New Trend in Topological Phases

: Geometric and Higher

Gil Young Cho

POSTECH

"Topological Phase" in Google Scholar

46,000 results (2014 ~)

Ex: Computational Groups @ KIAS

Prof. Kwon Park, Prof. Youngwoo Son

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My Small Contributions

Here and There

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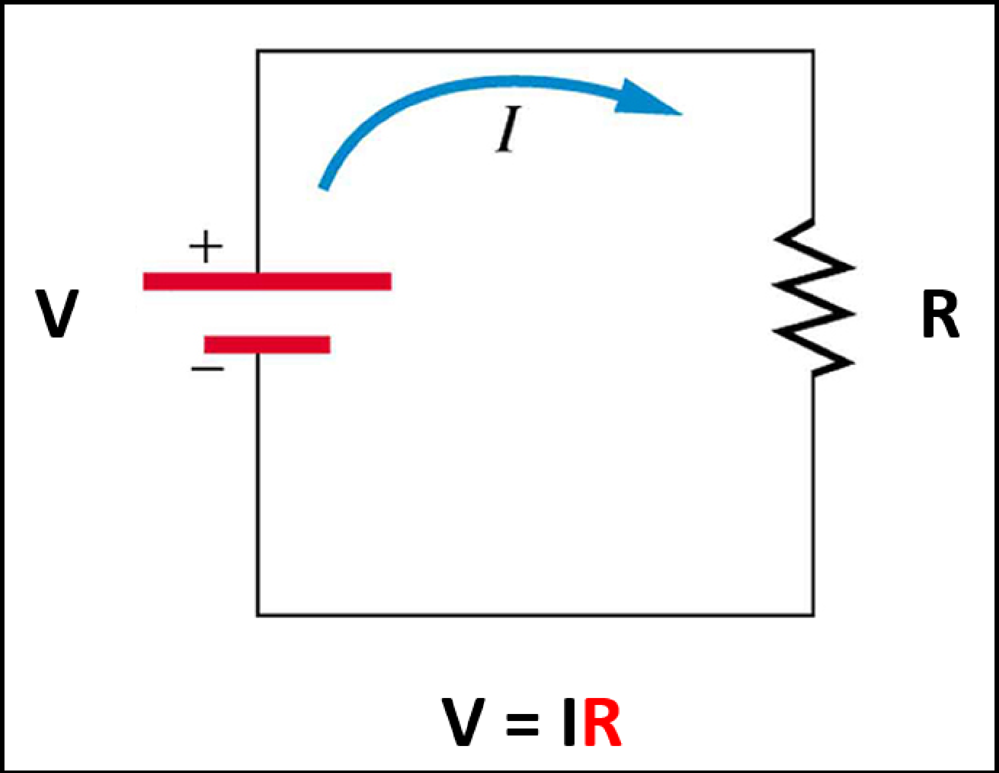
2. New Trend in Topological Phases

3. Outlooks and Conclusions

1. Introduction to Topological Phases

“Stuff” in Condensed Matter Physics

Some Solid “Rock”

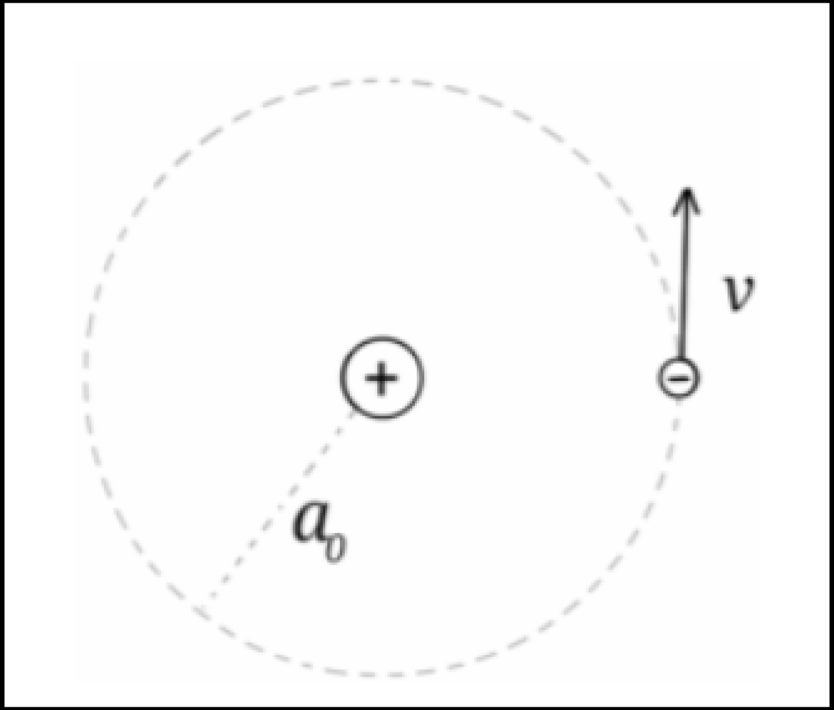


Measurement

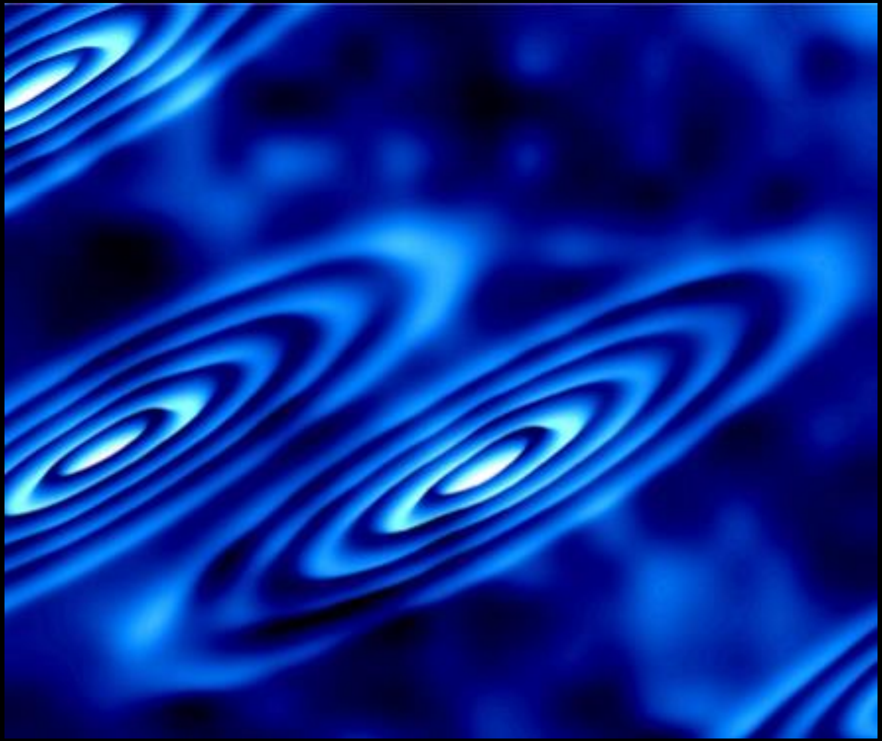
What can be *possibly surprising*?

“Topological Phase”

: **Phase of Matter** with certain **Shape of Quantum Wavefunctions**



Classical Electron

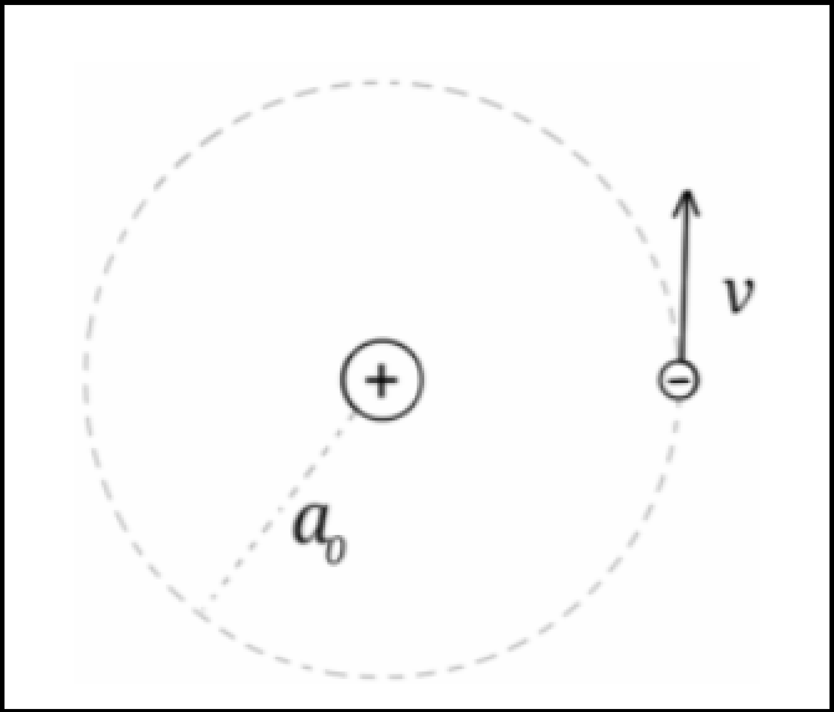


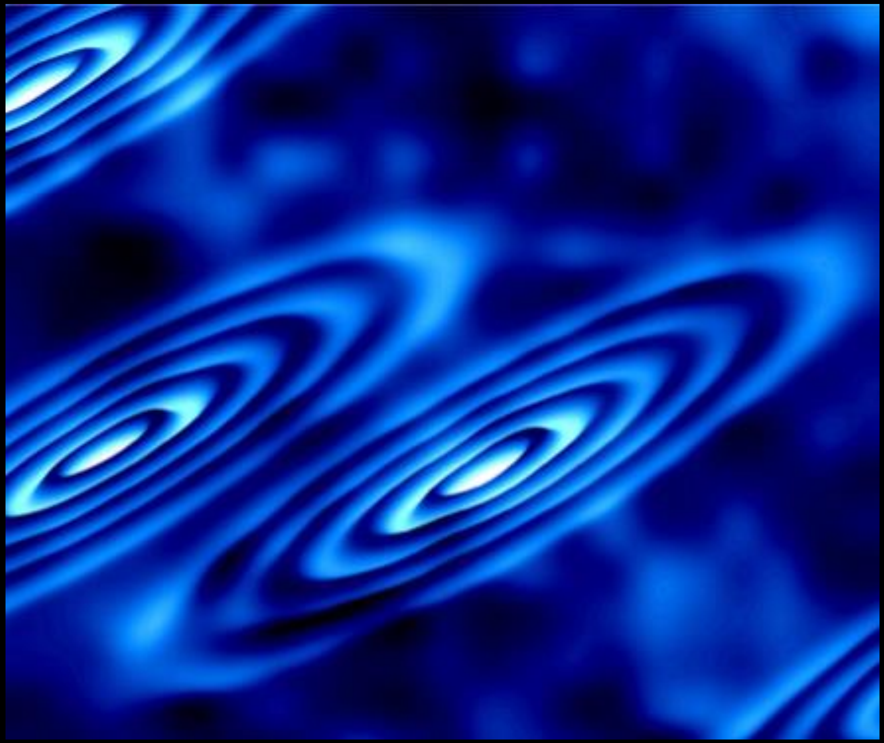
Electrons’ Orbits on Bismuth

[STM from Yazdani’s group (2016)]

"Topological Phase"

: **Phase of Matter** with certain **Shape of Quantum Wavefunctions**





Classical Electron

Electrons' Orbits on Bismuth

[STM from Yazdani's group (2016)]

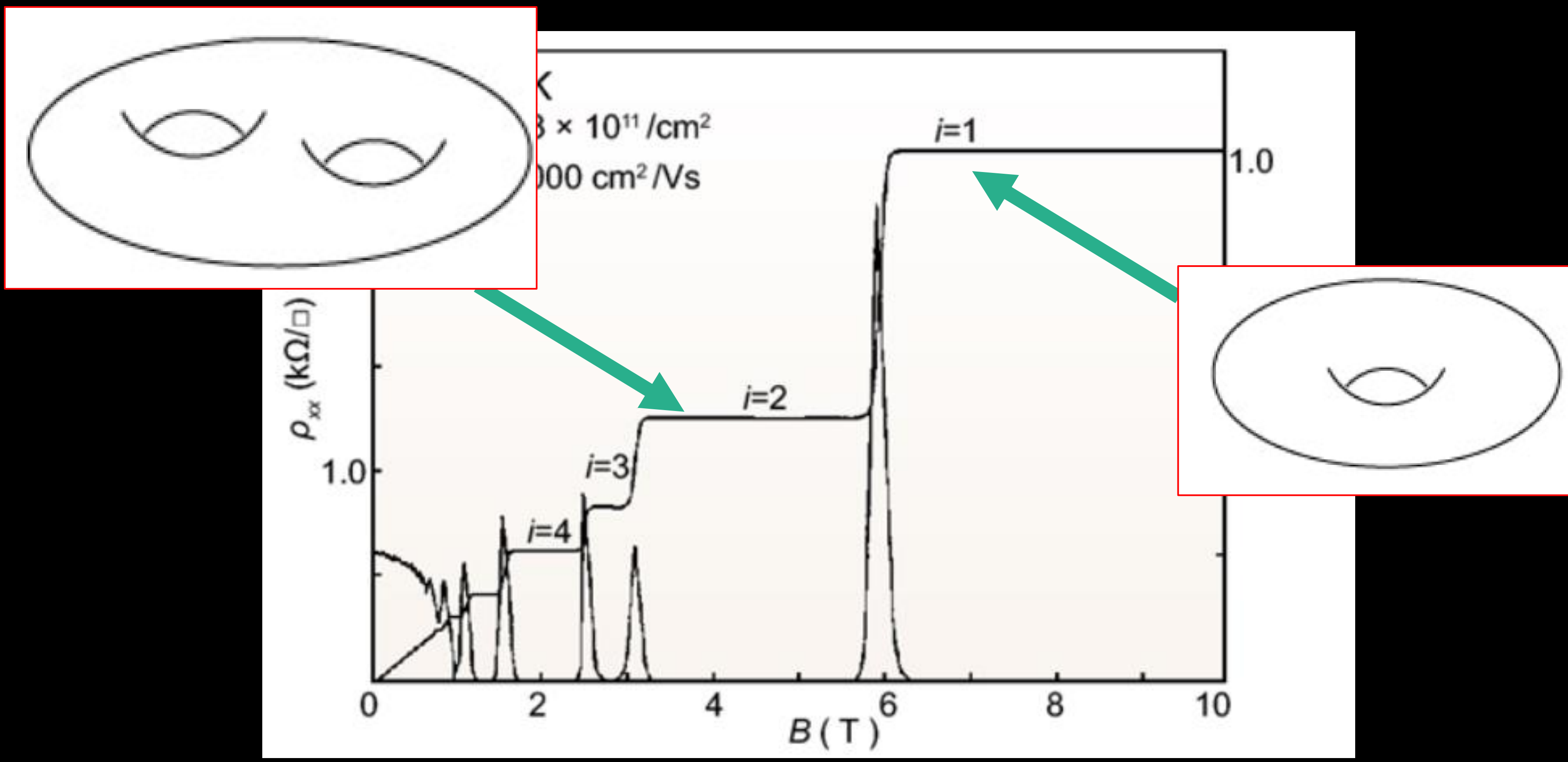
∃ **Striking Consequences** [e.g., Nobel Physics Prize (2016)]

Phenomenology of Topology of Quantum State

= “(Rough) Shape”

Consequences of Shapes/Topology

Ex: **2d electron gas under magnetic field**

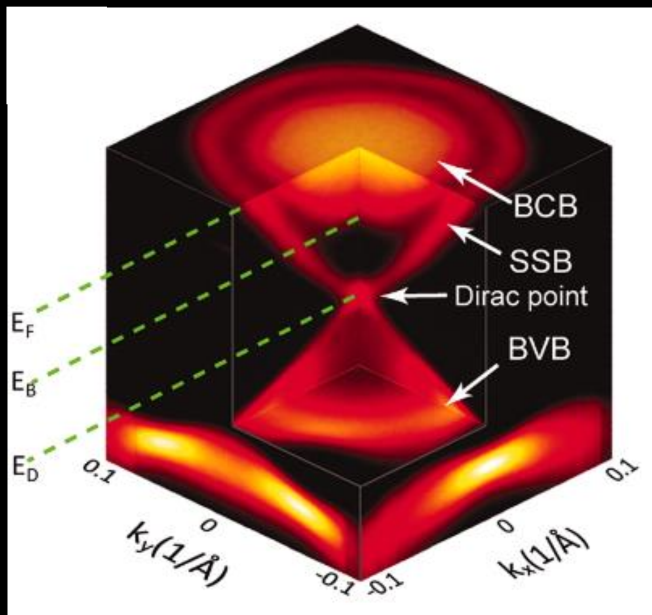


$$J_x = \rho_{xy} E_y \quad \text{and} \quad \rho_{xy} = \frac{h}{e^2} \times \frac{1}{N}$$

Ref. Review by MacDonald (1994)

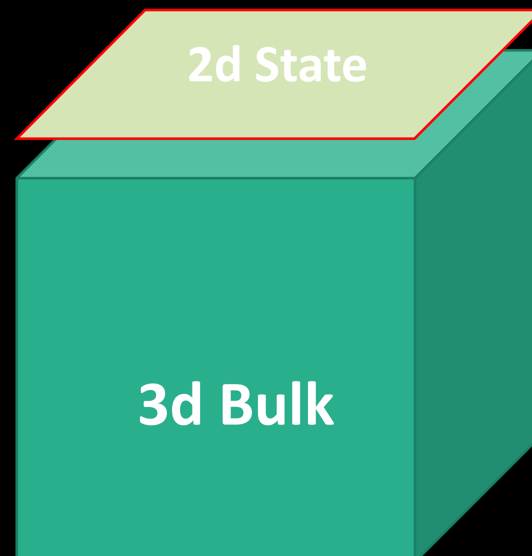
Consequences of Shapes/Topology

Ex: Anomalous Metallic Boundary States



Topological Band Insulator

Two-component Complex Fermion



2D state must be metallic

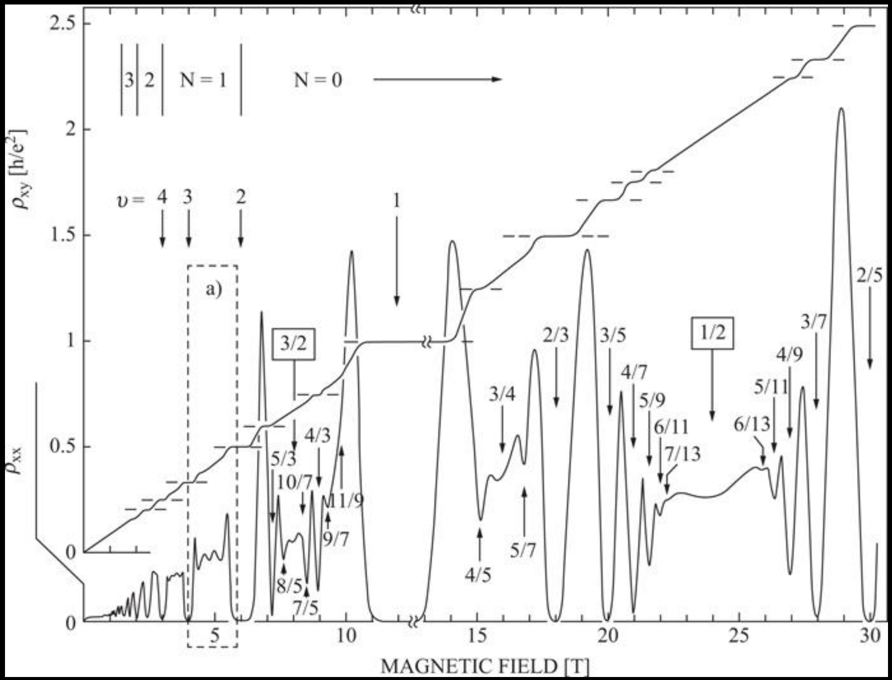
$$L = \bar{\Psi}\, i\gamma^{\mu} D_{\mu}\, \Psi$$

