Some results on vacuum decay

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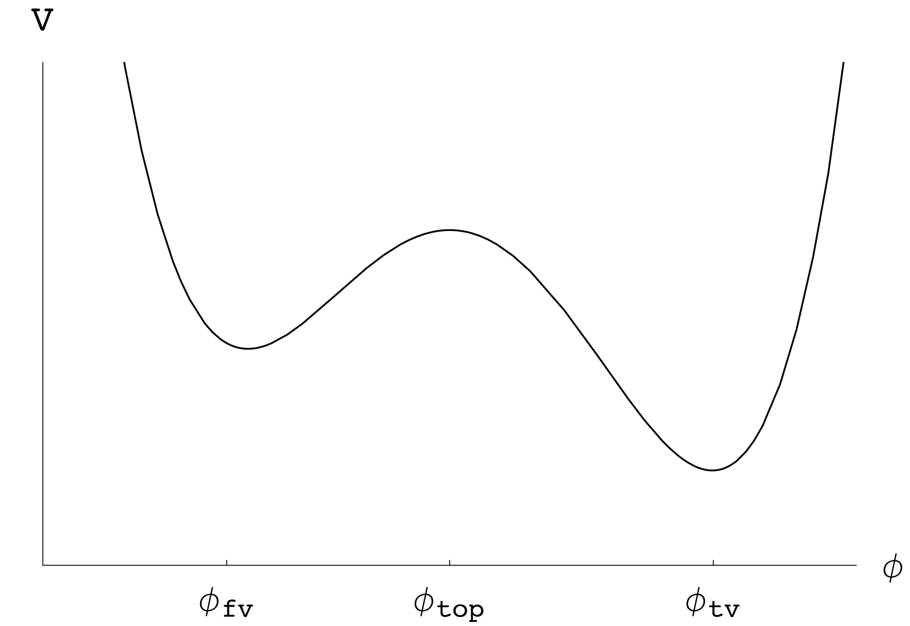
1) Review of basics

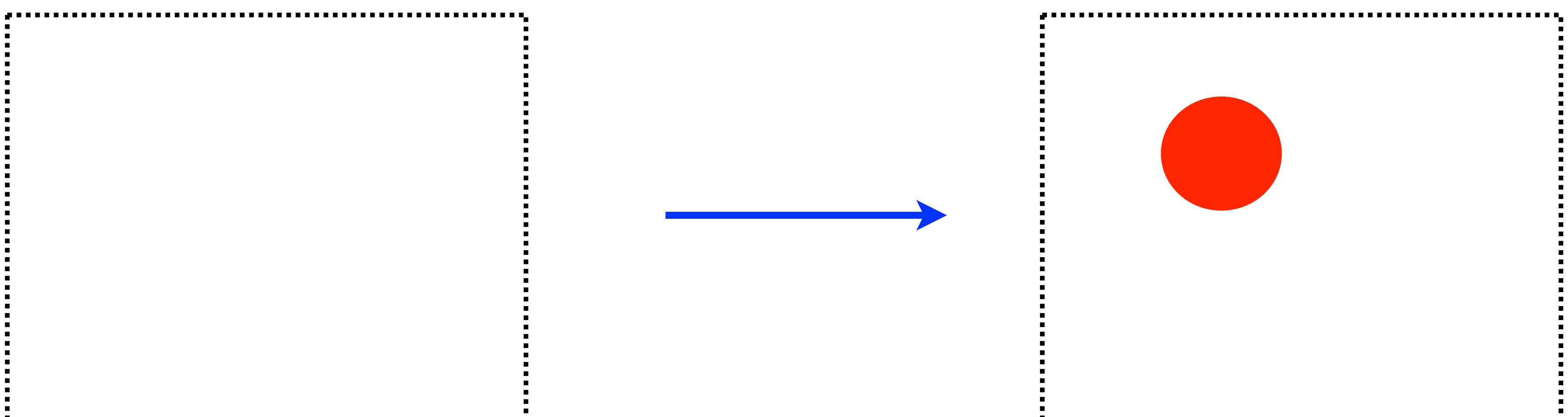
2) Decay to AdS — a looming threat
   False to true may be forbidden

3) de Sitter transitions — history (and future?)
   True to false always allowed

False vacuum ---- classically stable, decays by quantum tunneling -- bubble nucleation, expansion, and coalescence



Tunnel through barrier in $U[\phi(x)] = \int d^3x \left[\frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]$



Want: a) Γ = bubble nucleation rate per unit volume

b) configuration of bubble after nucleation

Decay by tunneling

1 d.o.f. : Rate ~ $A\,e^{-B}$, $\quad B = 2\int_{x_1}^{x_2} dx\,\sqrt{2m(V-E)}$

Many d.o.f.: Consider all paths, calculate B[path]

Minimizing path dominates

⇒ Solve $\delta B = 0$

⇒ Solve $\delta S_{\mathrm{Euc}} = 0$

⇒ Solve Euclidean equations of motion ⇒ bounce solution

$$x(\tau_{\mathrm{init}}) = x_1 \qquad x(\tau_{\mathrm{fin}}) = x_2$$

$$\dot{x}(\tau_{\mathrm{init}}) = \dot{x}(\tau_{\mathrm{fin}}) = 0$$

Append " τ -reversed" solution

Field theory

Coleman

$$S_E = \int d^3x d\tau \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right]$$

$$\frac{\partial^2 \phi}{\partial \tau^2} + \nabla^2 \phi = \frac{dV}{d\phi}$$

$\tau_{\text{init}} = -\infty$

$$\phi(\mathbf{x}, \tau_{\text{init}}) = \phi_{\text{fv}}$$

$$\left. \frac{\partial \phi}{\partial \tau} \right|_{\mathbf{x}, \tau_{\text{fin}}} = 0$$

