Majorana Multipole Response of Topological Superconductors

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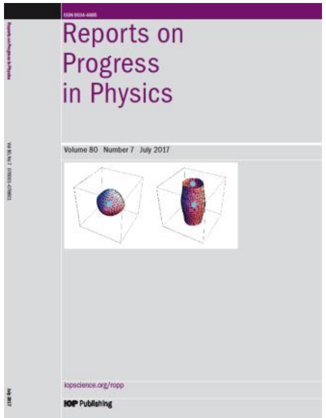

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A review paper on topological SCs with Yoichi Ando

MS, Ando, Rep. Prog. Phys. 80, 076501 (17)

Outline

1. Motivation

2. Anisotropic magnetic response of helical MFs

MS-Fujimoto, PR B79, 094504 (2009)
Mizushima-MS-Machida, PRL, 109, 165301 (2012)

3. Majorana multipole response of helical MFs

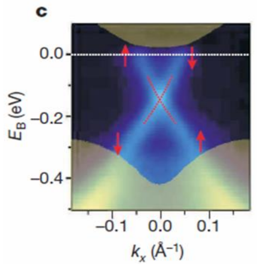
Kobayashi-Yamakage-Tanaka-MS, arXiv:1812.01857

Motivation

MFs were originally proposed as elementary particles, but now we know that they can be emergent excitations in electron or atomic systems.

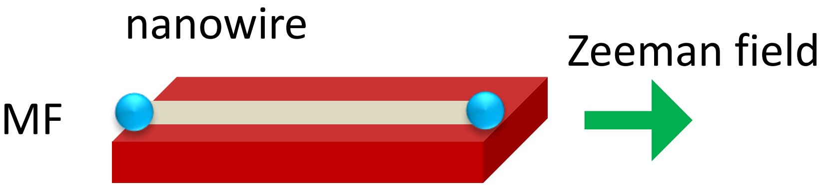
Majorana Fermions in various S-wave SCs

- Dirac fermion + s-wave condensate MS(03), Fu-Kane (08)



Hsieh et al

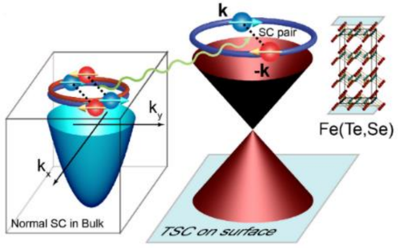
- S-wave superconducting state with Rashba SO + Zeeman field MS-Takahashi-Fujimoto (09), J. Sau et al (10)



Lutchyn et al (10), Oreg et al (10)

- MFs in iron-based SCs

G. Xu et al PRL (16)…



[P. Zhang et al Science (18)]

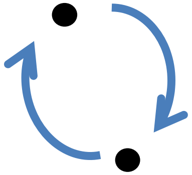
These emergent MFs in condensed matter physics share some properties with elementary Majorana particles in high energy physics

- Both obey the Dirac equation with self-charge-conjugation condition

$$\mathcal{H}(\boldsymbol{k}) = \sum_{i=1}^{d} \gamma_i k_i \qquad \psi(\boldsymbol{k}) = \mathcal{C}\psi(-\boldsymbol{k})$$

charge-conjugation

- Zero modes exhibit non-Abalian anyon statistics



However, there is an essential difference between them

CPT theorem

- ➢ CPT is a fundamental symmetry of relativistic QFT

  C: charge conjugation P: parity (inversion) T: time-reversal
- ➢ Any reasonable relativistic QFT is invariant under CPT

- Elementary MFs should respect CPT inv. since they should respect Lorentz inv.
- This means that **elementary MFs are self-conjugate under CPT,** not merely under C

➡ **This fundamental invariance of elementary MFs gives a strong constraint in electromagnetic responses**

Electromagnetic response of elementary MFs

Kayser-Goldhaber (83)

General form of one-particle EM-coupling for spin-1/2 relativistic fermions

$$(q = p_f - p_i)$$

$$\langle p_f | j_\mu A^\mu | p_i \rangle = i \bar{u}_{p_f} \left[F \gamma_\mu + M \sigma_{\mu\nu} q^\nu + i E \sigma_{\mu\nu} q^\nu \gamma_5 + G(q^2 \gamma_\nu - \not{q} q_\nu) \gamma_5 \right] u_{p_i} A^\mu$$

electric charge

magnetic dipole $\propto \boldsymbol{B}$

electric dipole $\propto \boldsymbol{E}$

toroidal moment

self-conjugation condition under CPT

$$\boxed{\langle p_f | j_\mu^{\mathrm{MF}} A^\mu | p_i \rangle = i \bar{u}_{p_f} \left[0 + 0 + 0 + G(q^2 \gamma_\nu - \not{q} q_\nu) \gamma_5 \right] u_{p_i} A^\mu}$$

- Charge neutral condition forMFs (F=0)
- Electro-magnetic dipole momenta of MFs vanish (M=E=0)

Elementary MFs only show moderate EM responses

However, emergent MFs are not subject to such a strong constraint.

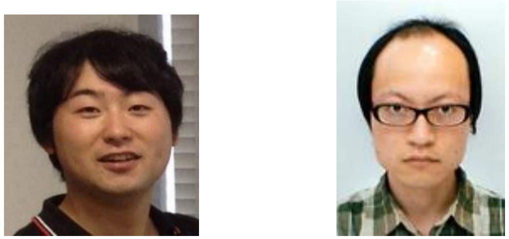
- Emergent MFs only have approximate Lorentz invariance.
- They are self-conjugate just under C(=PHS), not under CPT.

