Calculation of $B \rightarrow D^{*} \ell \bar{\nu}$ semileptonic decay form factors by lattice QCD

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CKM matrix

- In the Standard Model, the weak interaction with charged-currents are

\[ -\frac{g}{\sqrt{2}} \left( \bar{u}_L \bar{c}_L \bar{t}_L \right) \gamma^\mu W^+_{\mu L} V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}, \quad (1) \]

- The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3 by 3 unitary matrix parametrized by three mixing angles and the CP-violating phase.

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (2) \]

- Consistency of SM: check unitarity of \( V_{\text{CKM}} \).
The indirect CP violation parameter in the neutral kaon system $\varepsilon_K$ depends on $\rho$ and $\eta$: it gives constraint on the apex of the unitarity triangle.

$$\varepsilon_K \equiv \frac{A[K_L \to \pi\pi(I = 0)]}{A[K_S \to \pi\pi(I = 0)]}$$

Standard model evaluation of $|\varepsilon_K^{SM}|$ using inputs determined from lattice QCD ($\hat{B}_K$, exclusive $|V_{cb}|$, and etc.) has a strong tension ($\sim 4\sigma$) with the experimental value [Weonjong Lee, et al., PRD 98, 094505 (2018)].

$$|\varepsilon_K^{Exp}| = (2.228 \pm 0.011) \times 10^{-3}, \quad |\varepsilon_K^{SM}| = (1.57 \pm 0.16) \times 10^{-3},$$

The largest error of $|\varepsilon_K^{SM}|$ comes from the uncertainty of $V_{cb}$.

$$\varepsilon_K^{SM} = \sqrt{2}\exp(i\theta)\sin(\theta)\left(C_\varepsilon X_{SD} \hat{B}_K + \cdots \right) + \cdots$$

$$X_{SD} = \tilde{\eta}\lambda^2 |V_{cb}|^4 (1 - \bar{\rho}) + \cdots$$

Around 30% of total errors of $|\varepsilon_K^{SM}|$ is from the uncertainty of $|V_{cb}|$. 
Current status of $V_{cb}$

- The determination of $V_{cb}$: Inclusive vs Exclusive

| Determination                  | $|V_{cb}| \times 10^{-3}$ | memo                          |
|-------------------------------|---------------------------|-------------------------------|
| Exclusive($\bar{B} \to D^*\ell\bar{\nu}$) | 39.3(7)                   | FNAL/MILC + HFLAG             |
| Inclusive($\bar{B} \to X_c\ell\bar{\nu}$)  | 42.5(9)                   | HFLAG                         |


- Determination of CKM matrix elements with lattice QCD

Experiment = known factors $\times V_{CKM} \times$ Hadronic matrix element
Determinations of $|V_{cb}|$ using lattice QCD

Two kinds of exclusive determination of $|V_{cb}|$ by lattice QCD simulation.

- $\bar{B} \to D^* \ell \bar{\nu}$ decay: 3-point Green function. Lattice simulation at zero recoil is the best target to determine $|V_{cb}|$.
  - The decay rate depends on a single form factor $h_{A_1}$
  - Discretization error related to $\mathcal{O}(\Lambda_{QCD}/m_Q)$ corrections vanish by Luke’s theorem.

- $\bar{B} \to D \ell \bar{\nu}$ decay: 3-point Green function. Because of the phase-space suppression around zero recoil, precise experimental data is available only at non-zero recoil.
| $V_{cb}$ | from the exclusive decay $\bar{B} \rightarrow D^* \ell \bar{\nu}$ |

**Experiment:**

$$\frac{d\Gamma}{d\omega}(\bar{B} \rightarrow D^* \ell \bar{\nu}) = \frac{G_F^2 m_D^3}{48\pi^3} (m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{EW}|^2 |V_{cb}|^2 \chi(w)|F(w)|^2$$

where $w \equiv v_B \cdot v_{D^*}$, where $v_B$ and $v_{D^*}$ are meson velocities.

