross-section $\sigma_{\mathcal{N},lim}$ varies many orders of magnitude depending on effective couplings.
- In most cases, it is driven by,
  - **Xenon target:** C_1, C_3, C_5, C_6, C_8, C_{11}, C_{12}, C_{13}, and C_{15}
  - **Fluorine target:** C_4, C_7, C_9, C_{10}, C_{14}
- Out of 14 considered experiments, there are 9 experiments which provide the most stringent bounds on effective couplings:

  XENON1T, PandaX-II, CDMSLite, PICASSO, PICO-60, CRESST-II, DAMA0, DarkSide-50
- It is due to the complementarity between different targets, combinations of count rate and energy thresholds.
- For all the couplings the future experiments could improve the limits by two to three order of magnitudes.

Summary

- We extended our analysis to the relativistic operators.
- There are two cases,
  - The operators $\mathcal{Q}_{1,q}^{(5)}$, $\mathcal{Q}_{2,q}^{(5)}$, $\mathcal{Q}_{1,q}^{(6)}$, $\mathcal{Q}_{2,q}^{(6)}$, $\mathcal{Q}_{1}^{(7)}$, $\mathcal{Q}_{2}^{(7)}$, $\mathcal{Q}_{5,q}^{(7)}$, $\mathcal{Q}_{6,q}^{(7)}$ and $\mathcal{Q}_{10,q}^{(7)}$ follow the SI scaling of cross section and constrained by DS50 and Xenon1T.
  - The operators $\mathcal{Q}_{3,q}^{(6)}$, $\mathcal{Q}_{4,q}^{(6)}$, $\mathcal{Q}_{3}^{(7)}$, $\mathcal{Q}_{4}^{(7)}$, $\mathcal{Q}_{7,q}^{(7)}$, $\mathcal{Q}_{8,q}^{(7)}$, and $\mathcal{Q}_{9,q}^{(7)}$ follow the SD scaling of cross section and constrained by PICO60, PICASSO, and Xenon1T.
- In all models with exception of $\mathcal{Q}_{7,q}^{(7)}$ and $\mathcal{Q}_{8,q}^{(7)}$ the expected rate is driven by one of the NR operator with an accuracy of 60%.
- The bounds on some of the relativistic models look inconsistent with the validity of the effective theory, so for such models the sensitivity of present experiments does not put any meaningful constraint.
- We released a Python tool called **NRDD-constraints** using which it is very easy to test any relativistic model against DM direct detection.

Namaste!