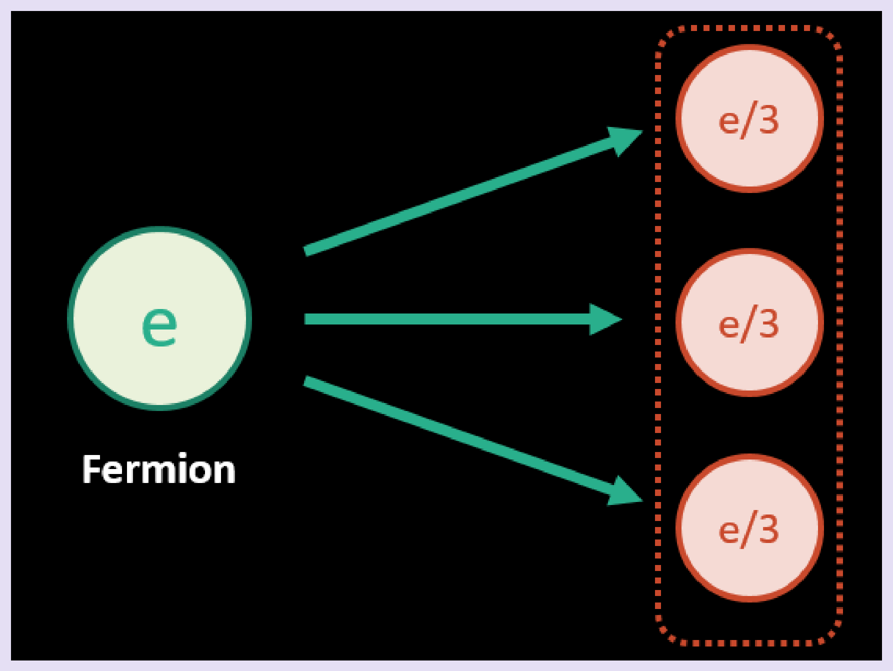
[Parity Anomaly, Redlich (1994)]

[cf. Hsieh, GYC, and Ryu (2016); Witten (2016)]

Consequences of Shapes/Topology

Ex: Fractionalization/Deconfinement





"Emergent Particles" = "fractional charge" + "fractional statistics"

[neither Boson nor Fermion]

(maybe) Useful in Quantum Computation

Ref. Review by Wen (1994)

Robustness of Topological Phases

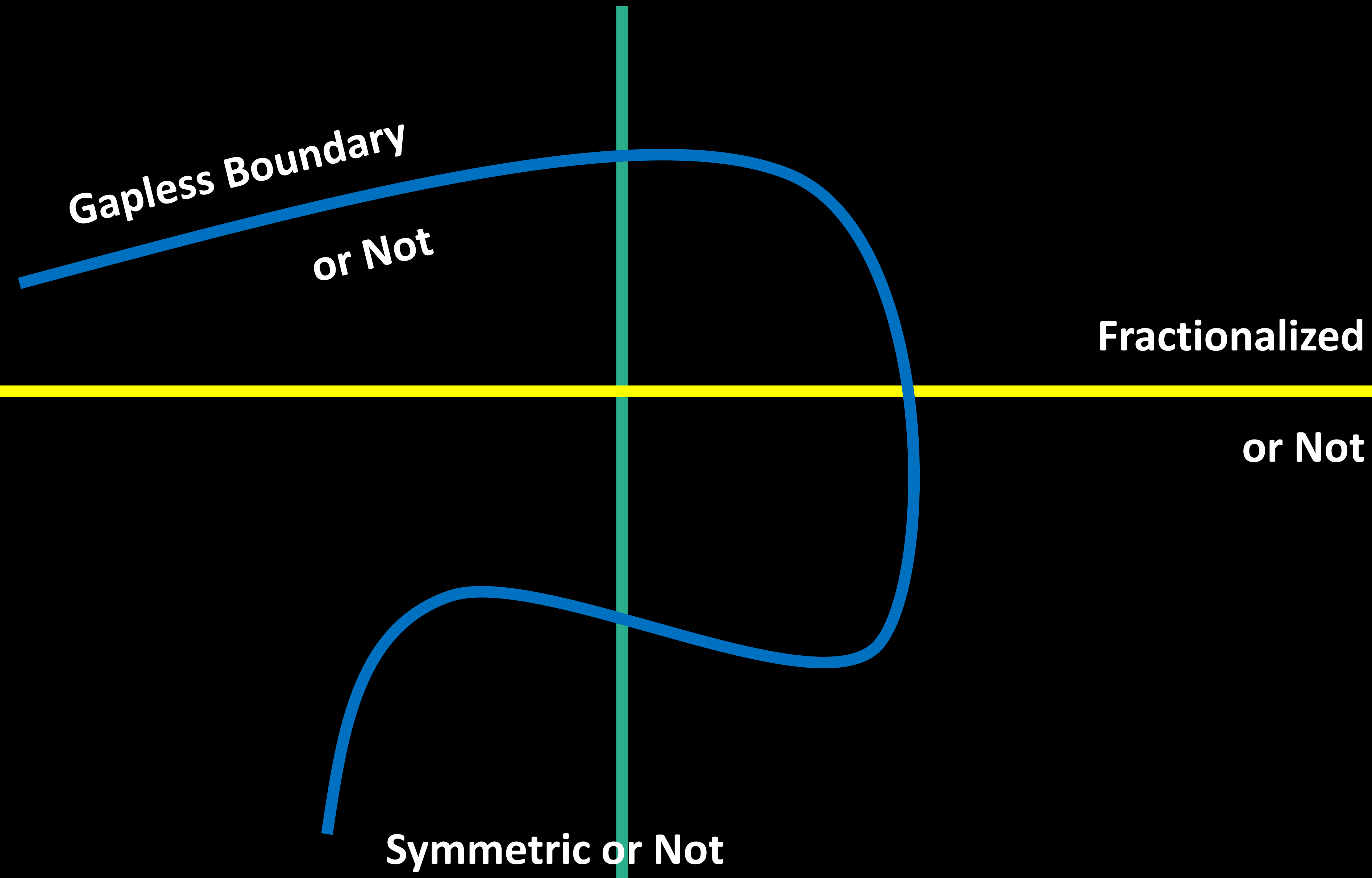
: Independent of “geometry”

“Only the global topology (of space, wavefunction, Hamiltonian) matters.”

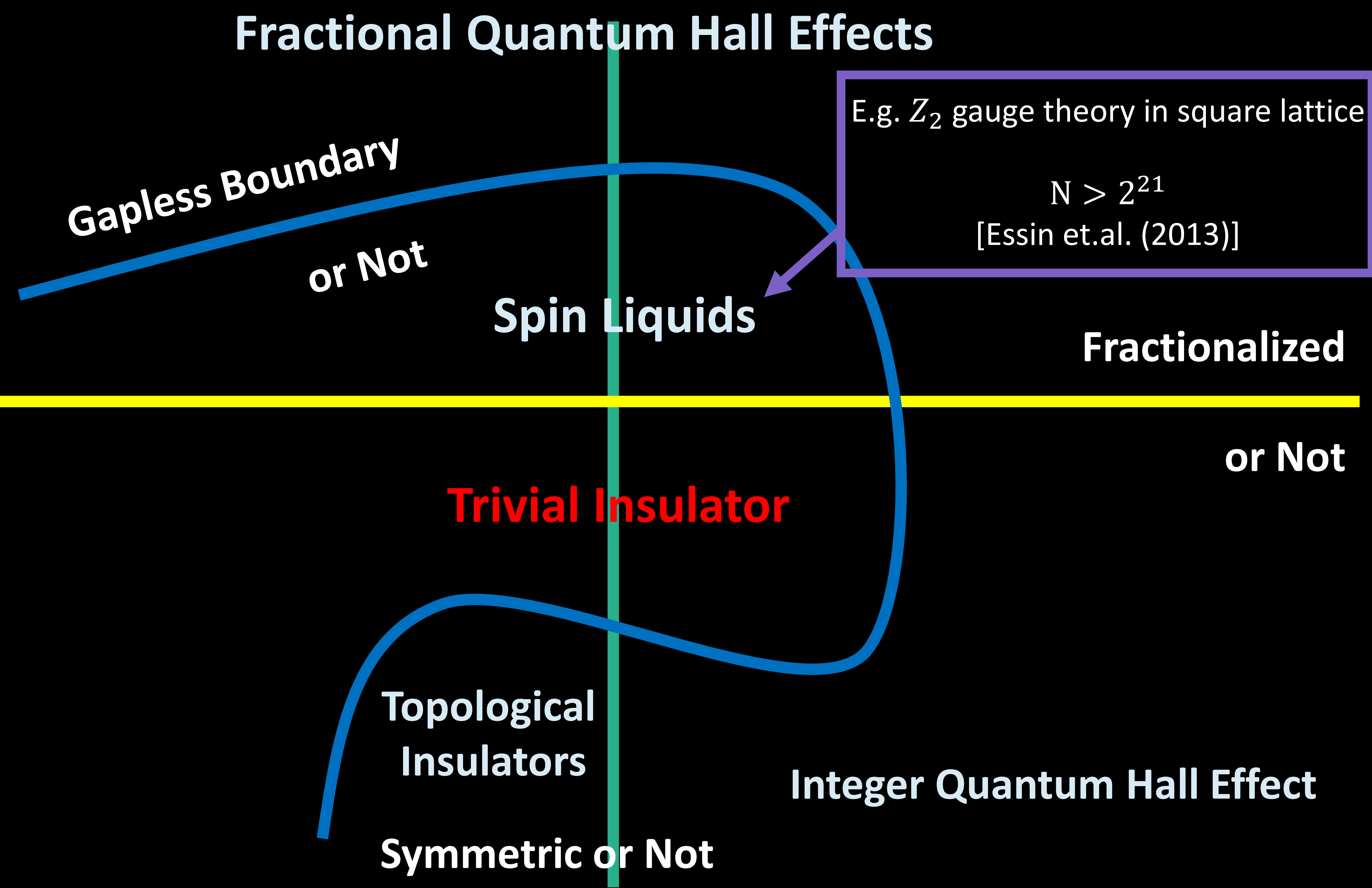
[disorders, chemical details, surface conditions etc, shouldn’t matter]

So how many such states are there?

So, how many different topological phases are there?



So, how many different topological phases are there?



How many topological insulators/superconductors are there?

Symmetry				d							
AZ	Θ	Ξ	Π	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Free Electrons in Various Dims.

$d = D + 1$	$\Omega_d^{\text{Spin}}(pt)$	$\Omega_d^{\text{Pin}^-}(pt)$	$\Omega_d^{\text{Pin}^+}(pt)$	$\Omega_d^{\text{Spin}}(B\mathbb{Z}_2)$
1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}_2^2$
2	$\mathbb{Z}_2$	$\mathbb{Z}_8$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$
3	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_8$
4	$\mathbb{Z}$	0	$\mathbb{Z}_{16}$	$\mathbb{Z}$
5	0	0	0	0
6	0	$\mathbb{Z}_{16}$	0	0
7	0	0	0	$\mathbb{Z}_{16}$
8	$\mathbb{Z}^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2 \times \mathbb{Z}_{32}$	$\mathbb{Z}^2$
9	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	0	$\mathbb{Z}_2^4$
10	$\mathbb{Z}_2^2 \times \mathbb{Z}$	$\mathbb{Z}_2 \times \mathbb{Z}_8 \times \mathbb{Z}_{128}$	$\mathbb{Z}_2^3$	$\mathbb{Z}_2^4 \times \mathbb{Z}$

Interacting Electrons in Various Dims.

symmetry group	1+1D	2+1D	3+1D	4+1D
0	0	Z	0	Z_2
$U(1) \rtimes Z_2^T$	Z_2	Z_2	$2Z_2 + Z_2$	$Z \oplus Z_2 + Z$
Z_2^T	Z_2	0	$Z_2 + Z_2$	0
Z_n	0	Z_n	0	$Z_n + Z_n$
$U(1)$	0	Z	0	$Z + Z$
$SO(3)$	Z_2	Z	0	Z_2
$SO(3) \times Z_2^T$	$2Z_2$	Z_2	$3Z_2 + Z_2$	$2Z_2$
$Z_2 \times Z_2 \times Z_2^T$	$4Z_2$	$6Z_2$	$9Z_2 + Z_2$	$12Z_2 + 2Z_2$

Bosonic Topological States

Of course, there are:

Non-Equilibrium versions, too
[Non-Equilibrium Bosons/Fermions]

(Partial) Classifications for fractional states

Are we done ?

Are we done ?

No.

A lot to work on.

2. New Trend in Topological Phases

: Geometric and Higher

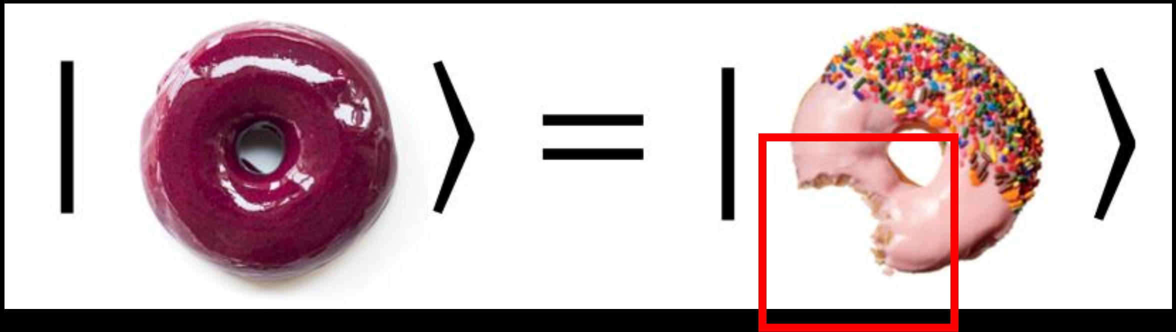
= "more details" than "topology" (Abused here)

E.g. curvature, torsion, and metric

E.g. crystal structure, and lattice defects

Q. Are “*Topological Phases*” truly “*Topological*”?

I.e.,



?

geometric features

No.

Illustrate this in:

Fractional Quantum Hall States

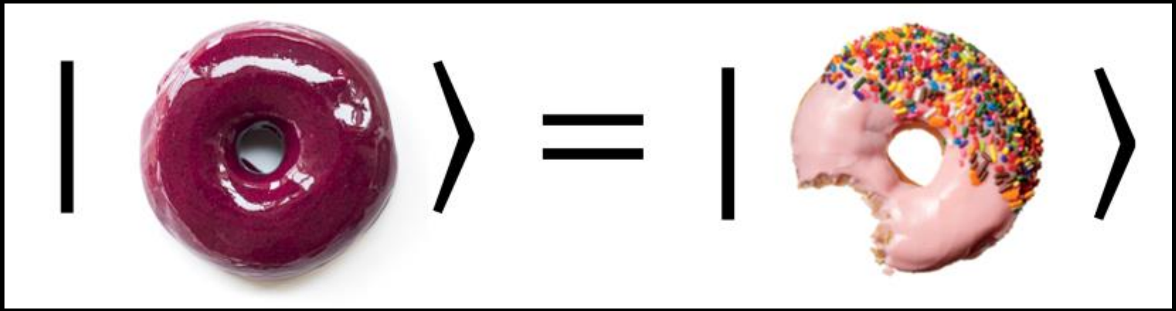
Fractional Quantum Hall States:

1. "**Birth place**" for the term "**Topological Phase**"

2. **2D Electrons** under uniform magnetic field

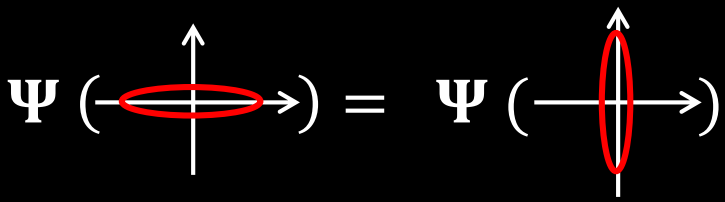
3. **Exotic:** Emergent fractional excitations with fractional statistics

4. **Topological**: **successful theoretical descriptions imply**

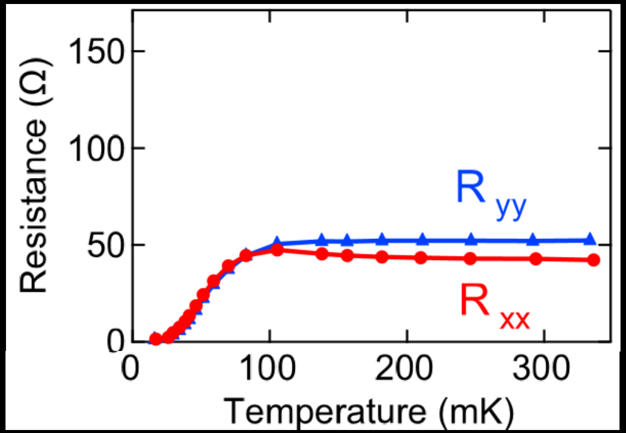


The wave-functions are independent of geometry !

