Strongly Correlated Quantum Materials
: Platform for new physics

Eun-Gook Moon

KAIST

KIAS colloquium

Apr. 10, 2019

Outline

1. Motivation

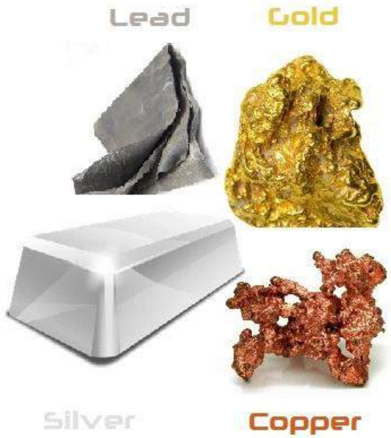
2. Review of Landau paradigms

3. Novel universality class of symmetry breaking transitions

4. Conclusion

Motivation

■ Condensed matter physics

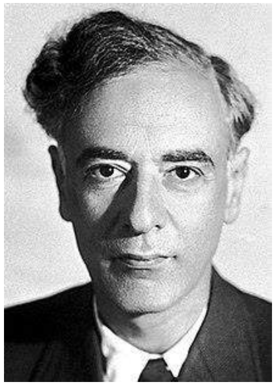


- made of electrons and ions on lattices

- many-body quantum mechanics ($N > 10^{23}$)

- Lots of materials

Current Paradigms



■ Landau’s two influential works

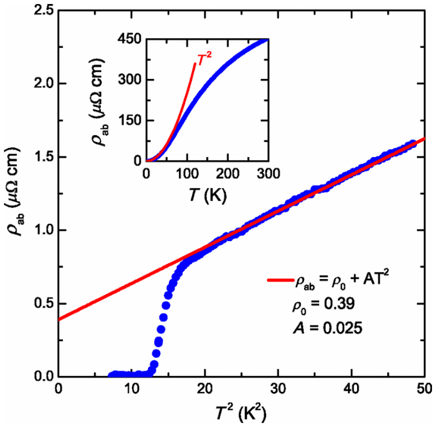
1. interacting many particles ~ (independent particle)^N

Landau’s Fermi liquid paradigm

ex) Many electrons ~ (electronic quasi-particle) ^{N}

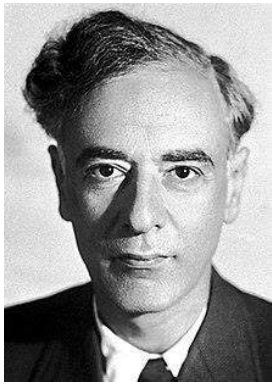
Theory of metals

$$\rho(T) = \rho_0 + A T^2$$



Hardy et. al. (2013)

Current Paradigms



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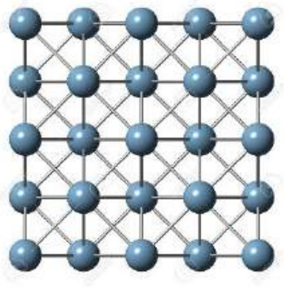
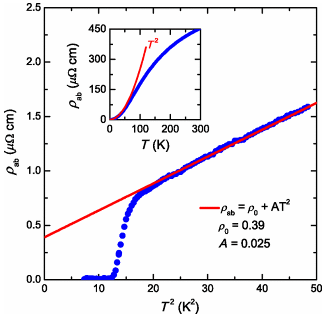
Landau's Fermi liquid paradigm

Energy

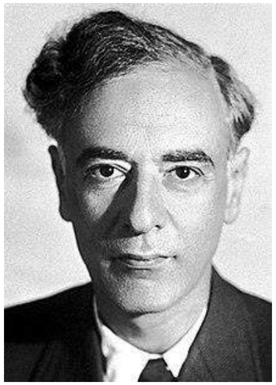
~ keV

< meV





Current Paradigms



■ Landau's two influential works

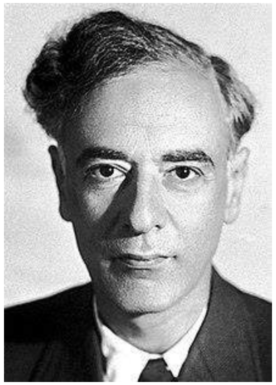
1. interacting many particles ~ (independent particle)^N

Landau's Fermi liquid paradigm

Energy

~ keV UV d.o.f : electrons

< meV IR d.o.f : electronic quasi-particles



Current Paradigms

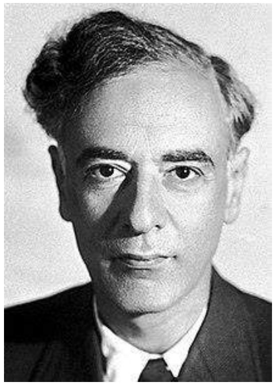
■ Landau’s two influential works

1. interacting many particles ~ (independent particle)^N

Landau’s Fermi liquid paradigm

UV d.o.f $\simeq$ IR d.o.f

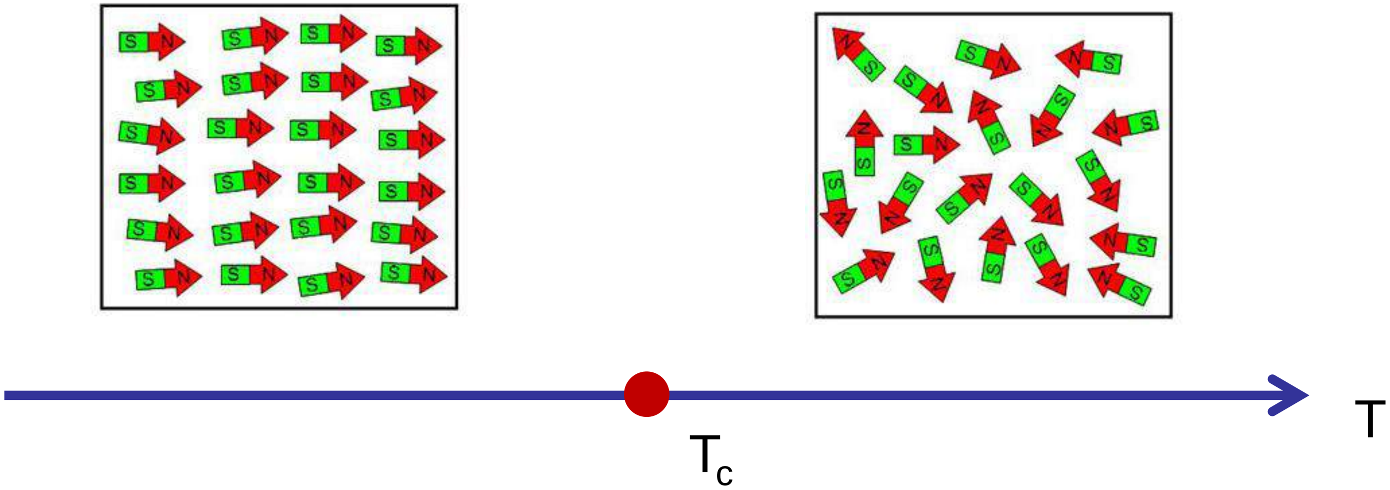
Current Paradigms



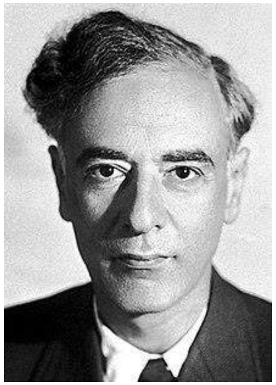
■ Landau’s two influential works

2. Symmetries characterize many-body phases

Landau’s symmetry breaking transition paradigm



$$F = E - TS$$



Current Paradigms

■ Landau's two influential works

2. Symmetries characterize many-body phases

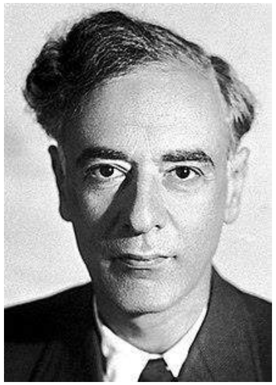
Landau's symmetry breaking transition paradigm

- order parameter : non-trivial rep. of a sym. group

- scale invariance at continuous transitions (universality)

$$C \sim |t|^{-\alpha}, \langle \Phi \rangle \sim |t|^{\beta}, \chi \sim |t|^{-\gamma}, \xi^{-1} \sim |t|^{\nu}, [\Phi] = \frac{1+\eta}{2}, \langle \Phi \rangle \sim h^{1/\delta}$$

Current Paradigms



- Landau's two influential works

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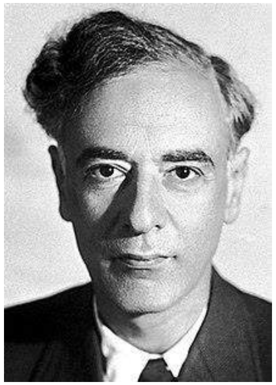
Z_2 symmetry in 2d
: Onsager's exact solution

	α	β	γ	δ	η	ν
d=2	0	$\frac{1}{8}$	$\frac{7}{4}$	15	$\frac{1}{4}$	1

Univ. class in $3d$	α	β	γ	ν	η	δ
Z_2 (Ising) [33]	0.11	0.33	1.24	0.63	0.036	4.79
$U(1)$ (XY) [33]	-0.015	0.35	1.32	0.67	0.038	4.78
Mean-field	0	0.5	1	0.5	0	3

Current Paradigms



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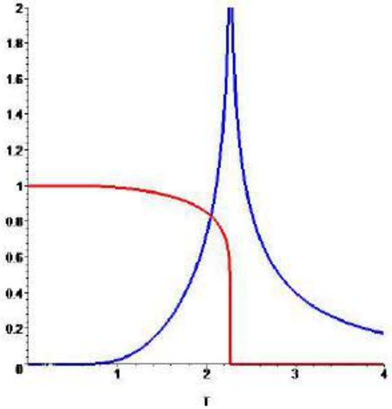
Z_2 symmetry breaking transitions

- Specific heat anomaly : jump or divergent

- Order parameter : sub-linear onset



https://www.researchgate.net/publication/228375244_Closed_Forms_What_They_Are_and_Why_We_Care

Current Paradigms

2-D Ising	0.125	$\nu=\nu'=1.0$	$\gamma=\gamma'=1.75$	0.25

ces for K_2CoF_4 and Rb_2CoF_4 determined by neutron measurements together with the retical values for the two-dimensional Ising lel.
K_2CoF_4 and Rb_2CoF_4, the two-dimensional

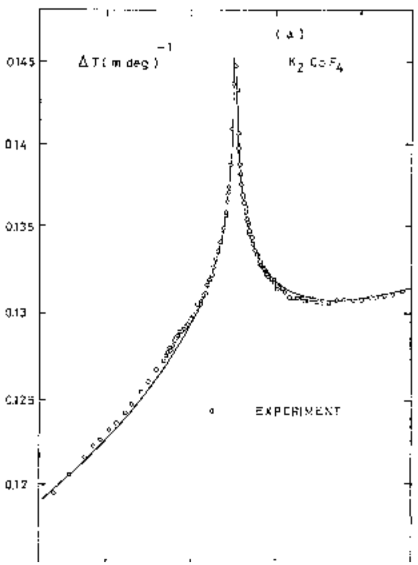
regarded as having $S=1/2$ Ising-like magn properties because the lowest Kramers dou lies below the first-excited Kramers dou with the energy separation of 400 K[4] whicl much higher than the transition temperatu

found from many experiments, such as on susceptibilities,[2] magnetic specific heats[3] direct neutron scattering,[4,5] that K_2NiF_4- pounds give a nearly ideal model for the two- al (2-*D*) magnetic system. In such 2-*D* phase transition is brought about by an y in the interaction parameter (i.e. Ising ...) and/or by the lattice dimensionality (i.e.

of the sublattice magnetization and spin corre function, from which critical exponents β, ν, γ were obtained by the least-squares fitting proc using an instrumental resolution function.

High quality single crystals were prepared mixtures of $CoCl_2$ + 4KF and a small amount KF·HF. A reddish violet transparent single cry

near behavior arise due to rounding.[12] In th



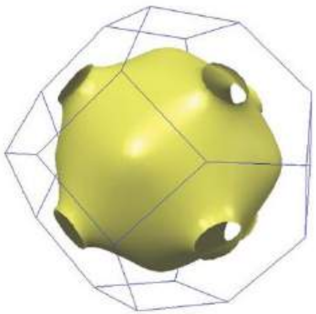
530 TWO-DIMENSIONAL

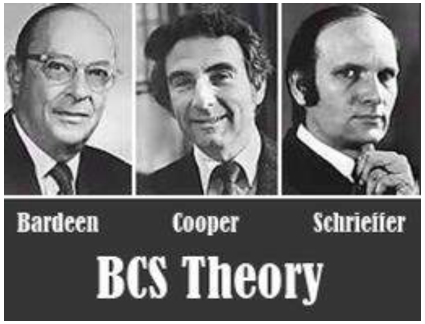
two-dimensional nature of spin correlations examined in this material. No extinction effects were found after inspection of the $(100)_M$ and $(300)_M$ reflections and of the agreement of twelve measured nuclear integrated intensities with the calculated ones. The sublattice magnetization normalized with the extrapolated intensity at 0 K is plotted in Figs. 1 and 2. Figure 1 shows that both the $(100)_M$ and $(300)_M$ normalized intensities, corrected for background contributions, fall on the same curve. Figure 2 shows that the magnetization of K_2CoF_4 obeys Onsager's formula[9] for the 2-*D* $S=\frac{1}{2}$ Ising model,[10]

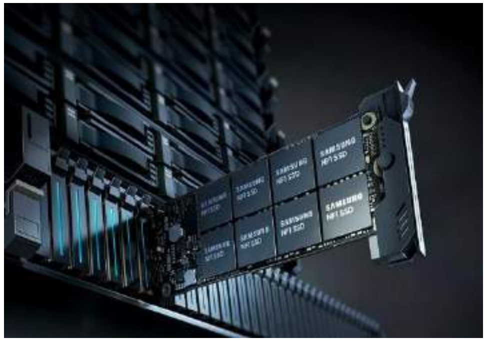
Perfect match between theory and experiment

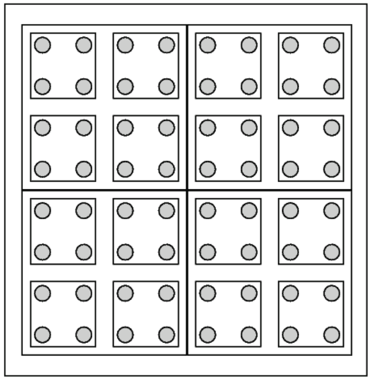
Landau Paradigms

■ Triumph of 20th century condensed matter physics









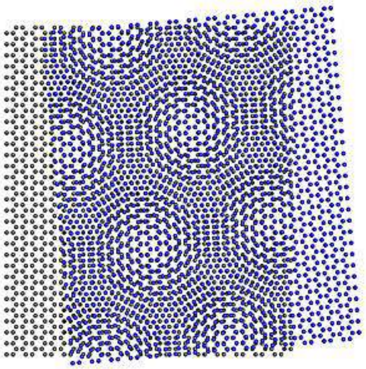
Renormalization group
: Kadanoff, Ginzburg, Wilson, Fisher,…

Phenomena beyond Landau paradigms

■ Strongly correlated quantum materials

$YB_2Cu_3O_7$

α-$RuCl_3$



Twisted bi-layer graphene

Many more …

Images from Google

Phenomena beyond Landau paradigms

- Strongly correlated quantum materials

$YB_2Cu_3O_7$

Phenomena beyond Landau paradigms

- Strongly correlated quantum materials

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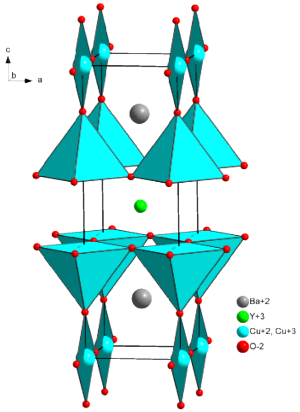
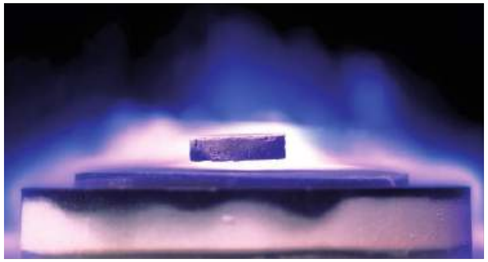




Mann, Nature 2011

High Tc Superconductivity

Phenomena beyond Landau paradigms

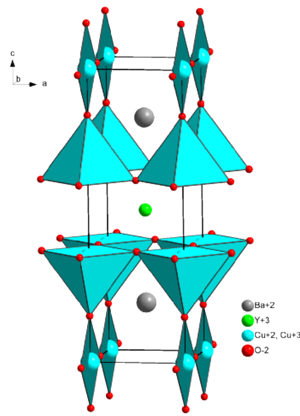
- Strongly correlated quantum materials

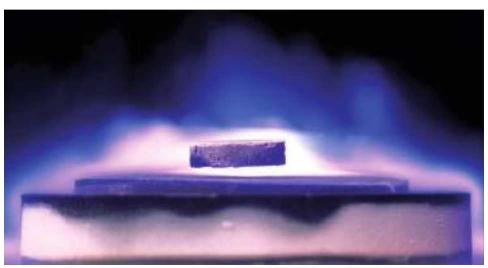
Energy

~ keV

< meV

$YB_2Cu_3O_7$





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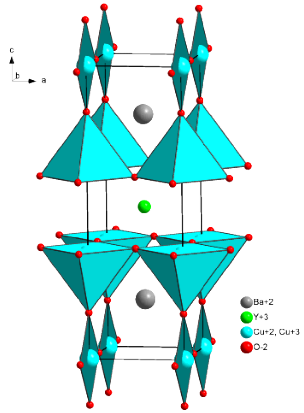
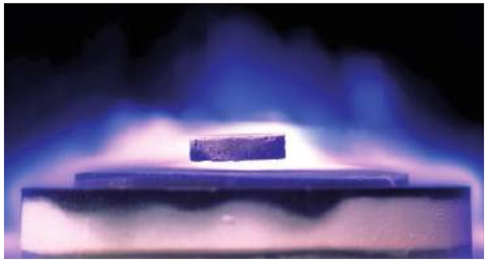




Mann, Nature 2011

High Tc Superconductivity

Energy

UV physics
- Electrons / spins/ ions on lattices
- (ex : DFT)

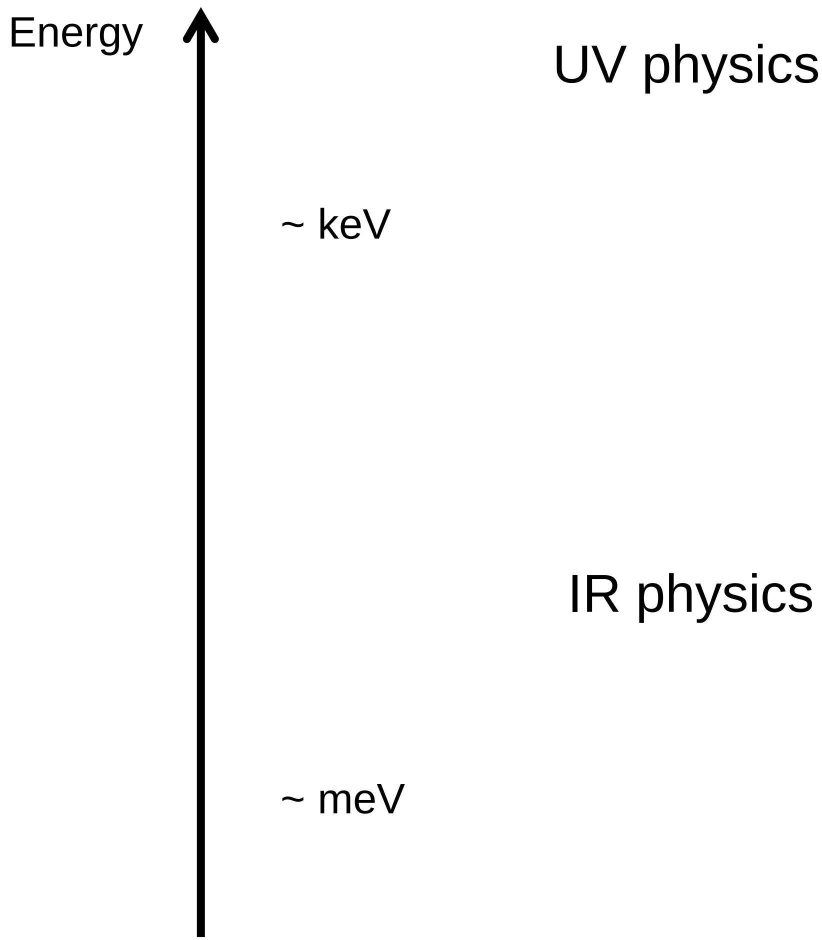
~ keV

IR physics
- *Mysterious* phenomena

< meV

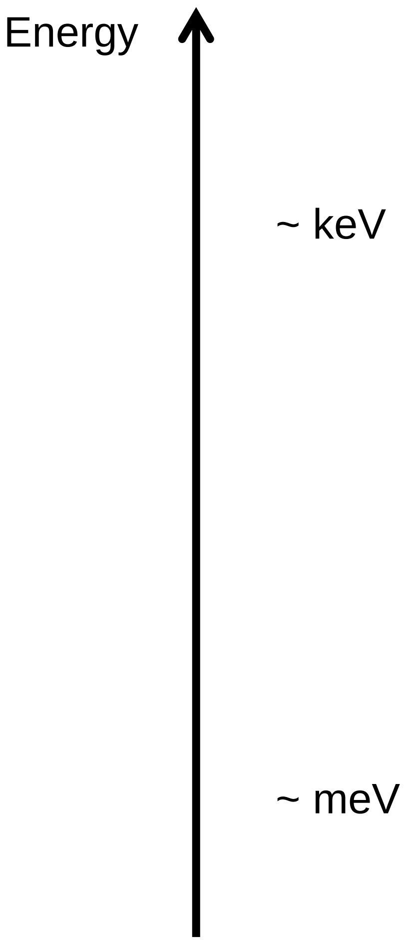
Phenomena beyond Landau paradigms

- Strongly correlated quantum materials

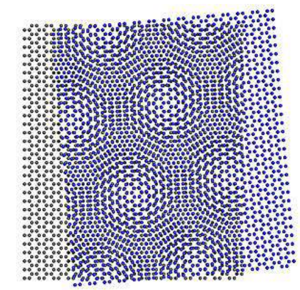


Phenomena beyond Landau paradigms

■ Strongly correlated quantum materials



UV physics



IR physics

New physics?
Ex) Majorana fermions?
emergent photons? SUSY?…

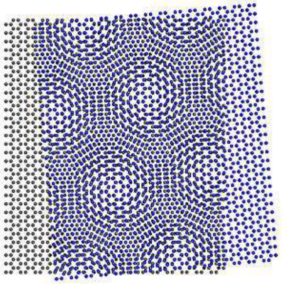
Phenomena beyond Landau paradigms

- Strongly correlated quantum materials

Energy

UV physics

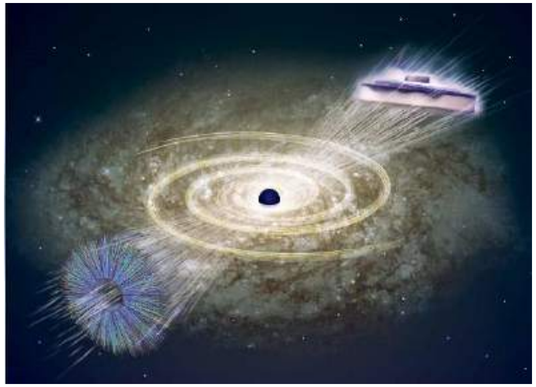
~ keV



IR physics

~ meV

New ideas!