→ emergence from barrier

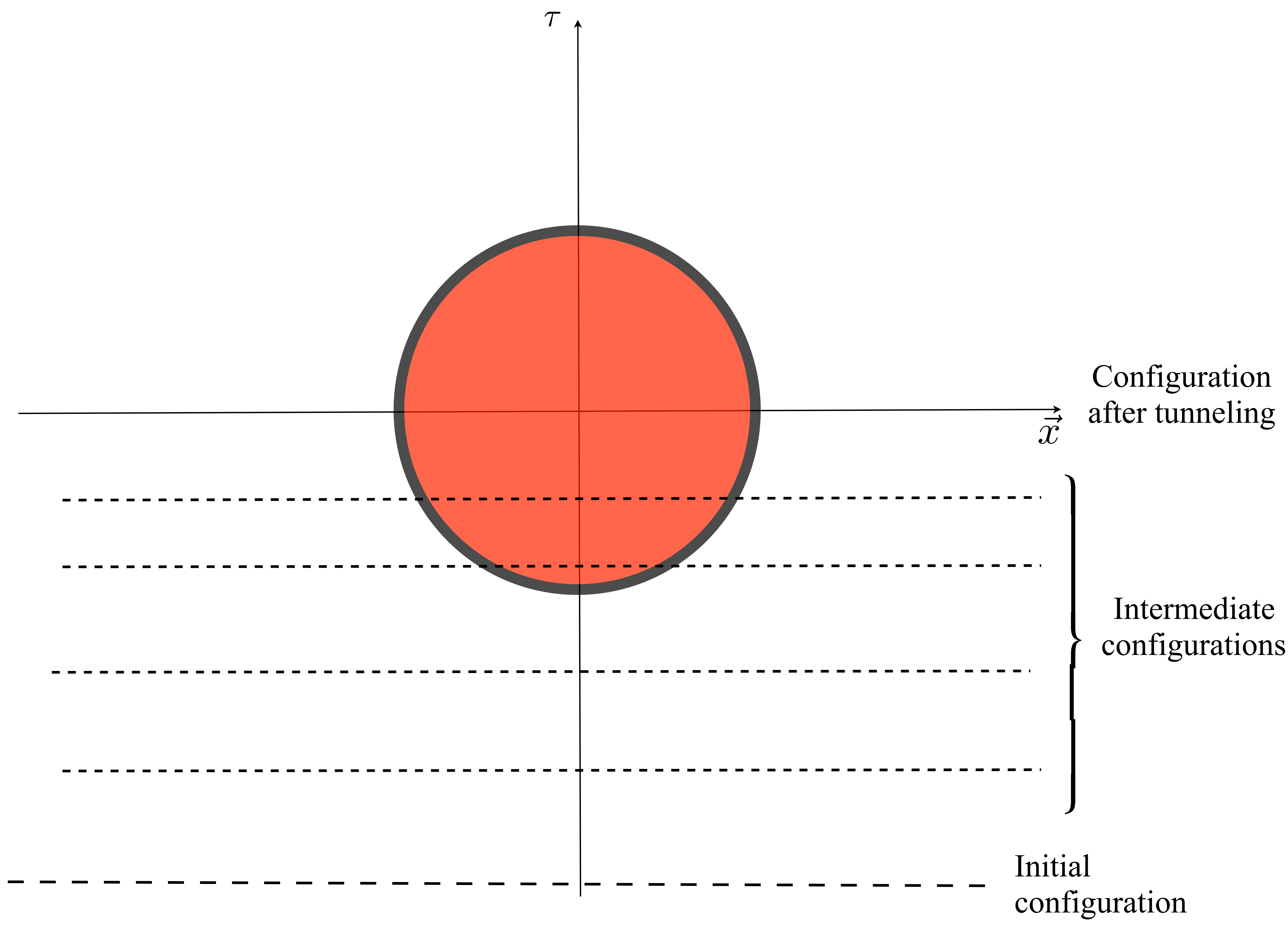
$\tau_{\text{fin}} = 0$

$$\phi(|\mathbf{x}| = \infty, \tau) = \phi_{\text{fv}}$$

→ finite energy configurations

$$0 \leq \tau < \infty \; : \qquad \phi(|\mathbf{x}|, \tau) = \phi(|\mathbf{x}|, -\tau)$$

$$B = S_E(\text{bounce}) - S_E(\text{fv})$$

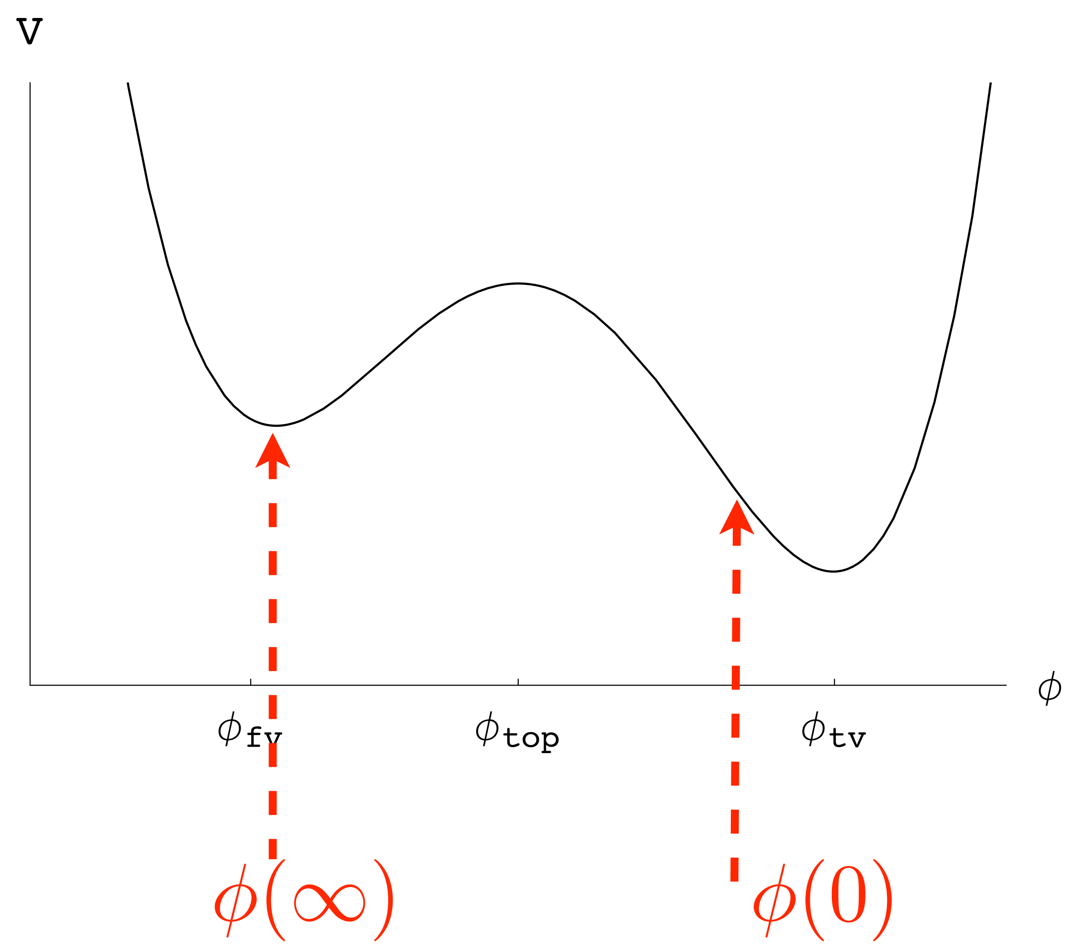

τ
$\vec{x}$
Configuration after tunneling
Intermediate configurations
Initial configuration

Formal O(4) symmetry: Look for O(4)-symmetric bounces

O(4)-symmetric bounces

$$\phi(\mathbf{x},\tau) = \phi(\sqrt{\mathbf{x}^2+\tau^2}) = \phi(s)$$

$$\frac{d^2\phi}{ds^2} + \frac{3}{s}\frac{d\phi}{ds} = \frac{dV}{d\phi} \qquad \dot{\phi}(0) = 0 \qquad \phi(\infty) = \phi_{\rm fv}$$