Rotation Symmetry of “Topological Phase”



[rotational symmetry]



Composite Particle Theory:

Composite Bosons:

Zhang, Hansson, Kivelson (1989)

[**cited: 1000 times**]

Wen (1992, 1995, 2004)

[**cited: 1200 times**]

Composite Fermions

Jain (1989)

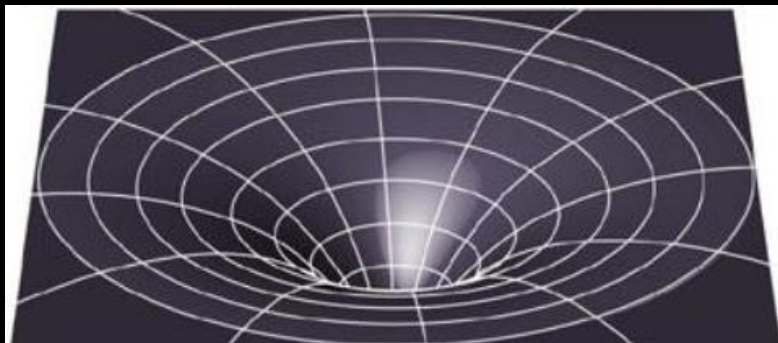
[**cited: 2200 times**]

Lopez, Fradkin (1991, 2013)

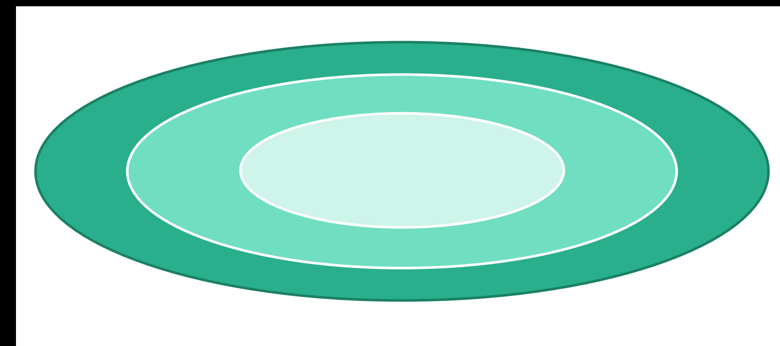
[**cited: 1700 times**]

..and many other papers & textbooks.

Numerically, ∃ Hidden Geometric Response



$\neq$



Curvature **Flat**

: Electric charge ∝ Curvature

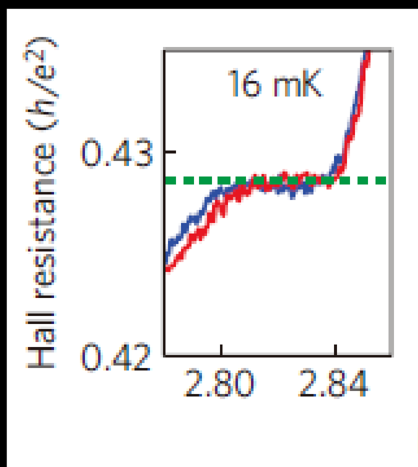
Ref. Wen, Zee (1992); GYC, You, Fradkin (2014)

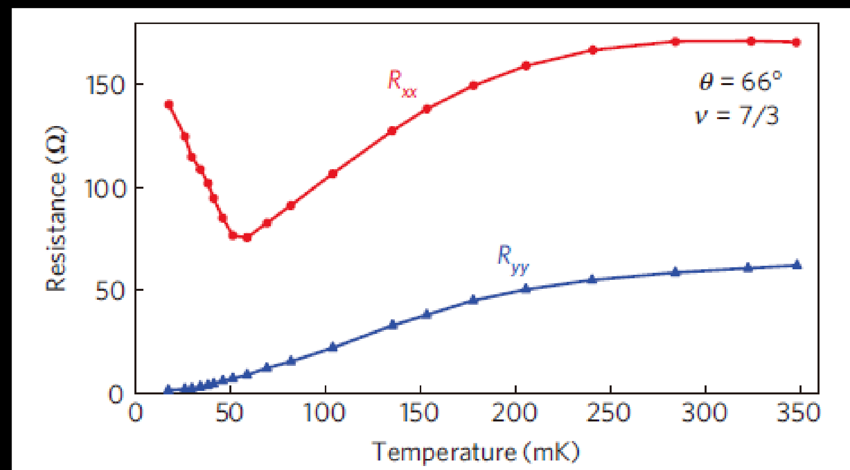
Quantum Hall States are Not Blind to Geometry

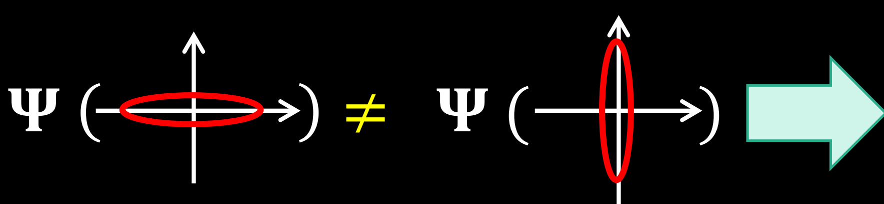
Experimental Finding of non-topological Quantum Hall State:

Evidence for a fractionally quantized Hall with anisotropic longitudinal transport

Jing Xia[1*†], J. P. Eisenstein[1], L. N. Pfeiffer[2] and K. W. West[2] [Nature Physics, 2011]







Not "*Topological*"

You, GYC, Fradkin, PRX (2014)

Obvious Contradiction to “being topological”

Composite Bosons:

Zhang, Hansson, Kivelson (1989)
[**cited: 1000 times**]

Wen (1992, 1995, 2004)
[**cited: 1200 times**]

Composite Fermions

Jain (1989)
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Lopez, Fradkin (1991, 2013)
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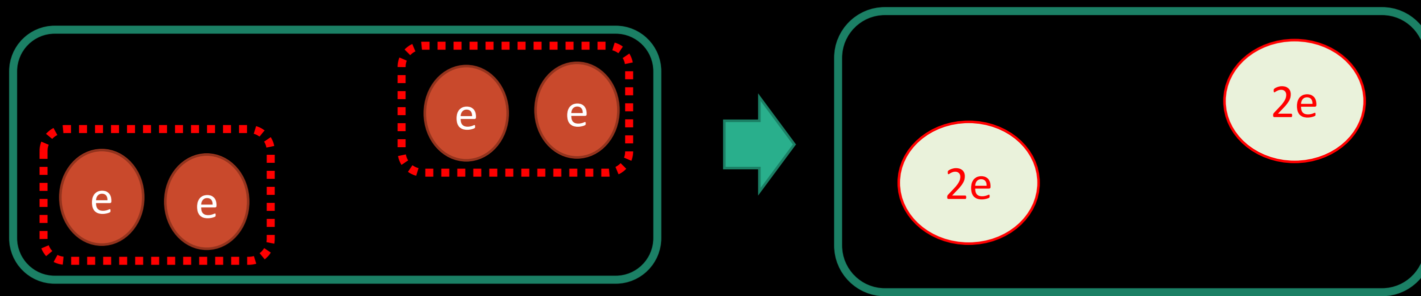
..and many other papers & textbooks.

Need to develop a new theory

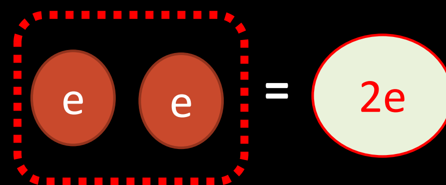
What is the composite particle theory?

Standard Composite Particle Theory

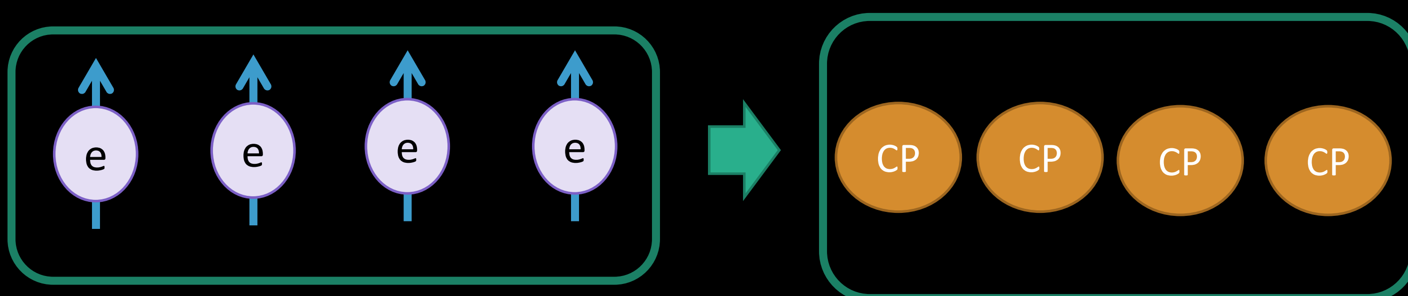
Landau-Ginzburg theory of BCS superconductors:



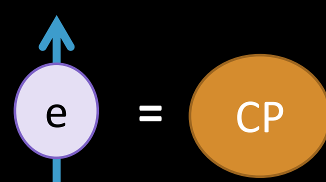
Cooper pair:



Composite particle theory: [Zhang, Hansson, Kivelson (1989), Wen (1992), (1995)]

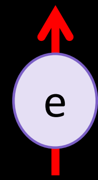


Flux attachment :



, "composite particle"

Composite Particle Theory:

This "composite particle" 

leads to "Chern-Simons Theory"

Electromagnetic gauge

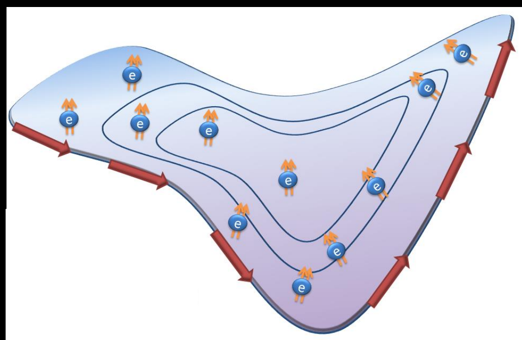
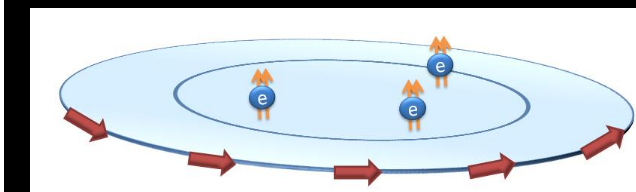
$$L = -\frac{k}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + A_\mu \boxed{\frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu b_\lambda} + \cdots$$

J^μ: electron current

Topological: no data about geometry g_{ij}

[Zhang, Hansson, Kivelson (1989), Wen (1992), (1995)]

I.E., Composite Particle Theories predict…

$\Big|$  $\Big\rangle$ $=$ $\Big|$  $\Big\rangle$

$$g_{ij} \neq \delta_{ij} \qquad\qquad g_{ij} = \delta_{ij}$$

[isotropic and flat]

Blind to Geometry = Wrong !

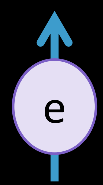
Cf. Hall conductance, excitation types, degeneracy

GYC, You, and Fradkin (2014); GYC, invited talk at APS MAR meeting (2015)

New Composite Particle Theory:

New theory:

Standard theory

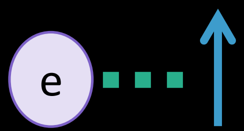


Point particle

Not working correctly!

New theory

[GYC, You, and Fradkin (2014, 2016)]



"Stick-like" object

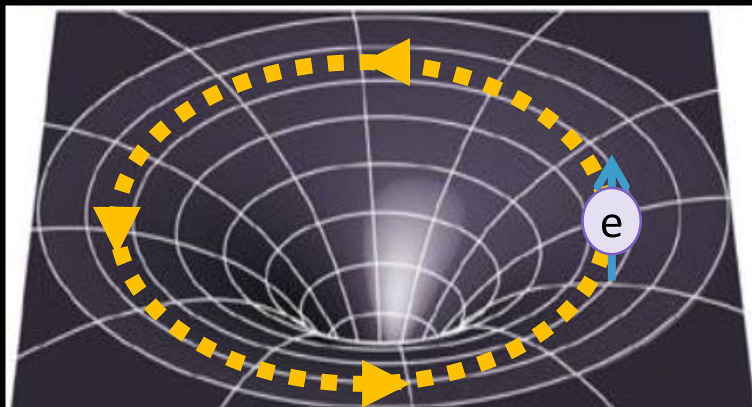
[cf. "Framing"]

Difference between the two?

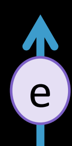
GYC, You, and Fradkin (2014); GYC, invited talk at APS MAR meeting (2015)

Curved space with curvature

Standard composite particle theory
[point particle]

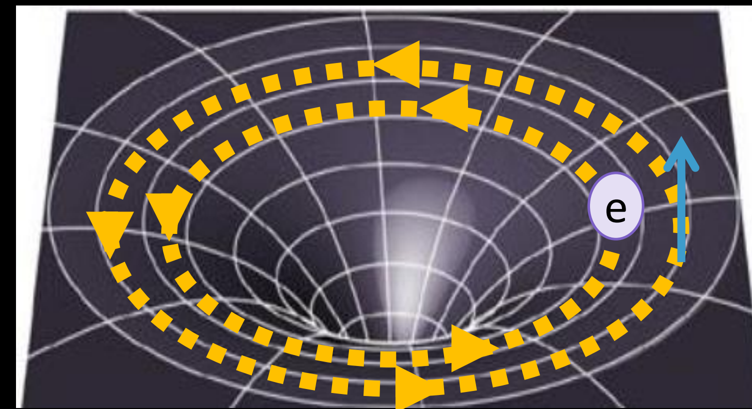


From the rest frame of the composite particle:

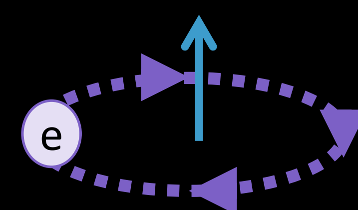


Nothing interesting happens.

New composite particle theory
[Stick-like structure]



[GYC, You, Fradkin (2014)]



Electron goes around the flux !
["internal rotation"]

Additional Berry phase from curvature of the geometry !

More formally, I can compute:

The quantum amplitude $P[C]$ **for the composite particle to move along curve C**

$$L = \frac{1}{4\pi \cdot \boldsymbol{k}} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda - j^\mu a_\mu$$

[Witten (1989), Polaykov (1988), Dunne, Jackiw, and Trugenberger (1989)]

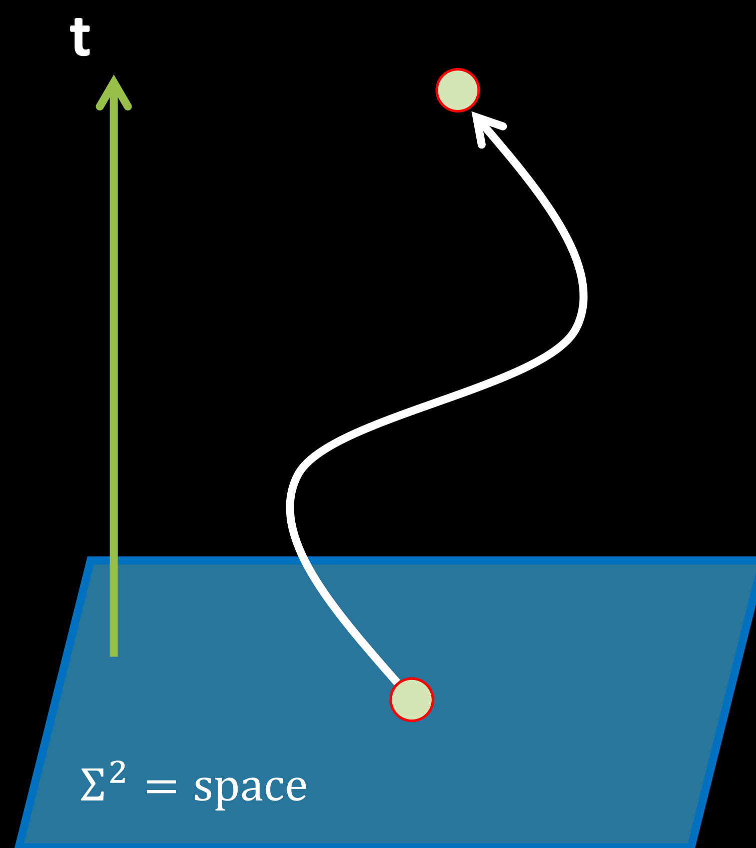
$$\text{P[C]} \propto [\text{Linking}] \cdot \mathbf{[Torsion]}$$

function of g_{ij}

$$\propto \exp\left(-i \int_C \boldsymbol{dx^\mu} \left[\boldsymbol{A_\mu} + \frac{\boldsymbol{k}}{\boldsymbol{2}} \boldsymbol{\omega_\mu}\right]\right)$$

…couple directly to $\boldsymbol{\omega_\mu}$ as like $\boldsymbol{A_\mu}$!

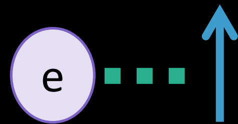
[**consistent with spin-statistics relations**]



Composite particle moving along C

GYC, You, and Fradkin (2014); GYC, invited talk at APS MAR meeting (2015)

New Composite Particle Theory: [GYC, You, Fradkin (2014)]

New "composite particle" 

leads to…

A geometric theory:

Electromagnetic gauge

$$L = -\frac{k}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \left(A_\mu + \frac{k}{2}\omega_\mu\right) \frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu b_\lambda + \cdots$$

J^μ: electron current

Spin connection: ω_μ

[a function of g_{ij}]

Explain: Electric Charge localized at Curvature

GYC, You, and Fradkin (2014)

New Composite Particle Theory: [GYC, You, Fradkin (2014)]

1) Explains:

- Curvature Response of quantum Hall states [“Spin” or “Shift”]

- Geometric Torsion Response [“Viscosity”]

- Quantized Thermal Hall Response [cf. Kitaev spin liquid, 5/2-filled non-Abelian states]

[Ref. Gromov, GYC, You, Abanov, Fradkin, PRL (2015)]

- Existence of Anisotropic Quantum Hall States

2) Predicts:

- New fractional excitation in Anisotropic Quantum Hall states

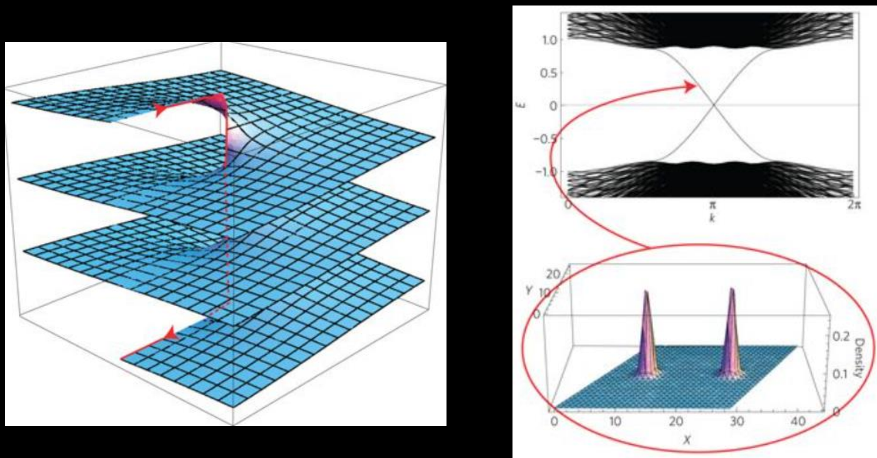
[Ref. GYC, Parrikar, You, Leigh, Hughes (2014); You, GYC, Fradkin (2016)]

- Transport Signature, Excitation Spectrum in Anisotropic Quantum Hall states

[Ref. You, GYC, Fradkin, PRX (2016)]

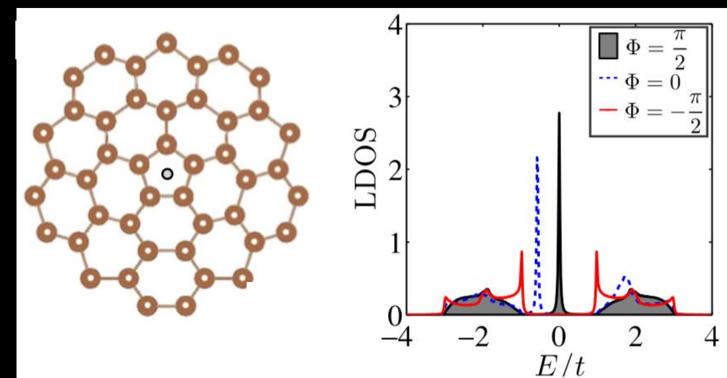
3) Many Topological Phases are "Geometric"

Screw Dislocation in Topological Band Insulator



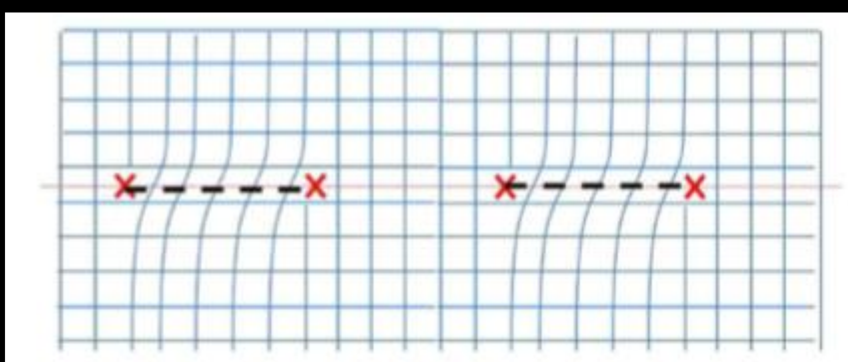
[Ran et.al., Nat. Phys. (2009)]

Disclination in Quantum Spin Hall Eff.