**Lattice QCD:** Calculate form factors from the matrix element.

$$\frac{1}{\sqrt{m_B m_{D^*}}} \langle D^*(p_{D^*}, \epsilon) | A_\mu | \bar{B}(p_B) \rangle = -ih_{A_1}(w)(w + 1)\epsilon^*_\mu + ih_{A_2}(w)(\epsilon^* \cdot v_B)v_{B\mu}$$

$$+ ih_{A_3}(w)(\epsilon^* \cdot v_B)v_{D^*\mu},$$

$$\frac{1}{\sqrt{m_B m_{D^*}}} \langle D^*(p_D, \epsilon) | V_\mu | \bar{B}(p_B) \rangle = h_V(w)\epsilon_{\mu\nu\alpha\beta} \epsilon^*_{\nu} v_{D^*}^\alpha v_{B}^\beta.$$ 

At zero recoil ($v_B = v_{D^*}$),

$$F(w = 1) \rightarrow h_{A_1}(w = 1).$$

(7)
2-point (Euclidean) Green’s function: extract meson’s energy spectrum

\[ C_{B}^{2\text{pt}}(p) = \sum_{x} e^{ip \cdot x} \langle O_{B}(t, x) O_{B}^\dagger(0) \rangle = \sum_{n} |\langle 0|O_{B}^\dagger|B_{n}\rangle|^{2} e^{-E_{n}(p)t} \]

\[ = |\langle 0|O_{B}^\dagger|\bar{B}\rangle|^{2} e^{-E_{0}(p)t} + |\langle 0|O_{B}^\dagger|B_{1}\rangle|^{2} e^{-E_{1}(p)t} + \ldots \] (8)

\[ O_{B}^\dagger \] (meson interpolating operator): creates \( B \)-meson as acting on vacuum.

3-point (Euclidean) Green’s function: extract hadronic matrix elements

\[ C_{A_{j}}^{B \rightarrow D^{*}}(t_{s}, t_{f}) = \sum_{x,y} \langle O_{D^{*}}(x, t_{f}) A_{j}^{cb}(y, t_{s}) O_{B}^\dagger(0, 0) \rangle \]

\[ = \langle 0|O_{D^{*}}|D^{*}\rangle \langle \bar{B}|O_{B}^\dagger|0\rangle \langle D^{*}|A_{j}|ar{B}\rangle e^{-M_{B}t_{s}} e^{-M_{D^{*}}(t_{f}-t_{s})} + \ldots \] (9)

where \( A_{j}^{cb} \) is flavor-changing axial current.

Do numerical analysis to extract the ground-state contribution out of excited-state contamination.
Path integral on lattice

- Calculate Green’s functions by performing path integral over discretized Euclidean space-time. For example,

\[
\langle O_2(x) O_1(0) \rangle = \frac{1}{Z} \int \prod dU d\bar{q} dq e^{-S_{QCD}^{\text{lat}}} O_2(x) O_1(0)
\]

(10)

The field variable \( U_\mu(x) \) is gauge link.

- The integral over fermionic Grassmann variables gives fermionic determinant. For example, if \( O_2 \equiv \bar{b} \gamma_5 d \) and \( O_1 \equiv \bar{d} \gamma_5 b \)

\[
\langle O_2(x) O_1(0) \rangle = \int \frac{1}{Z} \prod dU e^{-S_g^{\text{lat}}} \bar{b}(x) \gamma_5 d(x) \bar{d}(0) \gamma_5 b(0) \prod_q e^{-\sum_q \bar{q} D_q q}
\]

\[
= -\frac{1}{Z} \int \prod dU e^{-S_g^{\text{lat}}} \prod_q \det[D_q] \text{tr}[\gamma_5 D_d^{-1}(x, 0) \gamma_5 D_b^{-1}(0, x)]
\]

\[
\rightarrow \sum_{i=1}^{N_{\text{MC}}} w(U_i)(-\text{tr}[\gamma_5 D_d^{-1}(x, 0) \gamma_5 D_b^{-1}(0, x)])
\]

(11)

if \( \prod_q \det[D_q] \geq 0 \), one can use Monte Carlo simulation with importance sampling with weight factor \( w(U) \propto dU e^{-S_g^{\text{lat}}} \prod_q \det[D_q](U) \).
Lattice QCD simulations in three steps.