Conservation of energy

Set $U_{\text{fv}} = 0$

$$E(x_4 = -\infty) = 0 \qquad\qquad E(x_4 = 0) = \int_0^\infty dr\, r^2 \left(\frac{1}{2}\phi'^2 + U \right)$$

$$\text{At } x_4 = 0, \qquad \frac{d\phi}{dr} = \frac{d\phi}{ds}\,, \qquad \phi'' + \frac{3}{r}\,\phi' - \frac{dU}{d\phi} = 0$$

$$\begin{aligned} \frac{d}{dr}\left[r^3 \left(-\frac{1}{2}\phi'^2 + U \right) \right] &= r^3 \left(-\phi'\phi'' + \phi' \frac{dU}{d\phi} \right) + 3r^2 \left(-\frac{1}{2}\phi'^2 + U \right) \\ &= 3r^2\phi'^2 + 3r^2 \left(-\frac{1}{2}\phi'^2 + U \right) \\ &= 3r^2 \left(\frac{1}{2}\phi'^2 + U \right) \end{aligned}$$

$$E = \int_0^\infty dr\, r^2 \left(\frac{1}{2}\phi'^2 + U \right) = \frac{r^3}{3} \left(-\frac{1}{2}\phi'^2 + U \right) \Bigg|_0^\infty = 0$$

Thin-wall approximation

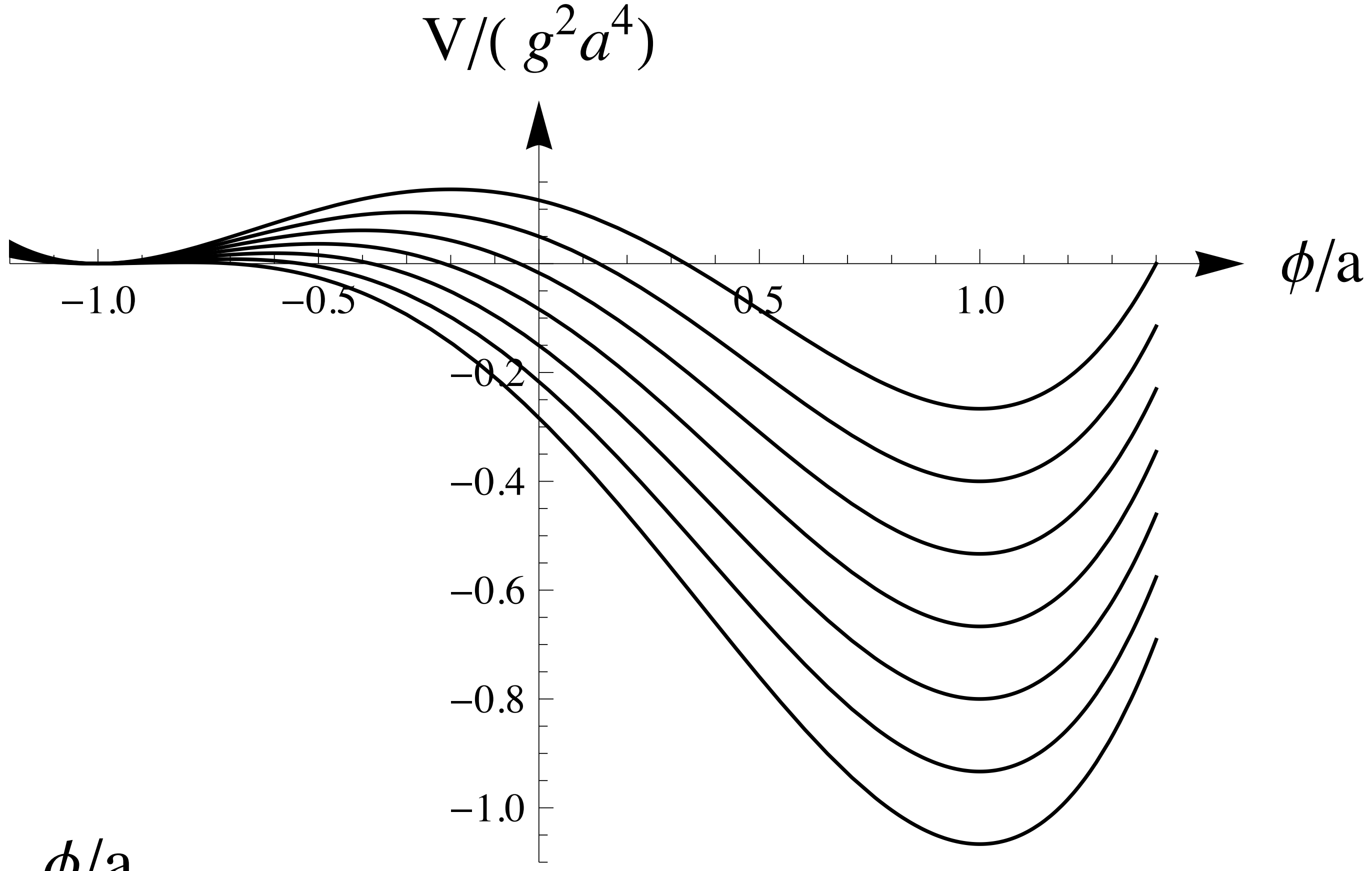
Suppose: $\epsilon \equiv U_{\rm tv} - U_{\rm fv} \ll U_{\rm top} - U_{\rm fv}$