J. Zannen, Nature (2007)

Phenomena beyond Landau paradigms

■ ∃ deconfined phases in quantum materials

ex) Kitaev's spin model:

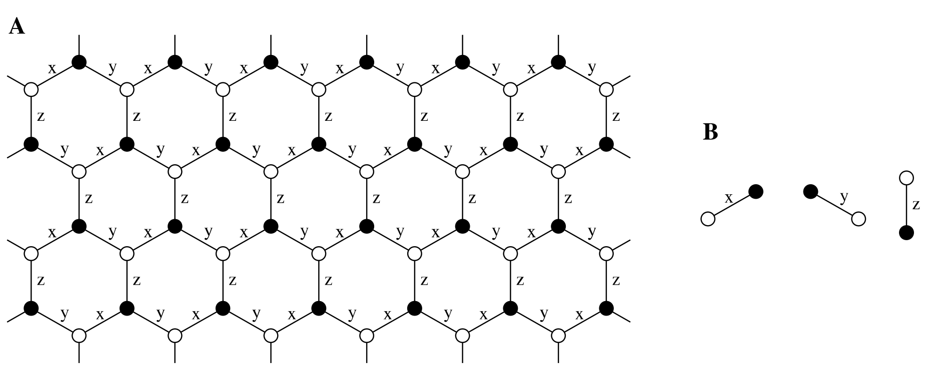
Anyons in an exactly solved model and beyond

Alexei Kitaev *

California Institute of Technology, Pasadena, CA 91125, USA

Received 21 October 2005; accepted 25 October 2005

$$H_K = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z,$$



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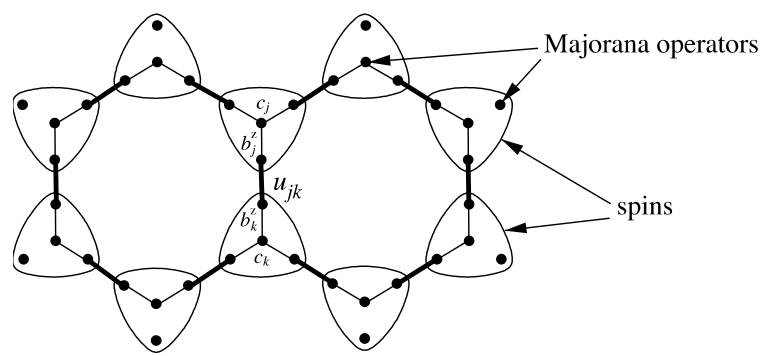
Kitaev representation
: spin ~ four Majorana fermions

$$\vec{\sigma} = ic\vec{b} \qquad c^\dagger = c, \quad \vec{b}^\dagger = \vec{b}, \quad c^2 = 1 = b_i^2$$



Phenomena beyond Landau paradigms

■ ∃ deconfined phases in quantum materials

ex) Kitaev's spin model:

Quantum spins $\xrightarrow{\text{exact mapping}}$ Majorana Fermions + Z_2 gauge

Phenomena beyond Landau paradigms

- ∃ deconfined phases in quantum materials

ex) Kitaev’s spin model:

Quantum spins	$\xrightarrow{\text{exact mapping}}$	Majorana Fermions + Z_2 gauge
Microscopic degrees of freedom (UV d.o.f)		Low energy degrees of freedom (IR d.o.f)

Phenomena beyond Landau paradigms

■ ∃ deconfined phases in quantum materials

ex) Kitaev's spin model:

Quantum spins $\xrightarrow{\text{exact mapping}}$ Majorana Fermions + Z_2 gauge

Microscopic degrees of freedom (UV d.o.f) — Low energy degrees of freedom (IR d.o.f)

cf) Quarks / Strings — Mesons / Baryons

UV d.o.f ≠ IR d.o.f

Phenomena beyond Landau paradigms

■ ∃ deconfined phases in quantum materials

ex) Kitaev's spin model:

Quantum spins $\xrightarrow{\text{exact mapping}}$ Majorana Fermions + Z_2 gauge

Majorana fermions are deconfined.

Phenomena beyond Landau paradigms

■ ∃ deconfined phases in quantum materials

ex) Kitaev's spin model:

Quantum spins	$\xrightarrow{\text{exact mapping}}$	Majorana Fermions + Z_2 gauge

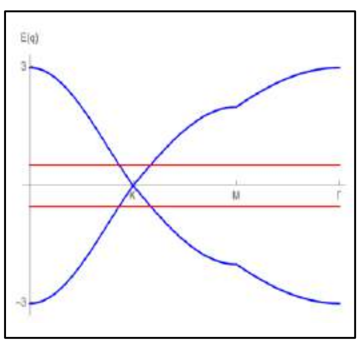
Majorana fermions are deconfined.
(= spins are fractionalized.)

Phenomena beyond Landau paradigms

■ ∃ deconfined phases in quantum materials

ex) Kitaev's spin model:

Quantum spins $\xrightarrow{\text{exact mapping}}$ Majorana Fermions + Z_2 gauge



c : mobile (dispersive)

$b_{x,y,z}$: immobile (flat band)

$$H_{eff} = it \sum_{\langle ij \rangle} \sigma_{ij} c_i c_j + \cdots$$

$$\sigma_{ij} \to \sigma_{ij}\eta_i\eta_j \quad c_i \to c_i\eta_i$$

$$(\sigma_{ij} = \pm 1, \eta_i = \pm 1)$$

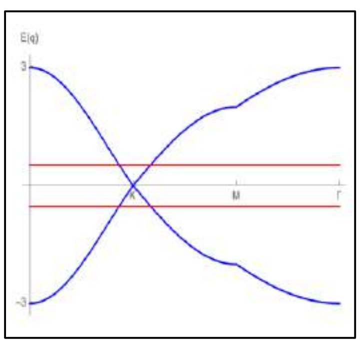
Thanks to J. Seong

Phenomena beyond Landau paradigms

■ ∃ deconfined phases in quantum materials

ex) Kitaev's spin model:

Quantum spins $\xrightarrow{\text{exact mapping}}$ Majorana Fermions + Z_2 gauge



Thanks to J. Seong

Deconfined (fractionalized) phase with Majorana fermions

➔

Beyond Landau paradigms

New Experiments

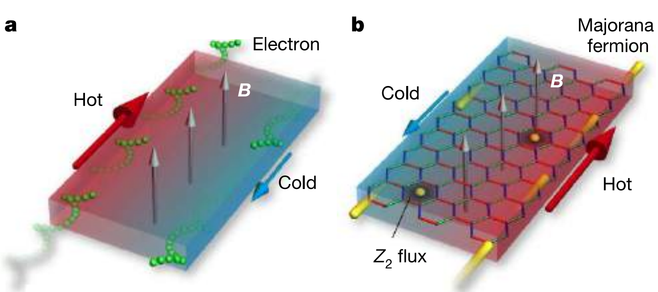
■ Discovery of new degrees of freedom

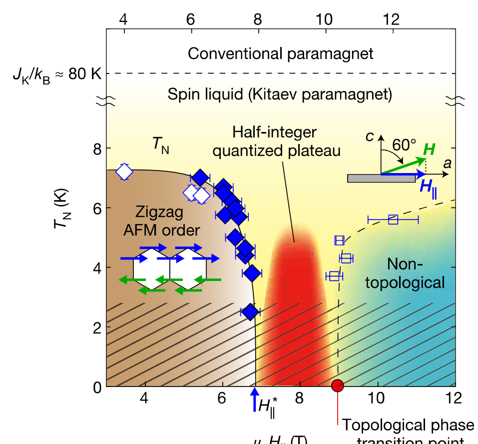
LETTER

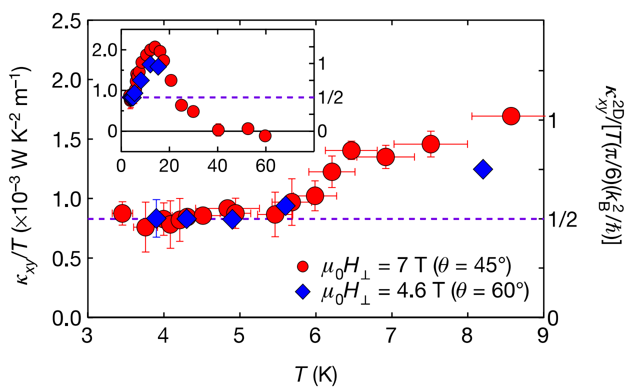
https://doi.org/10.1038/s41586-018-0274-0

Majorana quantization and half-integer thermal quantum Hall effect in a Kitaev spin liquid

Y. Kasahara[1], T. Ohnishi[1], Y. Mizukami[2], O. Tanaka[2], Sixiao Ma[1], K. Sugii[3], N. Kurita[4], H. Tanaka[4], J. Nasu[4], Y. Motome[5], T. Shibauchi[2] & Y. Matsuda[1]*





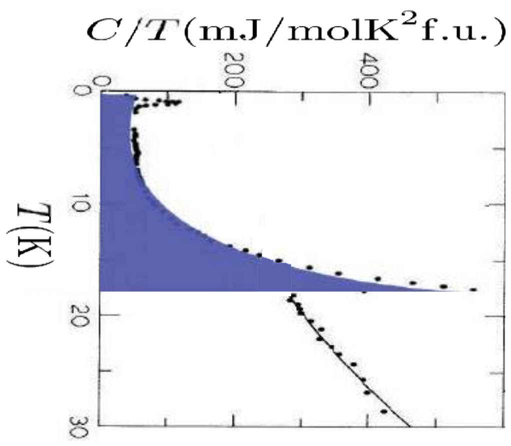


Phenomena beyond Landau paradigm

■ Exotic thermal phase transition

ex) Hidden order transition in URu_2Si_2

Large specific heat jump
but no (obvious) order-parameter



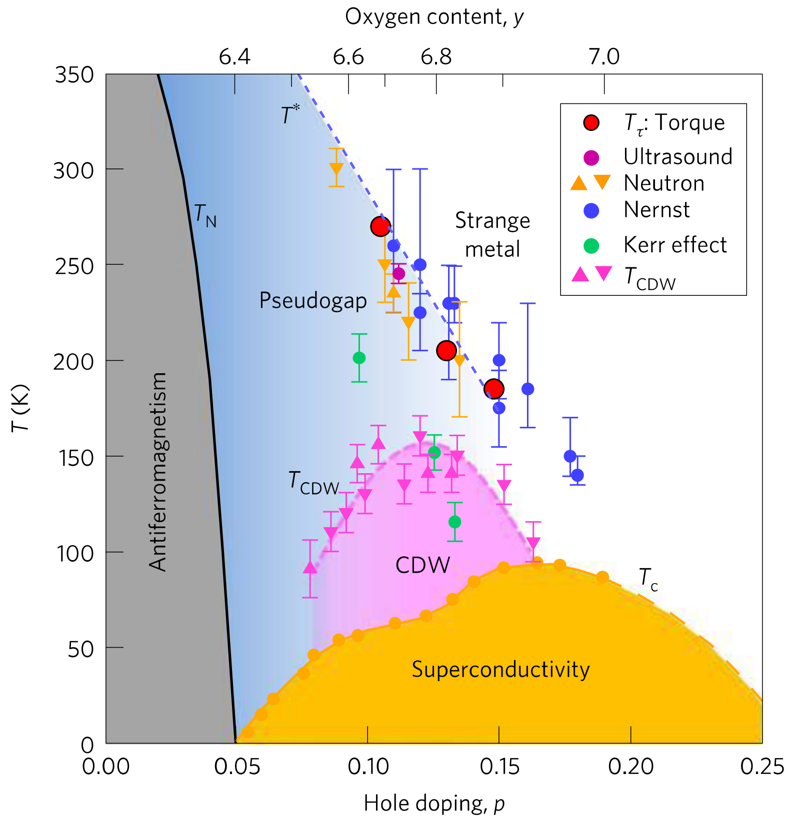
Hidden order behaviour in URu_2Si_2 (A critical review of the status of hidden order in 2014)

J.A. Mydosh[a*] and P.M. Oppeneer[b]

Phenomena beyond Landau paradigm

■ Exotic thermal phase transition

ex) Nematic transition at pseudogap T (cuprates)

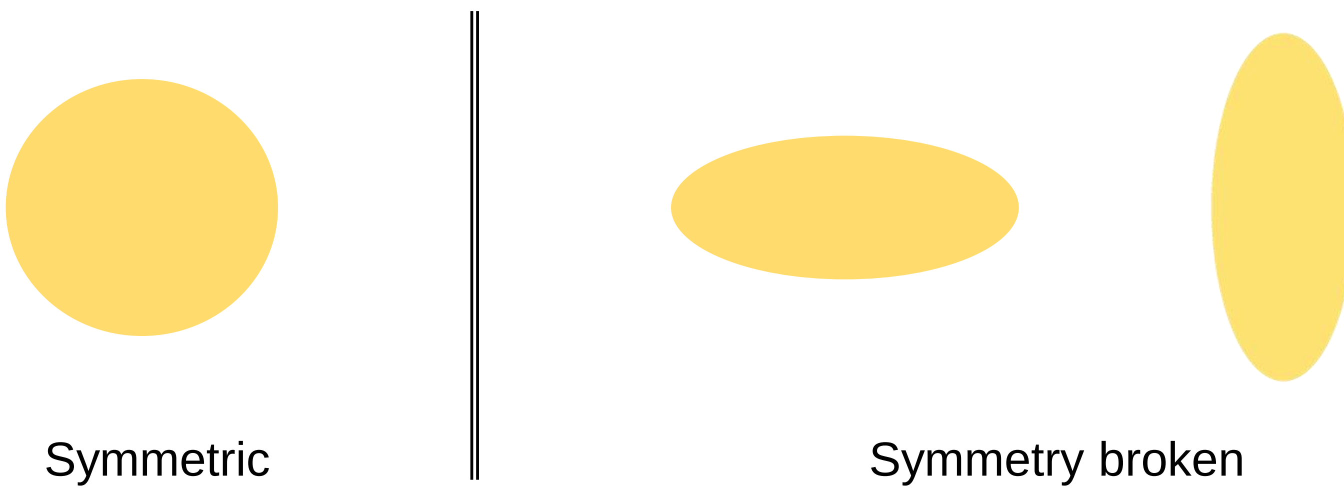


Phenomena beyond Landau paradigm

■ Exotic thermal phase transition

ex) Nematic transition at pseudogap T (cuprates)

- nematicy from C_4 symmetry : Z_2 symmetry

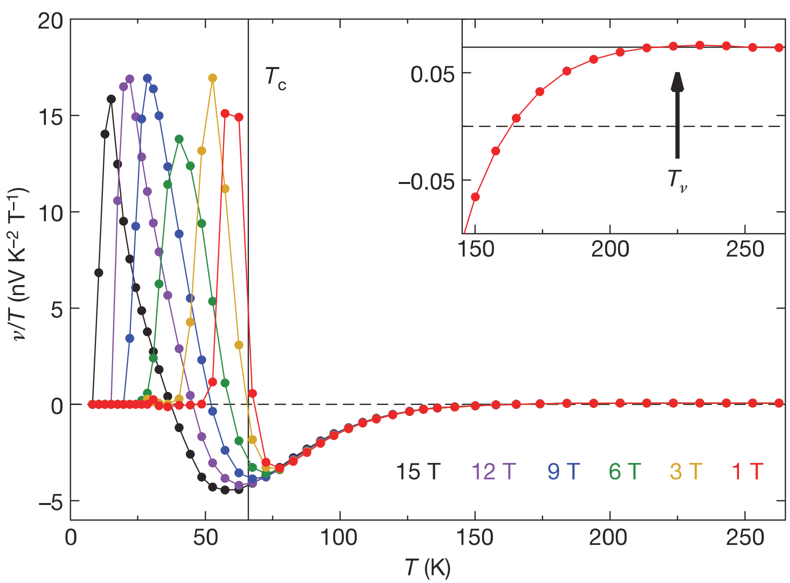


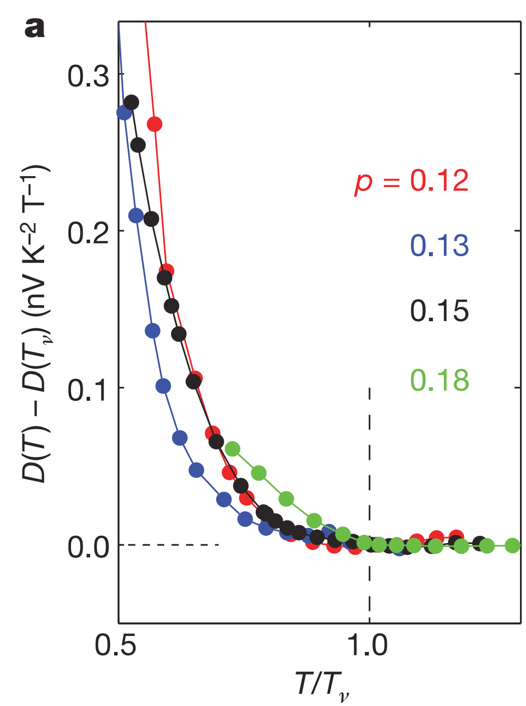
Phenomena beyond Landau paradigm

LETTERS

Broken rotational symmetry in the pseudogap phase of a high-T_c superconductor

R. Daou[1]†, J. Chang[1], David LeBoeuf[1], Olivier Cyr-Choinière[1], Francis Laliberté[1], Nicolas Doiron-Leyraud[1], B. J. Ramshaw[2], Ruixing Liang[2,3], D. A. Bonn[2,3], W. N. Hardy[2,3] & Louis Taillefer[1,3]





Phenomena beyond Landau paradigm

nature physics

LETTERS

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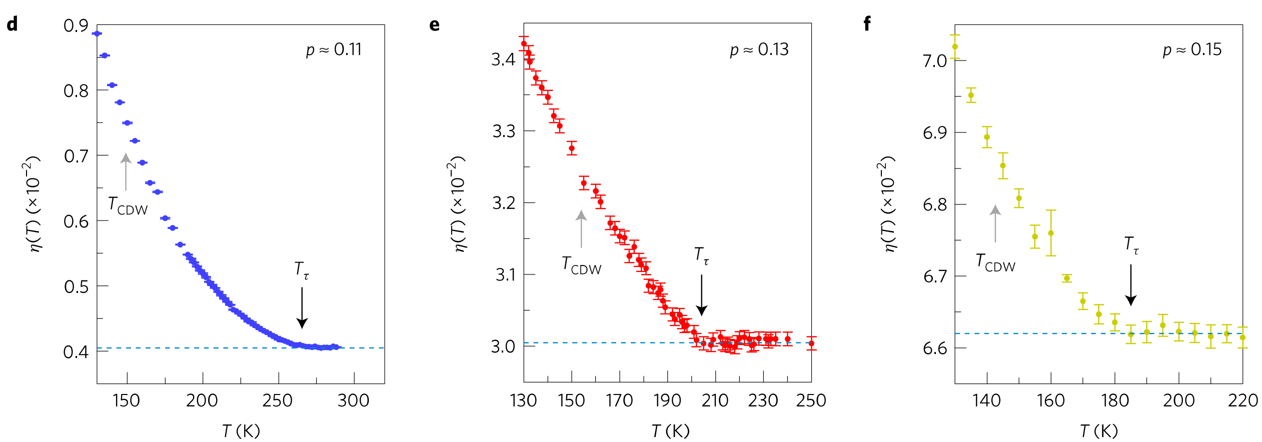
Thermodynamic evidence for a nematic phase transition at the onset of the pseudogap in $YBa_2Cu_3O_y$

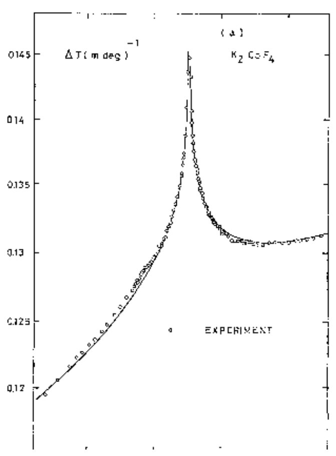
Y. Sato[1], S. Kasahara[1], H. Murayama[1], Y. Kasahara[1], E.-G. Moon[2], T. Nishizaki[3], T. Loew[4], J. Porras[4], B. Keimer[4], T. Shibauchi[5] and Y. Matsuda[1*]



Phenomena beyond Landau paradigm

Ising class

near behavior arise due to rounding.[13] In th

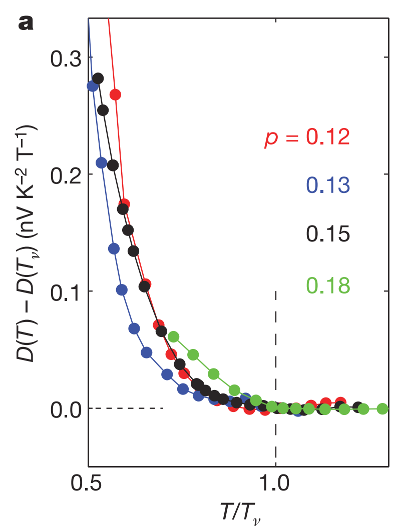


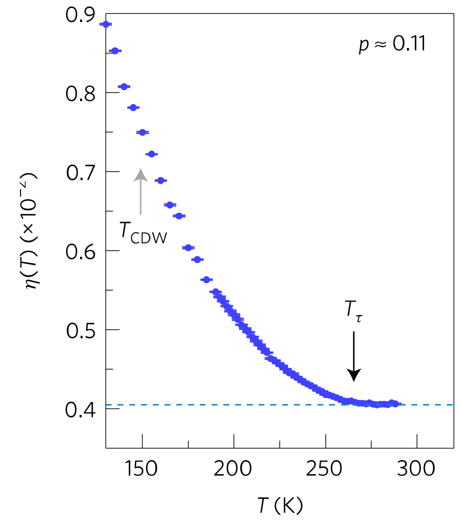
530 TWO-DIMENSIONAL

two-dimensional nature of spin correlations examined in this material. No extinction effects were found after inspection of the $(100)_M$ and $(300)_M$ reflections and of the agreement of twelve measured nuclear integrated intensities with the calculated ones. The sublattice magnetization normalized with the extrapolated intensity at 0 K is plotted in Figs. 1 and 2. Figure 1 shows that both the $(100)_M$ and $(300)_M$ normalized intensities, corrected for background contributions, fall on the same curve. Figure 2 shows that the magnetization of K_2CoF_4 obeys Onsager's formula[8] for the 2-D $S=\frac{1}{2}$ Ising model,[10]