◆ We can have different EM responses for emergent MFs

◆ In this talk, I will present a general theory of EM responses of emergent MFs in time-reversal invariant TSCs

Majorana multipole response in topological superconductors

Xiong-Yamakage-Kobayashi-MS-Tanaka, Crystal 2017, 7, 58
Kobayashi-Yamakage-Tanaka-MS, arXiv:1812.01857



Anisotropic magnetic response of MF

Helical Majorana fermions in TRI topological SCs show peculiar anisotropic magnetic response.

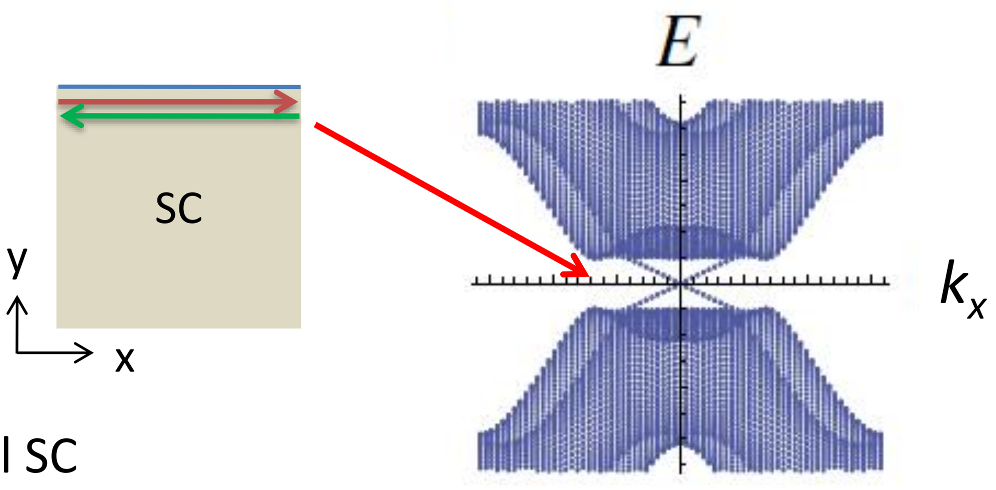
MS-Fujimoto (09)
Chung-Zhang (09)

2dim p-wave Rashba noncentrosymmetric SC

$$\Delta(\boldsymbol{k}) = i\boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}\sigma_y$$

$$\boldsymbol{d}(\boldsymbol{k}) \propto \lambda_{\mathrm{SO}}(-k_y, k_x, 0)$$

Helical Majorana fermion



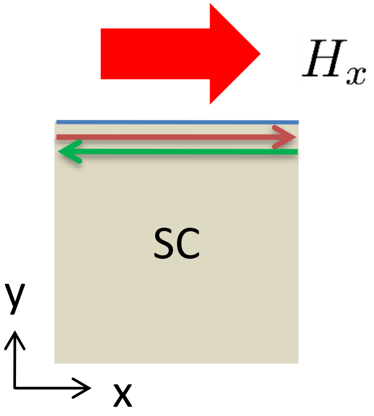
Non-trivial Z_2 topological number
2dim time-reversal invariant helical SC

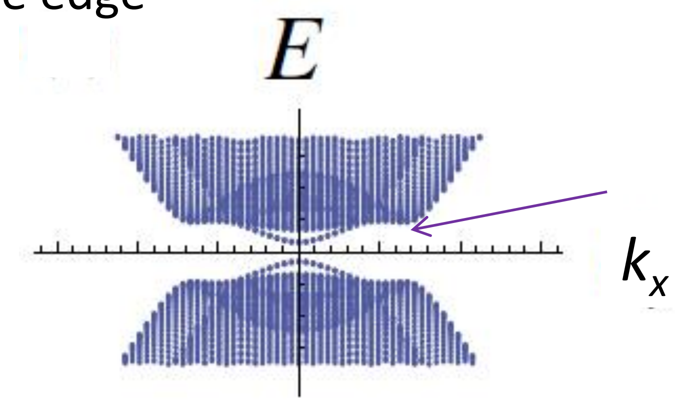
MS-Fujimoto (09), Y. Tanaka et al (09)

Under Zeeman fields, the helical MF shows anisotropic response.

MS-Fujimoto (09)

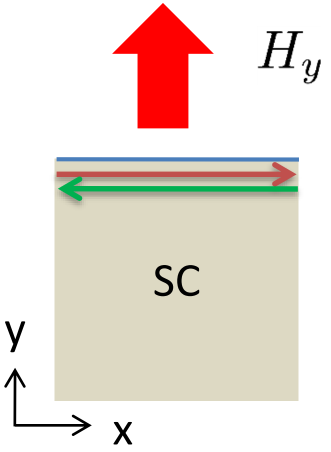
- Zeeman field **along** the edge

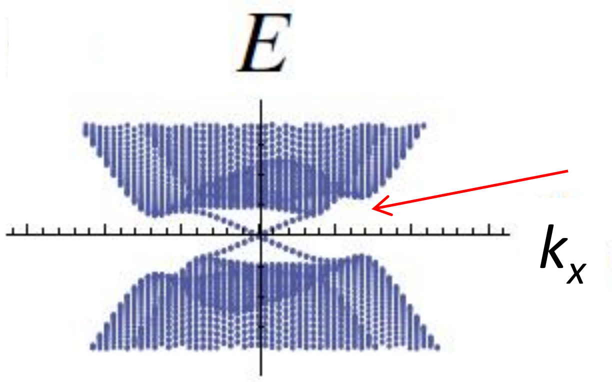




Gap opens due to TR breaking

- Zeeman field **normal** to edge



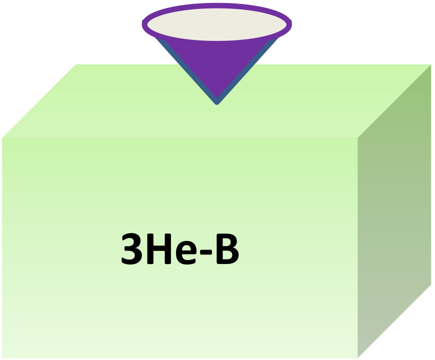


No gap opens in spite of TR breaking

A similar anisotropic magnetic response has been reported in 3dim time-reversal invariant SCs