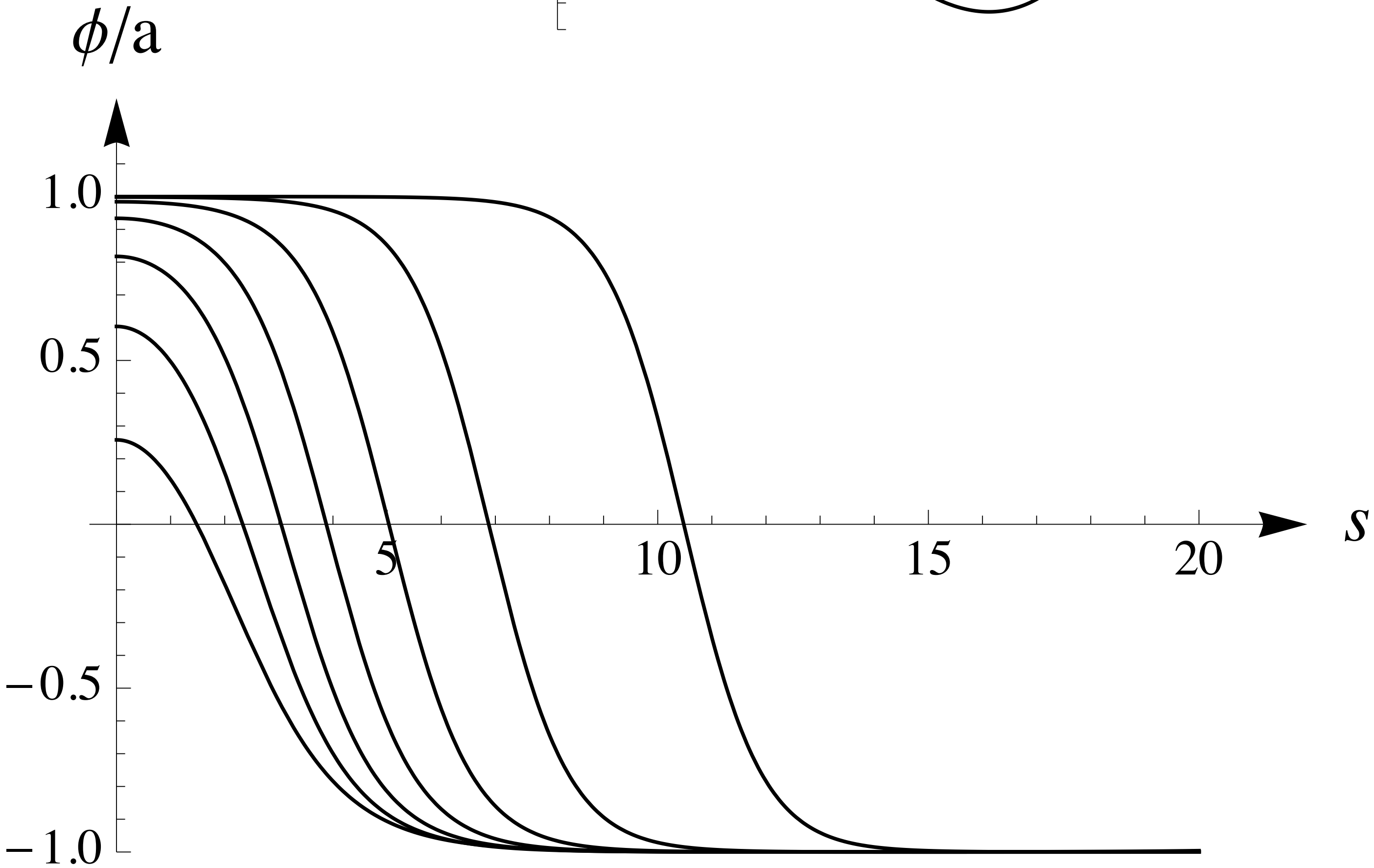
$$\phi(\xi) = \begin{cases} \phi_{\rm tv}\,, & 0 < \xi < R - \Delta \\ \phi_{\rm wall}(\xi)\,, & R - \Delta < \xi < R + \Delta \\ \phi_{\rm fv}\,, & R + \Delta < \xi, \infty \end{cases}$$

$$S_E(R) = \frac{1}{2}\pi^2 R^4 \epsilon + \pi^2 R^3 \sigma \qquad \sigma = \int_{\phi_{\rm fv}}^{\phi_{\rm tv}} d\phi\, \sqrt{2[U(\phi) - U({\rm fv})]}$$

$$R = \frac{3\sigma}{\epsilon} \qquad B = S_E = \frac{\pi^2}{2} R^3 \sigma = \frac{27\pi^2}{2} \frac{\sigma^4}{\epsilon^3} \gg 1$$

Beyond thin-wall





ϵ is irrelevant

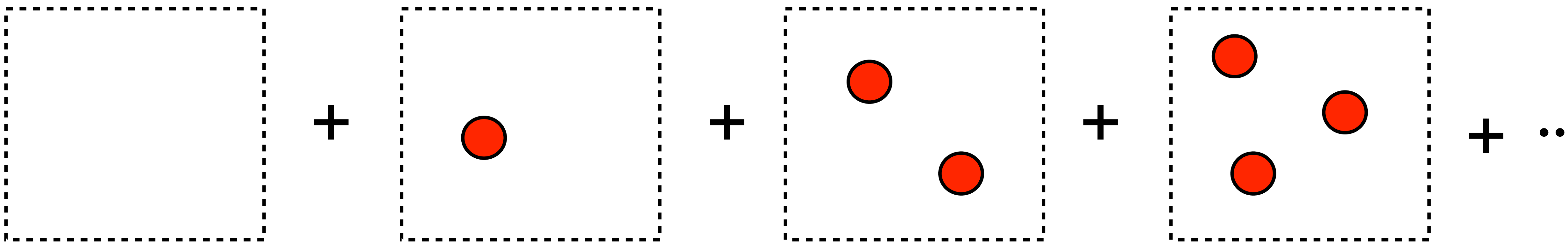
$R = ?$

Pre-exponential factor

Callan-Coleman

$$\langle \phi_{\rm fv}| e^{-HT} |\phi_{\rm fv}\rangle = \int [d\phi(\mathbf{x},\tau)\, e^{-S_E[\phi]} = \sum_n e^{-E_n T} \langle \phi_{\rm fv}|n\rangle\langle n|\phi_{\rm fv}\rangle$$

$$E_{\rm fv} = -\lim_{T\to\infty} \frac{1}{T} \ln \langle \phi_{\rm fv}| e^{-HT} |\phi_{\rm fv}\rangle$$



$$I_0 = \left[\det S''(\phi_{\rm fv})\right]^{-1/2} e^{-S_E(\phi_{\rm fv})}$$

$$I_1 = \frac{i}{2}\,\Omega T \left|\frac{\det' S''(\phi_{\rm bounce})]}{\det S''(\phi_{\rm fv})}\right|^{-1/2} J\, e^{-[S_E(\phi_{\rm bounce})-S_E(\phi_{\rm fv})]}\, I_0$$

$$\equiv i\Omega T K e^{-B}\, I_0$$

4 zero modes, 1 negative mode

$$\frac{d}{ds}\left(\ddot\phi + \frac{3}{s}\dot\phi - \frac{\partial V}{\phi}\right) = 0$$

$$\left(-\frac{d^2}{ds^2} - \frac{3}{s}\frac{d}{ds} + \frac{d^2V}{d\phi^2}\right)\dot\phi = -\frac{3}{s^2}\dot\phi \approx -\frac{3}{R^2}\dot\phi$$

$$I_n = \frac{1}{n!}\left(i\Omega T K e^{-B}\right)^n I_0$$

$$I = I_0 \sum_{n=0}^{\infty} \frac{1}{n!}\left(i\Omega T K e^{-B}\right)^n = I_0 \exp\left[i\Omega T K e^{-B}\right]$$

$$E_{\rm fv} = -\lim_{T\to\infty}\left(\frac{\ln\, I_0}{T}\right) - i\Omega K e^{-B}$$

Complex energy $\Longrightarrow$ unstable state

$$\Gamma = \frac{2\mathrm{Im}\, E_{\mathrm{fv}}}{\Omega} = 2Ke^{-B}$$

One, and only one, negative mode essential

Coleman: Additional negative modes $\Longrightarrow$ bounce of lower action

Including gravity -- Coleman-De Luccia

Add Euclidean Einstein-Hilbert term to action, solve Euclidean matter + metric equations