	α	β	γ	δ	η	ν
d=2	0	$\frac{1}{8}$	$\frac{7}{4}$	15	$\frac{1}{4}$	1

nematicity in cuprates





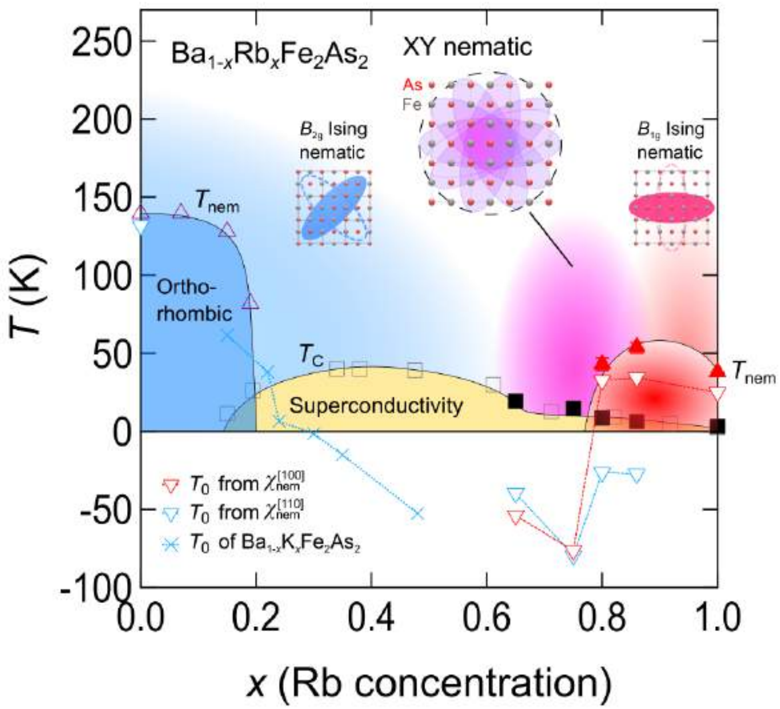
Super-linear onset

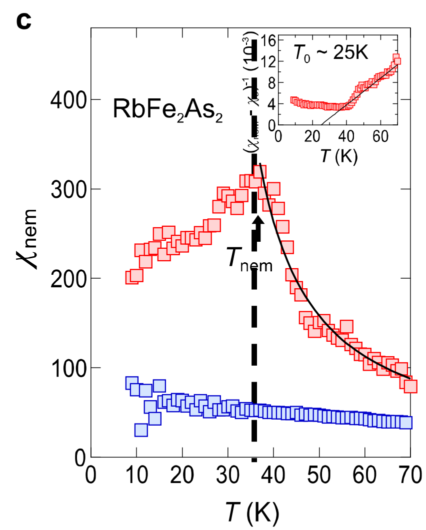
No discernable C(T) anomaly

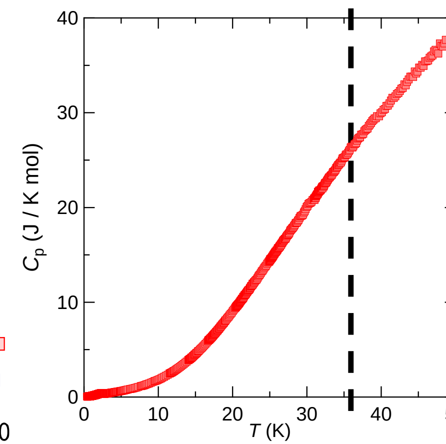
Phenomena beyond Landau paradigm

■ Exotic thermal phase transition

ex) Diagonal nematicity in $RbFe_2As_2$







Susceptibility kink

Shibauchi group (arXiv:1812.05267)

No discernable C(T) anomaly

Phenomena beyond Landau paradigm

- Intriguing phenomena in quantum materials

  ex1) Kitaev quantum spin liquid

  ex2) Hidden order transition in URu_2Si_2

  ex3) Nematic transition at pseudogap temp. of cuprates

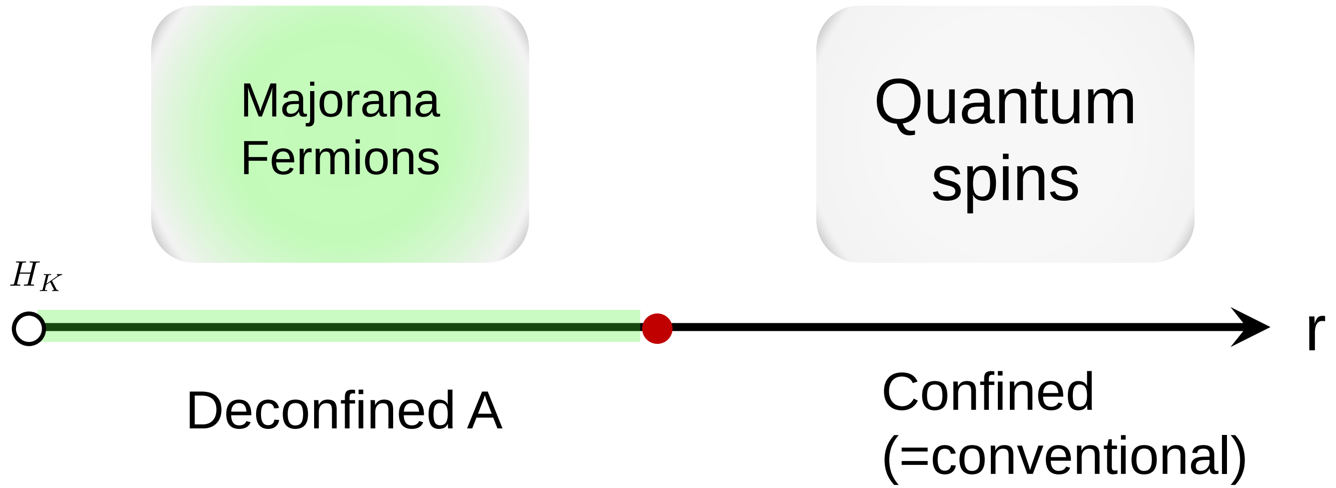
  ex4) Diagonal nematicity in $RbFe_2As_2$

……

New theory? New physics?

Theoretical Set-up I

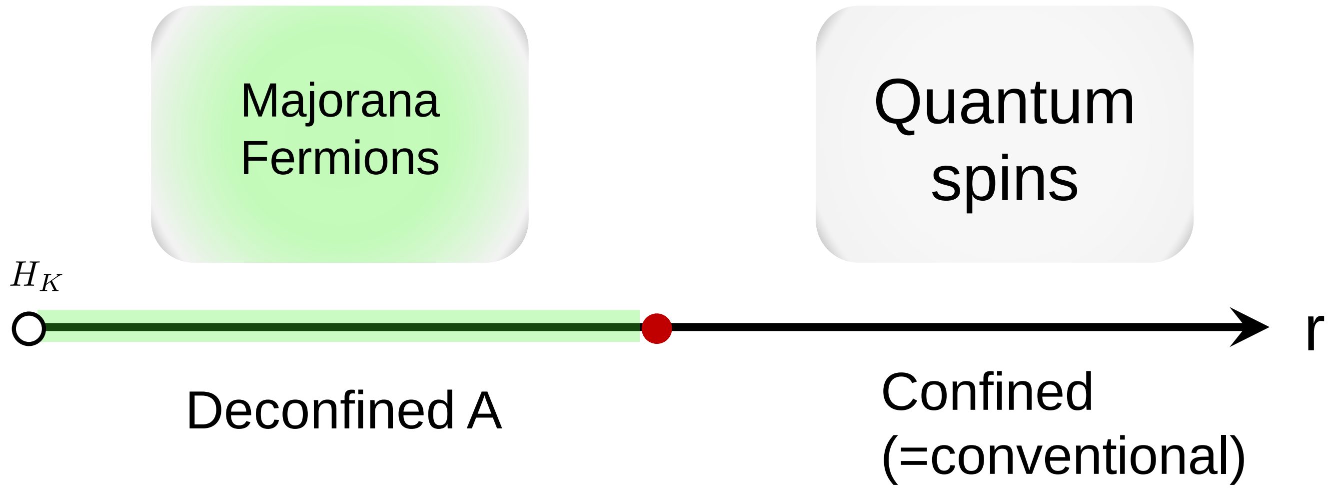
■ Schematic Phase diagram



$$H_K + r \sum_{\langle i,j \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Theoretical Set-up I

■ Schematic Phase diagram

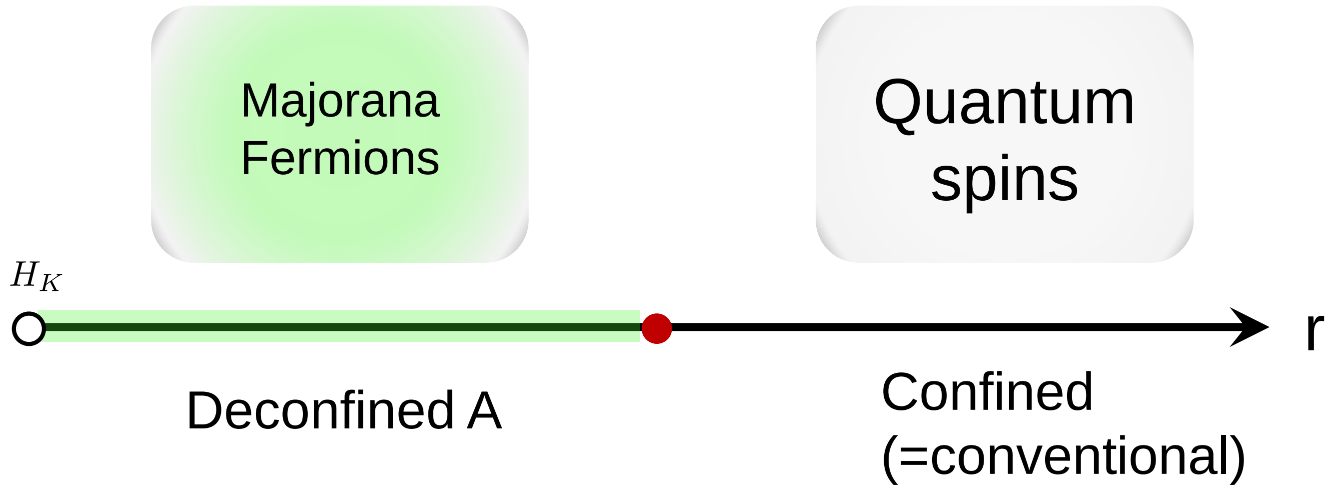


$$H_K + r \sum_{\langle i,j \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Q) possible phases?
constraints?
anomalies?
dualities?

Theoretical Set-up I

■ Schematic Phase diagram



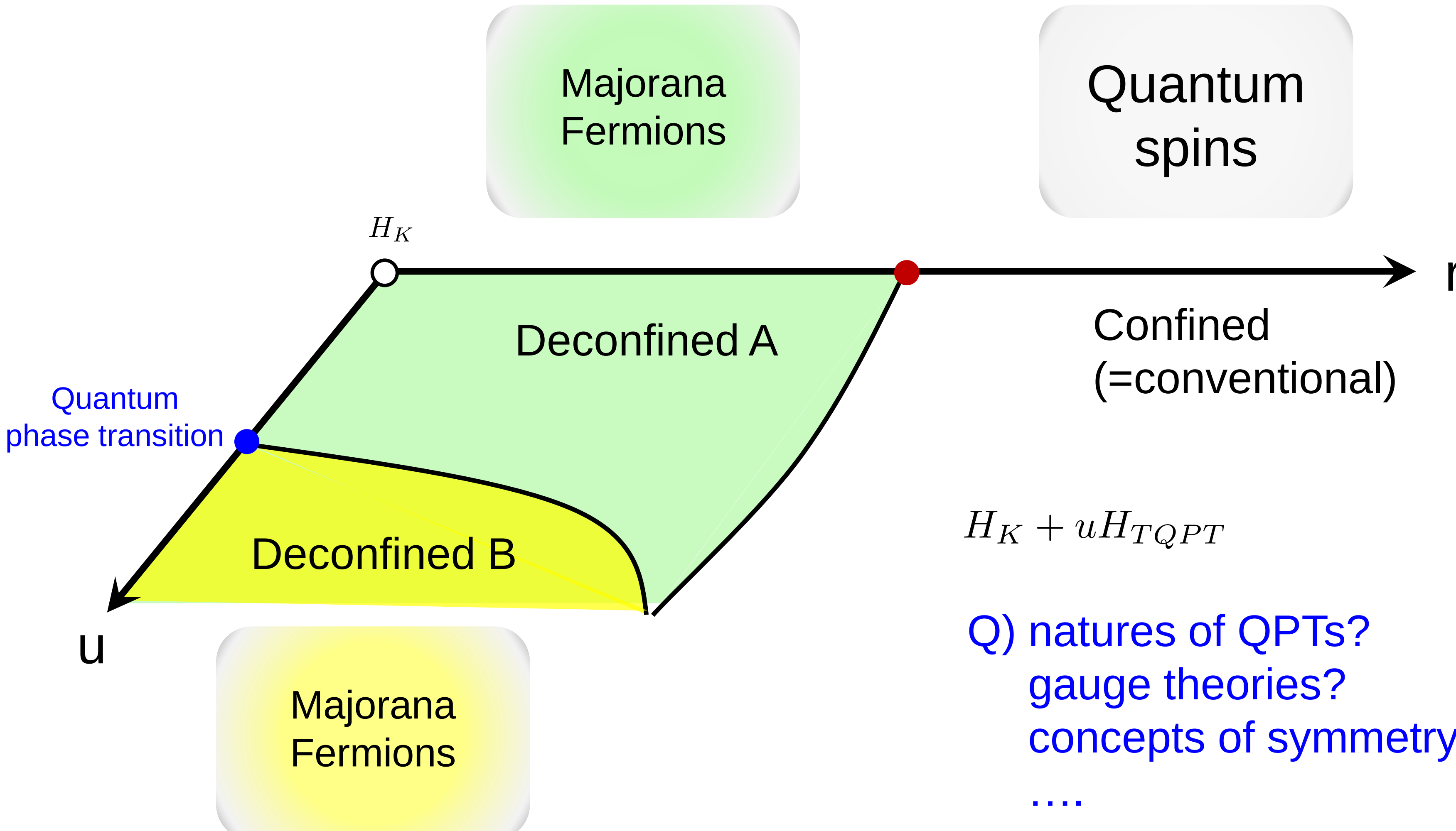
$$H_K + r \sum_{\langle i,j \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Quantum spin liquid
: massive entanglement

Magnetic ordering
: minimal entanglement

Theoretical Set-up I

- Schematic Phase diagram

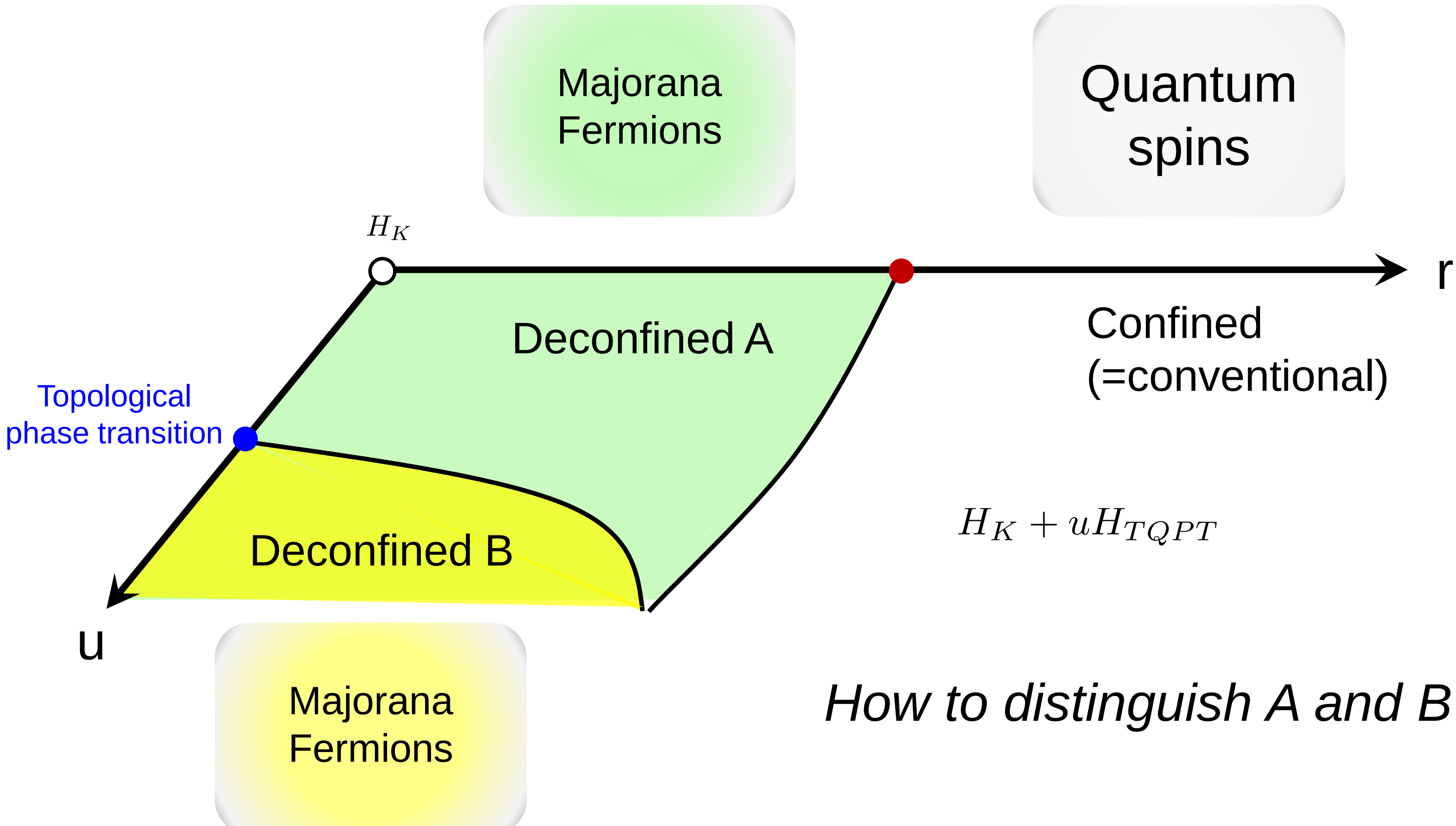


$$H_K + uH_{TQPT}$$

Q) natures of QPTs?
gauge theories?
concepts of symmetry?
….

Theoretical Set-up I

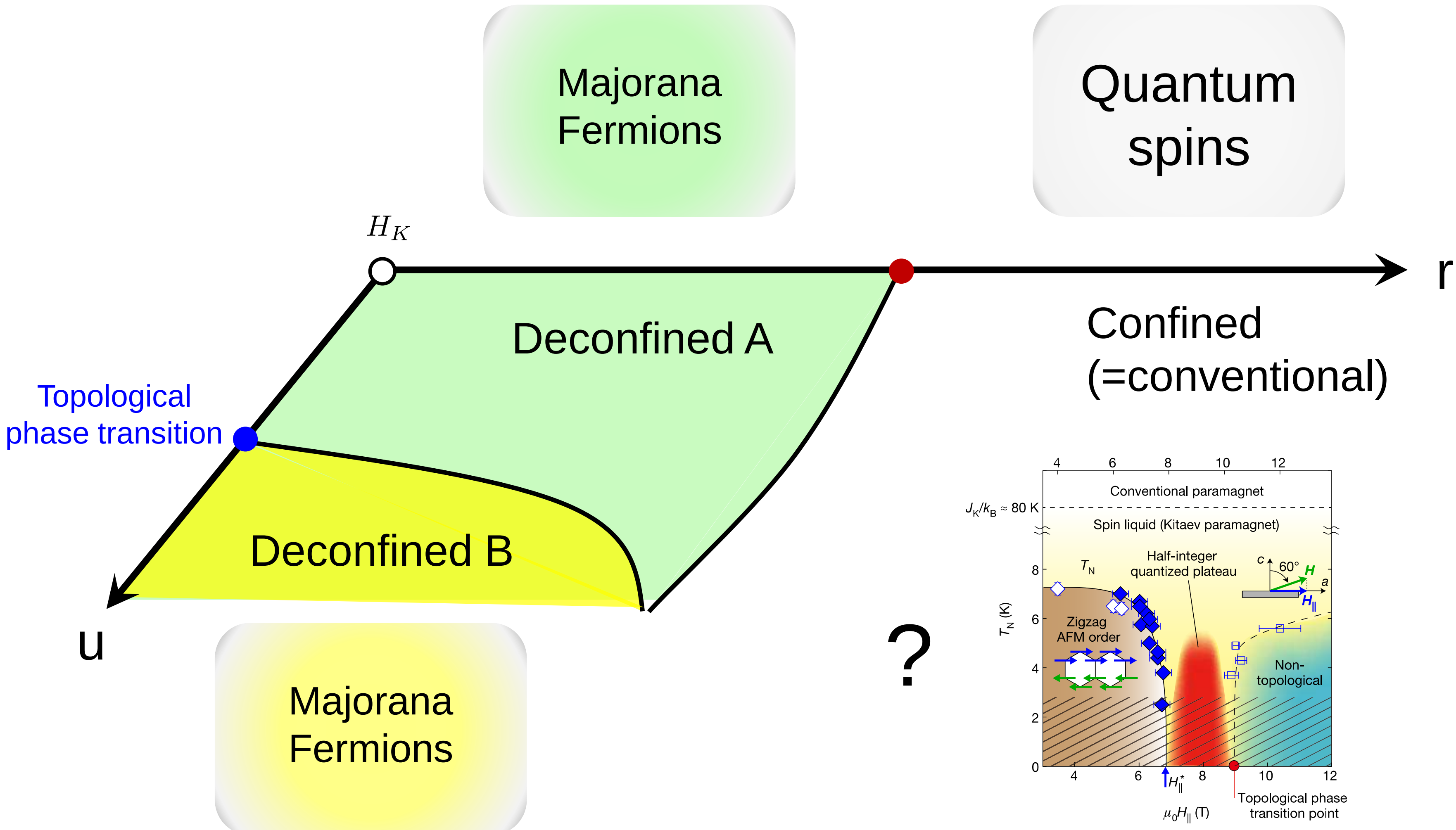
■ Schematic Phase diagram



How to distinguish A and B?