Chung-Zhang(09)
Shindou-Furusaki-Nagaosa(10)

Helical Majorana surface state in ^{3}He-B



$$\begin{bmatrix} \hat{\psi}_{\rightarrow} \\ \hat{\psi}_{\leftarrow} \\ \hat{\psi}^{\dagger}_{\rightarrow} \\ \hat{\psi}^{\dagger}_{\leftarrow} \end{bmatrix} = \sum_{\boldsymbol{k}_{\parallel}} (\hat{\gamma}_{\boldsymbol{k}} e^{i\boldsymbol{k}_{\parallel}\cdot\boldsymbol{r}_{\parallel}} + \hat{\gamma}^{\dagger}_{\boldsymbol{k}} e^{-i\boldsymbol{k}_{\parallel}\cdot\boldsymbol{r}_{\parallel}}) \begin{bmatrix} \cos\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \\ \cos\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\boldsymbol{k}}+\pi/2}{2} \end{bmatrix}$$

$$\times u_{\boldsymbol{k}} e^{\Delta z/\hbar v_{\mathrm{F}}} \sin(\sqrt{k_{\mathrm{F}}^2 - k_{\parallel}^2} z)$$

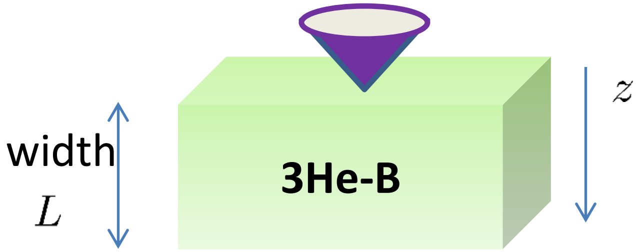
Spin density op.

$$\hat{I}_x = (\hat{\psi}^{\dagger}_{\rightarrow}\hat{\psi}_{\rightarrow} - \hat{\psi}^{\dagger}_{\leftarrow}\hat{\psi}_{\leftarrow})/2 = 0 \qquad \hat{I}_y = (\hat{\psi}^{\dagger}_{\rightarrow}\hat{\psi}_{\leftarrow} + \hat{\psi}^{\dagger}_{\leftarrow}\hat{\psi}_{\rightarrow})/2 = 0$$

$$\hat{I}_z = -i\hat{\psi}^{\dagger}_{\rightarrow}\hat{\psi}_{\leftarrow} \neq 0$$

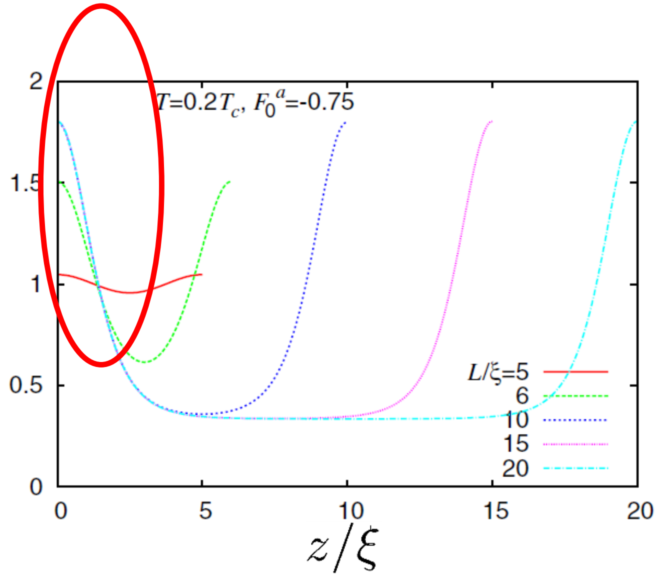
- MF behaves like **Ising spin (=magnetic dipole)**
- MF does **not couple to magnetic fields parallel to the surface**

Correspondingly, the system shows anisotropic surface spin-susceptibility.



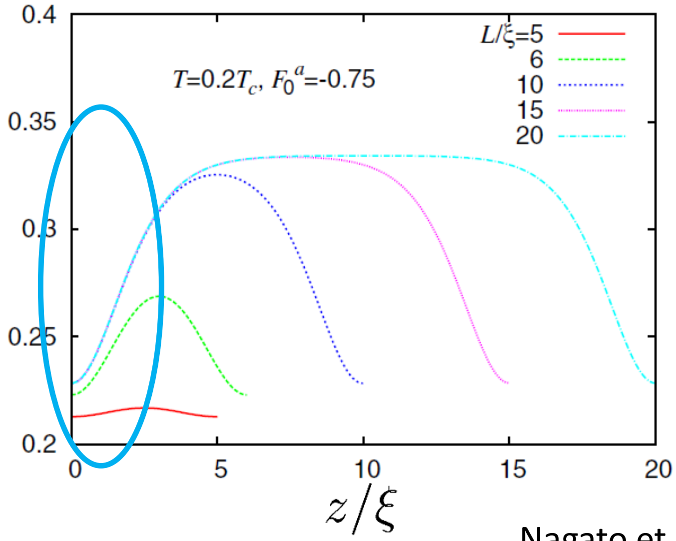
$$\frac{\chi_{zz}(z)}{\chi_N}$$

z-direction(normal to surface)



$$\frac{\chi_{xx}(z)}{\chi_N}$$

x-direction (along the surface)



Nagato et al , JPSJ (09)

positive

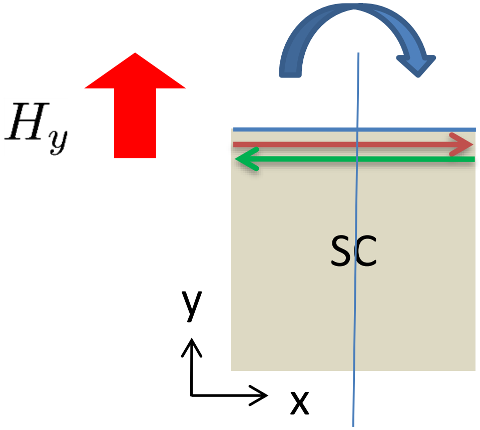
negative

➡ MF responds to only the Zeeman field normal to the surface.