- **Generate gauge configuration**: make gauge configurations to follow probability density $P(U) \propto w(U)$.
- **Measurements**: calculate Euclidean Green’s function.
  - Compute quark propagators: inverse of matrix with $4$ (spin) $\times 3$ (color) $\times V$ (lattice vol) rows and columns. (For $V = 48^3 \times 96$ hypercubic lattice, dimension of matrix is over 100 millions.)
  - Calculate correlation functions by contracting quark fields.
- **Extract energy spectrum, hadronic matrix elements, · · ·**

**Sea quark and valence quark**

- **Fermions in Lagrangian**: sea quark $\rightarrow$ fermion determinant
- **Fermions in the operator**: valence quark $\rightarrow$ propagator.
Current status of $h_{A_1}$

- The $h_{A_1}(w = 1)$ results from FNAL/MILC collaboration [PRD89,114504 (2014)] has error within 2%.

$$h_{A_1}(w = 1) = 0.906(4)(12)$$

(12)

Simulation details
- Valence $b$ and $c$ quark: Fermilab action
- Valence light quark: AsqTad improved action
- Sea light quark: $N_f = 2 + 1$ AsqTad improved action

- The dominant systematic error is from the discretization of the charm quark.

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(12)

Simulation details

- Valence $b$ and $c$ quark : Fermilab action → Oktay Kronfeld action
- Valence light quark : AsqTad improved action → HISQ action
- Sea light quark : $N_f = 2 + 1$ AsqTad improved action → $N_f = 2 + 1 + 1$ HISQ

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The Fermilab action [El-Khadra, Kronfeld, and Mackenzie, PRD55, 3933 (1997)]

\[ S_{\text{Fermilab}} \equiv S_0 + S_E + S_B, \]  
\[ S_0 \equiv a^4 \sum_x \bar{\psi}(x) \left[ m_0 + \gamma_4 D_{\text{lat},4} - \frac{a}{2} \Delta_4 + \zeta \left( \gamma \cdot D_{\text{lat}} - \frac{r_s a}{2} \Delta^{(3)} \right) \right] \psi(x) \]  
\[ S_E \equiv -\frac{1}{2} c_E \zeta a^5 \sum_x \bar{\psi}(x) \alpha \cdot E_{\text{lat}} \psi(x), \quad S_B \equiv -\frac{1}{2} c_B \zeta a^5 \sum_x \bar{\psi}(x) i\Sigma \cdot B_{\text{lat}} \psi(x), \]  
\[ \Delta^{(3)}, \Delta_4 : \text{discretized versions of } D^2, D_4^2. \]

- \( b \) and \( c \) quark: \( m_Q \sim 1/a \gg \Lambda_{\text{QCD}} \) for typical lattice spacing \((0.04 \text{fm} < a < 0.12 \text{fm})\) in simulations. One can describes the lattice action by the heavy-quark effective theory,

\[ \mathcal{L}_{\text{Fermilab}} \equiv \bar{h}(D_4 + m_1)h + \frac{1}{2m_2} \bar{h}D^2 h + \frac{Z_B}{2m_B} \bar{h}i\sigma \cdot B h + \cdots \]  

- The discretization error can be estimated by using power counting parameter \( \lambda \sim a\Lambda_{\text{QCD}} \sim \Lambda_{\text{QCD}}/(2m_q) \) and \( \alpha_s \).
Oktay-Kronfeld action

Extend the lattice action from $\mathcal{O}(\lambda)$ (Fermilab action) \(\rightarrow\) $\mathcal{O}(\lambda^3)$ (OK action).

[Oktay and Kronfeld, PRD78, 014504 (2008)]

- $S_{\text{OK}} \equiv S_0 + S_B + S_E + S_6 + S_7$.

\[
S_6 \equiv c_1 a^6 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_{\text{lat},i} \Delta_i \psi(x) + c_2 a^6 \sum_x \bar{\psi}(x) \{\gamma \cdot D_{\text{lat}}, \Delta^{(3)}\} \psi(x) \\
+ c_3 a^6 \sum_x \bar{\psi}(x) \{\gamma \cdot D_{\text{lat}}, i \Sigma \cdot B_{\text{lat}}\} \psi(x) \\
+ c_{\text{EE}} a^6 \sum_x \bar{\psi}(x) \{\gamma_4 D_{\text{lat},4}, \alpha \cdot E_{\text{lat}}\} \psi(x),
\]

(16)

\[
S_7 \equiv a^7 \sum_x \bar{\psi}(x) \sum_i \left[ c_4 \Delta_i^2 \psi(x) + c_5 \sum_{j \neq i} \{i \Sigma_i B_{\text{lat},i}, \Delta_j\} \right] \psi(x).
\]