Theoretical Set-up I

■ Schematic Phase diagram



Theoretical Set-up I

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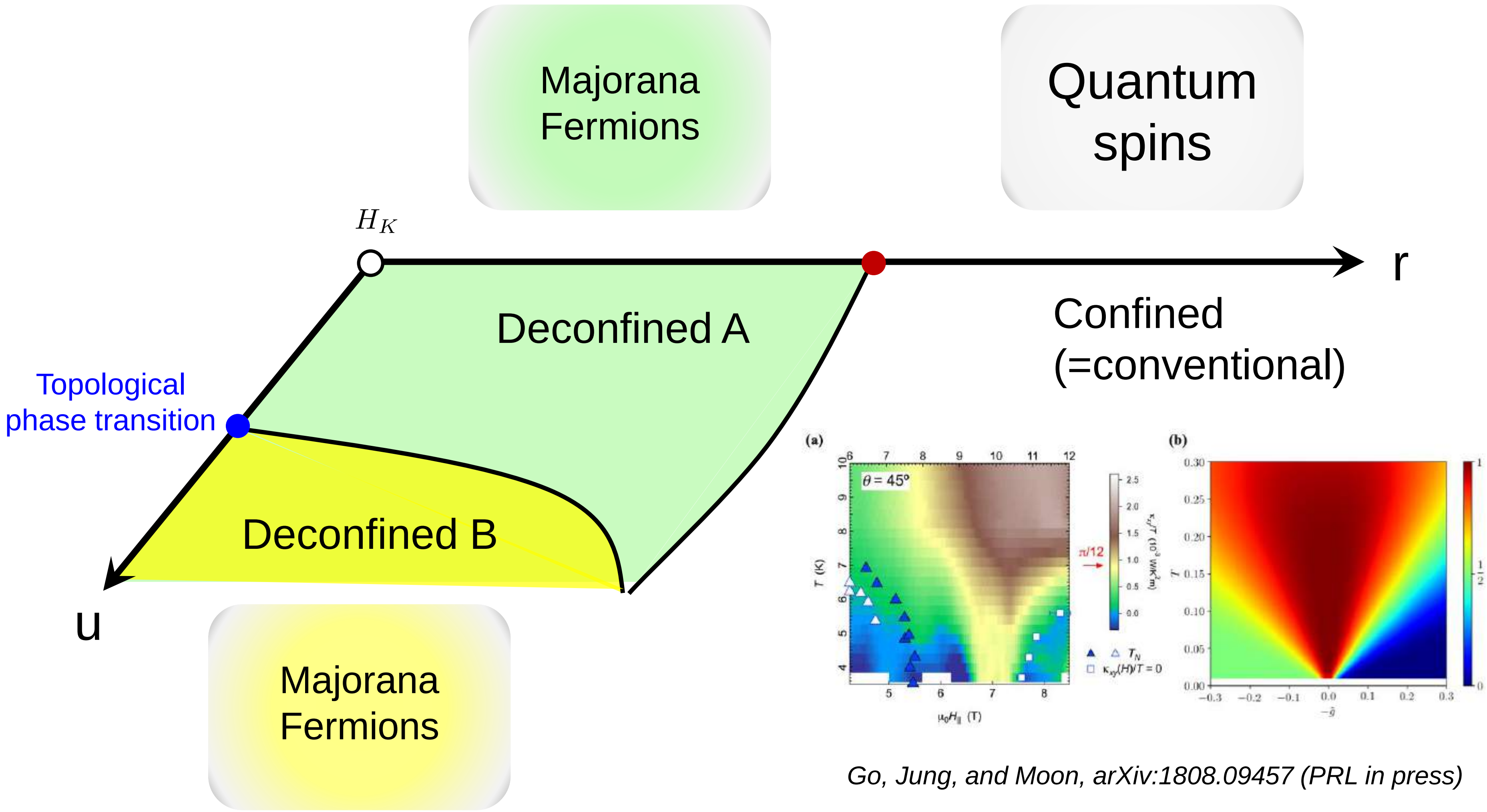
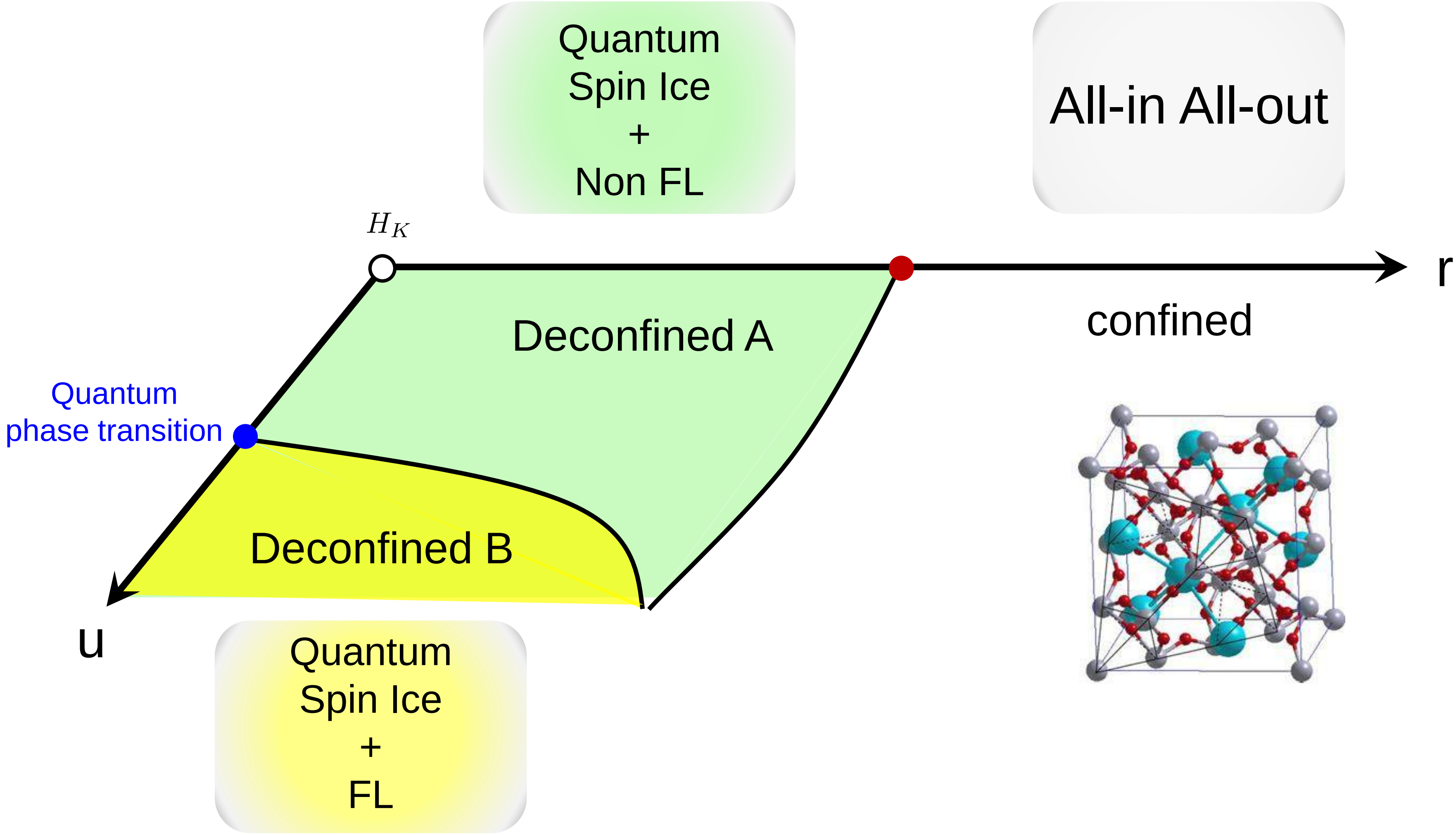


Go, Jung, and Moon, arXiv:1808.09457 (PRL in press)

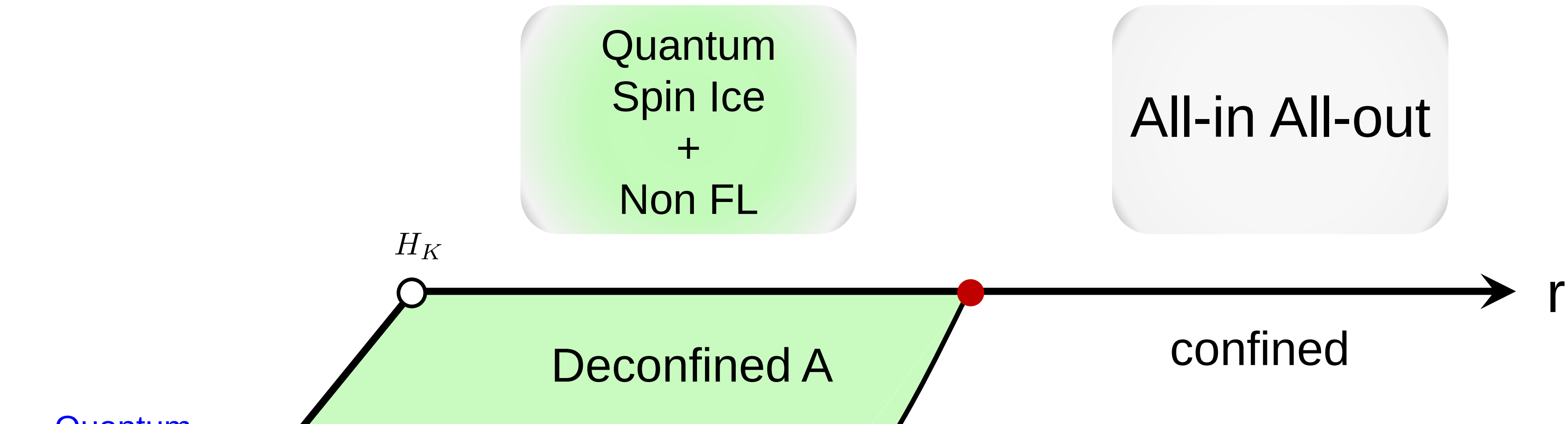
Theoretical Set-up I

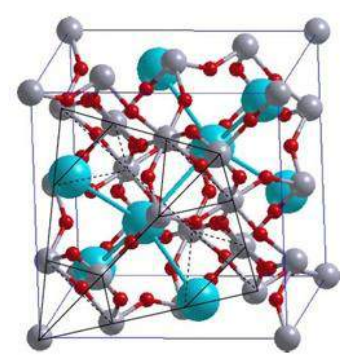
■ Schematic Phase diagram



Theoretical Set-up I

- Schematic Phase diagram

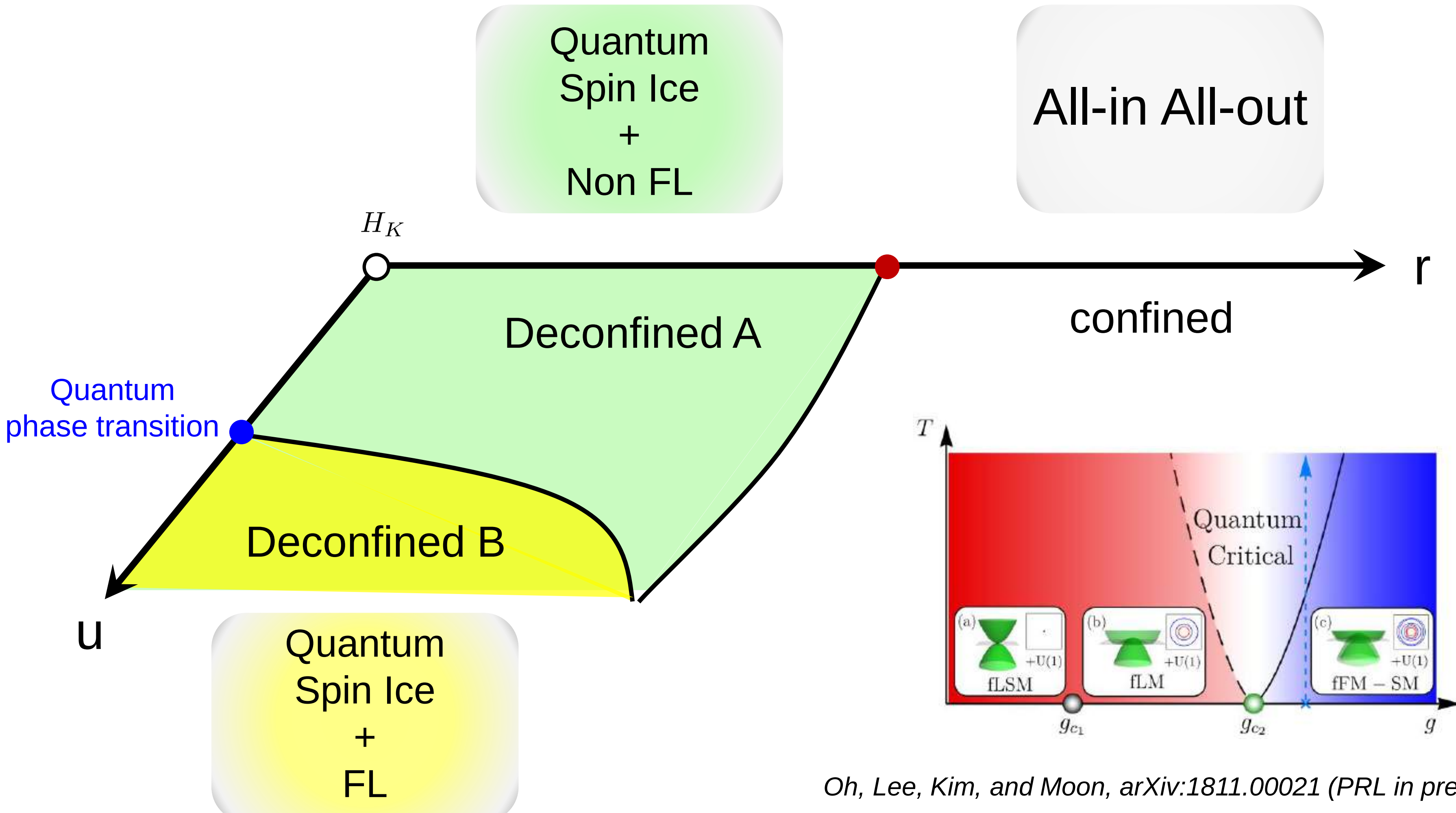




Emergent photons and monopoles

Theoretical Set-up I

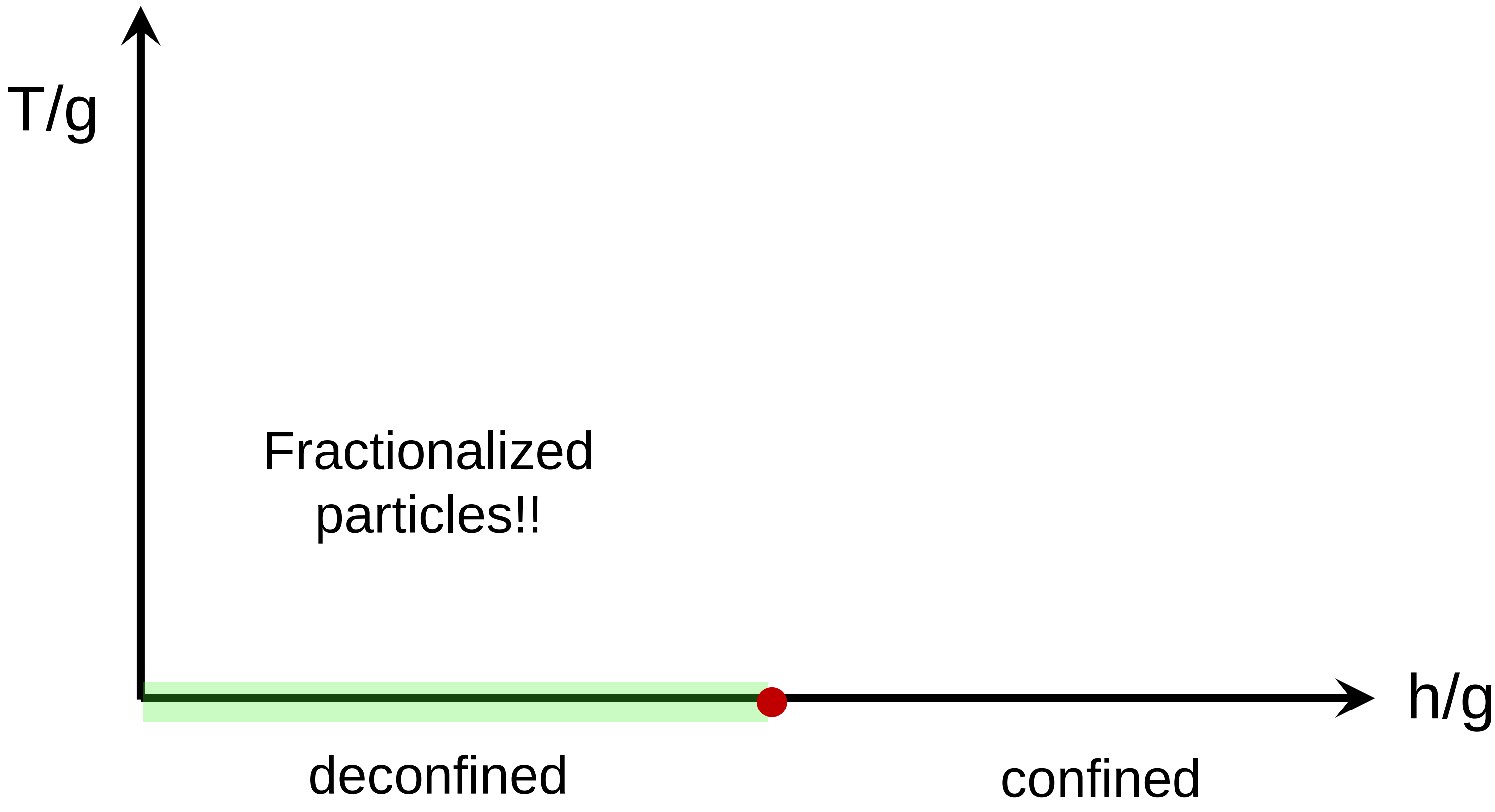
■ Schematic Phase diagram



Oh, Lee, Kim, and Moon, arXiv:1811.00021 (PRL in press)