A hint of these anisotropic behaviors is magnetic crystalline sym.

Mizushima-MS-Machida (12) Shiozaki-MS (14)

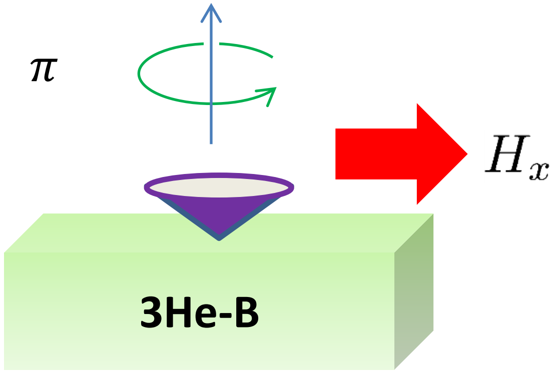
Rashba SC



TRS, mirror reflection

magnetic mirror reflection
(TRS+mirror reflection)

3He-B



TRS, rotation

magnetic two-fold rotation
(TRS+two-fold rotation)

- TRS can remain partially as magnetic symmetry.
- The remaining magnetic sym can stabilize gapless helical MFs under magnetic fields

Actually, one can define top. # by using these magnetic symmetries

BdG Hamiltonian

symmetric momentum under mirror/C_2-rot.

$$\boxed{w_{\rm M1D} = \frac{i}{4\pi}\int dk_\perp\, {\rm tr}\left[\Gamma_{\rm M}\mathcal{H}^{-1}(k_\perp,\boldsymbol{k}^0_{||})\partial_{k_\perp}\mathcal{H}(k_\perp,\boldsymbol{k}^0_{||})\right]}$$

$$\Gamma_{\rm M} = \underline{\mathcal{U}\mathcal{T}}\mathcal{C} \leftarrow \text{PHS}$$

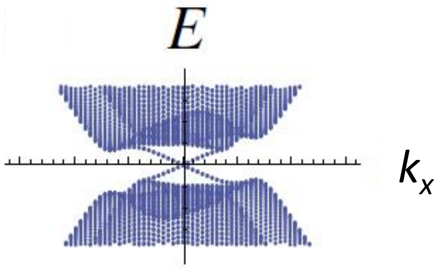
magnetic mirror/ C_2-rot.

For Rashba SC

$$\mathcal{U} = \mathcal{M}_y$$

$$w_{\rm M1D} = \underline{2}$$

spin-degeneracy

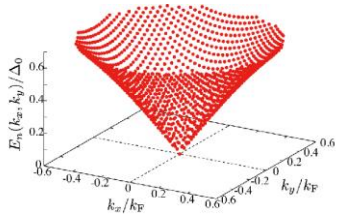


For 3He-B

$$\mathcal{U} = C_2$$

$$w_{\rm M1D} = \underline{2}$$

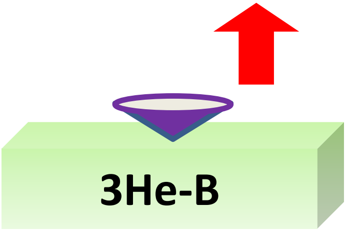
spin-degeneracy



Therefore, the magnetic winding # naturally explain why helical MFs can stay gapless even under magnetic fields

Question

1. How can we evaluate magnetic response more systematically?
2. Can we have magnetic response other than Ising (= magnetic dipole) behavior ?



Both show Ising behavior

$$\Delta E \propto B$$

To address these questions, we develop a general theory of quantum response of MFs

Basic idea

For elementary MFs, Lorentz and CPT inv. determine EM response …

Lorentz inv.

$$\langle p_f | j_\mu A^\mu | p_i \rangle = i\bar{u}_{p_f}\left[F\gamma_\mu + M\sigma_{\mu\nu}q^\nu + iE\sigma_{\mu\nu}q^\nu\gamma_5 + G(q^2\gamma_\nu - \not{q}\, q_\nu)\gamma_5\right] u_{p_i} A^\mu$$

⇓

CPT inv.

$$\langle p_f | j_\mu^{\mathrm{MF}} A^\mu | p_i \rangle = i\bar{u}_{p_f}\left[0 + 0 + 0 + G(q^2\gamma_\nu - \not{q}\, q_\nu)\gamma_5\right] u_{p_i} A^\mu$$

- For emergent MFs, we cannot use them, but we can use **crystalline symmetry and PHS (charge-conjugation)** instead

How to evaluate quantum operator

$$\hat{O} = \hat{c}^\dagger_\sigma(x) O_{\sigma,\sigma'} \hat{c}_{\sigma'}(x) = \frac{1}{2}\hat{\Psi}^\dagger(x)\mathcal{O}\hat{\Psi}(x)$$

hermitian

Nambu base

$$\mathcal{O} = \begin{pmatrix} O & 0 \\ 0 & -O^T \end{pmatrix} \quad \hat{\Psi} = \begin{pmatrix} \hat{c}_\sigma \\ \hat{c}^\dagger_\sigma \end{pmatrix}$$