[Ruegg et.al., PRL (2013)]

Parafermion @ Dislocation in quantum Hall states

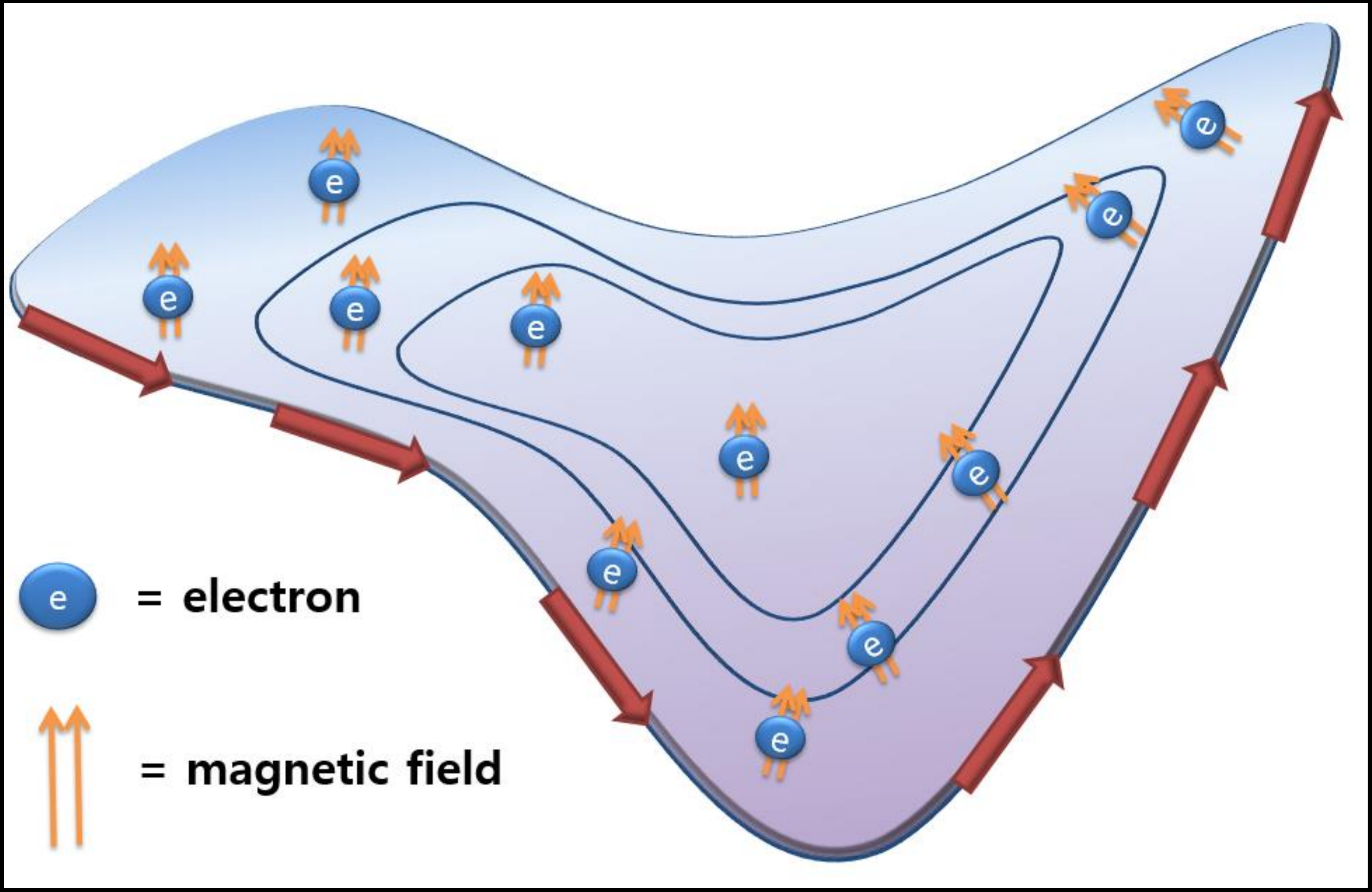


e.g. $\gamma^4 = 1$ instead of Majorana $\gamma^2 = 1$

[Barkeshli and Qi, PRX (2012)]

Cf. Dislocation in Weyl semimetals

Topological States are not only sensitive to "geometry"



But it reveals **new physics of topological states**

Zooming-Out:

Can "Geometry" play a *more essential role*?

⇨ **Physics from "*Crystal*" & "*Crystal Symmetry*" ?**

Zooming-Out:

Can "Geometry" play a *more essential role*?

➡ **Physics from "*Crystal*" & "*Crystal Symmetry*" ?**

(Maybe) Lattice is better than QFT.

Focusing more on crystals…

A New Class of Matter: *Higher-Order Topology*

& Definition of "Multipoles" in Crystal

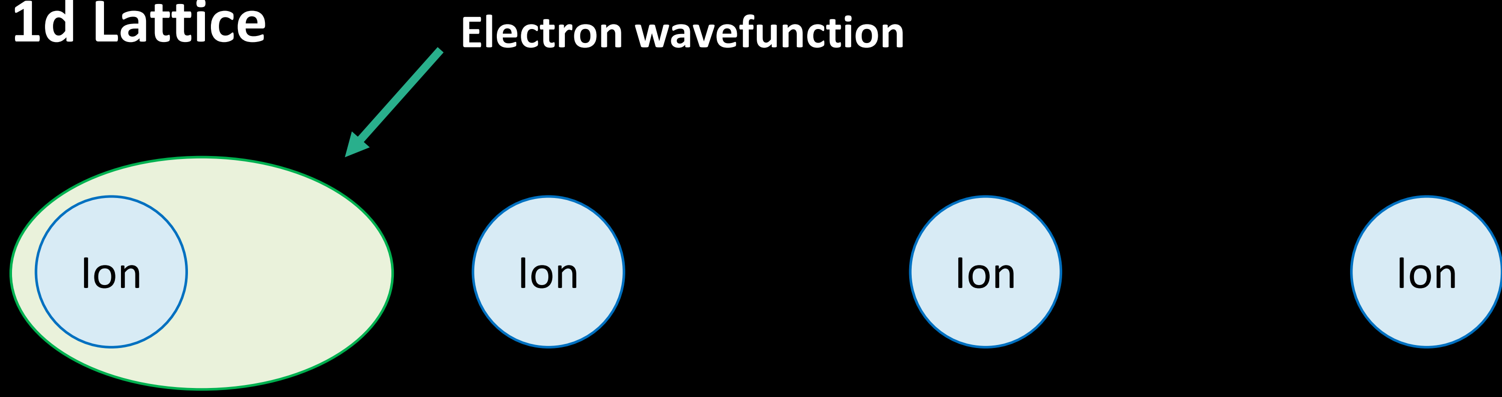
Byungmin Kang

Hyun-Jung Kim

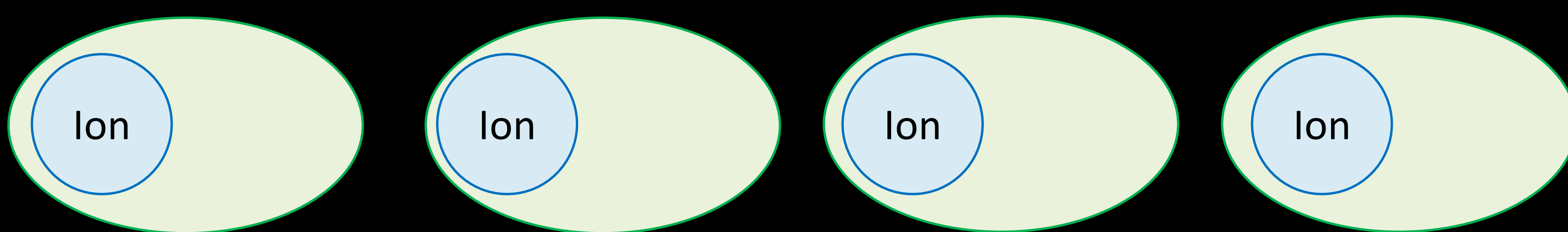
Simple-Minded Picture on Crystals

Electron in Crystal:

Ex: 1d Lattice



Due to translation symmetry:



How do we characterize the electron distribution?

Electron in Open Space:

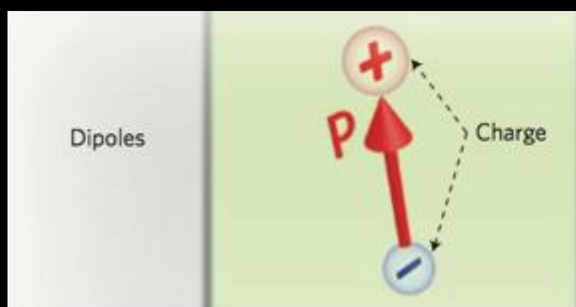
Multipole expansion

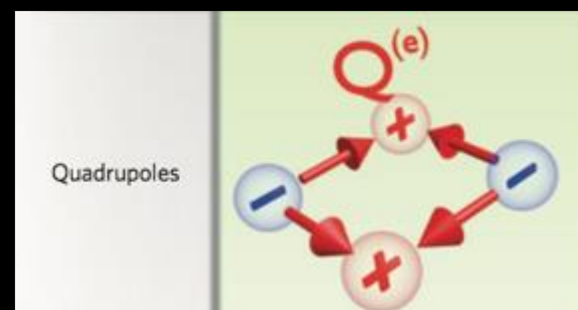
From Wikipedia, the free encyclopedia

$$q_{\text{tot}} \equiv \sum_{i=1}^{N} q_i$$

$$P_\alpha \equiv \sum_{i=1}^{N} q_i r_{i\alpha}$$

$$Q_{\alpha\beta} \equiv \sum_{i=1}^{N} q_i \left(3 r_{i\alpha} r_{i\beta} - \delta_{\alpha\beta} r_i^2\right)$$

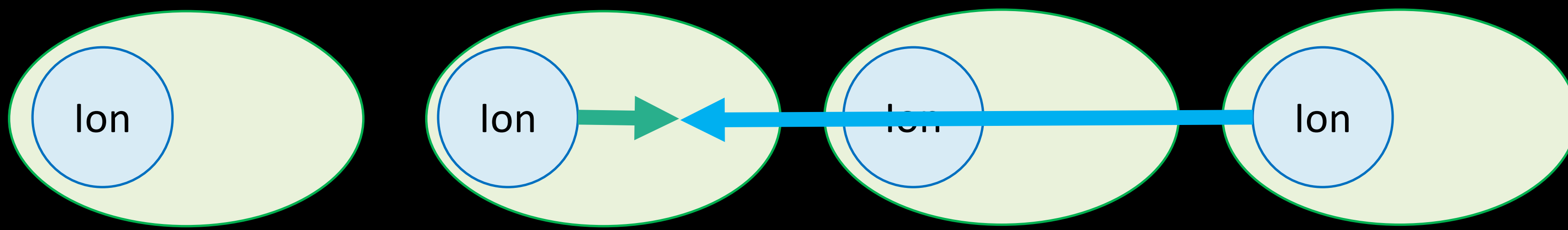




Nothing Particularly Interesting.

Can I do the same for crystal?

Multipoles in Crystal:



Dipole? $P_x = \sum x q_x \rightarrow \sum \langle\psi|\hat{x}|\psi\rangle q_x$? **Doomed to fail.**

VOLUME 80, NUMBER 9 PHYSICAL REVIEW LETTERS 2 MARCH 1998

Quantum-Mechanical Position Operator in Extended Systems

Raffaele Resta

of the nuclear potential acting on the electrons. Since the position operator is ill defined, so is its expectation value, whose observable effects in condensed matter are related to macroscopic polarization. For the crystalline

“ambiguity” in x

Fortunately, “Formula for Polarization”

VOLUME 80, NUMBER 9 PHYSICAL REVIEW LETTERS 2 MARCH 1998

Quantum-Mechanical Position Operator in Extended Systems

Raffaele Resta

$$U_1 = \exp\left(\frac{2\pi i}{L}\sum x\,\hat{N}(x)\right) \longleftrightarrow \langle \mathrm{GS}|U_1|\mathrm{GS}\rangle = \exp(2\pi i P_x)$$

$|GS\rangle =$ generic many-body ground state on a ring of length L

…consistent with:

p_x

$\delta Q = \vec{p}\cdot\hat{n} = -p_x$ $\delta Q = \vec{p}\cdot\hat{n} = p_x$

[Cf. No generic proof so far]

Why do I care about "multipoles"?

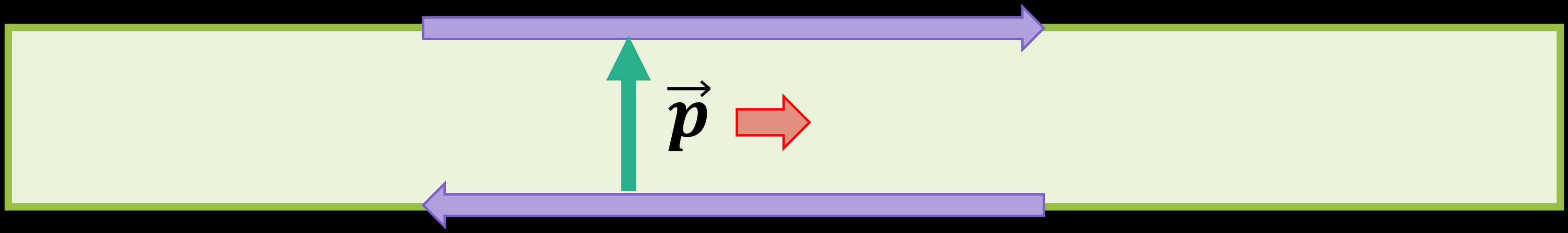
"Multipole = Building Blocks for Topological States"

Ex: dipole

1) Topological Band Index: $p_x = \frac{1}{2\pi} \oint A_k \mod 1$

[flux in momentum space]

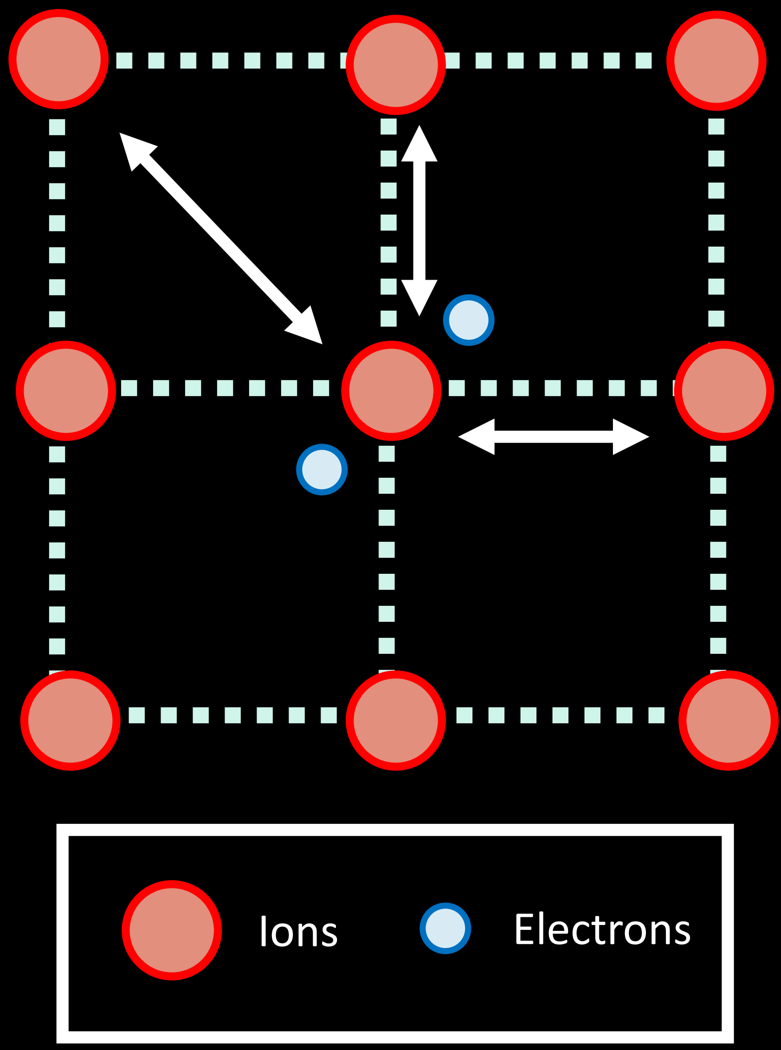
2) Moving Dipole = Quantum Hall Effect



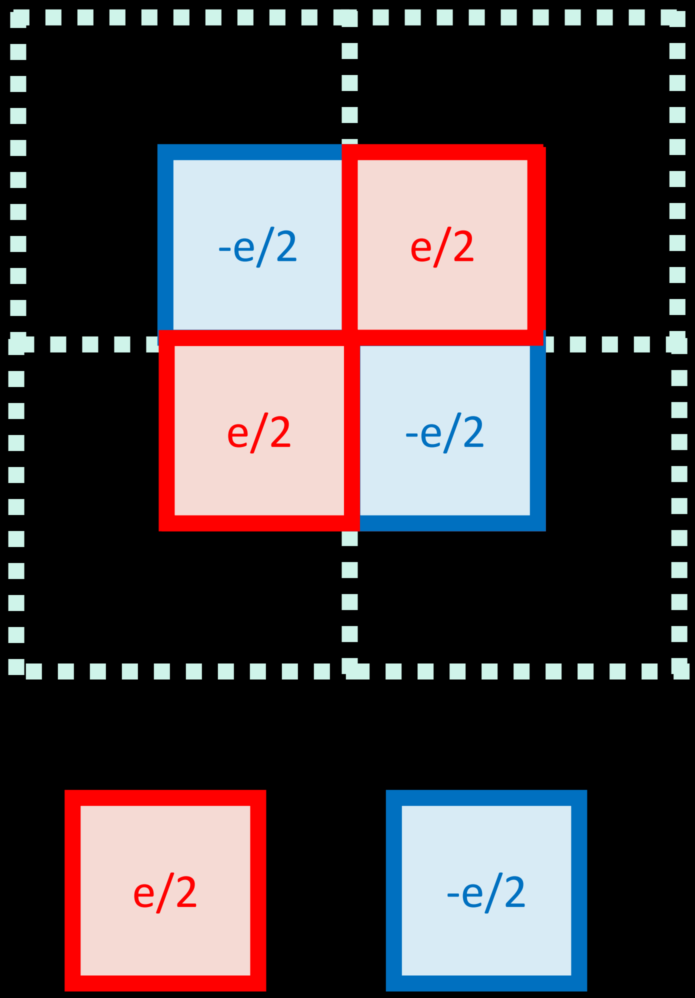
1-dim edge state

What do we expect for quadrupole Q_{xy}, e.g., $Q_{xy} = e/2$?