Assume O(4) symmetry: $ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega_3^2$

$$S = 2\pi^2 \int_{\xi_{min}}^{\xi_{max}} d\xi \left\{ \rho^3 \left[\frac{1}{2}\phi'^2 + U(\phi) \right] + \frac{3}{\kappa}(\rho^2\rho'' + \rho\rho'^2 - \rho) \right\} - \frac{6\pi^2}{\kappa}\, \rho^2\rho' \Big|_{\xi=\xi_{min}}^{\xi=\xi_{\max}}$$

$$S = 2\pi^2 \int_{\xi_{min}}^{\xi_{max}} d\xi \left\{ \rho^3 \left[\frac{1}{2}\phi'^2 + U(\phi) \right] - \frac{3}{\kappa}(\rho\rho'^2 + \rho) \right\}$$

$$\phi'' + \frac{3\rho'}{\rho}\,\phi' = \frac{dU}{d\phi}$$

$$\rho'^2 = 1 + \frac{\kappa}{3}\rho^2 \left[\frac{1}{2}\phi'^2 - U(\phi) \right]$$

Boundary Conditions

$\phi' = 0 \text{ if } \rho = 0$

Minkowski or AdS false vacuum: ρ has one zero R^4

$$\rho(0) = 0\,, \qquad \phi'(0) = 0\,, \qquad \phi(\infty) = \phi_{\rm fv}$$

de Sitter false vacuum: ρ has two zeros S^4

$$\rho(0) = \rho(\xi_{\rm max}) = 0\,, \qquad \phi'(0) = \phi'(\xi_{\rm max}) = 0$$

CDL Thin-wall approximation

Need ρ approximately constant in wall

Need ϵ small

$$B_{\rm outside} \; = \; 0$$

$$B_{wall} \; = \; 2\pi^2\bar{\rho}^3\sigma \equiv 4\pi^2\bar{\rho}^3 \int d\xi \; [U(\phi) - U(\phi_{\rm fv})]$$

$$B_{\rm inside} \; = \; \frac{12\pi^2}{\kappa^2}\left\{\frac{1}{U(\phi_{\rm tv})}\left[\left(1 - \frac{\kappa}{3}\bar{\rho}^2 U(\phi_{\rm tv})\right)^{3/2} - 1\right] - (\phi_{\rm tv} \to \phi_{\rm fv})\right\}$$

$$\sigma = \int_{\phi_{\rm fv}}^{\phi_{\rm tv}} d\phi \, \sqrt{2[U(\phi) - U({\rm fv})]}$$

Minkowski or AdS vacuum to AdS vacuum

Increasing surface tension requires larger bounce.
No CDL thin-wall bounce at all unless

$$\sigma = \int_{\phi_{\rm fv}}^{\phi_{\rm tv}} d\phi \sqrt{2[U(\phi) - U({\rm fv})]}$$

$$\sigma < \frac{2}{\sqrt{3\kappa}} \left(\sqrt{|U_{\rm tv}|} - \sqrt{|U_{\rm fv}|} \right)$$

Otherwise. no bounce → decay is quenched.

Why?

Euclidean approach: Must balance positive wall action against negative volume contribution. Can't be done in AdS if bounce is too large.

Lorentzian approach: Conservation of energy

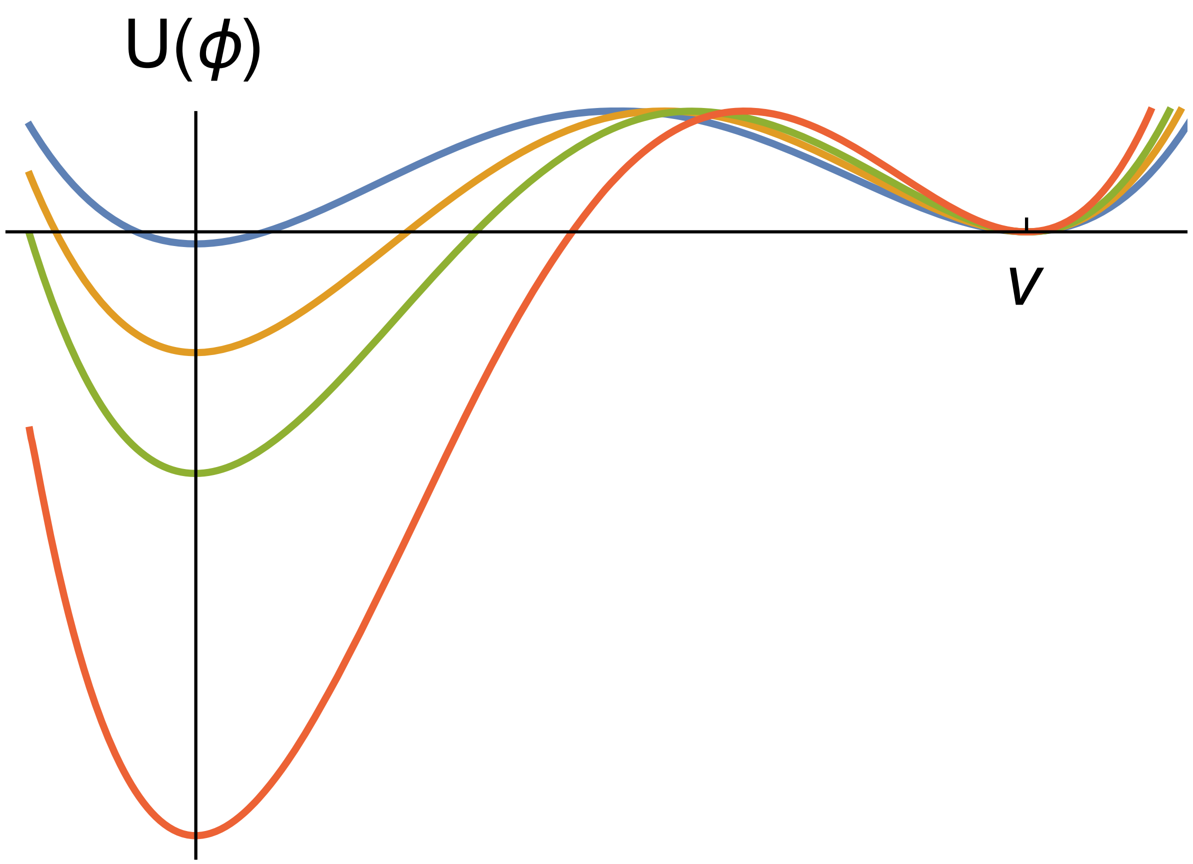
$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega_2^2$$

$$A(r) = \left[1 - \frac{2G_N \mathcal{M}(r)}{r}\right]^{-1} \qquad \mathcal{M}(r) = 4\pi \int_0^r ds\, s^2 \tilde{\rho}(s)$$