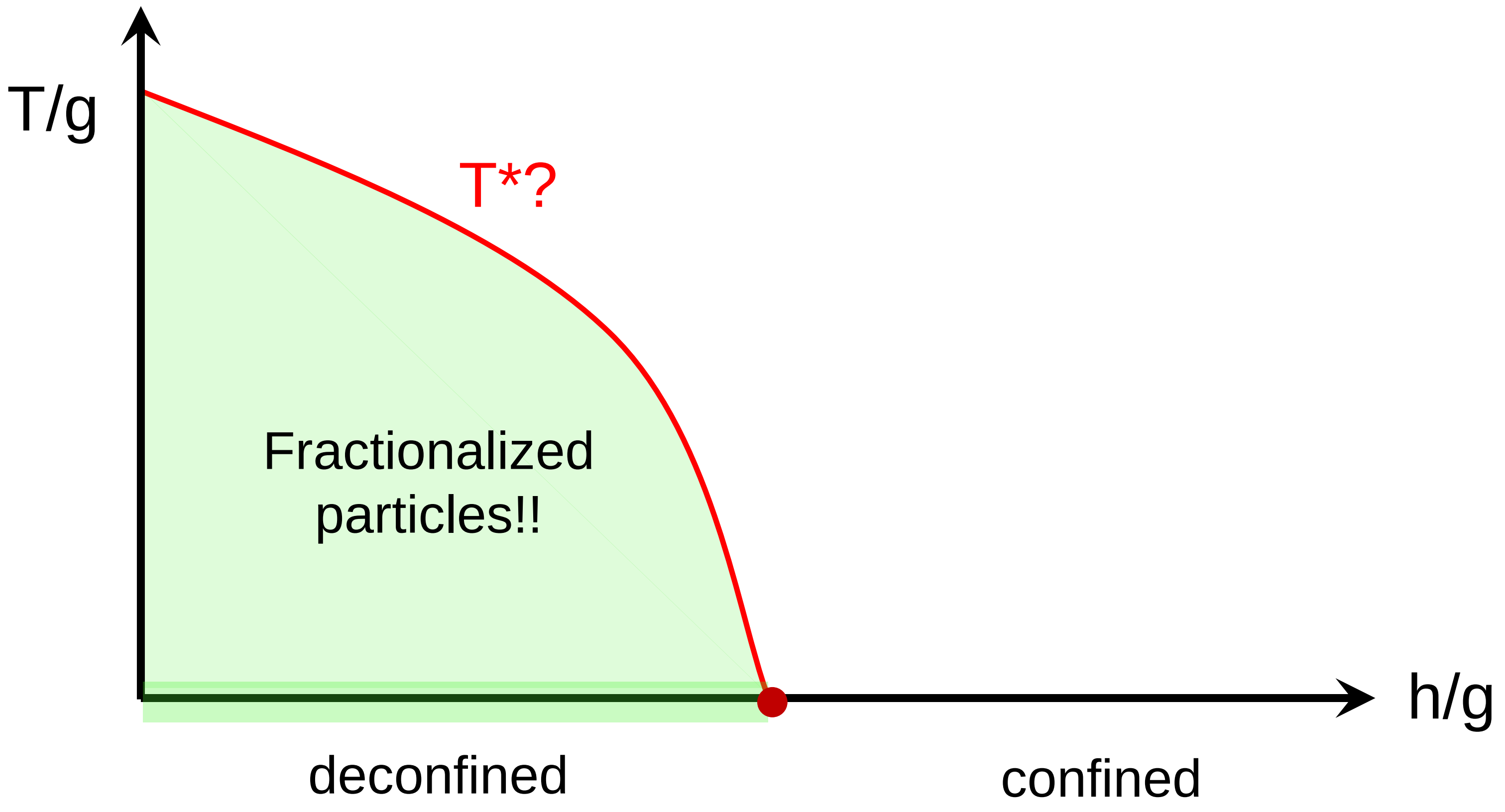
Theoretical Set-up II

■ Schematic Phase diagram



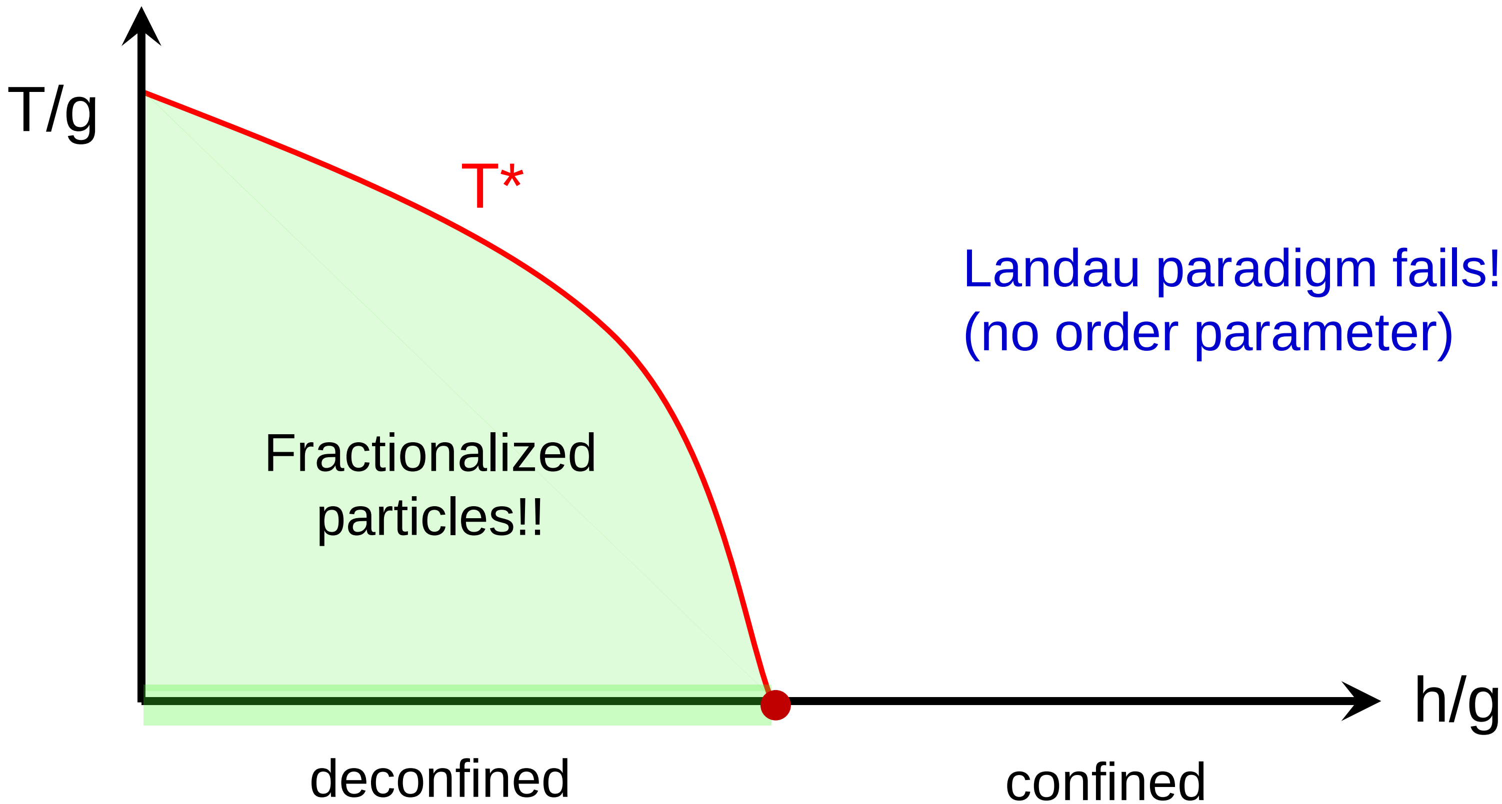
Theoretical Set-up II

- Schematic Phase diagram



Theoretical Set-up II

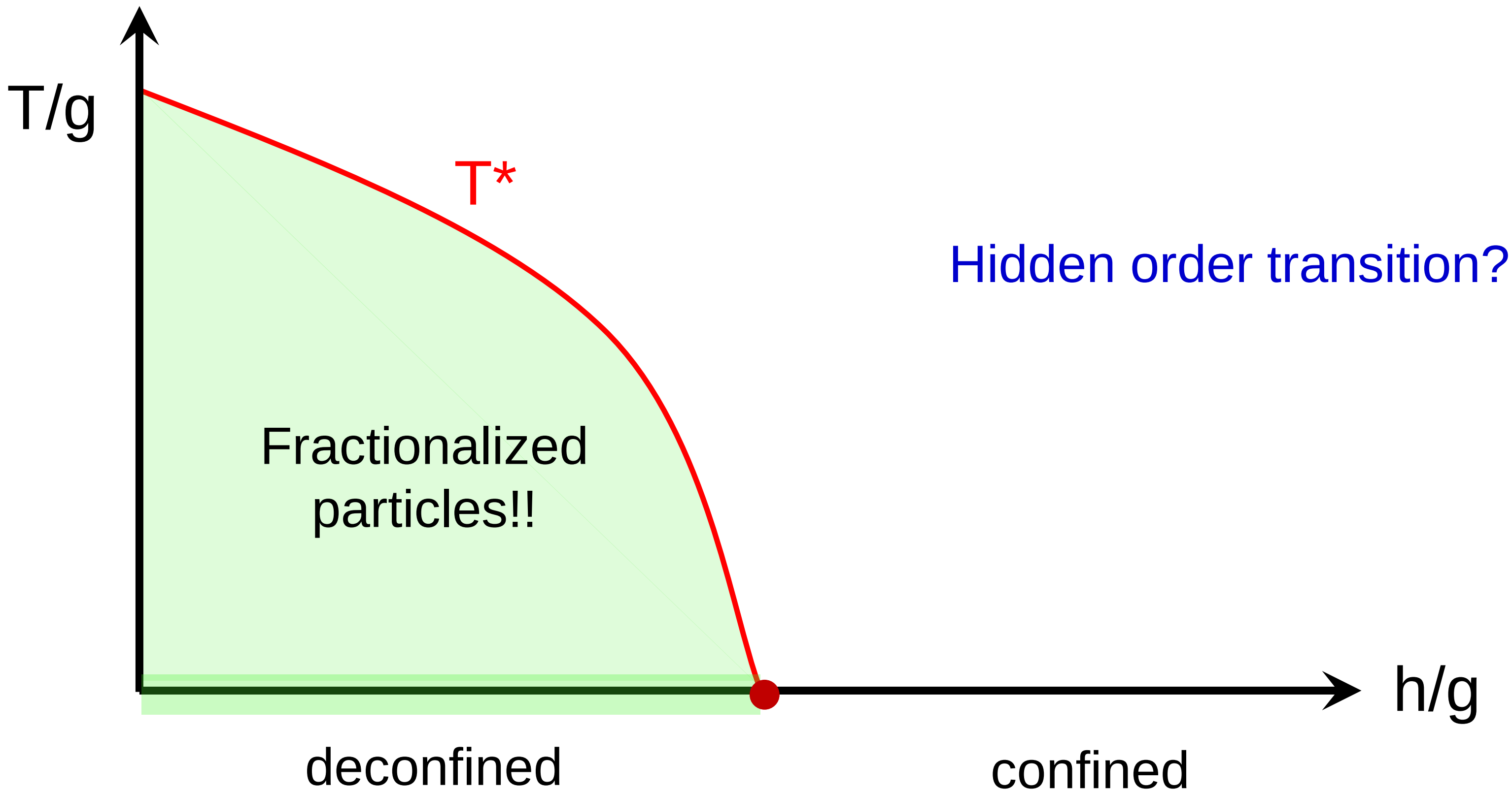
■ Schematic Phase diagram



Landau paradigm fails!
(no order parameter)

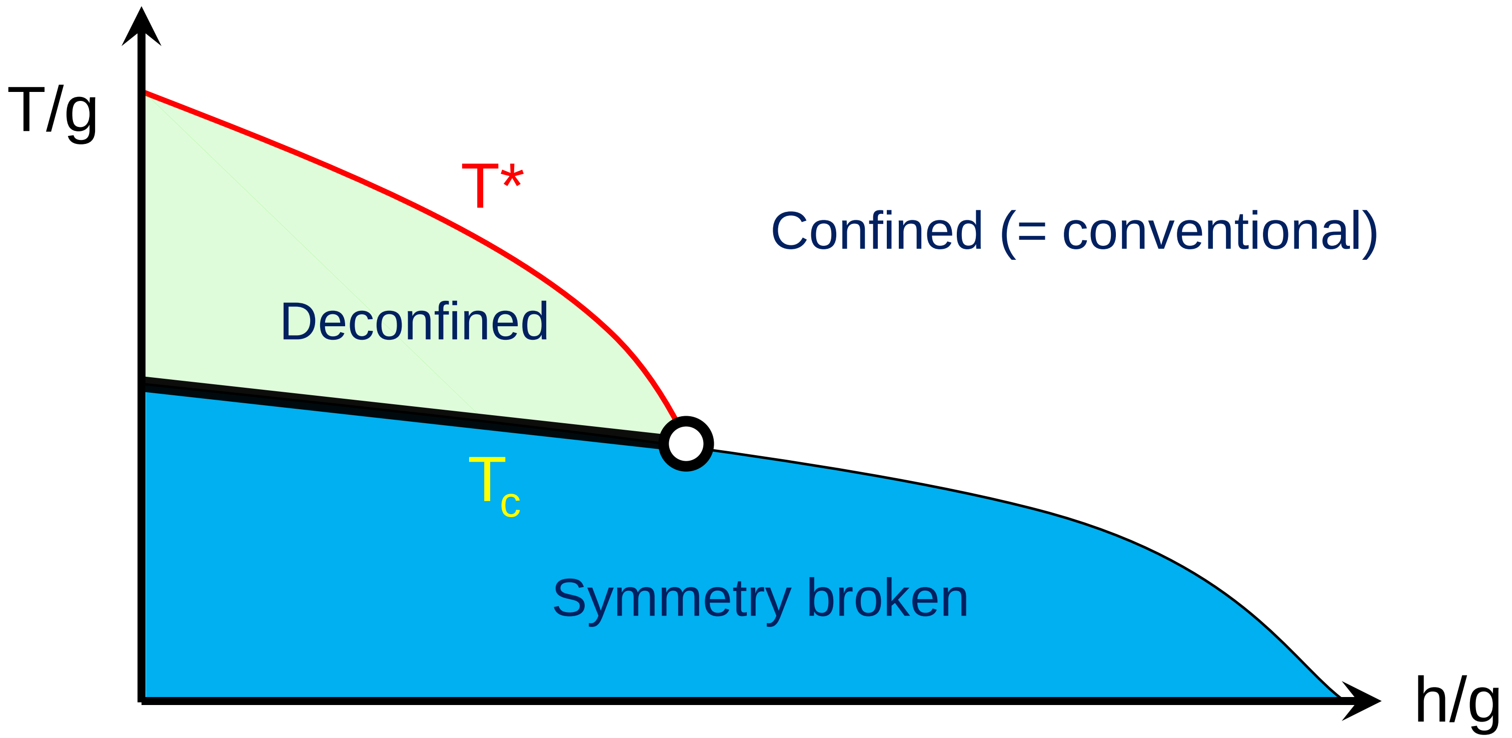
Theoretical Set-up II

■ Schematic Phase diagram



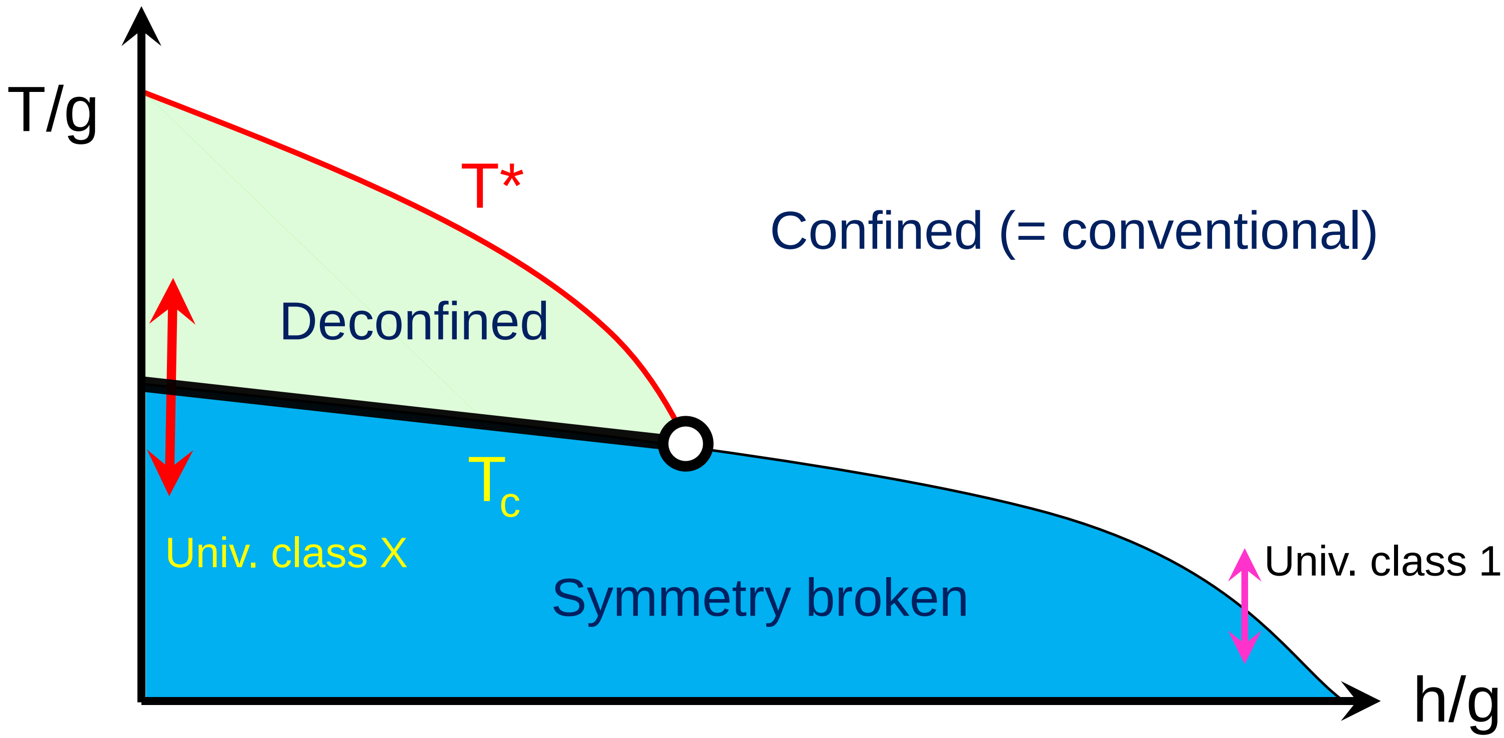
Theoretical Set-up II

■ Schematic Phase diagram



Theoretical Set-up II

■ Schematic Phase diagram



Univ. class X $\stackrel{?}{=}$ *Univ. class 1*

References

arXiv.org > cond-mat > arXiv:1812.05621

Condensed Matter > Strongly Correlated Electrons

Deconfined Thermal Phase Transitions with Z_2 Gauge Structures

Eun-Gook Moon

arXiv.org > cond-mat > arXiv:1811.00021

Condensed Matter > Strongly Correlated Electrons

Multi-scale Quantum Criticality driven by Kondo-lattice Coupling in Pyrochlore Systems

Hanbit Oh, Sangjin Lee, Yong Baek Kim, Eun-Gook Moon

arXiv.org > cond-mat > arXiv:1808.09457

Condensed Matter > Strongly Correlated Electrons

Vestiges of Topological Phase Transitions in Kitaev Quantum Spin Liquids

Ara Go, Jun Jung, Eun-Gook Moon

arXiv.org > cond-mat > arXiv:1803.00578

Condensed Matter > Strongly Correlated Electrons

Exotic Z_2 Symmetry Breaking Transitions in 2D Correlated Systems

Sangjin Lee, Jun Jung, Ara Go, Eun-Gook Moon

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nature physics

LETTERS

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arXiv.org > cond-mat > arXiv:1805.00276

Condensed Matter > Superconductivity

Diagonal Nematicity in the Pseudogap Phase of $HgBa_2CuO_{4+\delta}$

H. Murayama, Y. Sato, R. Kurihara, S. Kasahara, Y. Mizukami, Y. Kasahara, H. Uchiyama, A. Yamamoto, E.-G. Moon, J. Cai, J. Freyermuth, M. Greven, T. Shibauchi, Y. Matsuda

Summary of results

- Existence of a thermal deconfinement transition in 2d

- New universality class of sym. breaking transitions

  - c=1 Z_2 sym. breaking transition in 2d

  - Z_2 and U(1) sym. breaking transitions in 3d deconfined phases are in the same universality class

Univ. class in $3d$	α	β	γ	ν	η	δ
Z_2 (Ising) [33]	0.11	0.33	1.24	0.63	0.036	4.79
$U(1)$ (XY) [33]	−0.015	0.35	1.32	0.67	0.038	4.78
Mean-field	0	0.5	1	0.5	0	3
DC-Z_N/DC-U(1)	−0.015	0.83	0.35	0.67	1.47	1.43

DC-U(1) in $3d$	↵	β	γ	□	ℋ	δ
Z_2 gauge	−0.015	0.83	0.35	0.67	1.47	1.43
Z_3 gauge	−0.015	1.42	−0.83	0.67	3.23	0.42
Z_4 gauge	−0.015	2.09	−2.17	0.67	5.22	−0.04

Outline

1. Motivation

2. Review of Landau paradigms

3. Novel universality class of symmetry breaking transitions

4. Conclusion

Review of Landau Paradigm

- Symmetry and order parameter

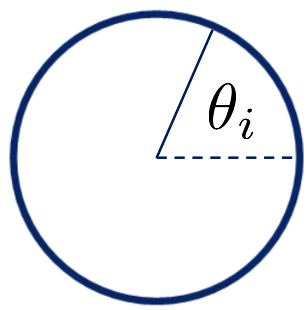
  Z_N sym. = U(1) sym. + Z_N Potential

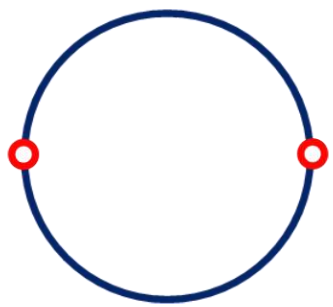
Review of Landau Paradigm

■ Symmetry and order parameter

Z_N sym. = U(1) sym. + Z_N Potential

ex : N=2





$$\theta_i = 0, \pi$$

Review of Landau Paradigm

- Symmetry and order parameter

Z_N sym. = U(1) sym. + Z_N Potential

- Landau functional : Z_N clock model

$$F_L = -\tilde{J} \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(N\theta_i)$$

Review of Landau Paradigm

■ Symmetry and order parameter

Z_N sym. = U(1) sym. + Z_N Potential

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$$F_L = -\tilde{J} \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(N\theta_i)$$

- order parameter $\Phi_i = \cos(\theta_i)$

$$\theta_i = \frac{2\pi}{N} n, \quad n = 0, \cdots, N-1$$

Review of Landau Paradigm

■ Symmetry and order parameter

Z_N sym. = U(1) sym. + Z_N Potential

- Landau functional : Z_N clock model

$$F_L = -\tilde{J}\sum_{\langle i,j\rangle}\cos(\theta_i-\theta_j) - u\sum_i \cos(N\theta_i)$$

- order parameter

$$\Phi_i = \cos(\theta_i)$$

$$\theta_i = \frac{2\pi}{N}n, \quad n = 0, \cdots, N-1$$

- field theory

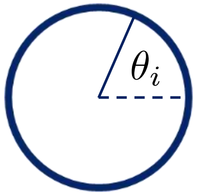
(Wilson-Fisher)

$$S = \int d^dx \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{u}{4}\phi^4$$

Review of Landau Paradigm

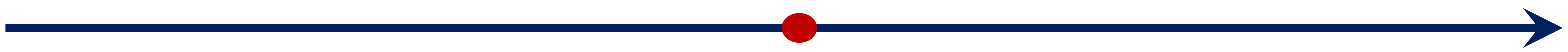
- Kosterlitz-Thouless transition

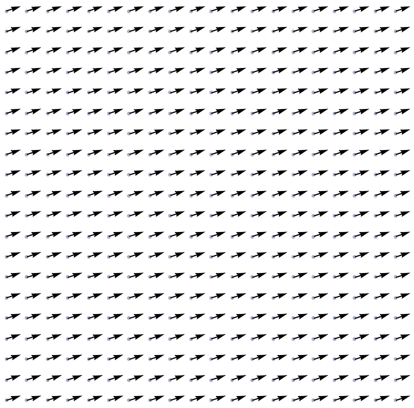
$$H_{KT} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)$$



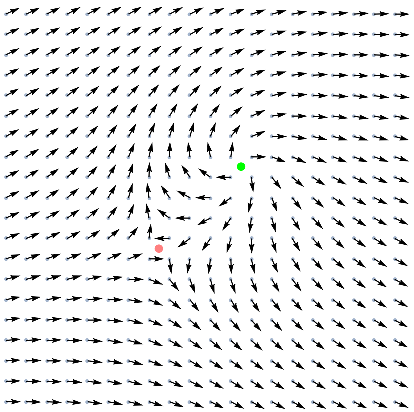
Vortex Free

2pi vortex proliferations

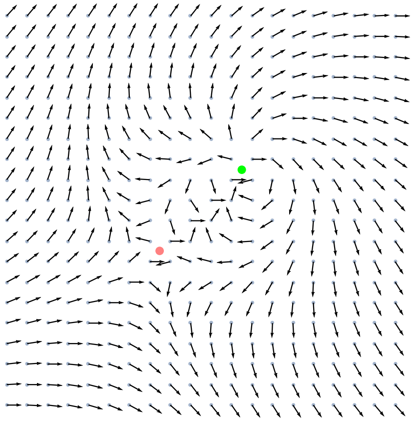




No vortex



2π vortex - antivortex

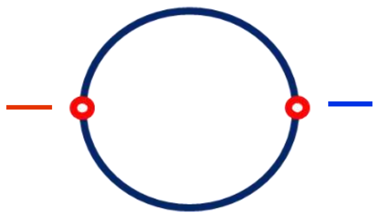


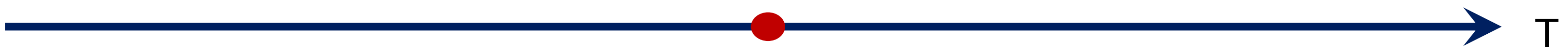
4π vortex - antivortex

Review of Landau Paradigm

- Ising transition

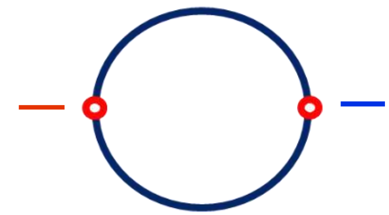
$$H_{Z_2} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$