Quadrupolar Insulator



Electronic Wavefunction



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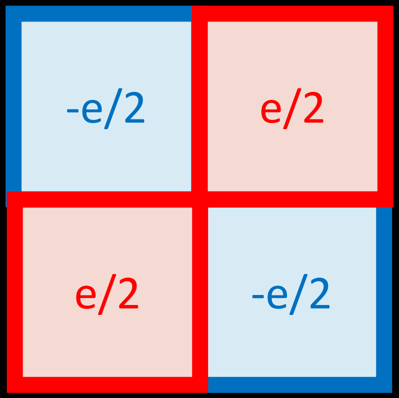
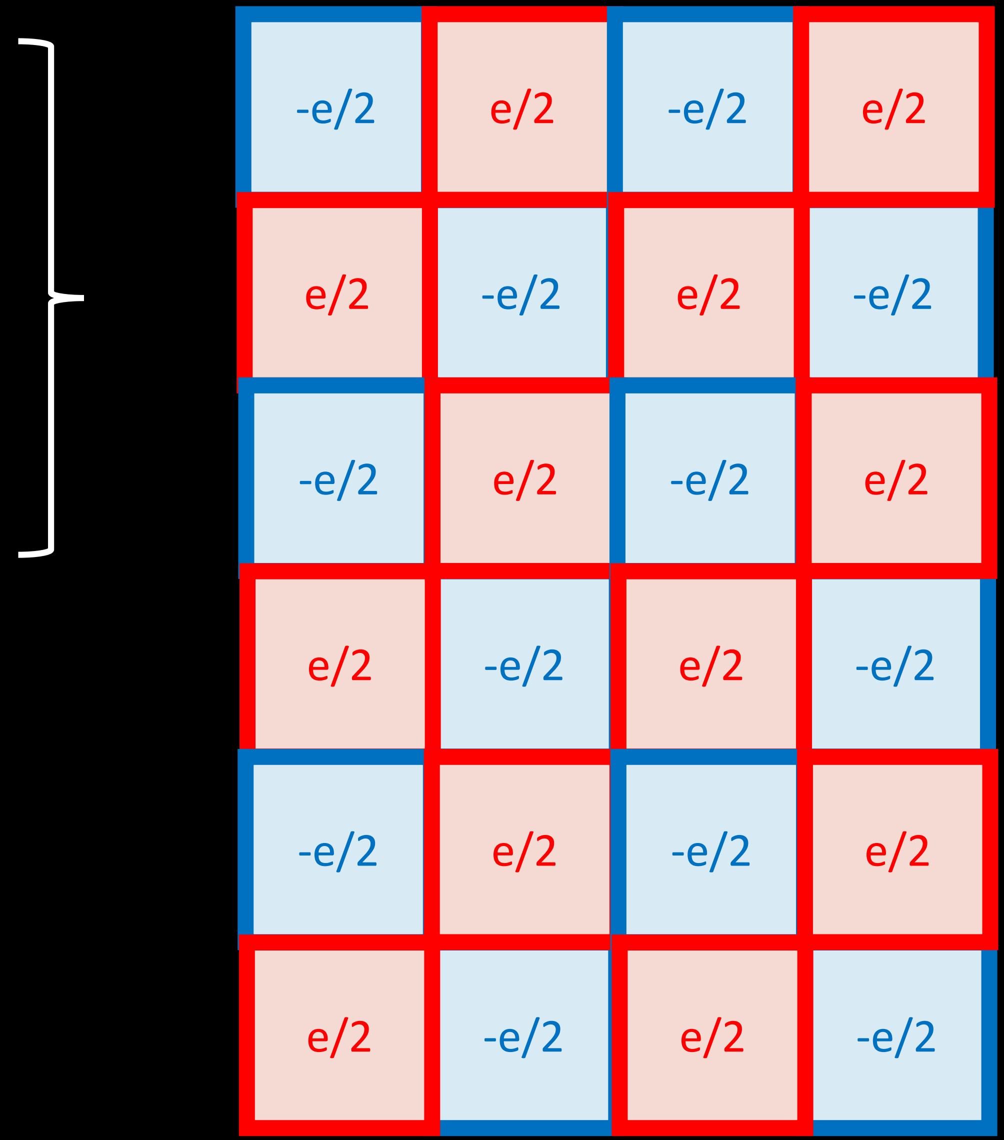


Quadrupole Moment Per Unit Cell



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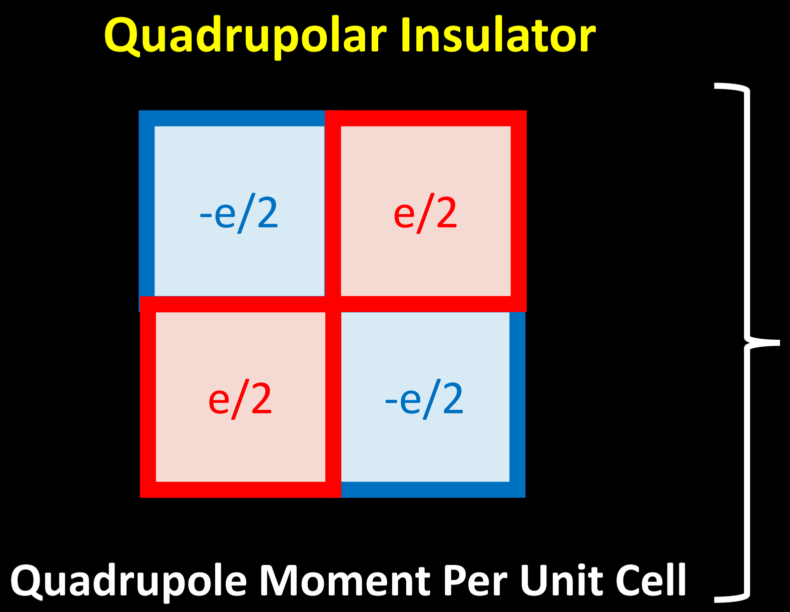

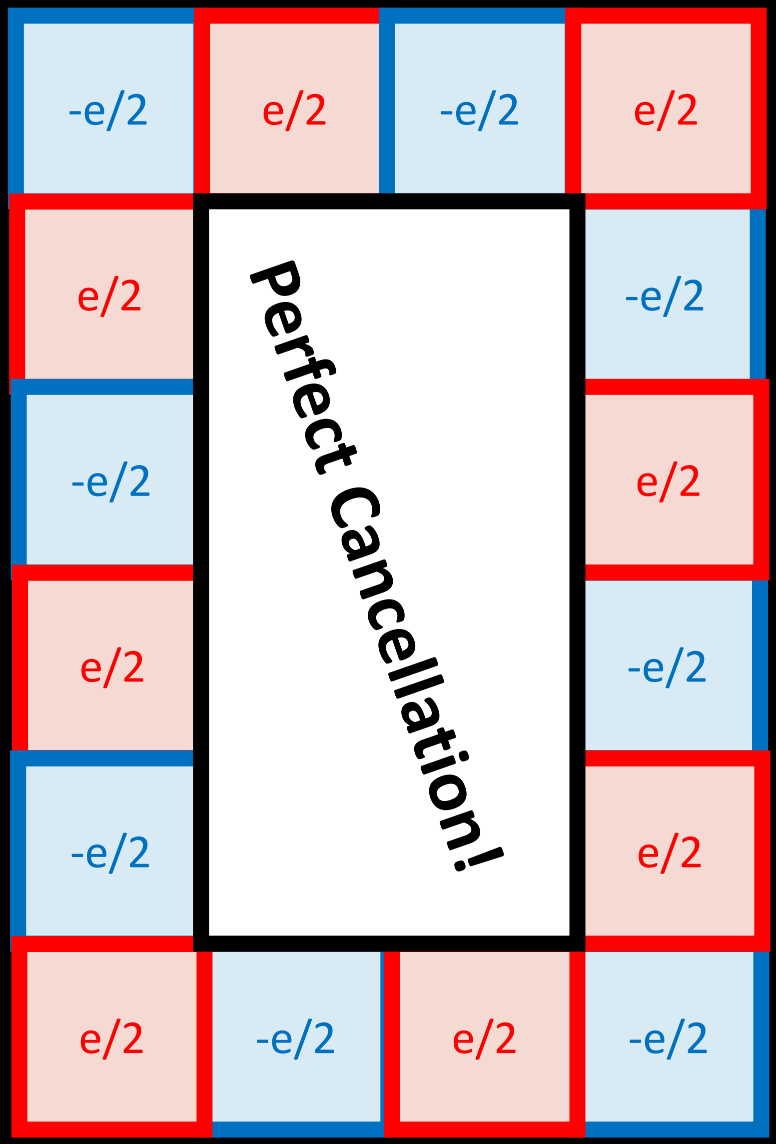

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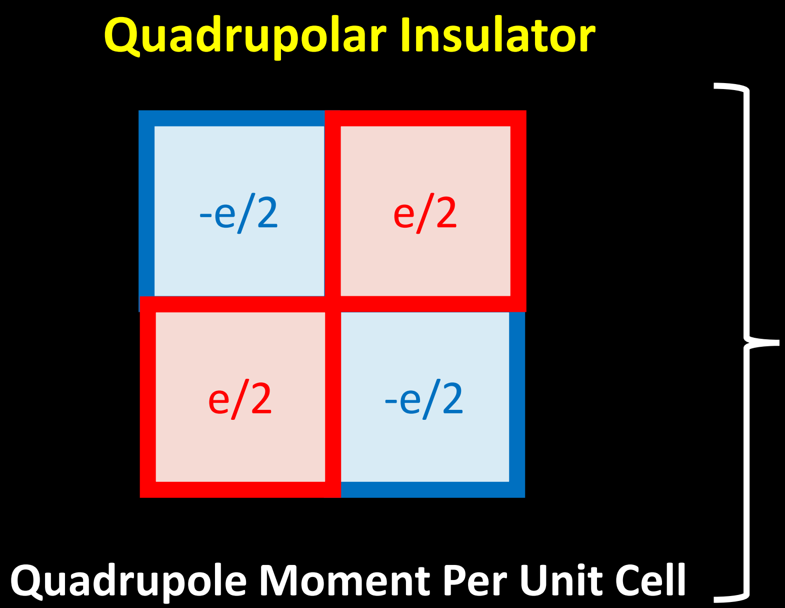
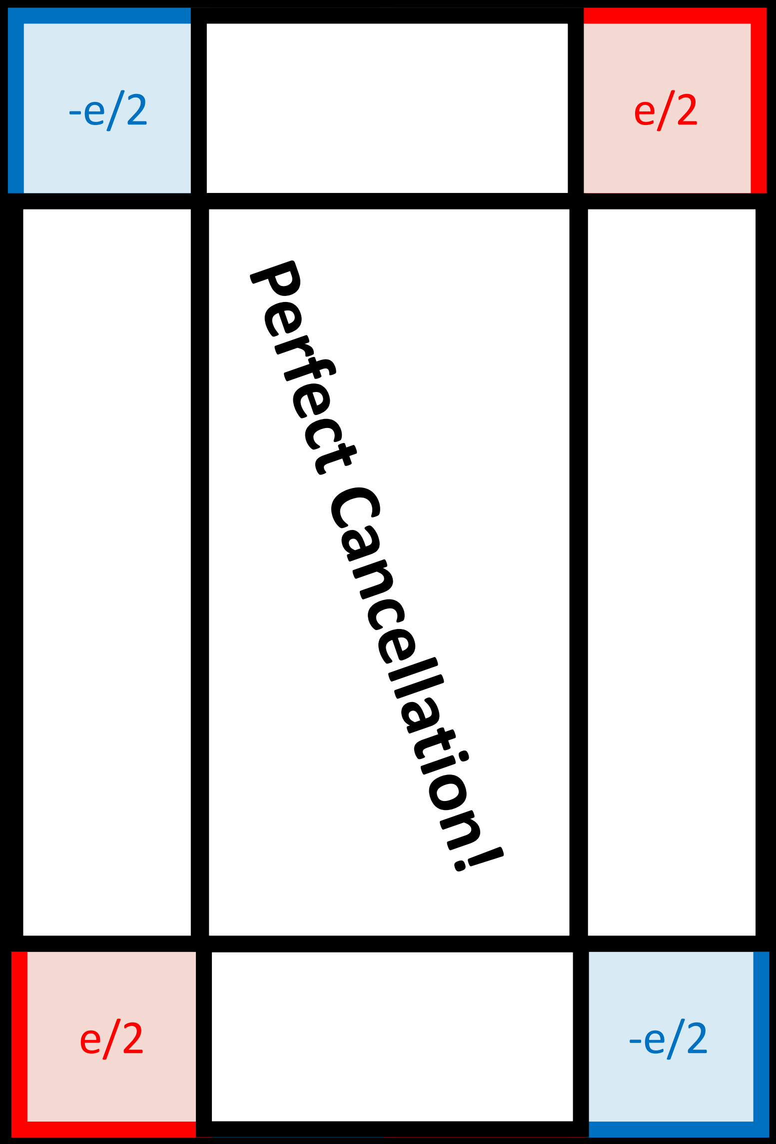
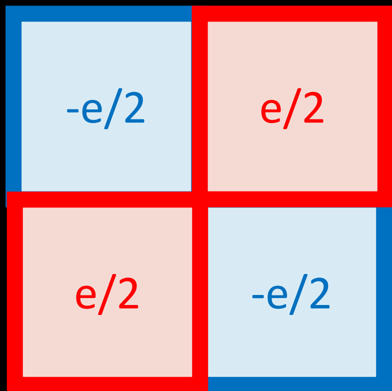


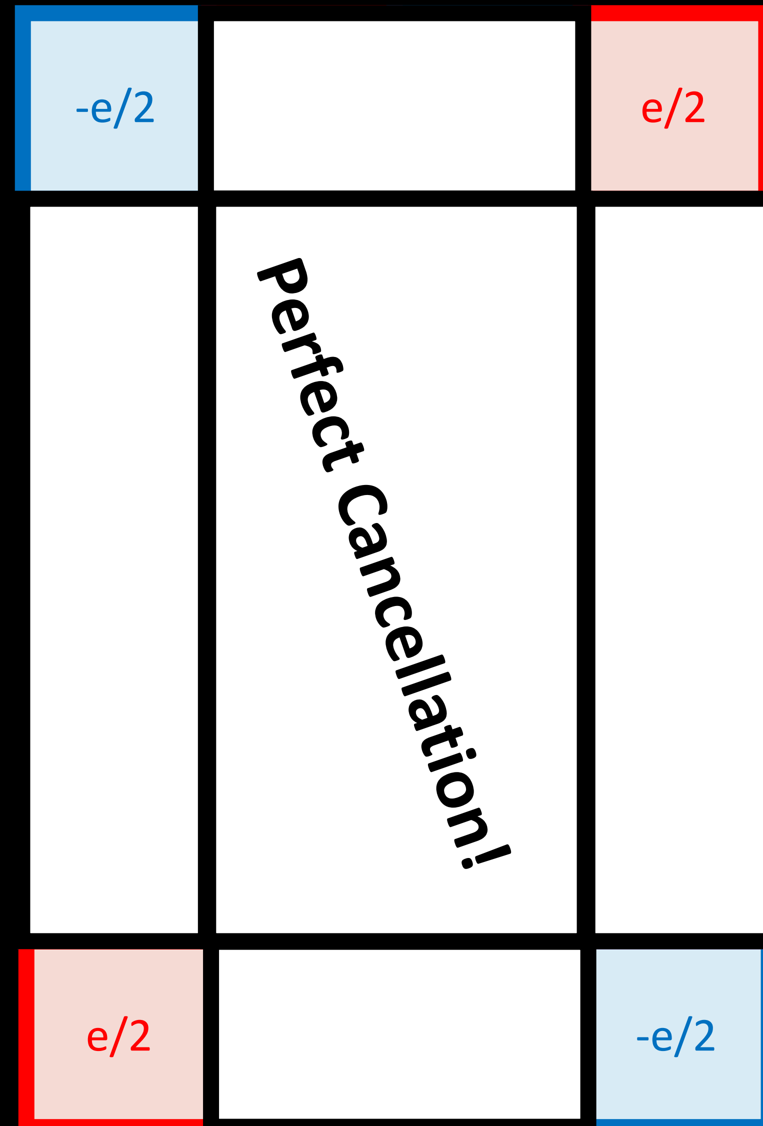


What do we expect for quadrupole Q_{xy}, e.g., $Q_{xy} = e/2$?