(17)

- Coefficients $c_i$ are fixed by matching dispersion relation, interaction with background field, and Compton scattering amplitude of on-shell quark through the tree level.
Current improvement

- **Simulation with the Fermilab action**: current improvement through $O(\lambda)$. Improved currents are constructed by improved quark fields [El-Khadra, Kronfeld, and Mackenzie, PRD55, 3933]

$$V_\mu = \bar{\Psi}_c \gamma_\mu \Psi_b, \quad A_\mu = \bar{\Psi}_c \gamma_\mu \gamma_5 \Psi_b,$$

where

$$\Psi_f = e^{m_{1f}a/2}(1 + d_{1f}a\gamma \cdot D_{\text{lat}})\psi_f, \quad f = b, c$$

And determine $d_{1f}$ by matching conditions.

- **Simulation with the OK action**: current improvement through $O(\lambda^3)$ is required.
Current improvement

- Tree-level relation between QCD operator and HQET operator is given by Foldy-Wouthouysen-Tani transformation

\[
\bar{c}\gamma_\mu b \doteq \bar{h}_c \Gamma h_b - \bar{h}_c \gamma_\mu \frac{\gamma \cdot D}{2m_b} h_b + \bar{h}_c \frac{\gamma \cdot \Delta}{2m_c} \gamma_\mu h_b + \cdots \tag{20}
\]

- Taking FWT transformation through \(\mathcal{O}(1/m_q^3)\) as ansatz, we introduced improved quark field [Jaehoon Leem, arXiv:1711.01777],

\[
\psi(x) = e^{m_1 a/2} \left[ 1 + d_1 a \gamma \cdot D_{\text{lat}} + \frac{1}{2} d_2 a^2 \Delta^{(3)} + \frac{1}{2} i d_B a^2 \Sigma \cdot B_{\text{lat}} + \frac{1}{2} d_E a^2 \alpha \cdot E_{\text{lat}} \\
+ d_{EE} a^3 \{ \gamma_4 D_{4\text{lat}}, \alpha \cdot E_{\text{lat}} \} + d_{rE} a^3 \{ \gamma \cdot D_{\text{lat}}, \alpha \cdot E_{\text{lat}} \} \\
+ \frac{1}{6} d_3 a^3 \gamma_i D_{\text{lat}} i \Delta_i + \frac{1}{2} d_4 a^3 \{ \gamma \cdot D_{\text{lat}}, \Delta^{(3)} \} + d_5 a^3 \{ \gamma \cdot D_{\text{lat}}, i \Sigma \cdot B_{\text{lat}} \} \\
+ d_6 a^3 [\gamma_4 D_{4\text{lat}}, \Delta^{(3)}] + d_7 a^3 [\gamma_4 D_{4\text{lat}}, i \Sigma \cdot B_{\text{lat}}] \right] \psi(x). \tag{21}
\]

- The term with \(d_3\) is necessary to remedy rotational symmetry breaking of lattice quark.
We calculate four-quark matrix elements for matching.

\[
\langle q(\eta_2, p_2) c(\eta_c, p_c) | \bar{\Psi}_c \Gamma \Psi_b | b(\eta_b, p_b) q(\eta_1, p_1) \rangle_{\text{lat}},
\]

(22)

where \( q \) represents a light spectator quark.

The matching condition implies [Jaehoon Leem, arXiv:1711.01777],

\[
\Psi = \left[ 1 - \frac{1}{2m_3} \gamma \cdot D + \frac{1}{8m_{D_\perp}^2} D^2 + \frac{i}{8m_{sB}^2} \Sigma \cdot B + \frac{1}{4m_{\alpha E}^2} \alpha \cdot E \right.
\]

\[
- \frac{\{\gamma_4 D_4, \alpha \cdot E\}}{8m_{\alpha EE}^3} - \frac{3\{\gamma \cdot D, D^2\}}{32m_{\gamma DD_\perp}^3} - \frac{3\{\gamma \cdot D, i \Sigma \cdot B\}}{32m_5^3} - \frac{\{\gamma \cdot D, \alpha \cdot E\}}{16m_{\alpha rE}^3}
\]

\[
+ \frac{[\gamma_4 D_4, D^2]}{16m_6^3} + \frac{[\gamma_4 D_4, i \Sigma \cdot B]}{16m_7^3} + w_1 \sum_i \gamma_i D_i^3 + w_2 [\gamma \cdot D, D^2] h + \cdots,
\]

(23)

The coefficients \( m_i \) and \( w_i \) : analytic functions of \((m_0 a, r_s, \zeta, c_i)\) and \(d_i\).