- Connection to relativistic effective theory: 1203.3542

j	$\mathcal{L}^j_{\rm int}$	Nonrelativistic reduction	$\sum_i c_i \mathcal{O}_i$	P/T
1	$\bar\chi\chi\bar N N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
2	$i\bar\chi\chi\bar N\gamma^5 N$	$i\frac{\vec q}{m_N}\cdot\vec S_N$	$\mathcal{O}_{10}$	O/O
3	$i\bar\chi\gamma^5\chi\bar N N$	$-i\frac{\vec q}{m_\chi}\cdot\vec S_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$	O/O
4	$\bar\chi\gamma^5\chi\bar N\gamma^5 N$	$-\frac{\vec q}{m_\chi}\cdot\vec S_\chi\frac{\vec q}{m_N}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$	E/E
5	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu N$	$1_\chi 1_N$	$\mathcal{O}_1$	E/E
6	$\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q^2}{2m_N m_M}1_\chi 1_N + 2(\frac{\vec q}{m_\chi}\times\vec S_\chi + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$\frac{\vec q^2}{2m_N m_M}\mathcal{O}_1 - 2\frac{m_N}{m_M}\mathcal{O}_3 + 2\frac{m_N^2}{m_M m_\chi}(\frac{\vec q^2}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
7	$\bar\chi\gamma^\mu\chi\bar N\gamma_\mu\gamma^5 N$	$-2\vec S_N\cdot\vec v^\perp + \frac{2}{m_\chi}i\vec S_\chi\cdot(\vec S_N\times\vec q)$	$-2\mathcal{O}_7 + 2\frac{m_N}{m_\chi}\mathcal{O}_9$	O/E
8	$i\bar\chi\gamma^\mu\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$2i\frac{\vec q}{m_M}\cdot\vec S_N$	$2\frac{m_N}{m_M}\mathcal{O}_{10}$	O/O
9	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma_\mu N$	$-\frac{\vec q^2}{2m_\chi m_M}1_\chi 1_N - 2(\frac{\vec q}{m_N}\times\vec S_N + i\vec v^\perp)\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)$	$-\frac{\vec q^2}{2m_\chi m_M}\mathcal{O}_1 + 2\frac{m_N}{m_M}\mathcal{O}_5 - 2\frac{m_N}{m_M}(\frac{\vec q^2}{m_N^2}\mathcal{O}_4 - \mathcal{O}_6)$	E/E
10	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$4(\frac{\vec q^2}{m_M^2}\mathcal{O}_4 - \frac{m_N^2}{m_M^2}\mathcal{O}_6)$	E/E
11	$\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N\gamma^\mu\gamma^5 N$	$4i(\frac{\vec q}{m_M}\times\vec S_\chi)\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
12	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$-[i\frac{\vec q^2}{m_\chi m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_\chi)]\frac{\vec q}{m_M}\cdot\vec S_N$	$-\frac{m_N}{m_\chi}\frac{\vec q^2}{m_M^2}\mathcal{O}_{10} - 4\frac{\vec q^2}{m_M^2}\mathcal{O}_{12} - 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
13	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma_\mu N$	$2\vec v^\perp\cdot\vec S_\chi + 2i\vec S_\chi\cdot(\vec S_N\times\frac{\vec q}{m_N})$	$2\mathcal{O}_8 + 2\mathcal{O}_9$	O/E
14	$\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$4i\vec S_\chi\cdot(\frac{\vec q}{m_M}\times\vec S_N)$	$-4\frac{m_N}{m_M}\mathcal{O}_9$	O/E
15	$\bar\chi\gamma^\mu\gamma^5\chi\bar N\gamma^\mu\gamma^5 N$	$-4\vec S_\chi\cdot\vec S_N$	$-4\mathcal{O}_4$	E/E
16	$i\bar\chi\gamma^\mu\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4i\vec v^\perp\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N}{m_M}\mathcal{O}_{13}$	E/O
17	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu N$	$2i\frac{\vec q}{m_M}\cdot\vec S_\chi$	$2\frac{m_N}{m_M}\mathcal{O}_{11}$	O/O
18	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}N$	$\frac{\vec q}{m_M}\cdot\vec S_\chi[i\frac{\vec q^2}{m_N m_M} - 4\vec v^\perp\cdot(\frac{\vec q}{m_M}\times\vec S_N)]$	$\frac{\vec q^2}{m_M^2}\mathcal{O}_{11} + 4\frac{m_N^2}{m_M^2}\mathcal{O}_{15}$	O/O
19	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N\gamma_\mu\gamma^5 N$	$-4i\frac{\vec q}{m_M}\cdot\vec S_\chi\vec v_\perp\cdot\vec S_N$	$-4\frac{m_N}{m_M}\mathcal{O}_{14}$	E/O
20	$i\bar\chi i\sigma^{\mu\nu}\frac{q_\nu}{m_M}\gamma^5\chi\bar N i\sigma_{\mu\alpha}\frac{q^\alpha}{m_M}\gamma^5 N$	$4\frac{\vec q}{m_M}\cdot\vec S_\chi\frac{\vec q}{m_M}\cdot\vec S_N$	$4\frac{m_N^2}{m_M^2}\mathcal{O}_6$	E/E

- WIMPs response funtions: 1203.3542

$$
\begin{aligned}
R_M^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right] \\
R_{\Phi''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{q^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right)\right]\frac{q^2}{m_N^2} \\
R_{\Phi'' M}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'}\right]\frac{q^2}{m_N^2} \\
R_{\tilde{\Phi}'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \left[\frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right)\right]\frac{q^2}{m_N^2} \\
R_{\Sigma''}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{q^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\left. \frac{q^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{q^4}{m_N^4} c_6^\tau c_6^{\tau'} + v_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{q^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right] \\
R_{\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{1}{8}\left[\frac{q^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right] + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \right. \\
&\left. \frac{q^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{q^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{q^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{q^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right] \\
R_{\Delta}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(\frac{q^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right)\frac{q^2}{m_N^2} \\
R_{\Delta\Sigma'}^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}\right) &= \frac{j_\chi(j_\chi+1)}{3}\left(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}\right)\frac{q^2}{m_N^2}.
\end{aligned}
$$