Quadrupolar Insulator



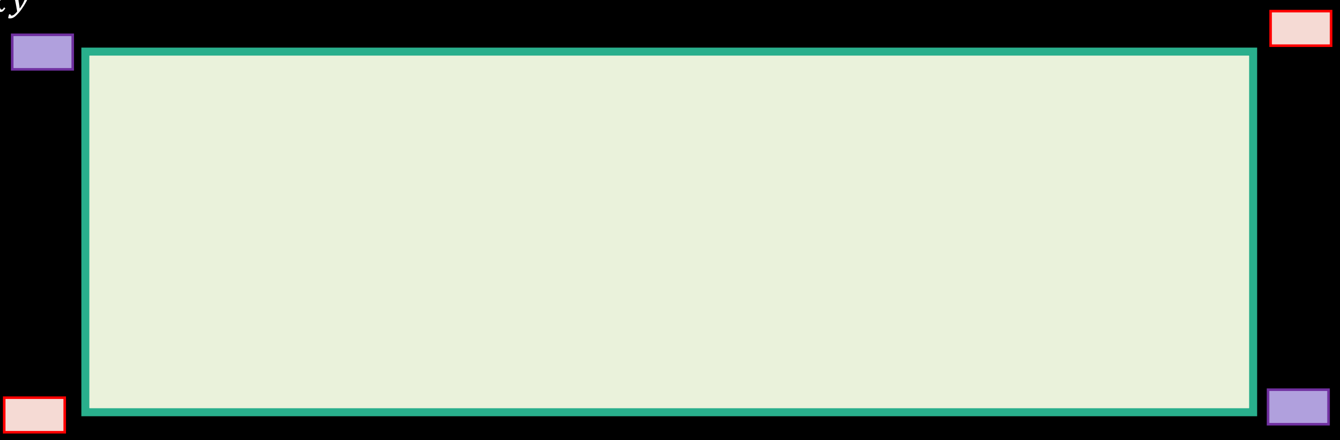
Quadrupole Moment Per Unit Cell



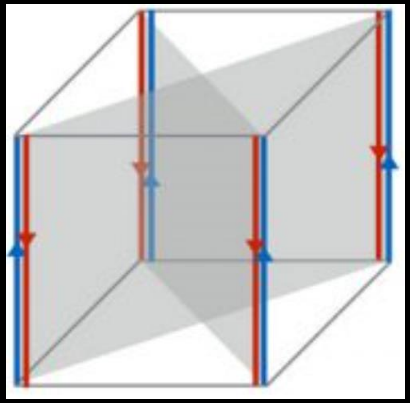
Topological Corner Charge

What do we expect for quadrupole Q_{xy}?

1) (Topological) Corner Charge:



2) Moving Quadrupole = Hinge State



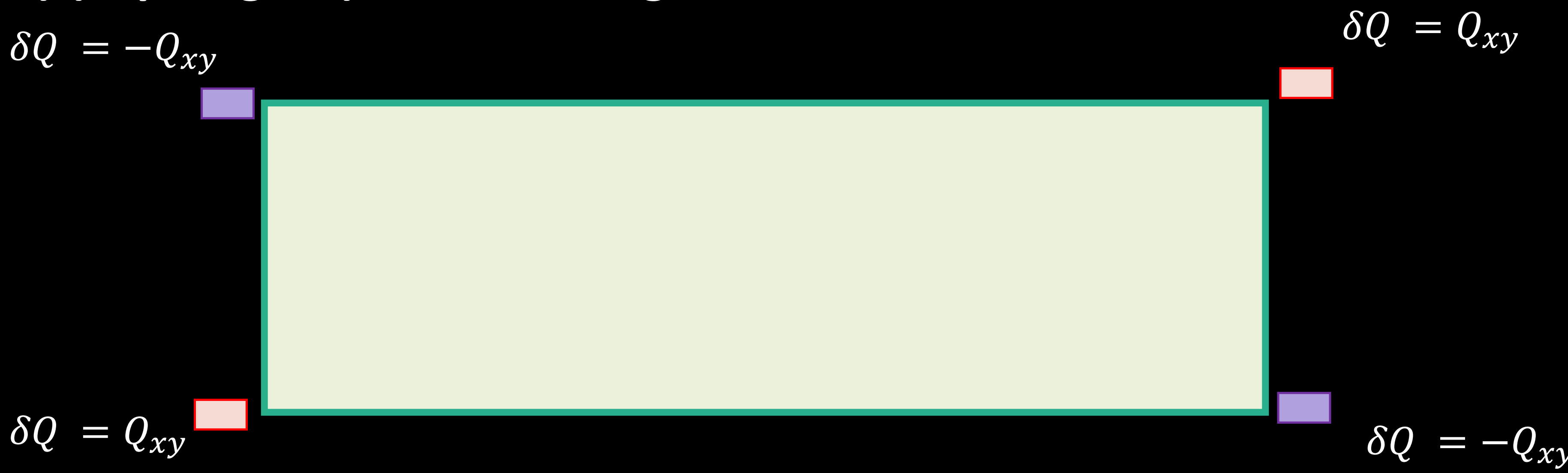
Hinge States

cf. Bismuth in Honeycomb Lattice

[Schindler et.al., Nat. Phys. (2018)]

What do we expect for quadrupole Q_{xy}?

1) (Topological) Corner Charge:



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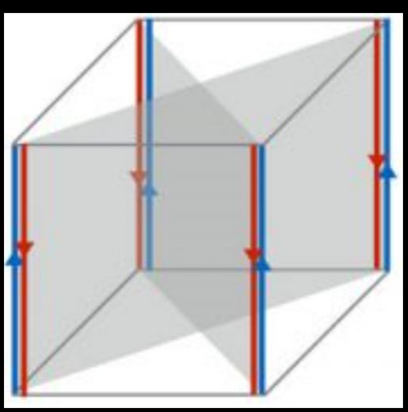


Hinge States

cf. Bismuth in Honeycomb Lattice

[Schindler et.al., Nat. Phys. (2018)]

"Higher-Order Topological Insulator"

New Generation: "Higher-Order Topological Insulator"

RESEARCH

TOPOLOGICAL MATTER

Quantized electric multipole insulators

Wladimir A. Benalcazar,[1] B. Andrei Bernevig,[2] Taylor L. Hughes[1*]

[Science, 2017]

SCIENCE ADVANCES | RESEARCH ARTICLE

MATERIALS SCIENCE

Higher-order topological insulators

Frank Schindler,[1] Ashley M. Cook,[1] Maia G. Vergniory,[2,3*] Zhijun Wang,[4] Stuart S. P. Parkin,[5]
B. Andrei Bernevig,[4,2,6†] Titus Neupert[1†]

[Science, 2018]

Reflection-Symmetric Second-Order Topological Insulators and Superconductors

Josias Langbehn, Yang Peng, Luka Trifunovic, Felix von Oppen, and Piet W. Brouwer
Phys. Rev. Lett. **119**, 246401 – Published 11 December 2017

[PRL, 2017]

Observation of a phononic quadrupole topological insulator

Marc Serra-Garcia, Valerio Peri, Roman Süsstrunk, Osama R. Bilal, Tom Larsen, Luis Guillermo Villanueva & Sebastian D. Huber

[Nature, 2018]

Higher-Order Topology in Bismuth

Frank Schindler,[1] Zhijun Wang,[2] Maia G. Vergniory,[3,4,5] Ashley M. Cook,[1] Anil Murani,[6]
Shamashis Sengupta,[7] Alik Yu. Kasumov,[6,8] Richard Deblock,[6] Sangjun Jeon,[9] Ilya Drozdov,[10]
Hélène Bouchiat,[6] Sophie Guéron,[6] Ali Yazdani,[9] B. Andrei Bernevig,[9] and Titus Neupert[1]

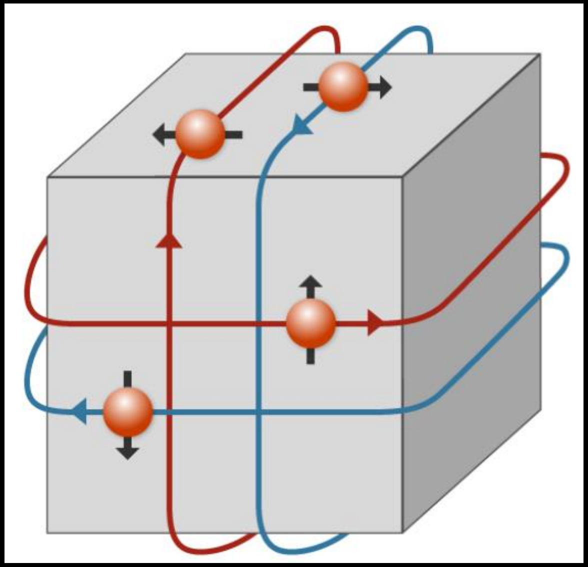
[Nat. Phys., 2018]

...and so on.

Higher-Order Topology?

In three dimensional systems:

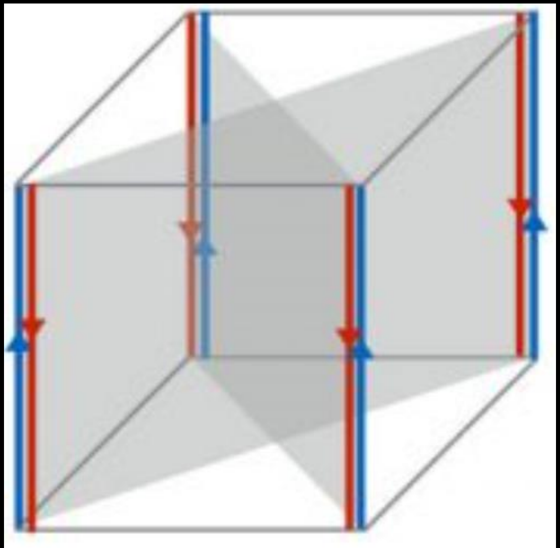
Typical Topological Insulators



- **Entire Surface is Metallic**
- **Don't need Crystal Symmetry**

E.g. Bi_2Se_3

Higher-Order Topological Insulators



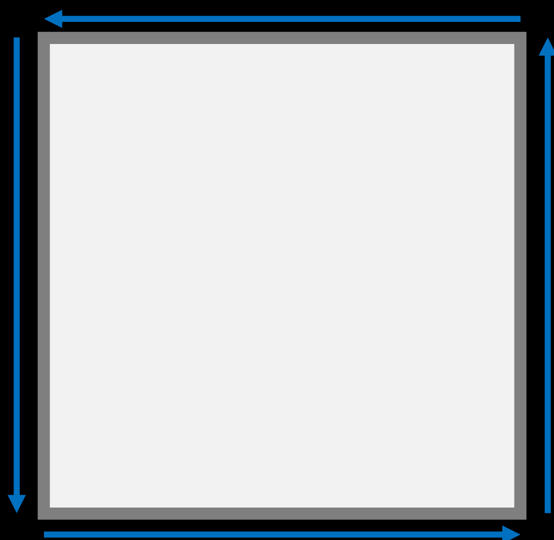
- **Lower-Dimensional Edge is Metallic**
- **Crystal Symmetry, Curvature etc**

E.g. Bismuth

Higher-Order Topology?

In two dimensional systems:

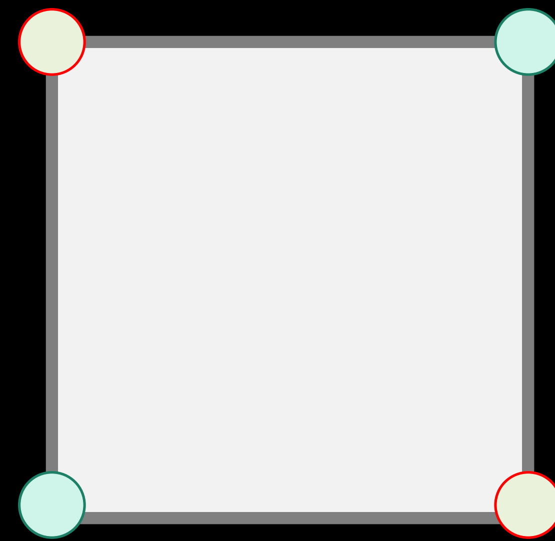
Typical Topological Insulators



2D Topological Insulators

E.g. QHEs or HgTe

Higher-Order Topological Insulators



Corner States

E.g., ?

…Generated from electric "Multipoles"

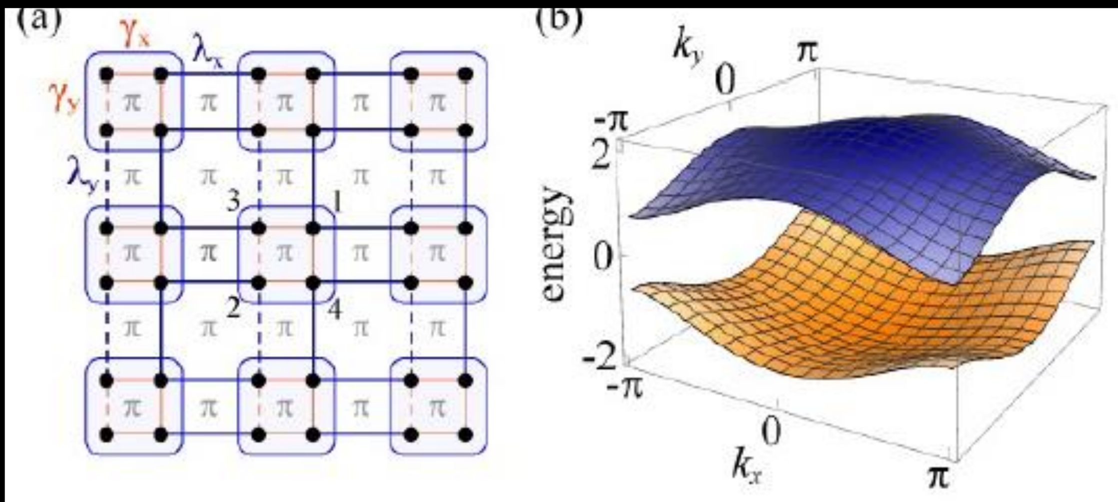
Do we have a good definition of Quadrupoles ?

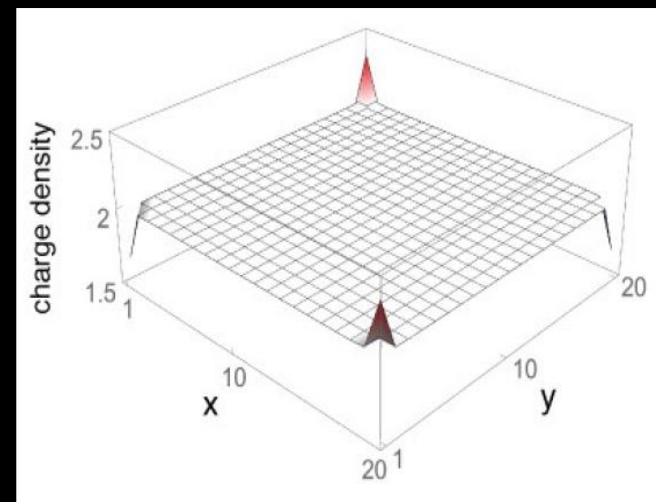
Model of quadrupole moment

Quantized electric multipole insulators

Wladimir A. Benalcazar,[1] B. Andrei Bernevig,[2] Taylor L. Hughes[1*]

[Science (2017)]





Tight-binding Model

(1) Gapped Spectrum

(2) Corner Charge $Q_c = \pm\frac{1}{2}$

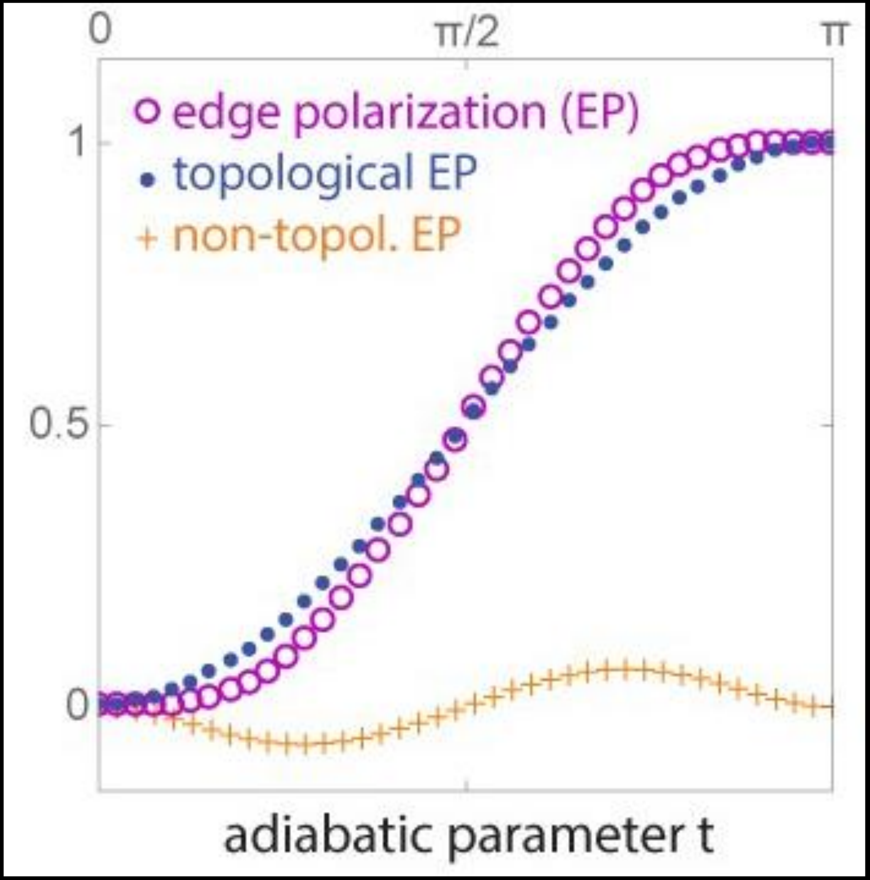
Characterization of Q_{xy}?

Quantized electric multipole insulators

Wladimir A. Benalcazar,[1] B. Andrei Bernevig,[2] Taylor L. Hughes[1*]

[Science (2017)]

..designed so-called "Wannier-sector polarization" (or nested Wilson loop)



tries. Hence, although the Wannier-sector polarization does not describe the precise value of the edge polarization and corner charge when there is a bulk contribution to the edge polarization, it does correctly describe the

Wannier-sector Polarization

≠ Physical Quadrupole Moment

Better Measure/Definition ?