First, use PHS of the Nambu spinor $\hat{\Psi}^\dagger(x)\tau_x = \hat{\Psi}^t(x)$

$$\hat{O} = \frac{1}{2}\hat{\Psi}^t(x)\left[\tau_x \mathcal{O}\right]\hat{\Psi}(x)$$

Then, perform mode expansion of quantum field,

$$\Psi(x) = \sum_a \hat{\gamma}^{(a)} |u_0^{(a)}\rangle + \cdots ,$$

gapless MF

Substituting this, we can extract the contribution of gapless MFs as

$$\hat{O}_{\rm MF} = \frac{1}{2}\sum_{ab}\hat{\gamma}^{(a)}\hat{\gamma}^{(b)}\langle u_0^{*(a)}|\tau_x\mathcal{O}|u_0^{(b)}\rangle$$

PHS

$$|Cu_0^{(a)}\rangle = \tau_x |u_0^{*(a)}\rangle$$

$$\hat{O}_{\rm MF} = \frac{1}{4}\sum_{ab}\hat{\gamma}^{(a)}\hat{\gamma}^{(b)}{\rm tr}\left[\mathcal{O}\rho^{(ab)}\right] \quad \rho^{(ab)} = \left(|u_0^{(b)}\rangle\langle Cu_0^{(a)}| - |u_0^{(a)}\rangle\langle Cu_0^{(b)}|\right)$$

From this form, we can derive conditions to obtain non-zero coupling b/w MFs and operator O

$$\hat{O}_{\mathrm{MF}} = \frac{1}{4}\sum_{ab} \hat{\gamma}^{(a)}\hat{\gamma}^{(b)} \mathrm{tr}\left[\mathcal{O}\rho^{(ab)}\right] \qquad \rho^{(ab)} = \left(|u_0^{(b)}\rangle\langle Cu_0^{(a)}| - |u_0^{(a)}\rangle\langle Cu_0^{(b)}|\right)$$

two conditions for non-zero MF operator

1. There must be zero modes $|u_0^{(a)}\rangle$

→ constraint for possible pairing sym of SCs

2. The operator O should have the same rep under crystalline sym as $\rho^{(ab)}$ has

→ Constraint for possible EM response

1. Zero modes

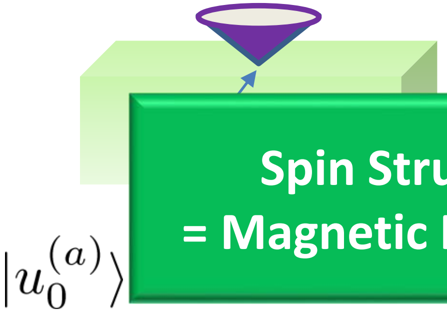
As mentioned before, the presence of zero modes is ensured by 1d magnetic winding #

$$w_{\mathrm{M1D}} = \frac{i}{4\pi}\int dk_{\perp}\mathrm{tr}[\Gamma_{\mathrm{M}}\mathcal{H}^{-1}(k_{\perp},\boldsymbol{k}_{\parallel})\partial_{k_{\perp}}\mathcal{H}(k_{\perp},\boldsymbol{k}_{\parallel})]$$

Actually, the spin-structure relevant to mag. response is specified by the generalized index theorem

MS-Tanaka-Yada-Yokoyama PRB (11)

Gapless MF is an eigenstate of Γ_{M}



Spin Structure
= Magnetic Response

For $w_{\mathrm{M1D}} > 0$ $\quad \Gamma_{\mathrm{M}}|u_0^{(a)}\rangle = |u_0^{(a)}\rangle$

$\mathrm{1D} < 0$ $\quad \Gamma_{\mathrm{M}}|u_0^{(a)}\rangle = -|u_0^{(a)}\rangle$

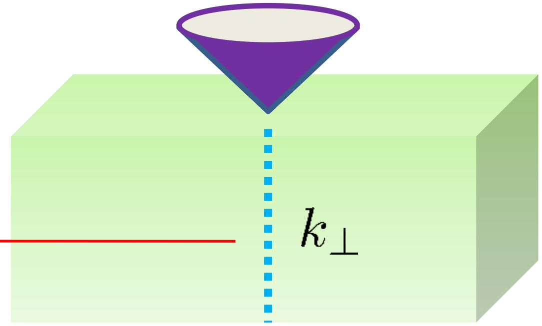
magnetic sym

$\Gamma_{\mathrm{M}} = \underline{\mathcal{UTC}}$

When can we obtain such 1d magnetic winding #?

From top table, we find that such 1d magnetic winding # can be obtained if the system has (emergent) AIII, BDI or CII sym.

	TRS	PHS	CS	d=1
A	0	0	0	0
AIII	0	0	1	**Z**
AI	1	0	0	0
BDI	1	1	1	**Z**
D	0	1	0	**Z_2**
DIII	-1	1	1	**Z_2**
AII	-1	0	0	0
CII	-1	-1	1	**2Z**
C	0	-1	0	0
CI	1	-1	1	0



$$w_{\mathrm{M1D}} = \frac{i}{4\pi}\int dk_\perp \mathrm{tr}\left[\Gamma_{\mathrm{M}}\mathcal{H}^{-1}\partial_{k_\perp}\mathcal{H}\right]$$

What determines the emergent AZ symmetry?

→ Symmetry of Cooper pairs

To see this, first I explain how the com. relation b/w PSH and crystalline sym is determined.