- Maxwellian velocity distribution:

$$
\begin{aligned}
f(\vec{v}_T, t) &= N \left(\frac{3}{2\pi v_{rms}^2} \right)^{3/2} e^{-\frac{3|\vec{v}_T + \vec{v}_E|^2}{2v_{rms}^2}} \Theta(u_{esc} - |\vec{v}_T + \vec{v}_E(t)|) \\
N &= \left[erf(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2} \right]^{-1},
\end{aligned}
$$

with $z = 3u_{esc}^2/(2v_{rms}^2)$. In the isothermal sphere model hydrothermal equilibrium between the WIMP gas pressure and gravity is assumed, leading to $v_{rms} = \sqrt{3/2} v_0$ with v_0 the galactic rotational velocity.

Six distinct nuclear response functions are defined as,

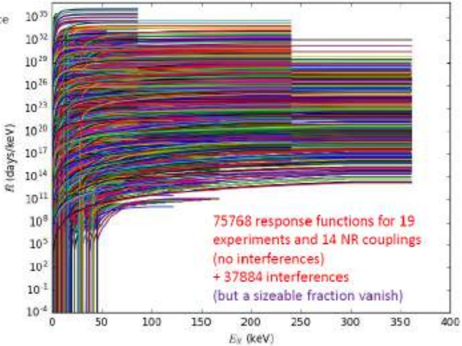
- M : vector-charge (**spin-independent part**, non-zero for all nuclei)
- Φ'' : vector-longitudinal, related to spin-orbit coupling $\sigma \cdot l$ (also spin-independent, non-zero for all nuclei)
- Σ', Σ'' : longitudinal and transverse components of nuclear spin, **their sum is the usual spin-dependent interaction**, require $j > 0$
- Δ : associated to orbital angular momentum operator l, requires $j > 0$
- $\tilde{\Phi}'$: related to the vector-longitudinal operator, transforms as a tensor under rotation, require $j > 1/2$

Tabulate full calculation of R response function for each:

1) Experiment
2) Energy bin/energy threshold/energy value
3) Isospin value (c_n/c_p=-1,0,1)
4) Nuclear target (including all stable isotopes)
5) Effective coupling
6) 4 terms including explicit velocity dependence

Isospin rotation with $r=c^n/c^p$:

$$R(r) = \frac{r(r+1)}{2} R(r=1) + (1-r^2) R(r=0) + \frac{r(r-1)}{2} R(r=-1)$$



Back Up

The rate can be written as

$$R = \sum_{k=1}^{N} \delta\bar{\eta}^k \times$$
$$\left\{ \bar{\mathcal{R}}_0 \left[E_R^{max}(v_k)\right] + (v_k^2 - \frac{\delta}{\mu_{\chi N}})\bar{\mathcal{R}}_1 \left[E_R^{max}(v_k)\right] \right.$$
$$\left. - \frac{m_N}{2\mu_{\chi N}^2}\bar{\mathcal{R}}_{1E} \left[E_R^{max}(v_k)\right] - \frac{\delta^2}{2m_N}\bar{\mathcal{R}}_{1E^{-1}} \left[E_R^{max}(v_k)\right] \right\}$$

In terms of four response functions that do not depend on the WIMP mass or mass splitting:

$$\bar{\mathcal{R}}_{0,1}(E_R) \equiv \int_0^{E_R} dE'_R \mathcal{R}_{0,1}(E'_R)$$
$$\bar{\mathcal{R}}_{1E}(E_R) \equiv \int_0^{E_R} dE'_R E'_R \mathcal{R}_1(E'_R)$$
$$\bar{\mathcal{R}}_{1E^{-1}}(E_R) \equiv \int_0^{E_R} dE'_R \frac{1}{E'_R} \mathcal{R}_1(E'_R)$$

that can be tabulated for later use.