We define:

Byungmin Kang

Quadrupole in **a crystal** is defined by:

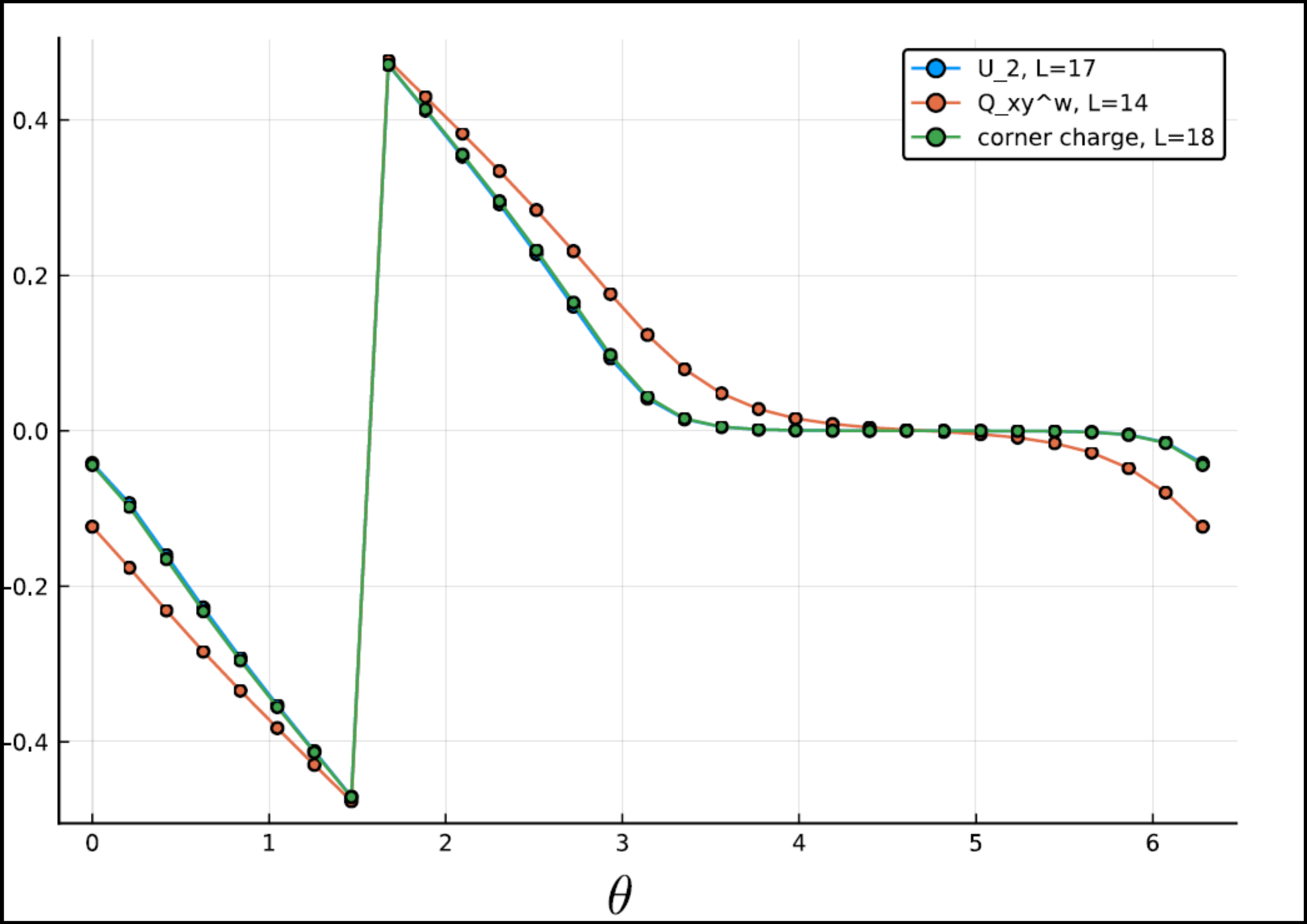
$$Q_{xy} = \frac{1}{2\pi} \mathrm{Im} \log \langle GS | U_2 | GS \rangle \quad \text{with} \quad U_2 = \exp\left(\frac{2\pi i}{L_x L_y} \sum xy \, \rho(x) \right)$$

Here: $|GS\rangle$ = many-body states on Torus

Essentially, $\langle U_2 \rangle = |\langle U_2 \rangle| \; \mathrm{Exp}\left(2\pi i Q_{xy}\right)$

[Byungmin Kang, K Shiozaki, and GYC (2018); W Wheeler, L Wagner, and T Hughes (2018)]

It seems working well.



[Byungmin Kang, K Shiozaki, and GYC (2018); W Wheeler, L Wagner, and T Hughes (2018)]

Further evidences

Higher Order Topological Insulators in Amorphous Solids

Adhip Agarwala,[1,2,*] Vladimir Juričić,[3,†] and Bitan Roy[2,‡]

[Amorphous, Disordered Fermionic (2019 Feb)]

Nonsymmorphic Topological Quadrupole Insulator in Sonic Crystals

Zhi-Kang Lin,[1] Hai-Xiao Wang,[2,1] Ming-Hui Lu,[3] and Jian-Hua Jiang[1,*]

[Nonsymmorphic, Bosonic (2019 Mar)]

Higher-order topological insulator out of equilibrium: Floquet engineering and quench dynamics

Tanay Nag,[1,2,*] Vladimir Juričić,[3,†] and Bitan Roy[2,‡]

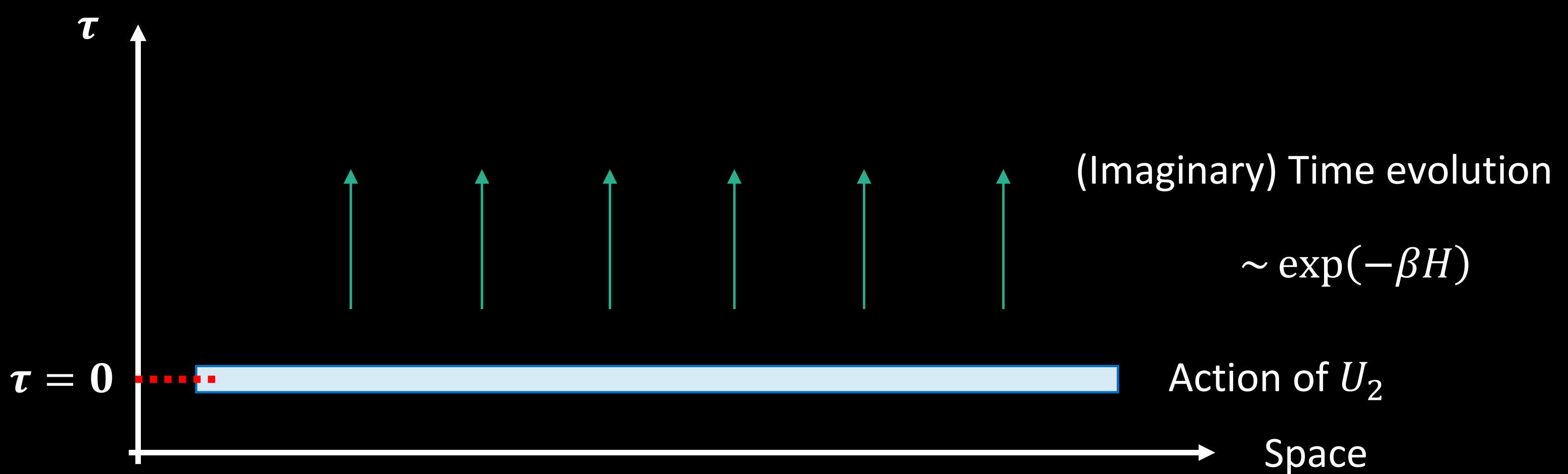
[Nonequilibrium, Floquet-driven (2019 April)]

[Works also for interacting spin models]

The formula passes several non-trivial tests !

Cf. Path-integral Interpretation of the overlap:

$$\langle \mathrm{GS}|\mathrm{U}_2|\mathrm{GS}\rangle = \frac{1}{Z}\,\mathrm{Tr}\,\mathrm{e}^{-\beta H}\mathrm{U}_2 \propto \exp\left(iS_{\mathrm{eff}}\left[A_\mu\right]\right)$$



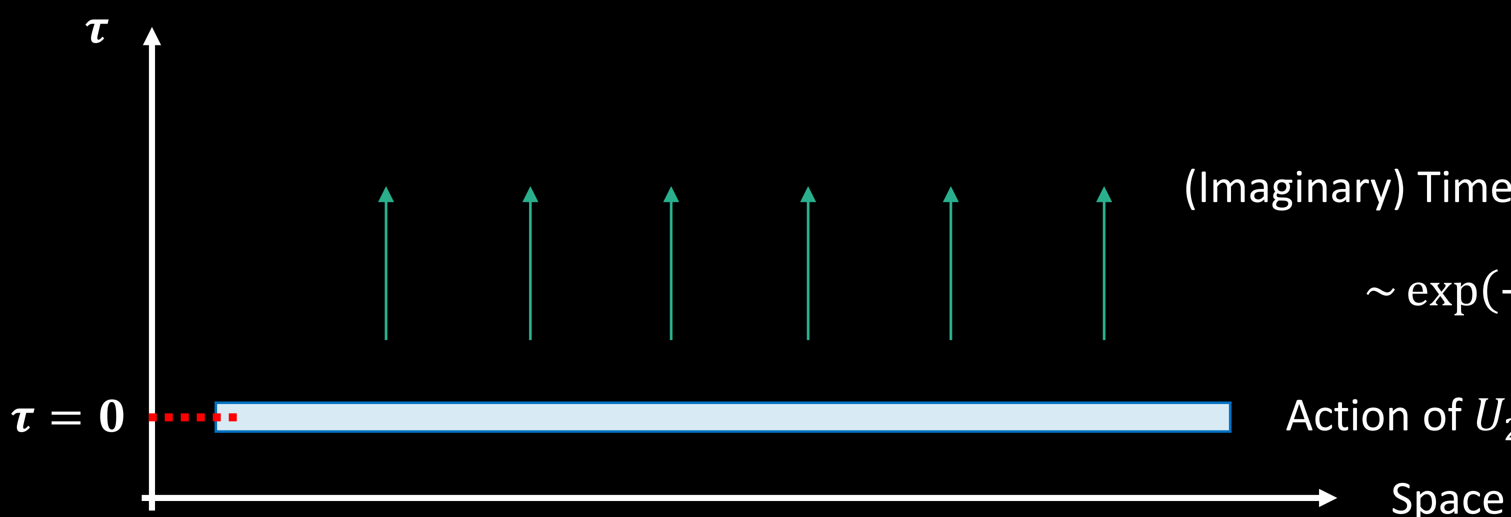
$$U_2 = \exp\left(\frac{2\pi i}{L_x L_y}\sum xy\,\widehat{N}\right) \Rightarrow A_\mu = \delta_{\mu 0}\delta(\tau)\frac{2\pi}{L_x L_y}xy$$

With $\mathrm{S}_{\mathrm{eff}} = \iiint \mathrm{d}\tau \mathrm{d}^2\mathrm{x}\; Q_{xy}\frac{\left[\partial_x E_y + \partial_y E_x\right]}{2}$, $\langle \mathrm{U}_2\rangle = e^{2\pi i Q_{xy}}$

[Byungmin Kang, K Shiozaki, and GYC (2018)]

Cf 2. Why Resta's formula works?

$$\langle \mathrm{GS}|\mathrm{U}_1|\mathrm{GS}\rangle = \frac{1}{Z}\,\mathrm{Tr}\,\mathrm{e}^{-\beta H}\mathrm{U}_1 \propto \exp\left(iS_{\mathrm{eff}}\left[A_\mu\right]\right)$$



$$U_1 = \exp\left(\frac{2\pi i}{L_x}\sum x\,\hat{N}\right) \Rightarrow A_\mu = \delta_{\mu 0}\delta(\tau)\frac{2\pi}{L_x}x,\ E_x = \frac{2\pi}{L_x}\delta(\tau)$$

With $S_{\mathrm{eff}} = \int d\tau \int dx\ P_x \cdot E_x$, $\langle \mathrm{U}_1\rangle = e^{2\pi i P_x}$!

[Byungmin Kang, K Shiozaki, and GYC (2018)]

Summary: Found a definition of multipoles

1. Consistent with the corner charge

2. Field-Theoretic Explanation (why it works)

3. Explain why Resta's formula works generically.

4. More Rigorous definition of Higher-Order Topology

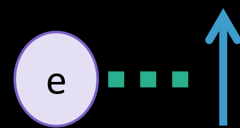
3. Conclusions and Outlooks

Part 1. Geometry in Quantum Hall States

1. Topological Phases see the Geometry
2. Interesting response to the Geometry

[e.g., curvature]

} Unexpected & Puzzling

3. New composite particle theory 

correctly explains the behavior.

Part 2. Multipoles & Higher-Order Topology

1. Proper Definitions of Multipoles are given.

2. Numerical/Field-Theoretical Proofs are given.

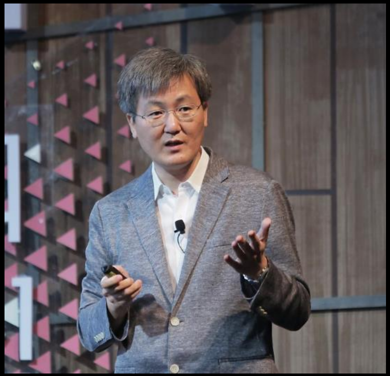
3. Material Realizations for the new “higher-order” topological phases

Q. What will be the corresponding anomaly (or, classification table) ?

Q. More Geometric Topological Phases?

Thank you !

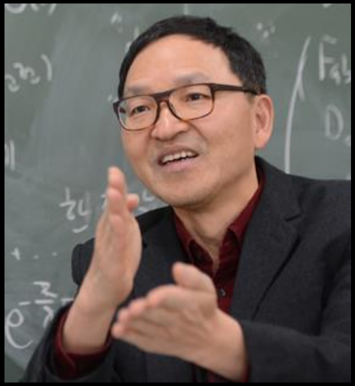
Teacher's day in Korea



Prof. Piljin Yi

Prof. Kwon Park

Prof. Sungjay Lee



Prof. Kimyeong Lee

Staffs @ School of Physics

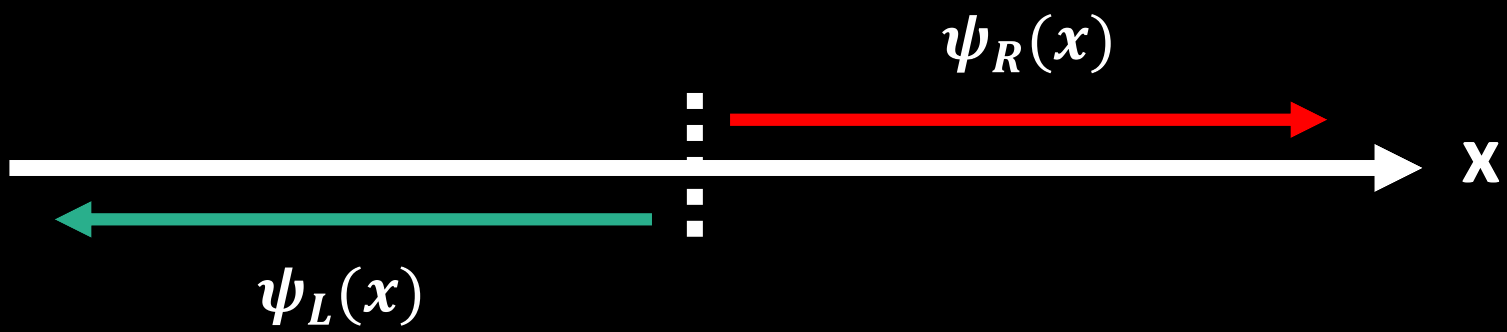
Brad Kwon

Sunmi Wee

JeongEun Yoon

Ex: Reflection-symmetric Theory

Imagine (1+1)d electron system:



With “reflection” symmetry $R_x: x \to -x$

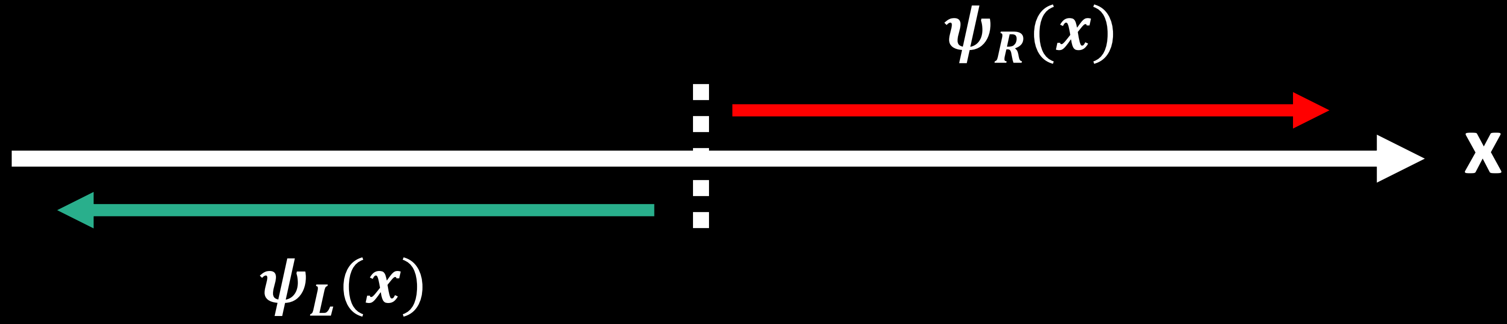
$$R_x: \psi_L(x) \to \psi_R^*(-x)$$

$$\psi_R(x) \to \psi_L^*(-x)$$

[Cf. $R_x: H[\psi_L, \psi_R] \to H[\psi_L, \psi_R]$ **is the symmetry of the action.]**

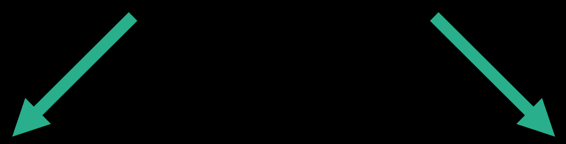
Hsieh, GYC, Ryu, Leigh (2014); GYC, Hsieh, Morimoto, and Ryu (2015)

Ex: Reflection-symmetric Theory



What can possibly go wrong with the symmetry ?

Gapless if reflection is intact



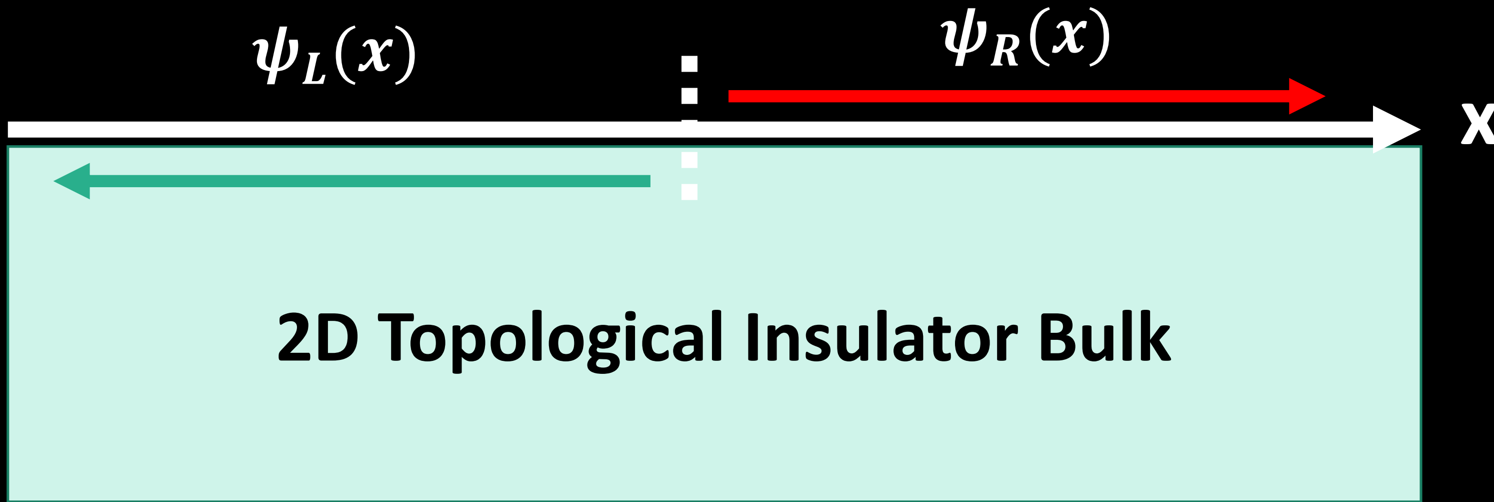
Condensed Matter Physicist	High-Energy Physicist
Edge of Topological Insulator	‘t Hooft Anomaly

Is it so here ?