Ex.) C_2-sym SC

C_2-even SC

$$\boxed{C_2\Delta(\boldsymbol{k})C_2^t = \Delta(C_2\boldsymbol{k})}$$

For Nambu space

particle

$$\tilde{C}_2 = \begin{pmatrix} C_2 & 0 \\ 0 & C_2^* \end{pmatrix}$$

hole

Particle and hole behave in the same way

⇒ $\tilde{C}_2\mathcal{C} = \mathcal{C}\tilde{C}_2$

commutation relation

C_2-odd SC

$$\boxed{C_2\Delta(\boldsymbol{k})C_2^t = -\Delta(C_2\boldsymbol{k})}$$

For Nambu space

$$\tilde{C}_2 = \begin{pmatrix} C_2 & 0 \\ 0 & -C_2^* \end{pmatrix}$$

Particle and hole behave in a different manner

⇒ $\tilde{C}_2 C = -C\tilde{C}_2$

anti-commutation relation

This difference results in the difference of the emergent AZ sym

C_2–even SC

$$\boxed{C_2\Delta(\boldsymbol{k})C_2^t = \Delta(C_2\boldsymbol{k})}$$

PHS

$$\tilde{C}_2\mathcal{C} = \mathcal{C}\tilde{C}_2$$

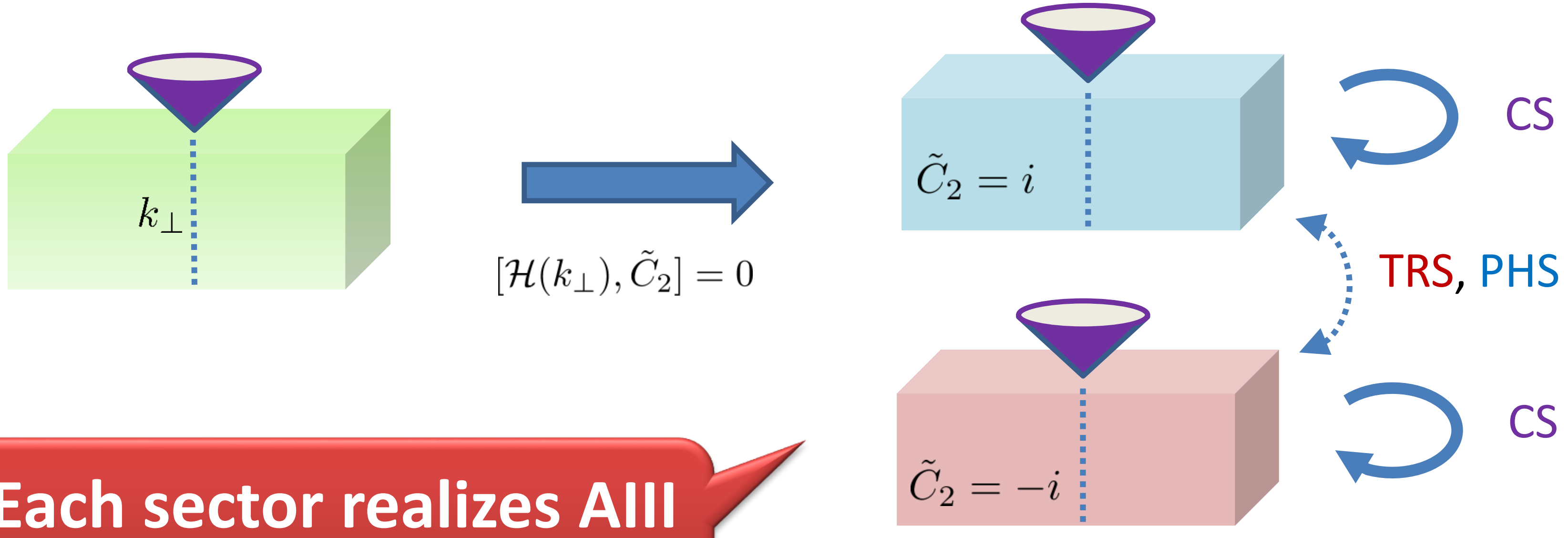
$$\tilde{C}_2\mathcal{T} = \mathcal{T}\tilde{C}_2$$

TRS

CS

$$\Gamma\tilde{C}_2 = \tilde{C}_2\Gamma$$

$$(\Gamma = \mathcal{T}\mathcal{C})$$



Each sector realizes AIII
MF can exist

On the other hand, a different result is obtained for C$_2$-odd SCs

For C$_2$–odd SC

$$\boxed{C_2\Delta(\boldsymbol{k})C_2^t = -\Delta(C_2\boldsymbol{k})} \quad\Longrightarrow$$