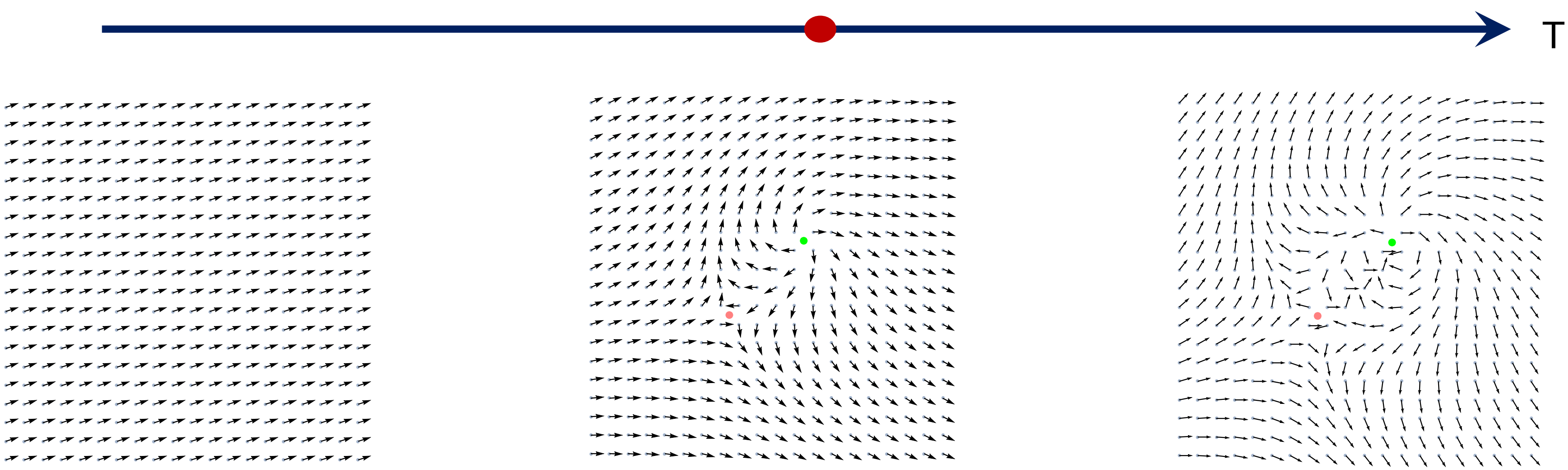


Review of Landau Paradigm

- Ising transition



$$H_{Z_2} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$



Review of Landau Paradigm

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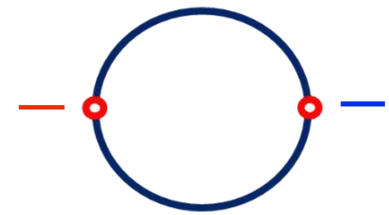

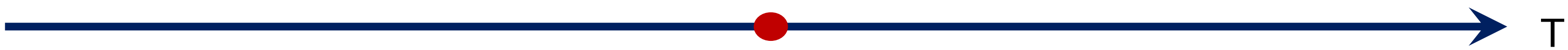

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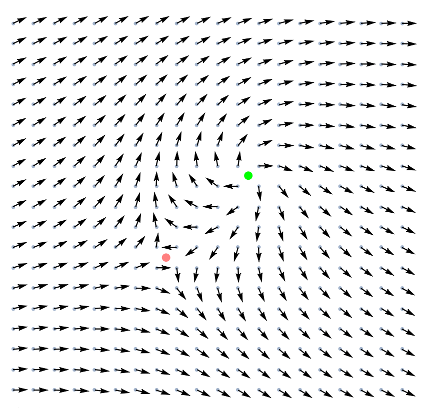
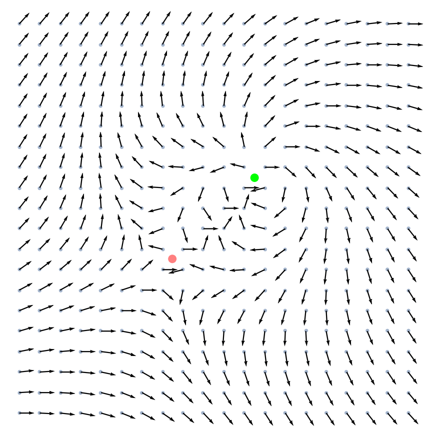
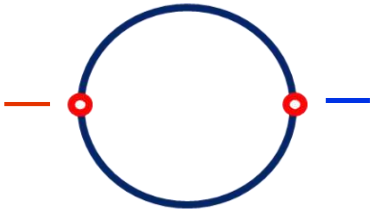


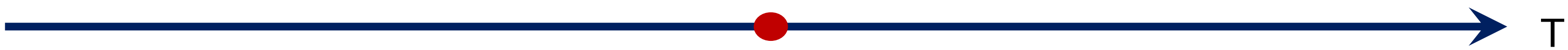


Review of Landau Paradigm

■ Ising transition

$$H_{Z_2} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$





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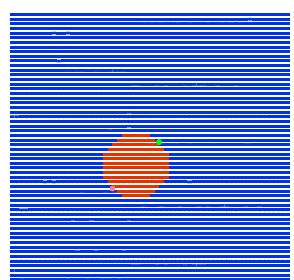
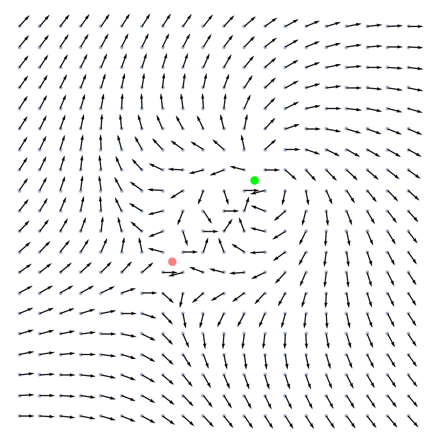


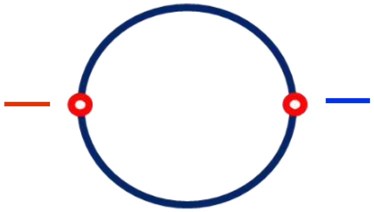
“2π” domain

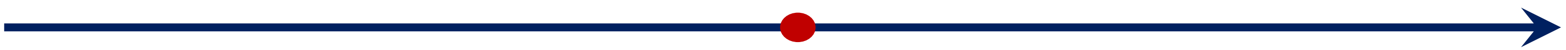


Review of Landau Paradigm

- Ising transition

$$H_{Z_2} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$





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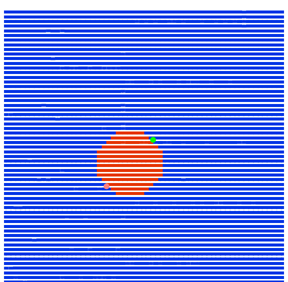
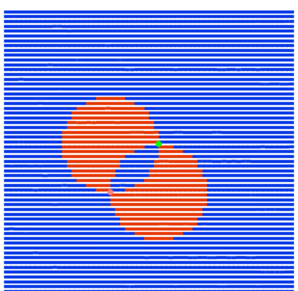


“2π” domain

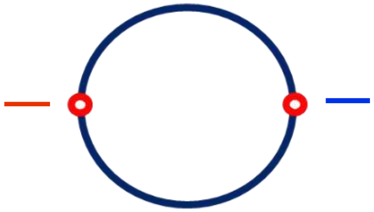


“4π” domain

Review of Landau Paradigm

- Ising transition

$$H_{Z_2} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$

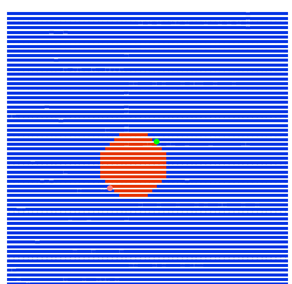


Domain free

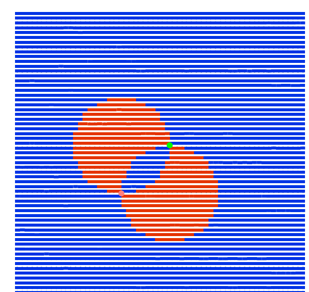
“2π” domain proliferations

T

No domain



“2π” domain

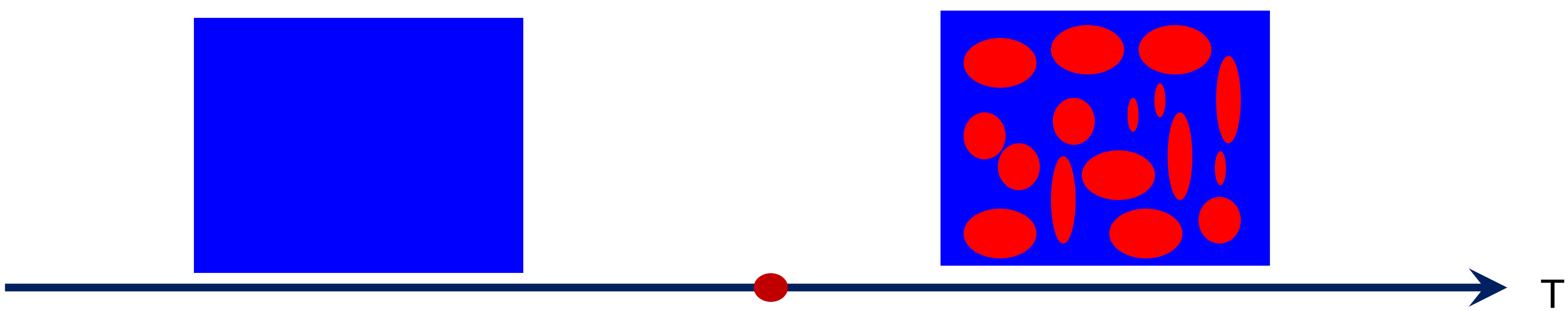


“4π” domain

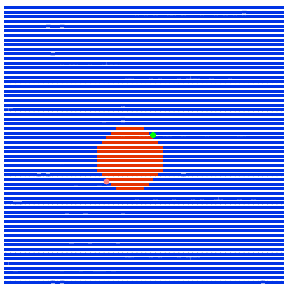
Review of Landau Paradigm

- Ising transition

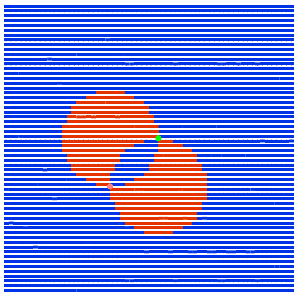
$$H_{Z_2} = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$



No domain



“2π” domain



“4π” domain

Review of Landau Paradigm

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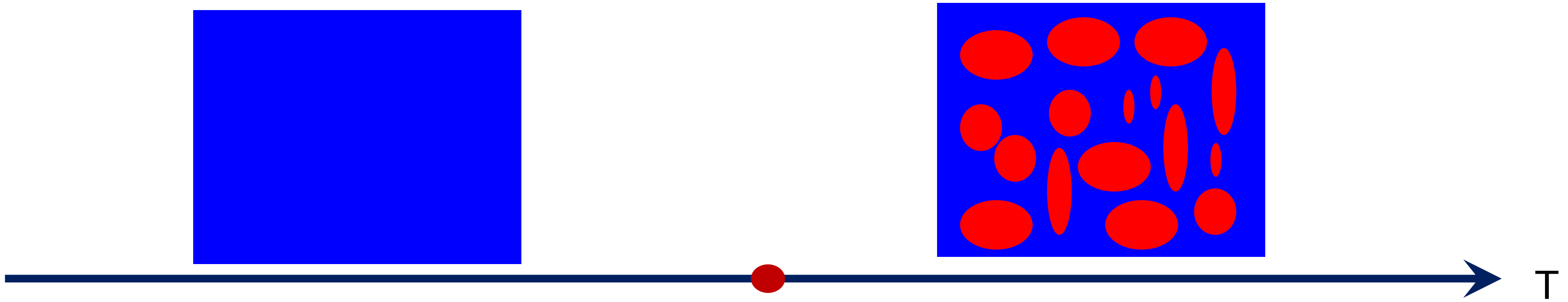


Symmetry breaking transition = 2π domain proliferation

Wilson-Fisher universality class

	α	β	γ	δ	η	ν
d=2	0	$\frac{1}{8}$	$\frac{7}{4}$	15	$\frac{1}{4}$	1

Univ. class in $3d$	α	β	γ	ν	η	δ
Z_2 (Ising) [33]	0.11	0.33	1.24	0.63	0.036	4.79

Outline

1. Motivation

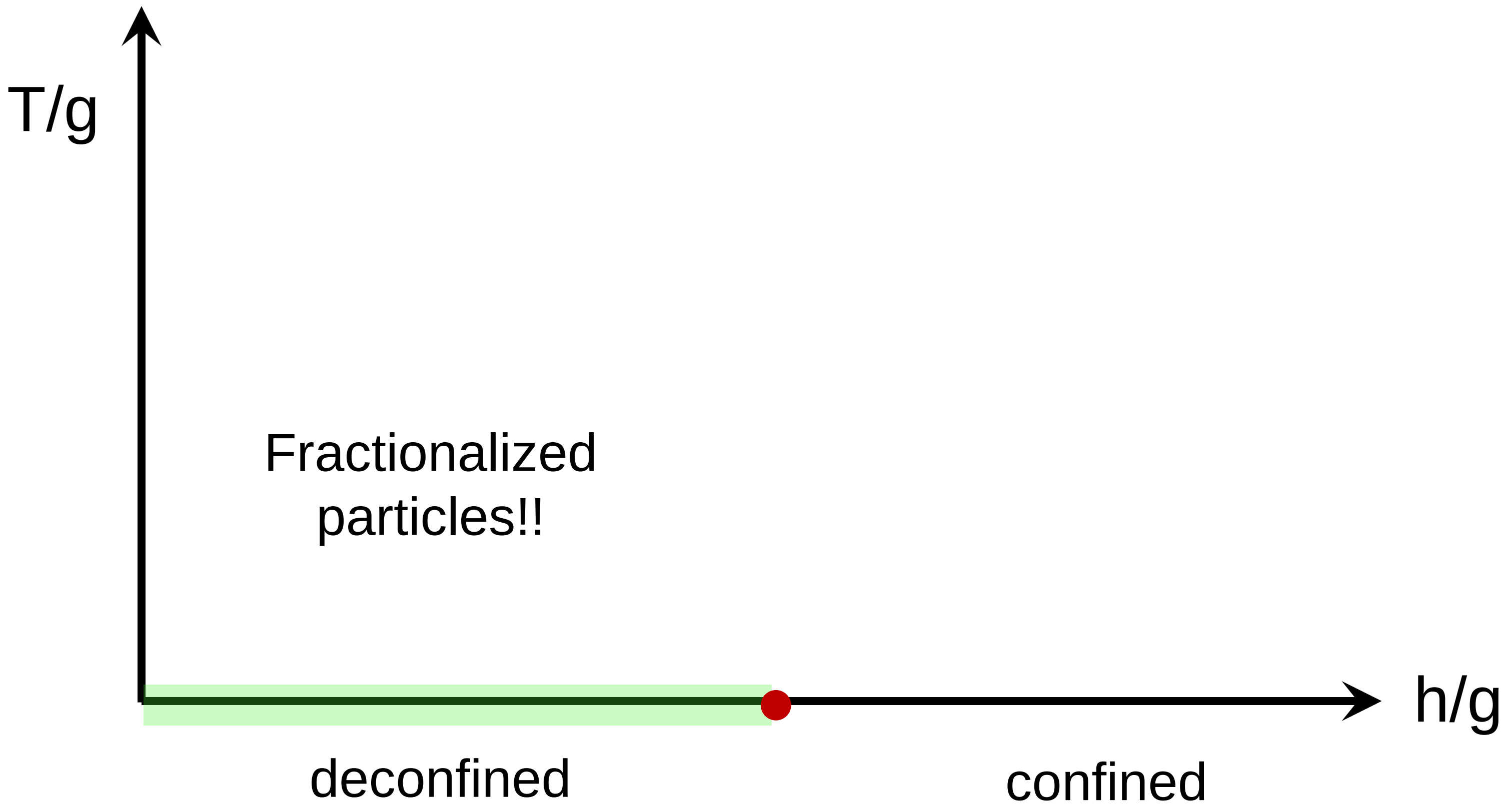
2. Review of Landau paradigms

3. Novel universality class of symmetry breaking transitions

4. Conclusion

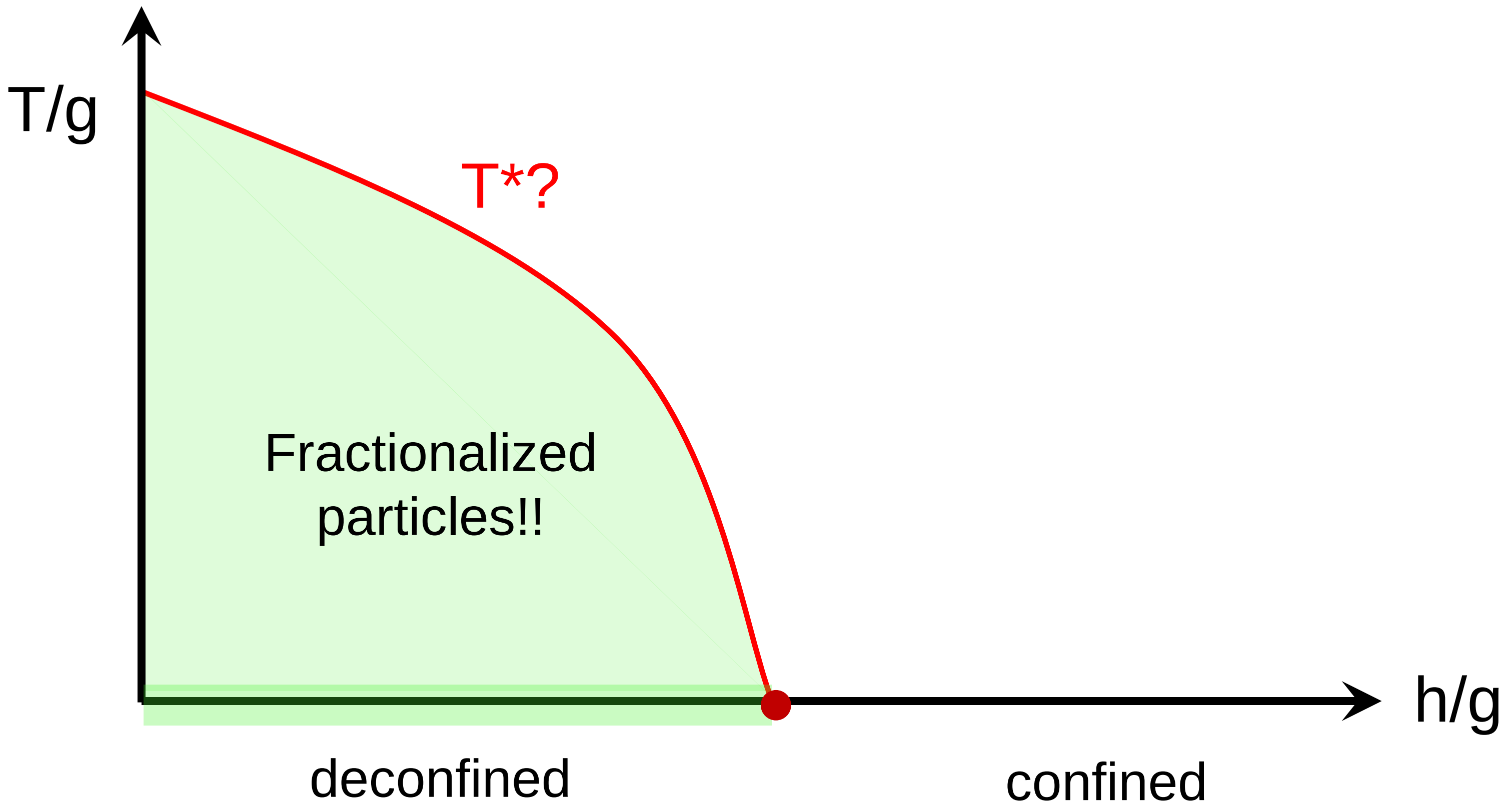
Theoretical Set-up

■ Schematic Phase diagram



Theoretical Set-up

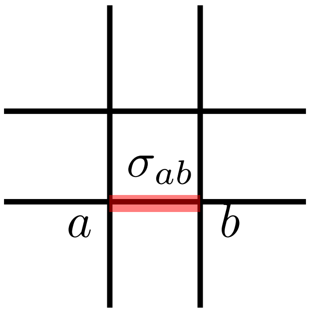
■ Schematic Phase diagram



List of Known Facts

■ ∃ thermal deconfinement transition

$$H_W = -g \sum_{I^*} \prod_{(ab)\in\Box} \sigma_{ab}$$



Z_2 gauge transformation

$$\sigma_{ij} \to \sigma_{ij}\eta_i\eta_j, \quad \eta_i = \pm 1$$

Wegner : ∃ thermal transition in 3d

Polyakov, Kogut, … : no thermal transition in 2d

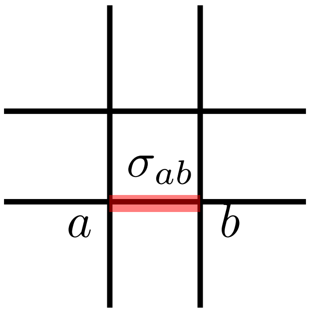
Fradkin and Shenker : Higgs = Confinement (fund.)

The Wegner model

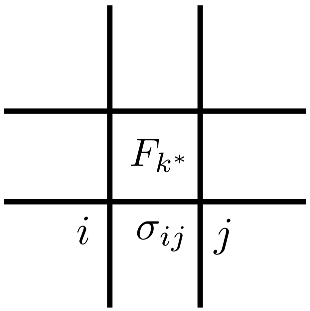
- Z_2 lattice gauge theory

$$H_W = -g \sum_{I^*} \prod_{(ab)\in\Box} \sigma_{ab}$$

$$= -g \sum_{k^*} F_{k^*}$$



$$F_{k^*} = \prod_{(ij)\in\Box} \sigma_{ij}$$



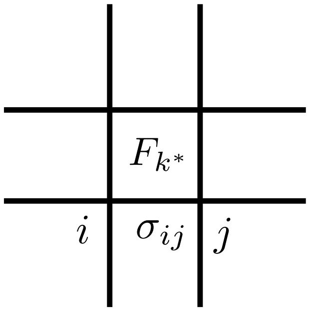
Z_2 gauge transformation $\quad \sigma_{ij} \to \sigma_{ij}\eta_i\eta_j, \quad \eta_i = \pm 1$

Wilson-loop operator $\quad W_{\mathcal{C}} = \prod_{(ij)\in\mathcal{C}} \sigma_{ij}$

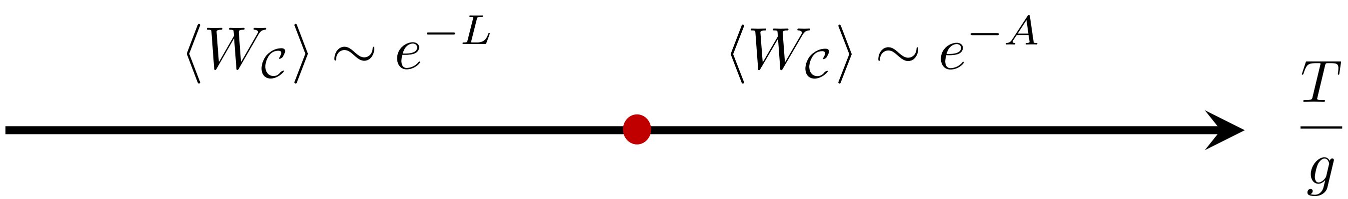
The Wegner model

- Kramers-Wannier duality transformation in 3d

$$H_W = -g \sum_k \prod_{(ij)\in\Box} \sigma_{ij} \equiv -g \sum_k F_{k^*}$$



dual to the 3d Ising model.