GYC, Hsieh, Morimoto, and Ryu (2015)

Ex: Reflection-symmetric Theory

1. It is indeed **the edge of the 2d reflection-symmetric Topological Insulator**



2. It has 't Hooft anomaly once it's put on a **Klein bottle.**

$$Z^{\text{Klein}}_{[a]} = \mathrm{Tr}_{a\otimes a}\left[(\mathcal{CP})e^{-2\pi i(b-1/2)F_V} q^{L_R}\bar{q}^{L_L}\right]$$

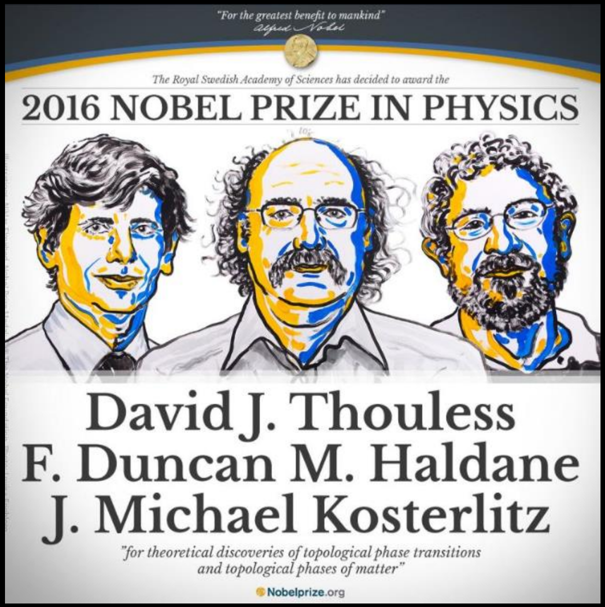
Not Invariant under **the Large Gauge Transformation**

(~ invisibility of the flux $\frac{2\pi e}{\hbar c}$)

GYC, Hsieh, Morimoto, and Ryu (2015); Yonekura, Tachikawa (2018)

Topological State *is* the Central Theme in Modern Physics

Nobel Prize (2016)



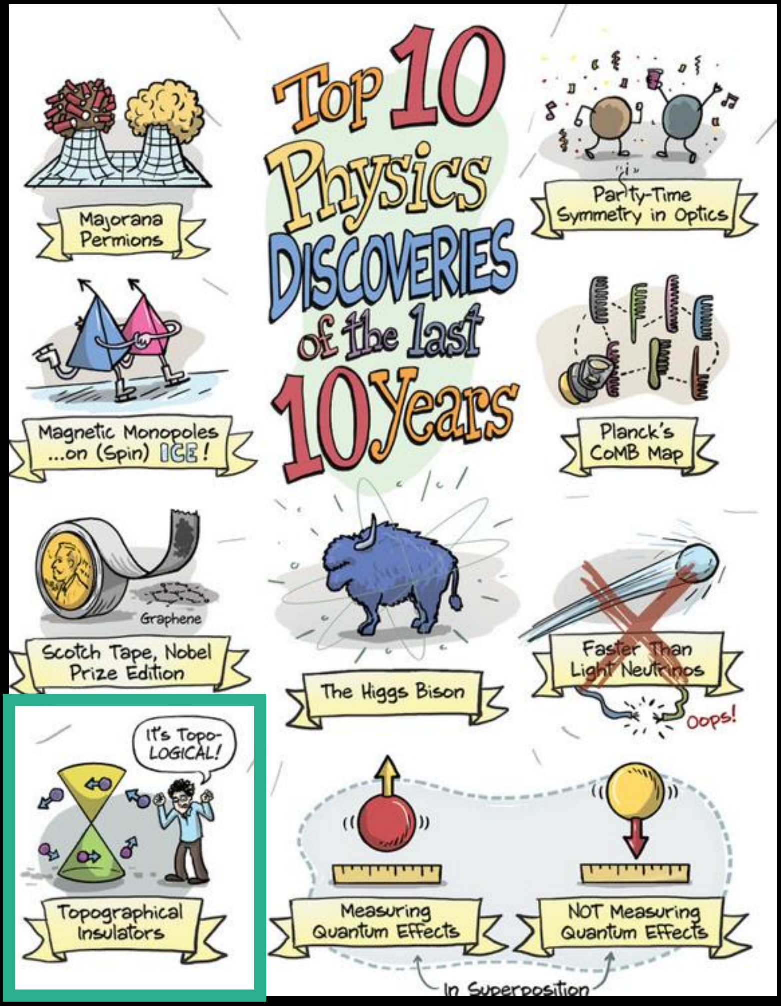
…and other prizes

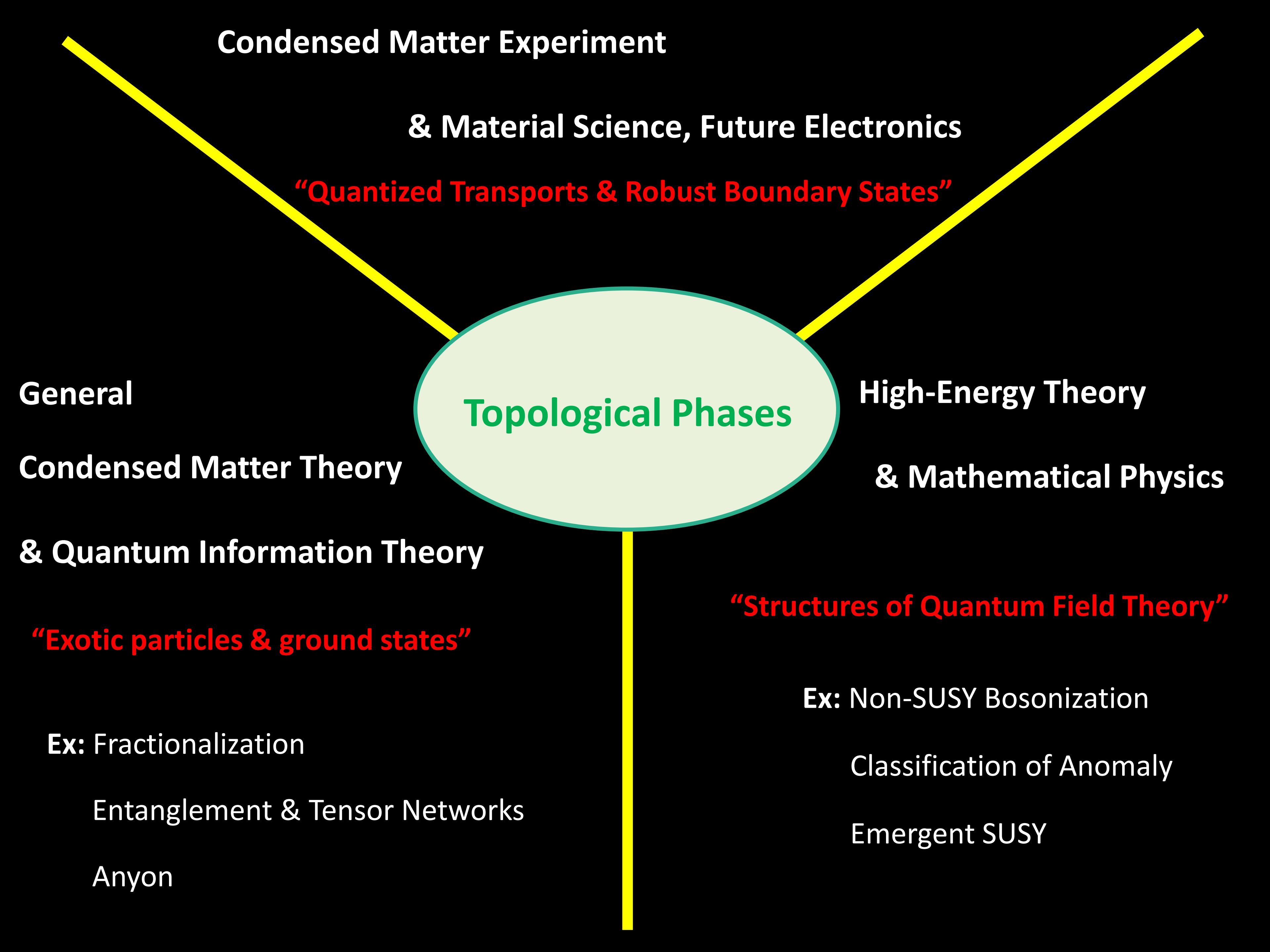
APS Buckley Prize (2017)

Dirac Medal (2015)

New Horizon Prize (2016, 2012)

nature physics




Condensed Matter Experiment
& Material Science, Future Electronics
"Quantized Transports & Robust Boundary States"
Topological Phases
General
Condensed Matter Theory
& Quantum Information Theory
"Exotic particles & ground states"
Ex: Fractionalization
Entanglement & Tensor Networks
Anyon
High-Energy Theory
& Mathematical Physics
"Structures of Quantum Field Theory"
Ex: Non-SUSY Bosonization
Classification of Anomaly
Emergent SUSY

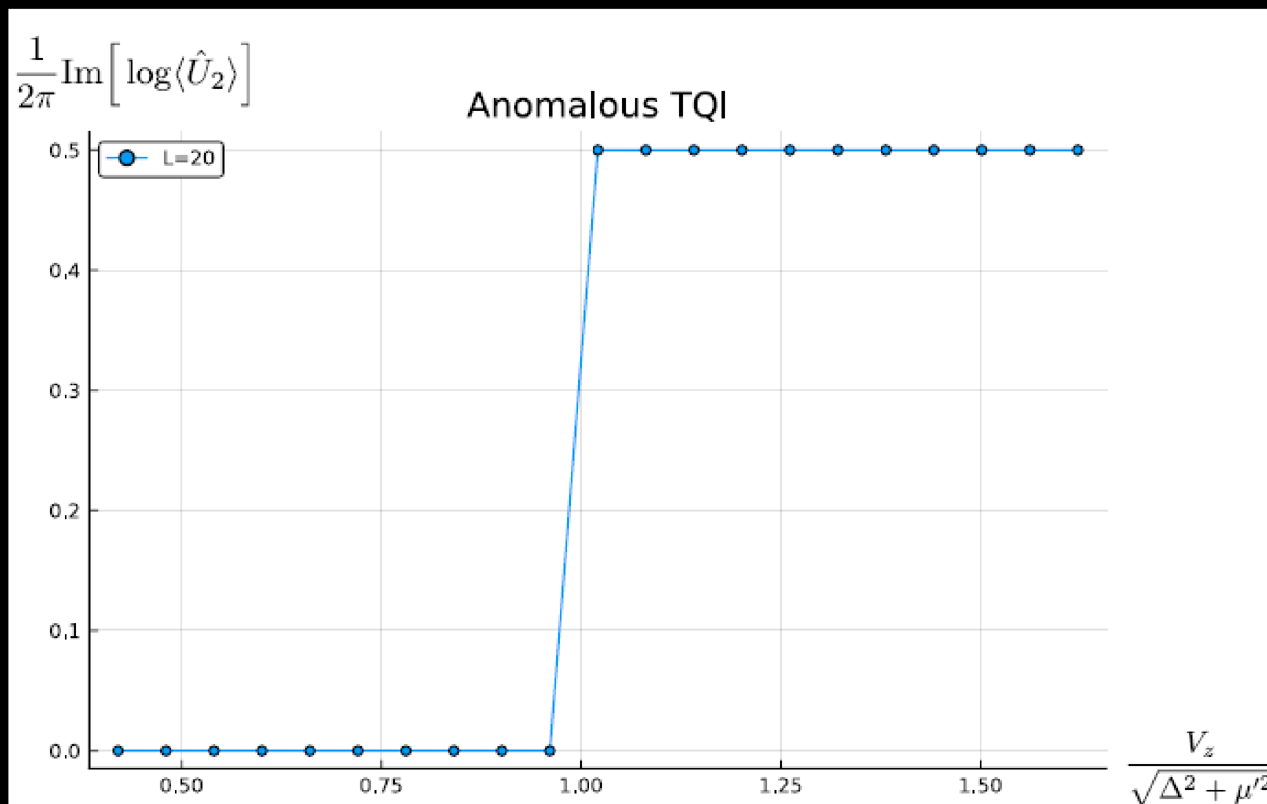
Another Test?

An anomalous higher-order topological insulator

S. Franca,[1] J. van den Brink,[1,2] and I. C. Fulga[1]

despite having a trivial topological invariant. We introduce a concrete example of an anomalous HOTI, which has a quantized bulk quadrupole moment and fractional corner charges, but a vanishing nested Wilson loop index. A new invariant able to capture the topology of this phase is then constructed. Our work shows that anomalous topological phases, previously thought to be unique to periodically driven systems, can occur and be used to understand purely time-independent HOTIs.

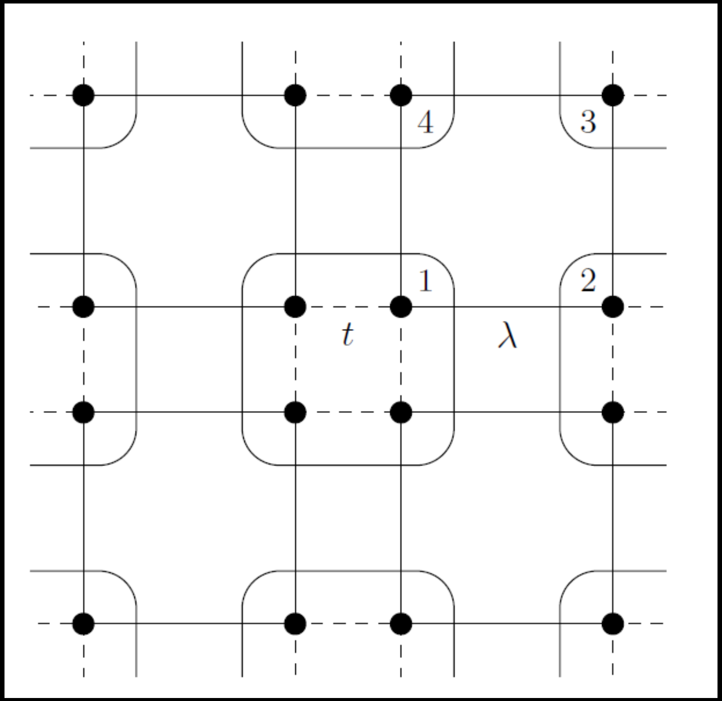
[Benalcazar et. al. Science, 2017]



[Byungmin Kang, K Shiozaki, and GYC (2018)]

Yet another application?

Byungmin Kang, K Shiozaki, and GYC (2018)



[Each dot is spin-1/2]

$$H_p = \lambda \sum_{a=x,y} (\sigma_1^a \sigma_2^a + \sigma_2^a \sigma_3^a + \sigma_3^a \sigma_4^a + \sigma_4^a \sigma_1^a)$$

[Ref. Dubinkin-Hughes (2018)]

At the exactly-soluble limits:

(1) $\lambda \neq 0$ and $t = 0$: Topological

- Dangling spin-$\frac{1}{2}$'s at the corners

$$\langle U_2 \rangle = -1$$

(2) $\lambda = 0$ and $t \neq 0$: Trivial

$$\langle U_2 \rangle = +1$$

Maybe… my next talk will be:

~~New~~ Newer Trend in Topological Phases

: Geometric and Higher

And More

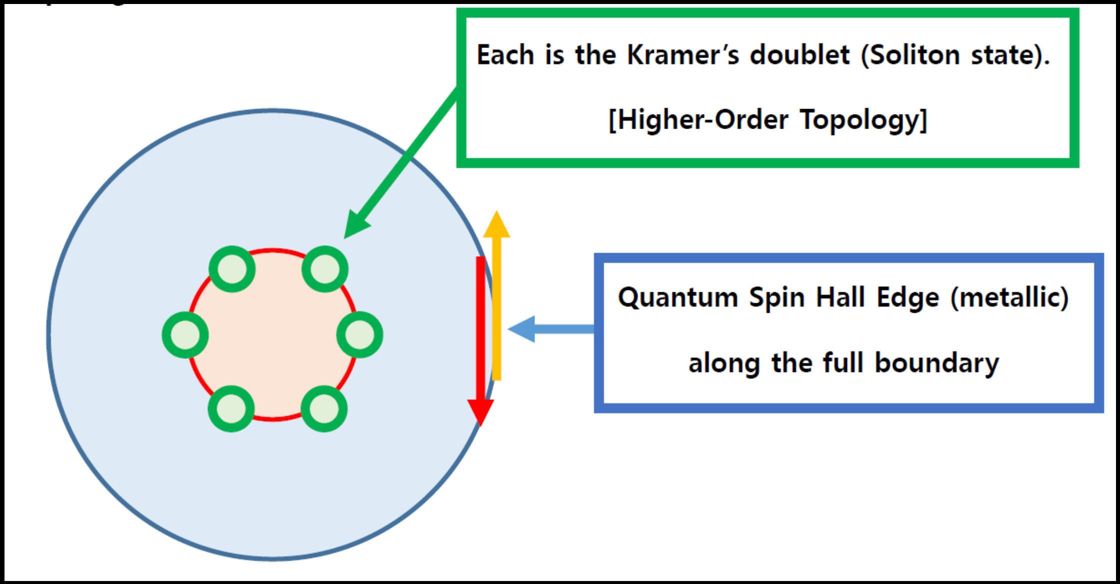
Gil Young Cho

POSTECH

Bismuth (111) thin films: Famous for being quantum spin Hall effect

	1BL	2	3	4	5	6	7	8	9	10
ν^{π}	1	-1	1	-1	1	1	-1	-1	1	-1
$\nu^{\pm\pi/3}$	-1	1	-1	1	-1	-1	1	1	-1	1
Z^2	1	1	1	1	1	1	1	1	1	1

How can the Higher-Order Topology appear? By "Relative Topology at Step Edge"



Corner is *hidden*

inside the QSH Flakes!

Cf. WTe2, SnTe, SnPb etc

[Note: this corner cannot be diagnosed by nested Wilson loop methods]