PHS

$$\tilde{C}_2\mathcal{C} = \underline{-}\mathcal{C}\tilde{C}_2$$

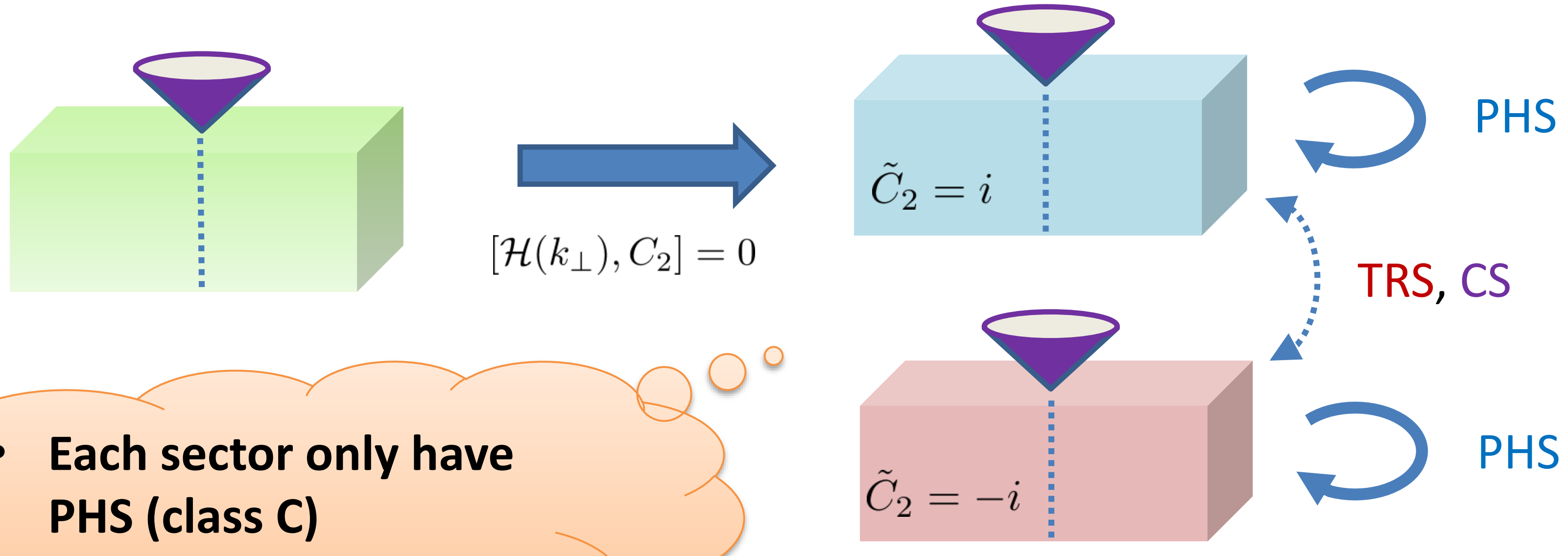
$$\tilde{C}_2\mathcal{T} = \mathcal{T}\tilde{C}_2$$

TRS

CS

$$\Gamma\tilde{C}_2 = \underline{-}\tilde{C}_2\Gamma$$

$$(\Gamma = \mathcal{T}\mathcal{C})$$



- **Each sector only have PHS (class C)**
- **No stable MF**

Pairing sym determines the existence of MFs

2. Possible Majorana op

O should be the same rep as $\rho^{(ab)}$

$$\hat{O}_{\rm MF} = \frac{1}{4}\sum_{ab}\hat{\gamma}^{(a)}\hat{\gamma}^{(b)}{\rm tr}\left[\mathcal{O}\rho^{(ab)}\right]$$

$$\rho^{(ab)} = \left(|u_0^{(b)}\rangle\langle Cu_0^{(a)}| - |u_0^{(a)}\rangle\langle Cu_0^{(b)}|\right)$$

We use the standard rep. theory to identify possible Os..

1. The representation of $|u_0^{(a)}>$ under crystalline sym. is the same as that of usual electrons (double point group rep.)

   ⇨ $\rho^{(ab)}$ is the product rep. of $|u_0^{(a)}>$

2. Calculating the character of $\rho^{(ab)}$, we can identify possible O coupling to MFs

3. By specifying the parity of O under T, magnetic or electric response is identified.

magnetic

$$\mathcal{T}\mathcal{O}\mathcal{T}^{-1} = -\mathcal{O}$$

electric

$$\mathcal{T}\mathcal{O}\mathcal{T}^{-1} = \mathcal{O}$$

In this manner, we complete the list of possible pairing sym and quantum operator O for MFs for each point group for surface state

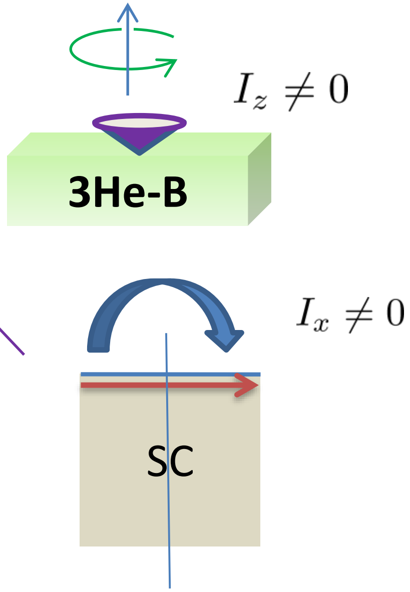
Kobayashi-Yamakage-Tanaka-MS (18)

PG	Δ_Γ	basis of Δ_Γ $(\times e^{-i\pi J_y})$	$\mathcal{U}$	$\mathcal{O}_\Gamma$	basis of $\mathcal{O}_\Gamma$
$\mathrm{C}_2, \mathrm{C}_4, \mathrm{C}_6$	A	$\boldsymbol{k} \cdot \boldsymbol{J}$	C_2	A	J_z
C_3	–	–	–	–	–
C_s	A	$k_x J_z, k_x J_y, k_y J_x, k_z J_x$	$\sigma_v(yz)$	A	J_x
C_{2v}	A_2	$k_z J_z$	C_2	A_2	J_z
	B_1	$k_x J_z, k_z J_x$	$\sigma_v(yz)$	B_1	J_x
	B_2	$k_y J_z, k_z J_y$	$\sigma_v(xz)$	B_2	J_y
C_{3v}	A_1	$k_z(J_x^3 - J_x J_y J_y - J_y J_x J_y - J_y J_y J_x)$	$\sigma_v(yz)$	A_1	$J_x^3 - J_x J_y J_y - J_y J_x J_y - J_y J_y J_x$
C_{4v}	A_2	$k_z J_z$	C_2	A_2	J_z
C_{6v}	A_2	$k_z J_z$	C_2	A_2	J_z
	B_1	$k_z(J_x^3 - J_x J_y J_y - J_y J_x J_y - J_y J_y J_x)$	$\sigma_v(yz)$	B_1	$J_x^3 - J_x J_y J_y - J_y J_x J_y - J_y J_y J_x$
	B_2	$k_z(J_y^3 - J_y J_x J_x - J_x J_y J_x - J_x J_x J_y)$	$\sigma_d(xz)$	B_2	$J_y^3 - J_y J_x J_x - J_x J_y J_x - J_x J_x J_y$