Require: At $x_4 = 0\,,$ $\mathcal{M}_{\rm bounce}(r = \infty) = 0$

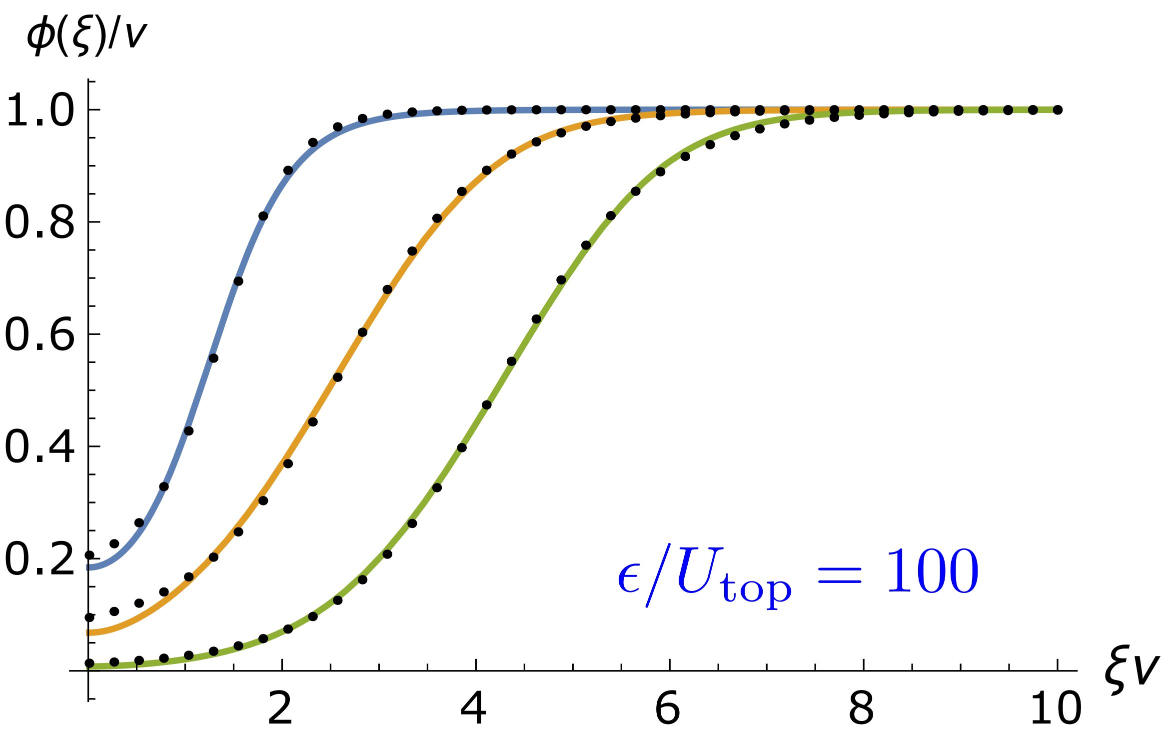
If not, no bounce, no tunneling

Beyond the thin-wall approximation

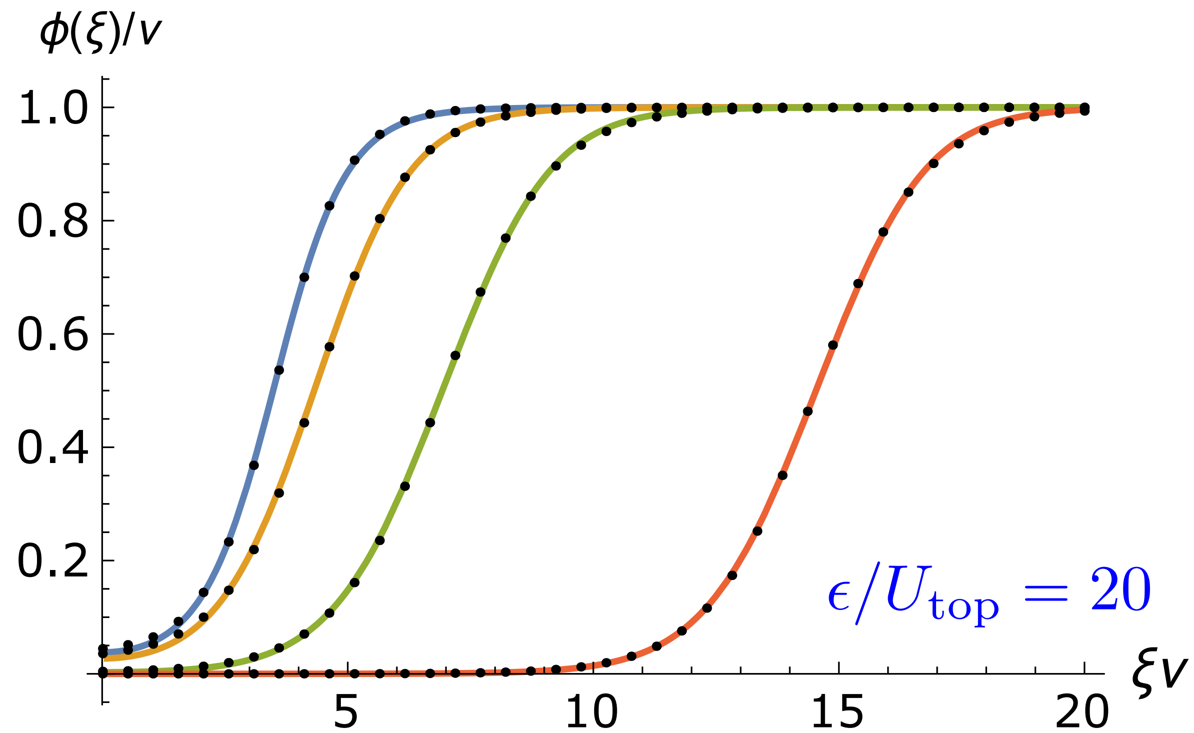


$$\phi_{\rm tv} = 0\,, \qquad \phi_{\rm fv} = v$$

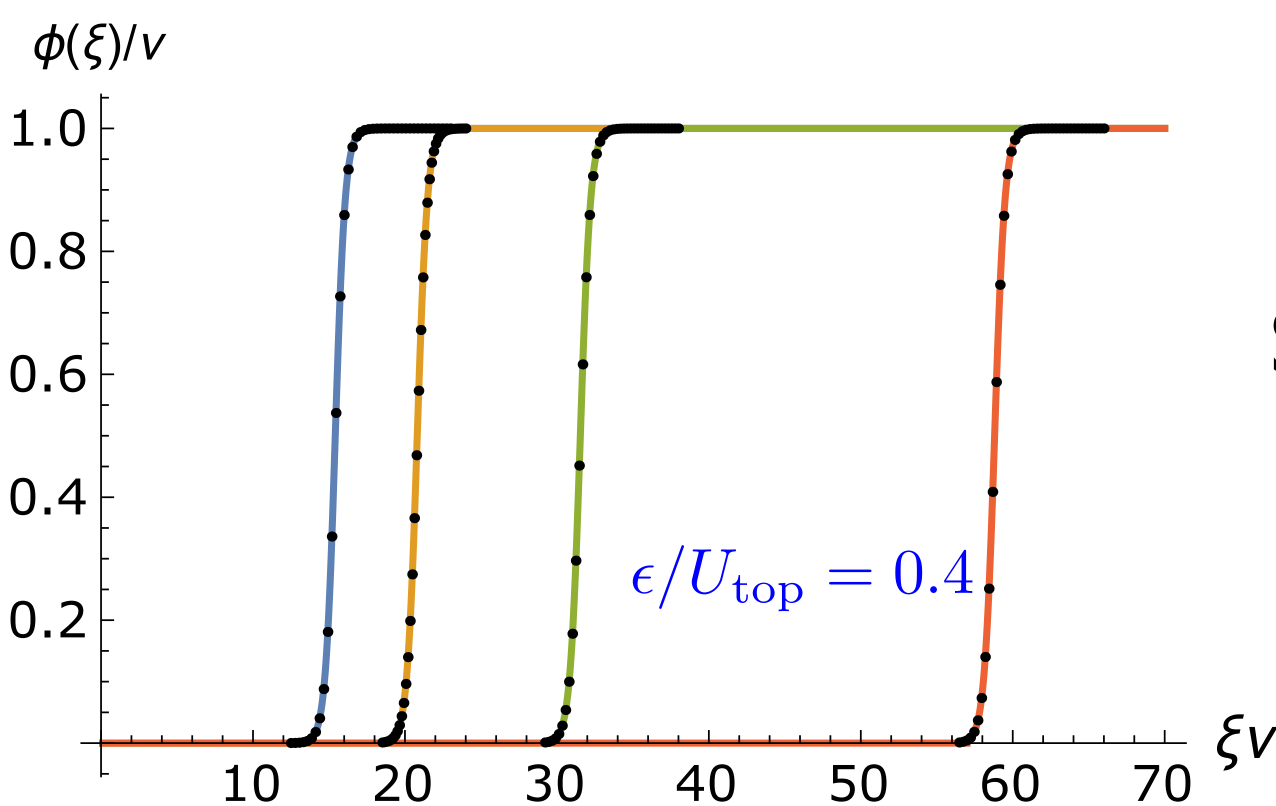
Fix U, vary $\beta = \sqrt{\kappa v^2} = \dfrac{\sqrt{8\pi}\ v}{M_{\rm Pl}}$



$\beta = 0,\ 5.747,$ and 5.763



$\beta = 0,\ 2.70,\ 3.33,$ and 3.38



$\beta = 0,\ 0.588,\ 0.714,$ and 0.739

Curves = fit to tanh

Stronger gravity:

- Kink moves to right
- Kink shape almost constant

$\epsilon/U_{\rm top} = 20$

β	B	$\xi_{\rm wall}$	$\rho_{\rm wall}$	$\ell_{\rm AdS}$	$\Delta\xi_{\rm wall}$	$\Delta\rho_{\rm wall}$	η	$\sigma_{\rm wall}$	b
0	54.2	4.32	4.318	∞	2.39	2.39	0.05013	0.0216	0.70
0.100	54.36	4.31	4.323	38.7	2.397	2.398	0.05014	0.0216	0.70
1.000	67.02	4.42	4.03	3.87	2.472	2.563	0.05053	0.0224	0.68
2.500	309.7	5.17	11.5	1.549	2.857	4.269	0.04094	0.0264	0.56
2.564	355.2	5.24	12.36	1.51	2.881	4.483	0.03955	0.0266	0.56
2.703	500.0	5.41	14.81	1.43	2.934	5.082	0.03598	0.0272	0.54
3.226	8678	6.88	63.9	1.201	3.165	16.77	0.01193	0.0295	0.48
3.367	1.1×10^6	9.54	725.2	1.15	3.236	173.1	9.9×10^{-4}	0.0302	0.46