Determine \( d_i \) by the conditions that nonphysical terms vanish \((w_i \rightarrow 0)\), and all the mass-like terms be identical to the physical mass.
Numerical Simulation

- Sea quarks: Highly-improved-staggered quark (HISQ) action with $N_f = 2 + 1 + 1$ flavors. [MILC collab., PRD87, 054505 (2013)]

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- Valence quark for ($u, d, s$): HISQ action
- Valence quark for $b$ and $c$: Oktay-Kronfeld action
- Tune the Oktay-Kronfeld action: tune the bare masses of $b$ and $c$ numerically by tuning the energy spectrum of lattice meson.
- Calculate 3-point Green’s functions and determine hadronic matrix elements with the tuned action and the improved current.
We calculate following 3-point Green’s functions

\[ C_{A_j}^{B \rightarrow D^*}(t, \tau) \equiv \sum_{x,y} \langle O_{D^*}(0) A^{cb}_j(y, t) O_B^\dagger(x, \tau) \rangle, \]

\[ C_{A_j}^{D^* \rightarrow B}(t, \tau) \equiv \sum_{x,y} \langle O_B(0) A^{bc}_j(y, t) O_{D^*}^\dagger(x, \tau) \rangle, \]

\[ C_{V_4}^{B \rightarrow B}(t, \tau) \equiv \sum_{x,y} \langle O_B(0) V^{bb}_4(y, t) O_B^\dagger(x, \tau) \rangle, \]

\[ C_{V_4}^{D^* \rightarrow D^*}(t, \tau) \equiv \sum_{x,y} \langle O_{D^*}(0) V^{cc}_4(y, t) O_{D^*}^\dagger(x, \tau) \rangle, \]

where \( O_B \) and \( O_{D^*} \) are meson interpolating operators with a HISQ light quark and an Oktay-Kronfeld heavy quark.

The currents are given by improved quark fields

\[ A^{cb}_j \equiv \bar{\Psi}_c \gamma_5 \gamma_j \Psi_b, \]

\[ V^{bb}_4 \equiv \bar{\Psi}_b \gamma_4 \Psi_b. \]
Control excited-state contamination

- Generate zero momentum meson propagators and do the multi-states fitting [Sungwoo Park, et al., Lattice 2018]

\[ C_{B(\text{or } D^*)}^{2pt}(t, 0) = |A_0|^2 e^{-M_0 t} \left( 1 + \frac{A_2}{A_0} \right)^2 e^{-\Delta M_2 t} + \frac{A_4}{A_0} \right)^2 e^{-(\Delta M_2 + \Delta M_4) t} + \ldots \]

\[ - (-1)^t \left( \frac{A_1}{A_0} \right)^2 e^{-\Delta M_1 t} - (-1)^t \left( \frac{A_3}{A_0} \right)^2 e^{-(\Delta M_1 + \Delta M_3) t} + \ldots \right) + (t \leftrightarrow T - t) \]

\( D^* \) 3+2 multistate fit

\( 5 \leq t \leq 18 \)

\( M_{D^*}^0 = 1.1280(43) \)

(a) \( m_{\text{eff}}(t) \equiv \frac{1}{2} \ln |C^{2pt}(t)/C^{2pt}(t + 2)| \). The horizontal line is the ground state mass.