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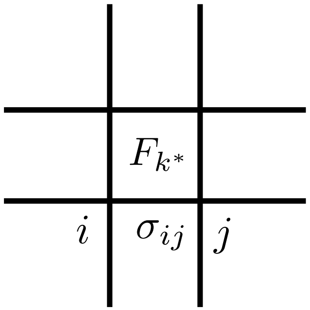
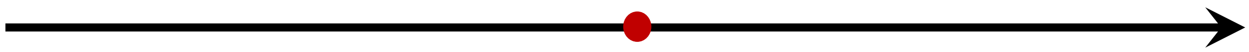


dual to the 3d Ising model.

$$\langle W_{\mathcal{C}} \rangle \sim e^{-L} \qquad \langle W_{\mathcal{C}} \rangle \sim e^{-A}$$

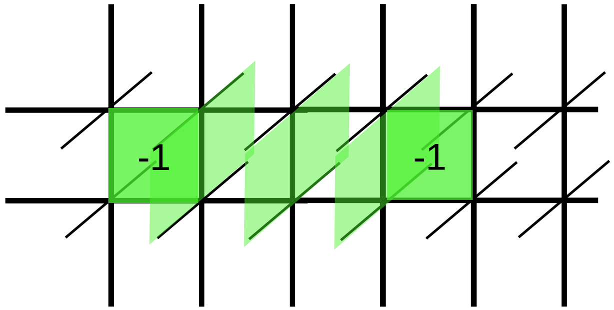


$$\frac{T}{g}$$

No symmetry change / topological transition

The Wegner model

- Topological defects



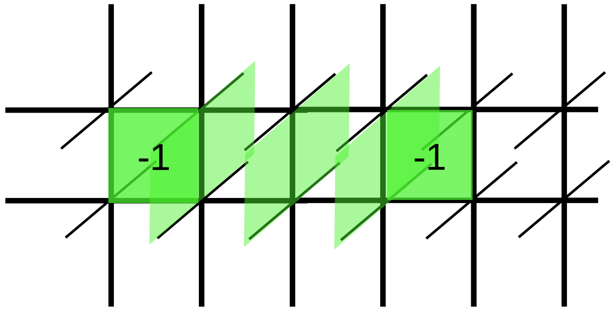
flux-line in 3d

$$f_{3d}(l) \sim gl - T \log \Omega(l)$$

Transition at $T_c \sim g$

The Wegner model

- Topological defects



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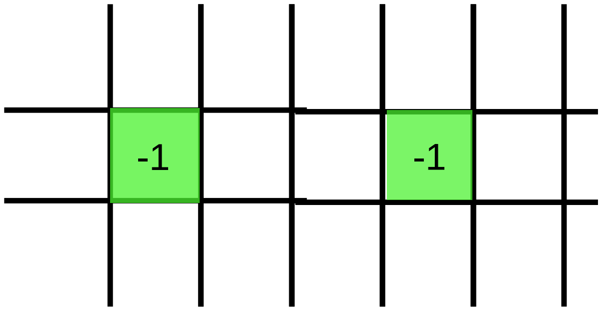


fluxes in 2d

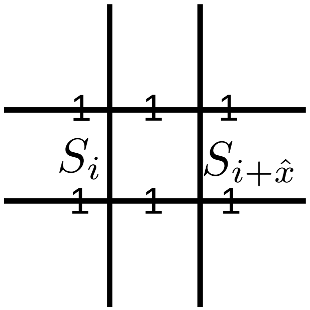
$$f_{2d}(l) \sim g - T\log\Omega(l)$$

no transition

The Wegner model

- Gauge fixing in 2d

$$\sigma_{ii+\hat{x}} = 1, \quad \sigma_{ii+\hat{y}} = S_i \qquad\qquad F_{i^*} = S_i S_{i+\hat{x}}$$



The Wegner model becomes

$$H_W = -g \sum_i S_i S_{i+\hat{x}}$$

$\Longrightarrow$ decoupled spin chains

$\Longrightarrow$ No thermal transition!

Facts

- ∄ thermal transition in 1d Ising spin-chain (text-book)

$$H_{Ising} = -J \sum_i S_i S_{i+1}$$

$$\Delta f \propto T e^{-2J/T}$$

no singular point

- ∃ thermal transition in 1d Dyson-Ising spin-chain

Received October 28, 1968

Abstract. Existence of a phase-transition is proved for an infinite linear chain of spins $\mu_j = \pm 1$, with an interaction energy

$$H = - \sum J(i-j)\, \mu_i \mu_j \,,$$

where $J(n)$ is positive and monotone decreasing, and the sums $\sum J(n)$ and $\sum (\log \log n)\, [n^3 J(n)]^{-1}$ both converge. In particular, as conjectured by Kac and Thompson, a transition exists for $J(n) = n^{-\alpha}$ when $1 < \alpha < 2$. A possible extension of these results to Heisenberg ferromagnets is discussed.

I. Introduction

We consider the one-dimensional Ising ferromagnet with sites labeled by an integer j taking all values from $-\infty$ to $+\infty$. At each site is a random variable μ_j taking the values ± 1, the total energy being

$$H = - \sum J(i-j)\, \mu_i \mu_j \,, \tag{1.1}$$

Dyson, CMP 12, 91 (1969)

Facts

- $\nexists$ thermal transition in 1d Ising spin-chain (text-book)

$$H_{Ising} = -J \sum_i S_i S_{i+1} \qquad \Delta f \propto T e^{-2J/T}$$

no singular point

- $\exists$ thermal transition in 1d Dyson-Ising spin-chain

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Dyson, CMP 12, 91 (1969)

New model

- Spin Hamiltonian with long range interaction

$$H_X = -J_r \sum_i \sum_{r>0} \frac{S_i S_{i+r\hat{x}}}{r^\omega}, \quad 1 < \omega \leq 2$$

Decoupled Dyson-Ising spin chains

$\Longrightarrow \exists$ thermal transition

$$F_{i^*} = S_i S_{i+\hat{x}} \qquad F_{i^*} F_{i^*+\hat{x}} = S_i S_{i+2\hat{x}} \qquad \prod_{j=0}^{r-1} F_{i^*+j\hat{x}} = S_i S_{i+r\hat{x}}$$

$$H_Z = -J_r \sum_i \sum_{r>0} \frac{\prod_{j=0}^{r-1} F_{i^*+j\hat{x}}}{r^\omega}$$

$\Longrightarrow \exists$ thermal transition in 2d! $\qquad f_{2F}(l) = g l^{2-\omega} - T \log \Omega(l)$

Perimeter Law

- Spin Hamiltonian with long range interaction

$$H_X = -J_r \sum_i \sum_{r>0} \frac{S_i S_{i+r\hat{x}}}{r^\omega}, \quad 1 < \omega \leq 2 \qquad H_Z = -J_r \sum_i \sum_{r>0} \frac{\prod_{j=0}^{r-1} F_{i^*+j\hat{x}}}{r^\omega}$$

$$\langle S_i \rangle \neq 0 \qquad\qquad \langle S_i \rangle = 0$$

Perimeter Law

- Spin Hamiltonian with long range interaction

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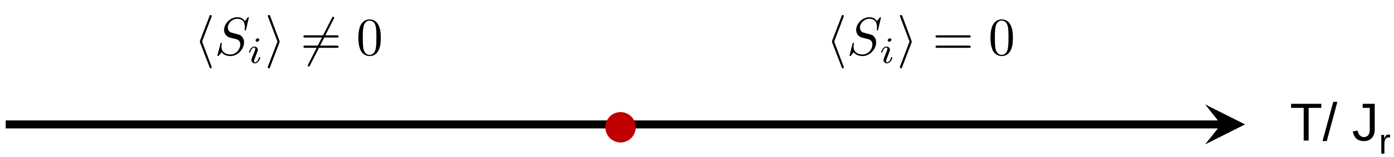

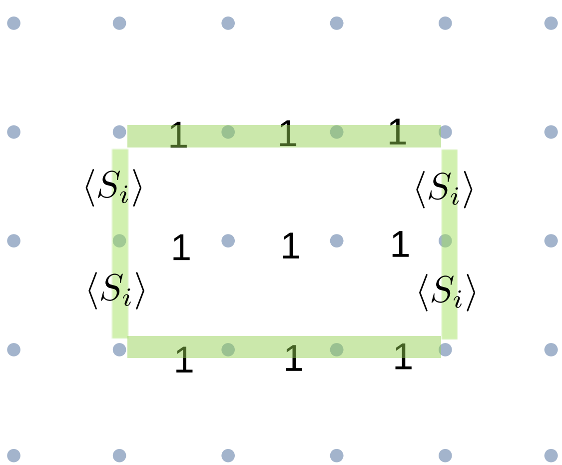

$$\langle W_C \rangle = \langle \prod_{(ab)\in C} \sigma_{ab} \rangle = (\langle S_i \rangle^2)^{L_y}$$

Discussion

- Thermal transition in 2d exists!

$$H_Z = -J_r \sum_i \sum_{r>0} \frac{\prod_{j=0}^{r-1} F_{i^*+j\hat{x}}}{r^\omega}$$

Strongly interacting flux models

Spatial symmetry is broken (x-dir)

Would be great to find a simpler model

$$H_{mZ} = -g \sum_i F_{i^*} - \sum_{i,j} J_{ij} F_{i^*} F_{j^*}$$

Discussion

■ Flux Hamiltonian with long range interaction

$$n_i \equiv \frac{1 - F_i}{2}, \quad \sum_i n_i \in 2\mathbb{Z}$$

$$H_F = g \sum_{i \neq j} J_{ij} n_i n_j + O(n_i^4), \quad J_{ij} > 0$$

$$\text{For } \ T \ll g, \ J_{ij} = \log(|i - j|)$$

Discussion

■ Flux Hamiltonian with long range interaction

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$$\text{For } \ T \ll g, \ J_{ij} = \log(|i-j|)$$

Ground state : flux-free state

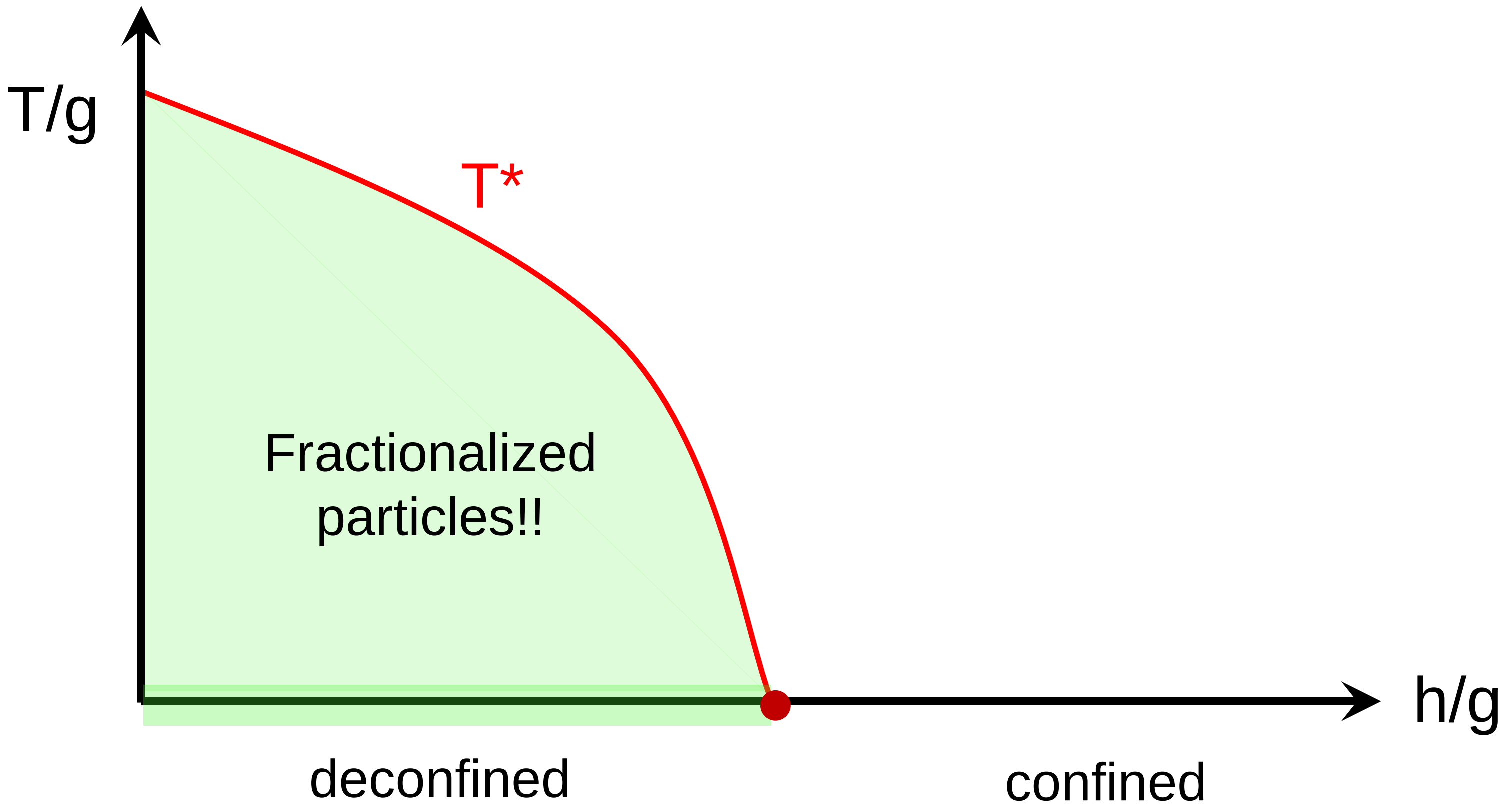
Excited state : two-flux state $\quad E_{2F}(r) = g \log(r)$

$$F_{2F}(r) = g\log(r) - T\log(r)$$

$\Longrightarrow$ KT-type free energy

Theoretical Set-up

■ Schematic Phase diagram



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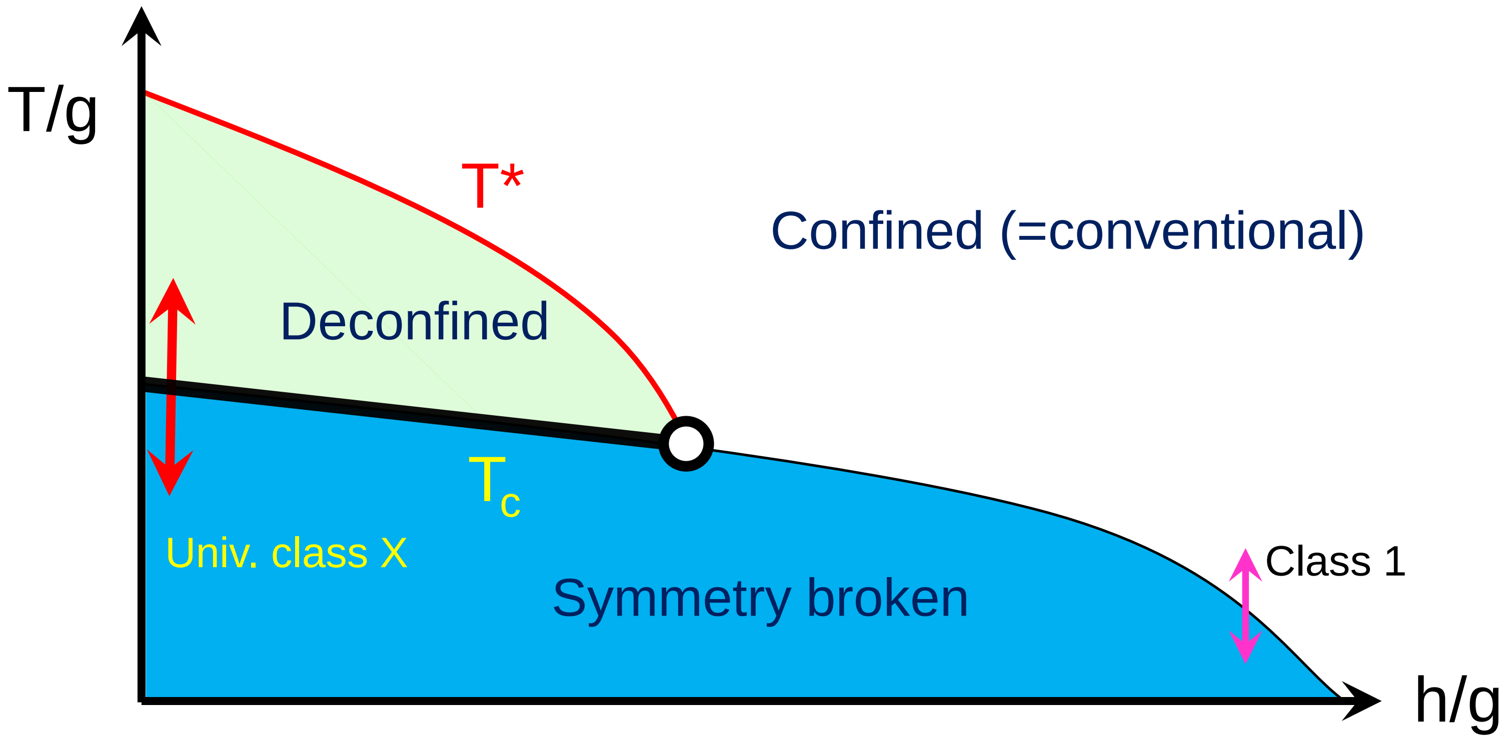

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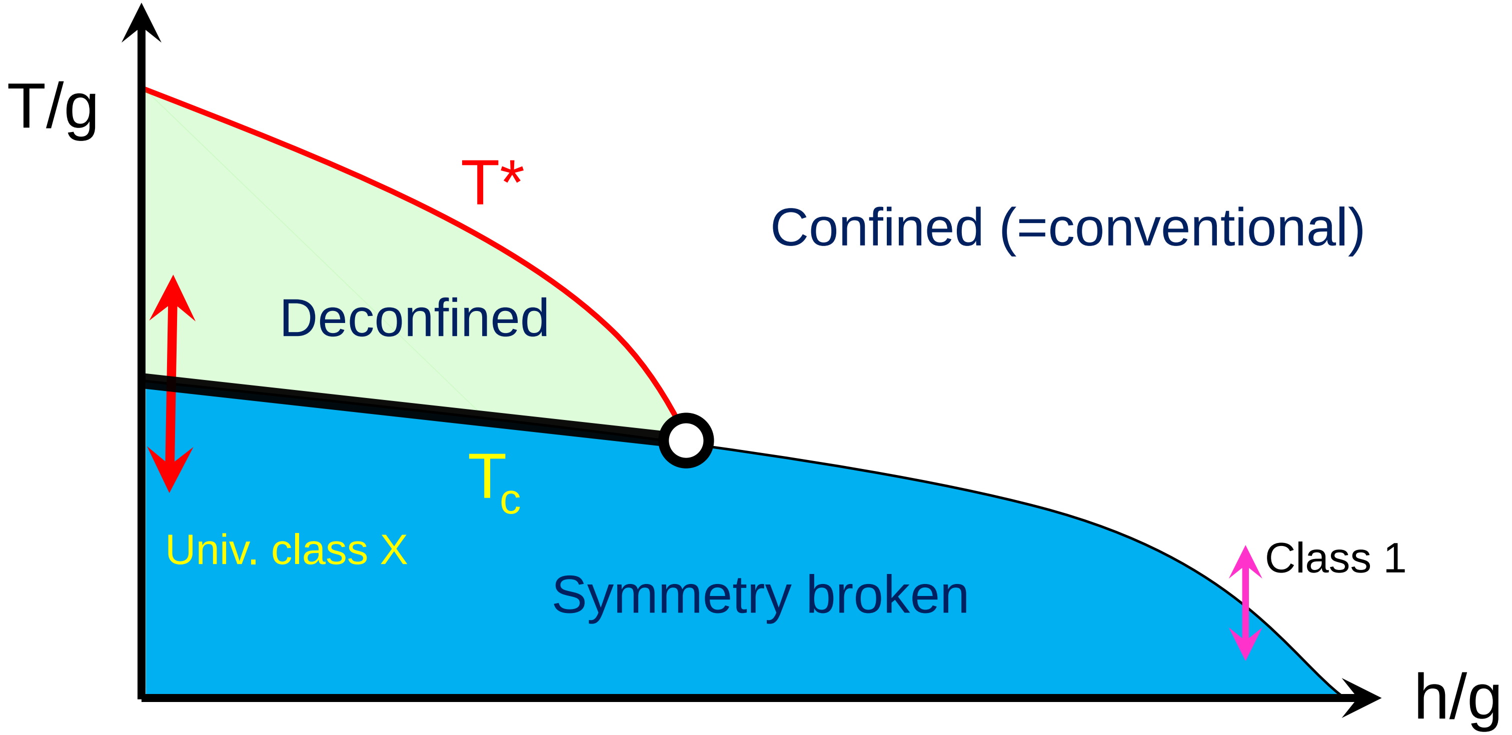
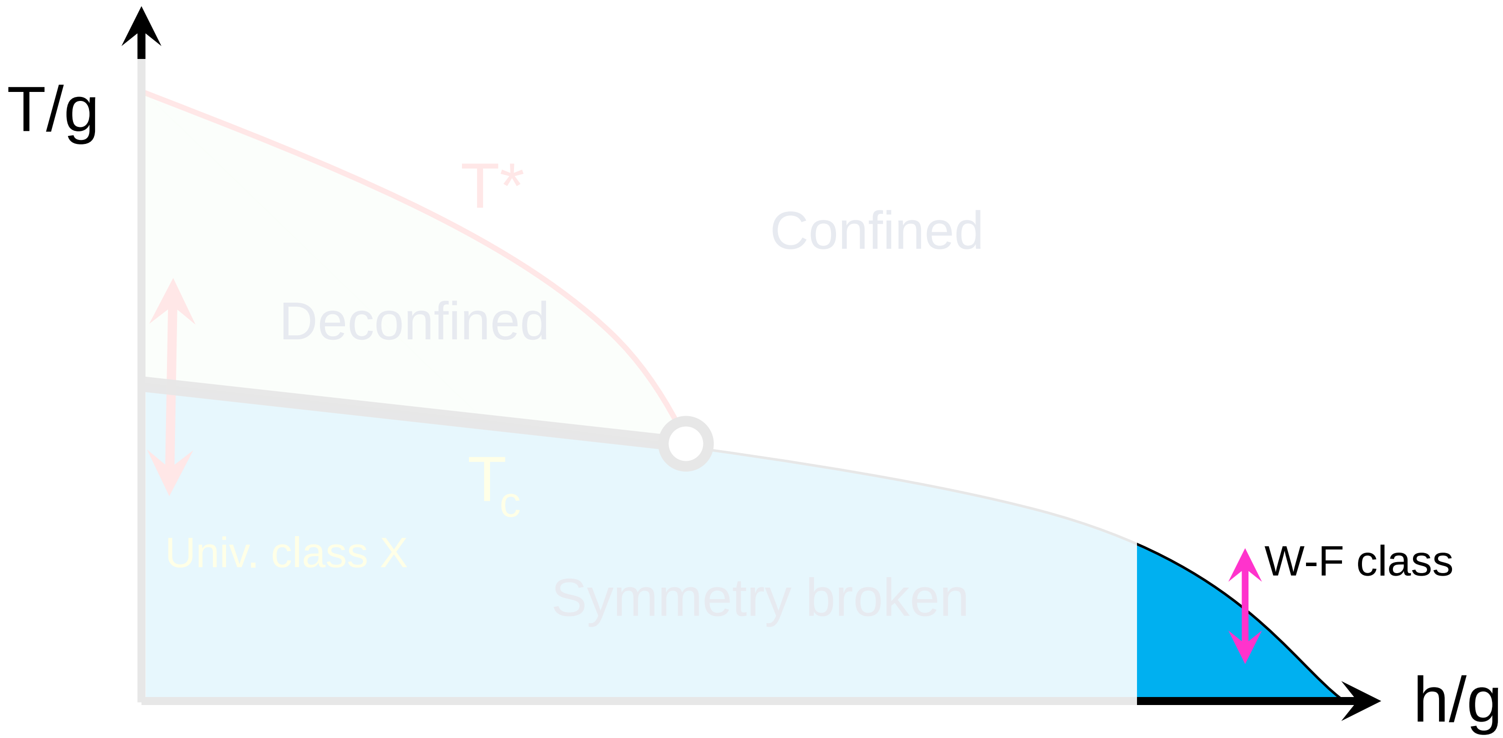