3.378	4.1×10^7	11.6	4427	1.146	3.242	1048	1.3×10^{-4}	0.0303	0.46
3.380	1.7×10^8	12.4	9084	1.146	3.243	2148	6.0×10^{-5}	0.0303	0.46

Analytic understanding

$$E = \frac{1}{2}\phi'^2 - U(\phi)$$

$$E' = -3\frac{\rho'}{\rho}\phi'^2$$

$$\frac{\rho'^2}{\rho^2} = \frac{1}{\rho^2} + \frac{\kappa}{3}E$$

Region I, $0 < \xi < \xi_1$ $\quad \phi \approx \phi_{\text{tv}}\,, \quad \rho \approx \ell \sinh(\xi/\ell)\,, \quad E \approx \epsilon$

Region III, $\xi_3 < \xi < \infty$ $\quad \phi \approx \phi_{\text{fv}}\,, \quad \rho \approx \rho(\xi_3) + (\xi - \xi_3)\,, \quad E \approx 0$

Region IIa, $\xi_1 < \xi < \xi_2$ $\quad \frac{\rho'^2}{\rho^2} = \cancel{\frac{1}{\rho^2}} + \frac{\kappa}{3}E$

Region IIb, $\xi_2 < \xi < \xi_3$ $\quad \frac{\rho'^2}{\rho^2} = \frac{1}{\rho^2} + \cancel{\frac{\kappa}{3}E}$

Region IIa:

$$\sqrt{E(\xi)} = \sqrt{\epsilon} - \sqrt{\frac{3\kappa}{4}} \int_{\xi_1}^{\xi} d\xi \; \phi'^2$$

$$\rho(\xi) = \rho(\xi_1) \exp\left[\int_{\xi_1}^{\xi} d\xi \sqrt{\frac{\kappa}{3}} \sqrt{E}\right]$$

$$\phi'' + \sqrt{3\kappa} \sqrt{\frac{1}{2}\phi'^2 - U(\phi)} \; \phi' = \frac{\partial U}{\partial \phi}$$

No dependence on ρ

Approx. constant kink shape.

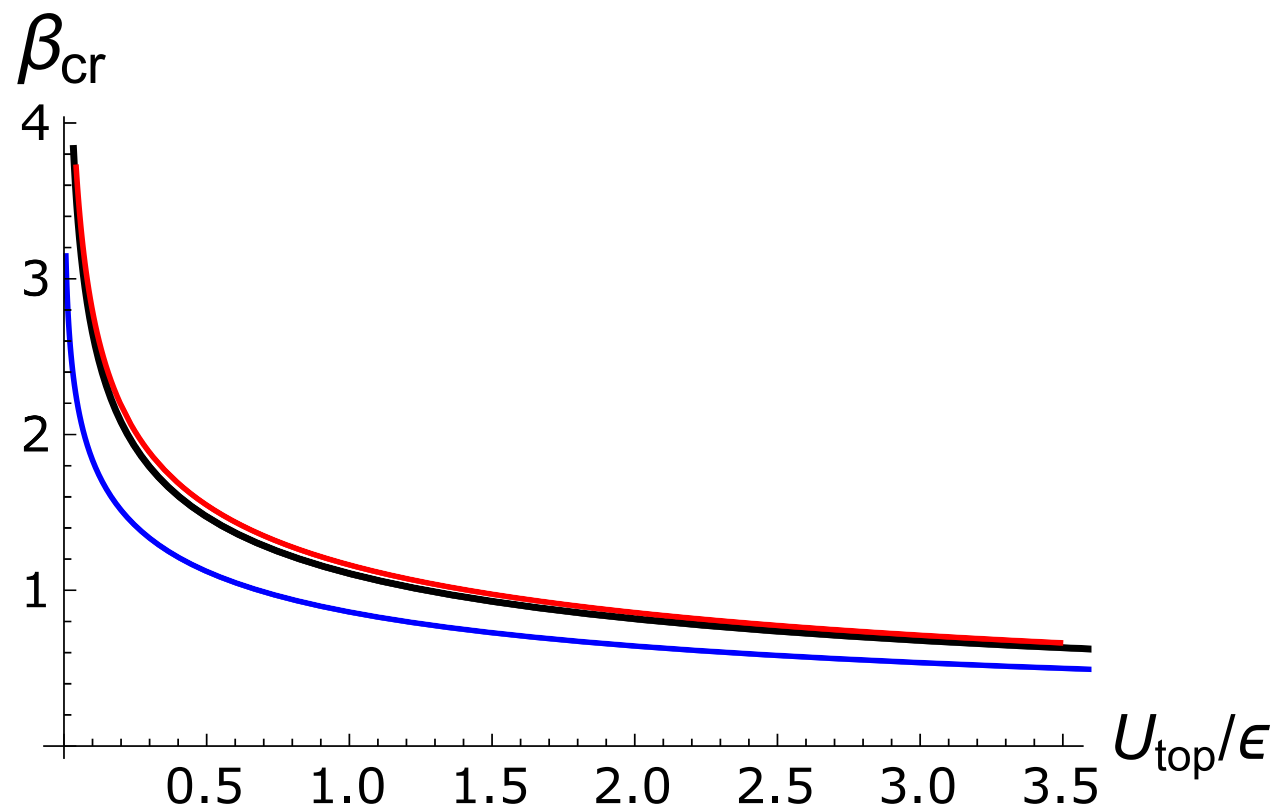
Increasing gravity: longer stay in Region I