point group

gap function

operator



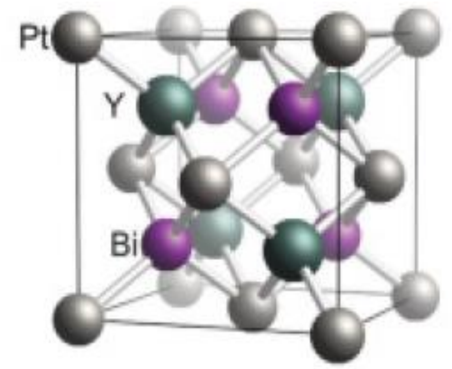
high spin Cooper pairs

magnetic octupole

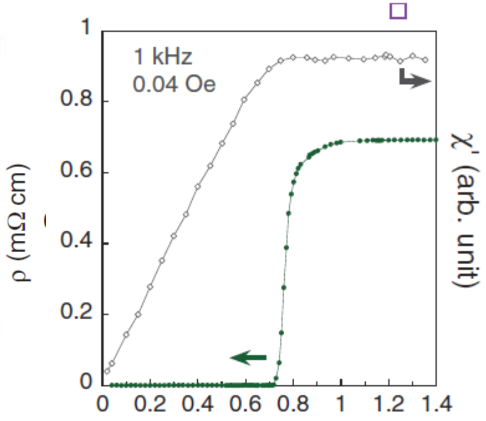
Our theory predicts magnetic octupole response in high spin TSC !!

Application to half-Heusler SCs

YPtBi



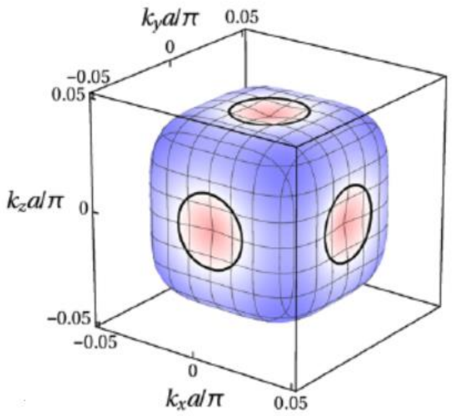
experiment



Butch et al (11) T_c=0.7K

- Noncentrosymmetric TSC
- High-spin (J=3/2) Cooper pair

theory



Brydon et al (16)

proposed gap fn.

$$\Delta(\boldsymbol{k}) \propto \left[\eta \mathbf{1}_{4\times 4} + \sum_i \underline{k_i(J_{i+1}J_iJ_{i+1} - J_{i+2}J_iJ_{i+2})}\right] e^{-iJ_y\pi}$$

high-spin Cooper pair

We can expect magnetic octupole response !!

The octupole response is confirmed by model calculation

Kobayashi-Yamakage-Tanaka-MS (18)

Model Brydon et al (16)

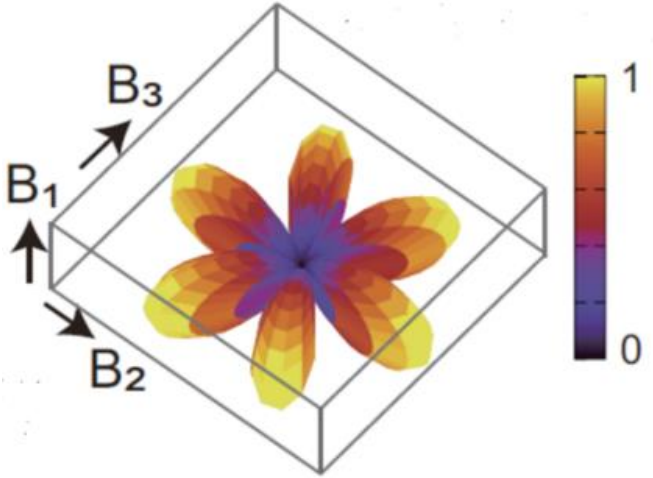
$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \mathcal{E}(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & -\mathcal{E}^t(-\boldsymbol{k}) \end{pmatrix}$$

J_i: 3/2 spin

$$\mathcal{E}(\boldsymbol{k}) = \alpha k^2 + \beta \sum_i k_i^2 J_i^2 + \gamma \sum_{i \neq j} k_i k_j J_i J_j + \delta \sum_i k_i (J_{i+1} J_i J_{i+1} - J_{i+2} J_i J_{i+2})$$

$$\Delta(\boldsymbol{k}) = \Delta_0 / \sqrt{1+\eta^2} \left[\eta \mathbf{1}_{4\times4} + \sum_i k_i (J_{i+1} J_i J_{i+1} - J_{i+2} J_i J_{i+2}) \right] e^{-iJ_y \pi}$$

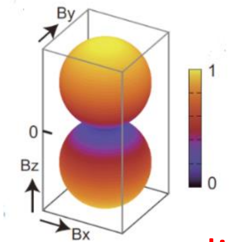
Magnetic response of MFs on [111]



$$\Delta E \propto |B_2|(B_2^2 - 3B_3^2)$$

octupole response

c.f) ^{3}He-B



dipole

Summary

1. In contrast to elementary Majorana particles, emergent MFs may exhibit richer magnetic structures.

2. We find a one-to-one correspondence between symmetry of Cooper pairs and rep. of magnetic response, which provides a novel way to identify unconventional SC.

3. Detection of magnetic octupole response of MFs is a direct evidence of high spin topological superconductivity.