Univ. Class 1 : order parameter

Univ. Class X : order parameter + gauge structure

Theoretical Set-up

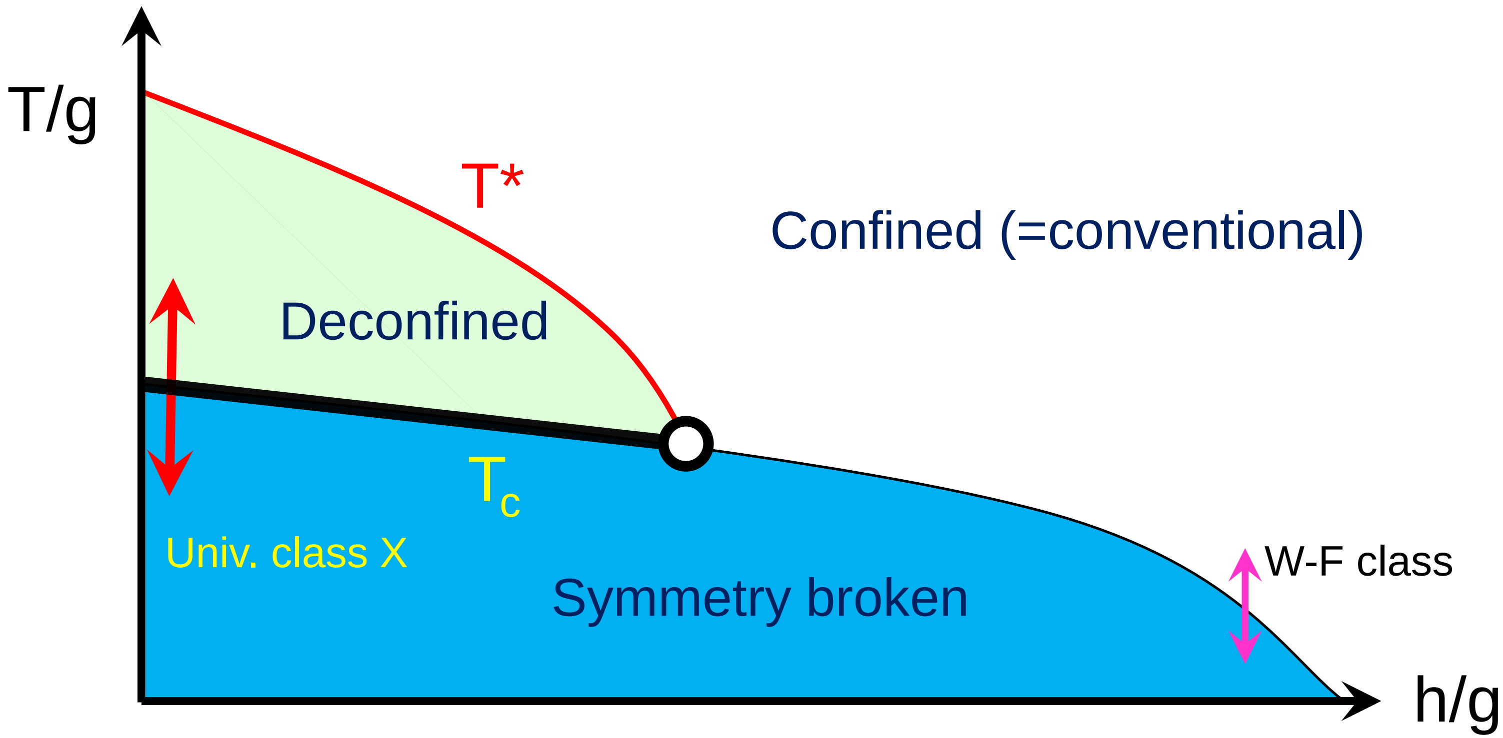
■ Schematic Phase diagram



Univ. class X $\overset{?}{=}$ Wilson-Fisher

Theoretical Set-up

■ Schematic Phase diagram (with symmetry)



Univ. class X $\stackrel{?}{=}$ *Wilson-Fisher*

Symmetry Breaking Transitions in Deconfined Phases

■ Z_2 **global** symmetry with Z_2 **gauge** field

$$H_s = -J \sum_{\langle i,j \rangle} \sigma_{ij} \cos\left(\frac{\theta_i - \theta_j}{2}\right) - u \sum_i \cos(2\theta_i) - g \sum_{i*} \prod_{(ab)\in\square} \sigma_{ab}$$

- Gauge transformation :

$$\sigma_{ij} \to \sigma_{ij}\eta_i\eta_j \quad \theta_i \to \theta_i + \pi(1-\eta_i)$$

cf : $$F_L = -\tilde{J} \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$

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- Confinement limit ($g \ll T, J, u$)

: Z_2 gauge fields are averaged away.
: Landau functional is restored.

$$F_L = -\tilde{J} \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(2\theta_i)$$

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Symmetry breaking ~ 2π vortex proliferation

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- Deconfinement limit (g ≫ T, J, u)

: Z_2 gauge fluxes (fields) are frozen (σ=+1).

$$H_{eff} = -J \sum_{\langle i,j \rangle} \cos(\frac{\theta_i - \theta_j}{2}) - u \sum_i \cos(2\theta_i)$$

Symmetry Breaking Transitions in Deconfined Phases

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- Deconfinement limit ($g \gg T, J, u$)

: Z_2 gauge fluxes (fields) are frozen (σ=+1).

Symmetry breaking ~ 4π vortex proliferation

Deconfinement Limit

$$H_{eff} = -J \sum_{\langle i,j \rangle} \cos(\frac{\theta_i - \theta_j}{2}) - u \sum_i \cos(N\theta_i)$$

Introduce the half-angle variable $\vartheta_i = \frac{\theta_i}{2}$

$$H_{eff} = -J \sum_{\langle i,j \rangle} \cos(\vartheta_i - \vartheta_j) - u \sum_i \cos(2N\vartheta_i)$$

Z_{2N} clock model : order para. = secondary operator

$$\Phi_i = \cos(\theta_i) = \cos(2\vartheta_i)$$

In 3d, Z_4 anisotropy term is irrelevant.

Hasenbusch and Vicari, PRB 84, 125136 (2011)

Discussion

- New universality class is obtained.

An order para. is a **secondary** operator.

$\alpha < 0$: Harris criteria applies (stable under disorder!)

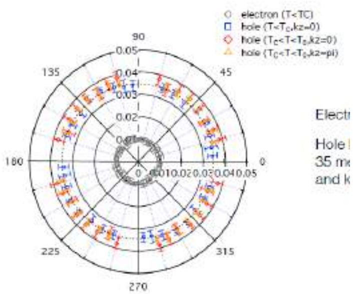
Quantum-classical mapping : frustrated magnetism

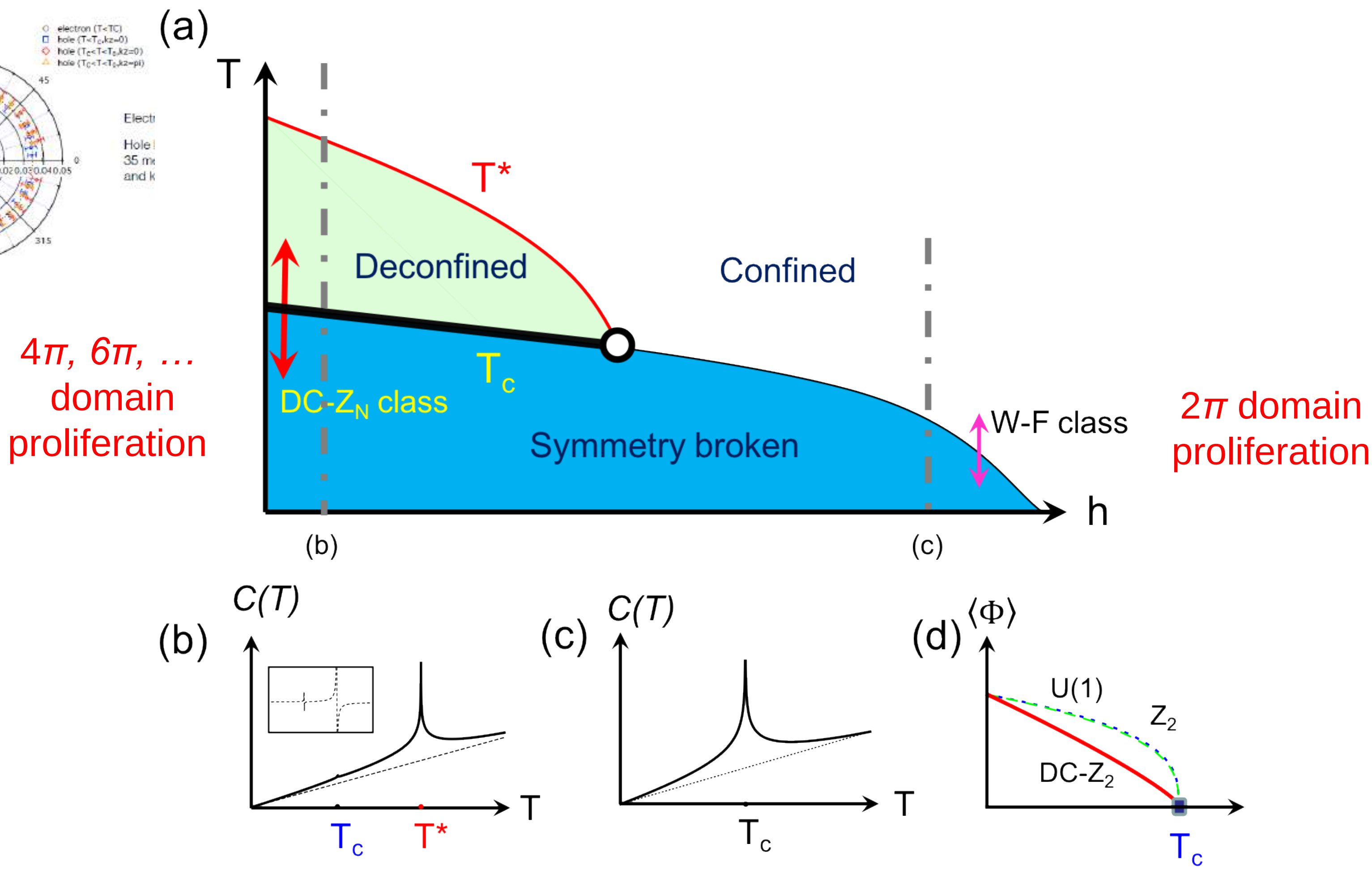
Univ. class in $3d$	α	β	γ	ν	η	δ
Z_2 (Ising) [33]	0.11	0.33	1.24	0.63	0.036	4.79
$U(1)$ (XY) [33]	−0.015	0.35	1.32	0.67	0.038	4.78
Mean-field	0	0.5	1	0.5	0	3
DC-Z_N/DC-U(1)	−0.015	0.83	0.35	0.67	1.47	1.43

DC-U(1) in $3d$	↵	β	γ	□	ℋ	δ
Z_2 gauge	−0.015	0.83	0.35	0.67	1.47	1.43
Z_3 gauge	−0.015	1.42	−0.83	0.67	3.23	0.42
Z_4 gauge	−0.015	2.09	−2.17	0.67	5.22	−0.04

Symmetry Breaking Transitions in Deconfined Phases

■ Schematic Phase diagram (with symmetry)





New Universality Class in 2d

- New universality class is obtained.

$$H_{eff} = -J \sum_{\langle i,j \rangle} \cos(\vartheta_i - \vartheta_j) - u \sum_i \cos(2N\vartheta_i)$$

Z_2 sym. breaking transition = Z_4 clock model (c=1).

New Universality Class in 2d

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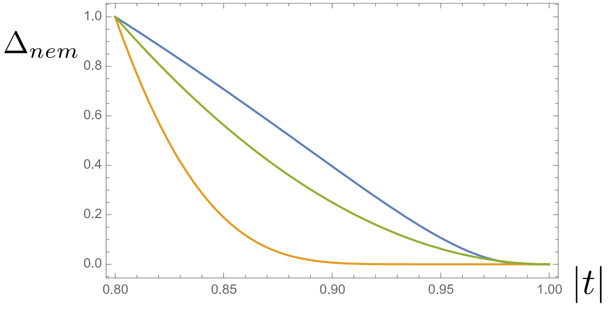
Z_2 sym. breaking transition = Z_4 clock model (c=1).

CFT with c=1 (similar to KT)

$$\xi_{KT} \sim e^{\frac{c}{\sqrt{|T-T_c|}}}$$

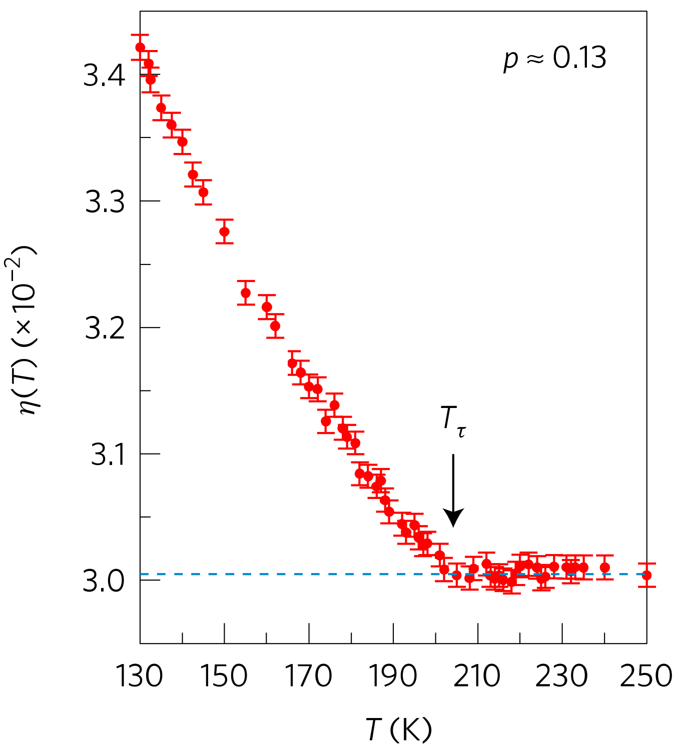
$$[\Delta_{Z_2}] = [e^{i\theta_i}] = [e^{i2\vartheta_i}] = 1/2$$

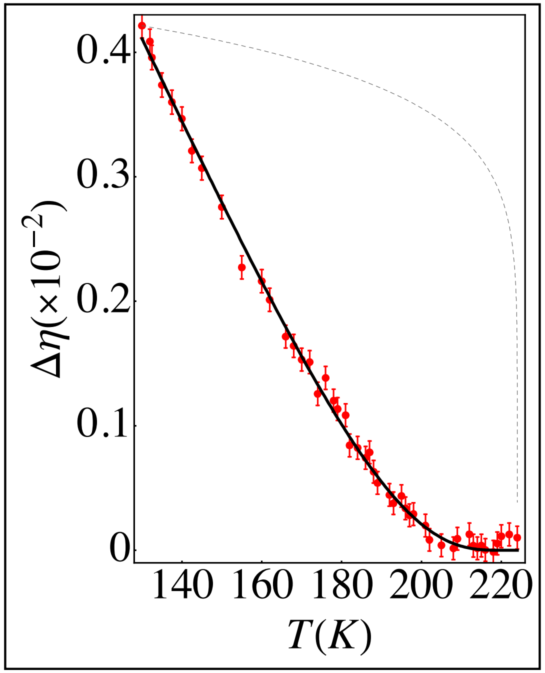
$$\Delta_{nem} \sim \xi^{-1/2}$$



Super-linear onset!

Discussion





$$\Delta_{nem,KT}(T) = \Delta_{nem}(T > T^*) + C_1 e^{-\frac{C_2}{\sqrt{|T-T^*|}}}$$

Fitting parameters : $C_1, \;\; C_2$

Outline

1. Motivation / Facts

2. Existence of a thermal deconfined transition in 2d

3. Novel universality class of symmetry breaking transitions

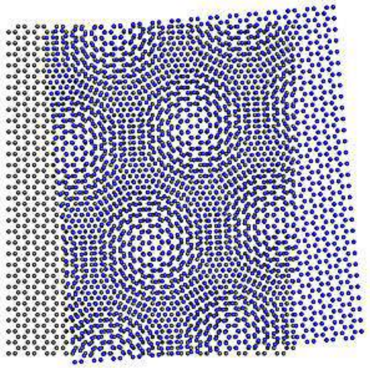
4. Conclusion

Conclusion

- Strongly correlated quantum materials

$YB_2Cu_3O_7$

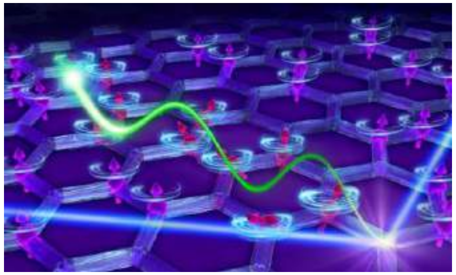
α-$RuCl_3$



Twisted bi-layer graphene

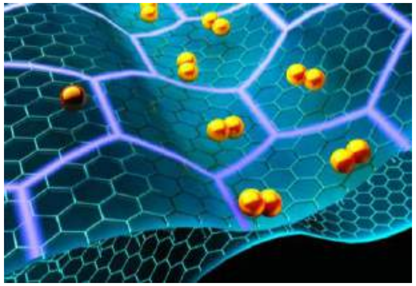
Mann, Nature 2011

High Tc Superconductivity



Phys. org (2017)

Majorana Fermions



Phys. org (2019)

Topological SC?

Conclusion

- Strongly correlated quantum materials

- Phenomena beyond Landau paradigms

Landau	Beyond Landau
UV d.o.f $\simeq$ IR d.o.f	UV d.o.f. $\neq$ IR d.o.f
Wilson-Fisher class	Universality class X

Conclusion

- Strongly correlated quantum materials

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- Platform for new physics!

Thank you for your attention!

Symmetry Breaking Transitions in Deconfined Phases

■ Excitations

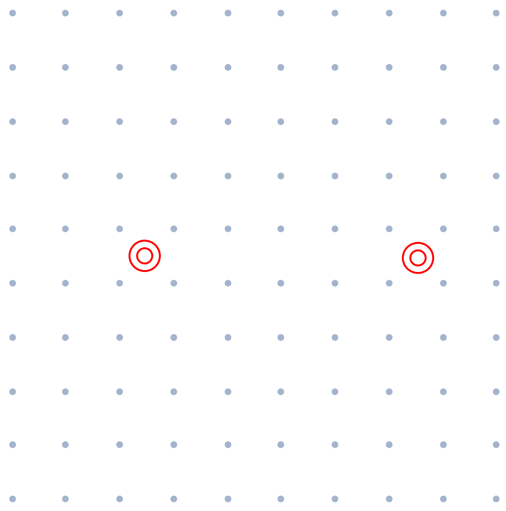
$$H_s = -J\sum_{ij}\sigma_{ij}\cos(\frac{\theta_i-\theta_j}{2}) - u\sum_i \cos(N\theta_i) + gV_F(\{F_{i^*}\})$$

- *Flux (vison) pair*

- 2π vortex of order parameter

- 4π vortex of order parameter

…



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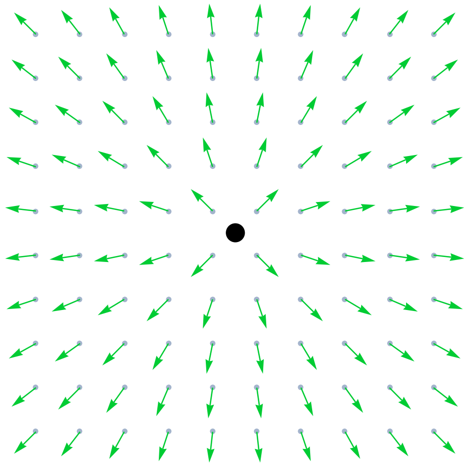

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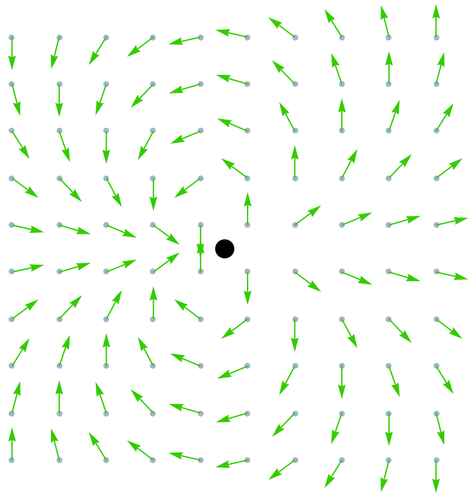

Symmetry Breaking Transitions in Deconfined Phases

- Two limits

$$H_s = -J \sum_{ij} \sigma_{ij} \cos(\frac{\theta_i - \theta_j}{2}) - u \sum_i \cos(N\theta_i) + gV_F(\{F_{i*}\})$$

- Confinement limit (g ≪ T, J, u)
  : gauge contributions are averaged away

- Deconfinement limit (g ≫ T, J, u)
  : gauge fluxes are frozen. (F =+1, σ=+1)

Symmetry Breaking Transitions in Deconfined Phases

■ Two limits

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- Confinement limit (g ≪ T, J, u)

$$H_{eff} = -\tilde{J} \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - u \sum_i \cos(N\theta_i) + \cdots$$

- Deconfinement limit (g ≫ T, J, u)

$$H_{eff} = -J \sum_{\langle i,j \rangle} \cos(\frac{\theta_i - \theta_j}{2}) - u \sum_i \cos(N\theta_i)$$

Symmetry Breaking Transitions in Deconfined Phases

- Two limits

$$H_s = -J \sum_{ij} \sigma_{ij} \cos(\frac{\theta_i - \theta_j}{2}) - u \sum_i \cos(N\theta_i) + g V_F(\{F_{i*}\})$$

- Confinement limit ($g \ll T, J, u$)

  Symmetry breaking ~ 2π vortex proliferation

- Deconfinement limit ($g \gg T, J, u$)

  Symmetry breaking ~ 4π vortex proliferation