(b) The excited state masses from the 3+2-states fit using empirical Bayesian priors. (PNDME collab., PRD95, 074508 (2017))
Control excited-state contamination

- Fit 3-point function including 2+1 states for $|B_m\rangle$ and $|D_n^*\rangle$ with $n, m = 0, 1, 2$.

$$C_{A_j}^{B\rightarrow D^*}(t_s, \tau) = A_0^{D^*} A_0^B \langle D_0^* | A_j^{cb} | B_0 \rangle e^{-M_{B_0}(\tau-t_s)} e^{-M_{D_0}^* t_s}$$

$$- A_0^{D^*} A_1^B \langle D_0^* | A_j^{cb} | B_1 \rangle (-1)^{\tau-t_s} e^{-M_{B_1}(\tau-t_s)} e^{-M_{D_0}^* t_s}$$

$$- A_1^{D^*} A_0^B \langle D_1^* | A_j^{cb} | B_0 \rangle (-1)^{t_s} e^{-M_{B_0}(\tau-t_s)} e^{-M_{D_1}^* t_s}$$

$$+ A_1^{D^*} A_1^B \langle D_1^* | A_j^{cb} | B_1 \rangle (-1)^{\tau} e^{-M_{B_1}(\tau-t_s)} e^{-M_{D_1}^* t_s}$$

$$+ A_2^{D^*} A_0^B \langle D_2^* | A_j^{cb} | B_0 \rangle e^{-M_{B_0}(\tau-t_s)} e^{-M_{D_2}^* t_s}$$

$$+ A_0^{D^*} A_2^B \langle D_0^* | A_j^{cb} | B_2 \rangle e^{-M_{B_2}(\tau-t_s)} e^{-M_{D_0}^* t_s}$$

$$- A_2^{D^*} A_1^B \langle D_2^* | A_j^{cb} | B_1 \rangle (-1)^{\tau-t_s} e^{-M_{B_1}(\tau-t_s)} e^{-M_{D_2}^* t_s}$$

$$- A_1^{D^*} A_2^B \langle D_1^* | A_j^{cb} | B_2 \rangle (-1)^{t} e^{-M_{B_2}(\tau-t_s)} e^{-M_{D_1}^* t_s}$$

$$+ A_0^{D^*} A_2^B \langle D_0^* | A_j^{cb} | B_2 \rangle e^{-M_{B_2}(\tau-t_s)} e^{-M_{D_0}^* t_s} + \ldots$$ (26)

- In the fitting, the 2pt amplitudes $A$ and masses $M$ are constant fixed from the 2-point function analysis.
Preliminary results for $h_{A_1}(1)$

- The zero recoil form factor $h_{A_1}(1)$ can be determined by

$$\frac{\langle D^*|A^j_{cb}|\bar{B}\rangle\langle \bar{B}|A^j_{bc}|D^*\rangle}{\langle D^*|V^4_{cc}|D^*\rangle\langle \bar{B}|V^4_{bb}|\bar{B}\rangle} = \left|\frac{h_{A_1}(1)}{\rho_{A_j}}\right|^2$$  \hspace{1cm} (27)

where $\rho_{A_j}^2 = \frac{Z_{A_j}^{j_{cb}} Z_{A_j}^{j_{bc}}}{Z_{V_{cc}}^{4} Z_{V_{bb}}^{4}} = 1 + \mathcal{O}(\alpha_s)$ is very close to 1.

- Setting $\rho_{A_j}^2 = 1$ [Sungwoo Park, et al., Lattice 2018]

![Graph showing data points and error bars for $|h_{A_1}(1)|$ vs $a$ [fm]. The graph includes data from FNAL/MILC '14, HPQCD '18, and HPQCD '18, $M_\pi \approx 310$ MeV, with $\mathcal{O}(\lambda^2)$-improved and $\mathcal{O}(\lambda^1)$-improved cases.]

[Fermilab-MILC collab., PRD 89, 114504 (2014)] [HPQCD collab., PRD 97, 054502 (2018)]
Summary

- We are calculating $\bar{B} \to D^* \ell \bar{\nu}$ semileptonic decay form factor at zero recoil using the Oktay-Kronfeld action for $b$ and $c$ quarks.

- We improved the lattice current through $O(\lambda^3)$ in the same level as the Oktay-Kronfeld action. (Numerical analysis with the $O(\lambda^3)$ terms in the current are under progress.)

- We controlled the excited-state contamination by multi-state fitting.

- The preliminary results with the improved current through $O(\lambda^2)$ is consistent with the results from FNAL/MILC and HPQCD.

- Our next targets are
  - Continuum-chiral extrapolation.
  - Calculate renormalization factor $\rho_{Aj}$.
  - Non-zero recoil study. ($\bar{B} \to D$ and $\bar{B} \to D^*$)
  - Calculate beyond the standard model contributions from scalar or tensor type currents.

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