$\rho(\xi_1)$ is larger

later transition IIa to IIb

Critical solution when IIb disappears:

$$\int_{\xi_1}^{\infty} d\xi \; \phi'^2 = \sqrt{\frac{4\epsilon}{3\kappa}}$$



Fix potential, increase gravity → tunneling always quenched

$$U_{\rm top}/\epsilon \gg 1 \quad [\text{CDL-TWA}], \qquad \beta_{\rm cr} \sim (U_{\rm top}/\epsilon)^{-1/2}$$

$$U_{\rm top}/\epsilon \ll 1\,, \quad \beta_{\rm cr} \sim (U_{\rm top}/\epsilon)^{-\alpha} \quad (\alpha \text{ model-dependent})$$

A new thin-wall regime?

For CDL thin-wall

$$\sigma < \frac{2}{\sqrt{3\kappa}}\left(\sqrt{|U_{\rm tv}|} - \sqrt{|U_{\rm fv}|}\right)$$

For new thin-wall ??

$$\sigma = ? \qquad \bar{\rho} = ?$$

$$\tilde{\sigma} < \frac{2}{\sqrt{3\kappa}}\left(\sqrt{|U_{\rm tv}|} - \sqrt{|U_{\rm fv}|}\right)$$

For new thin-wall:

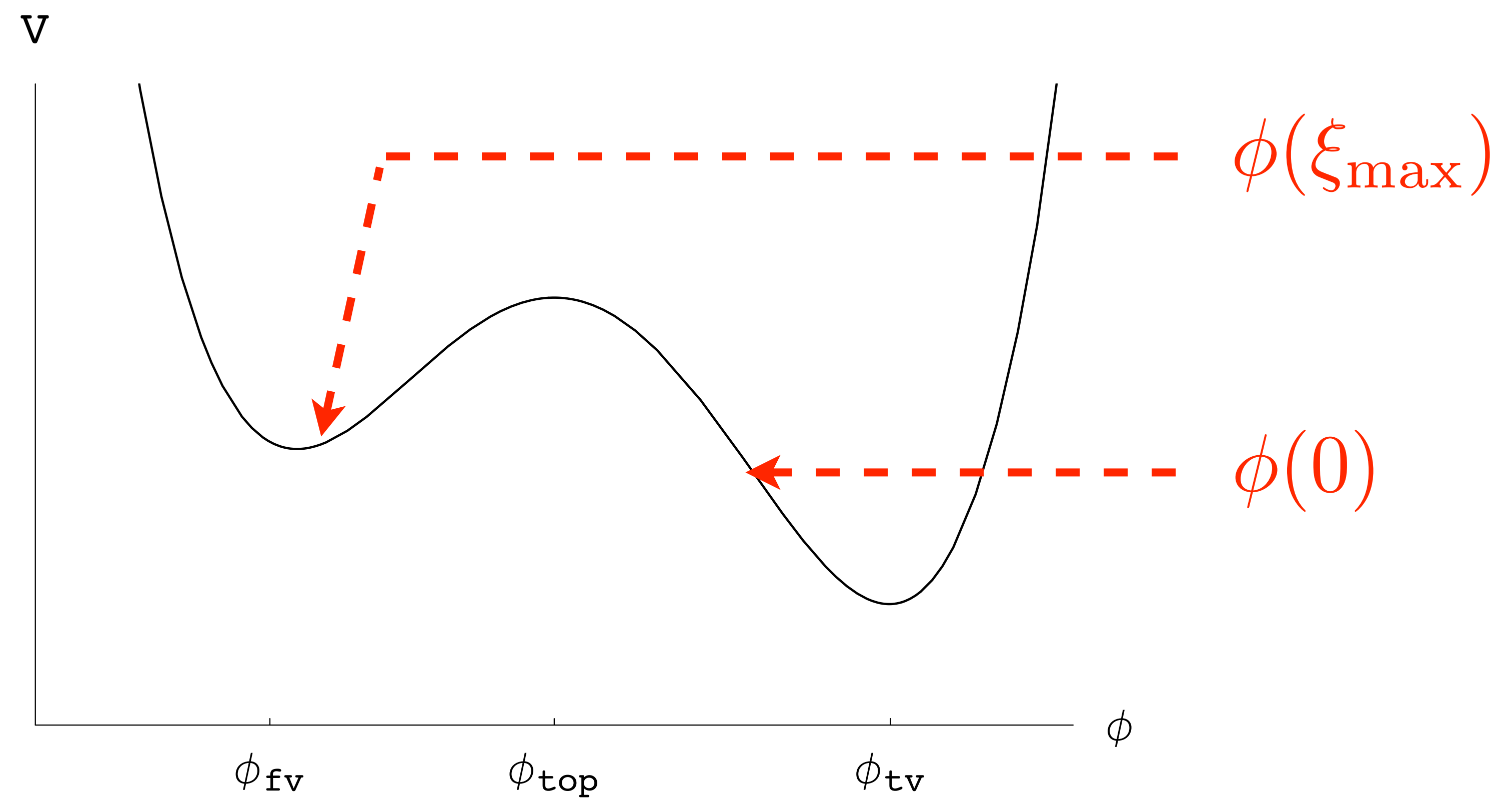
$$\rho(\xi) = \rho_1 \, e^{G(\xi)} \qquad G(\xi) = \sqrt{\frac{\kappa}{3}} \int_{\xi_1}^{\xi} d\xi \, \sqrt{\frac{1}{2}\phi'^2 - U(\phi)}$$

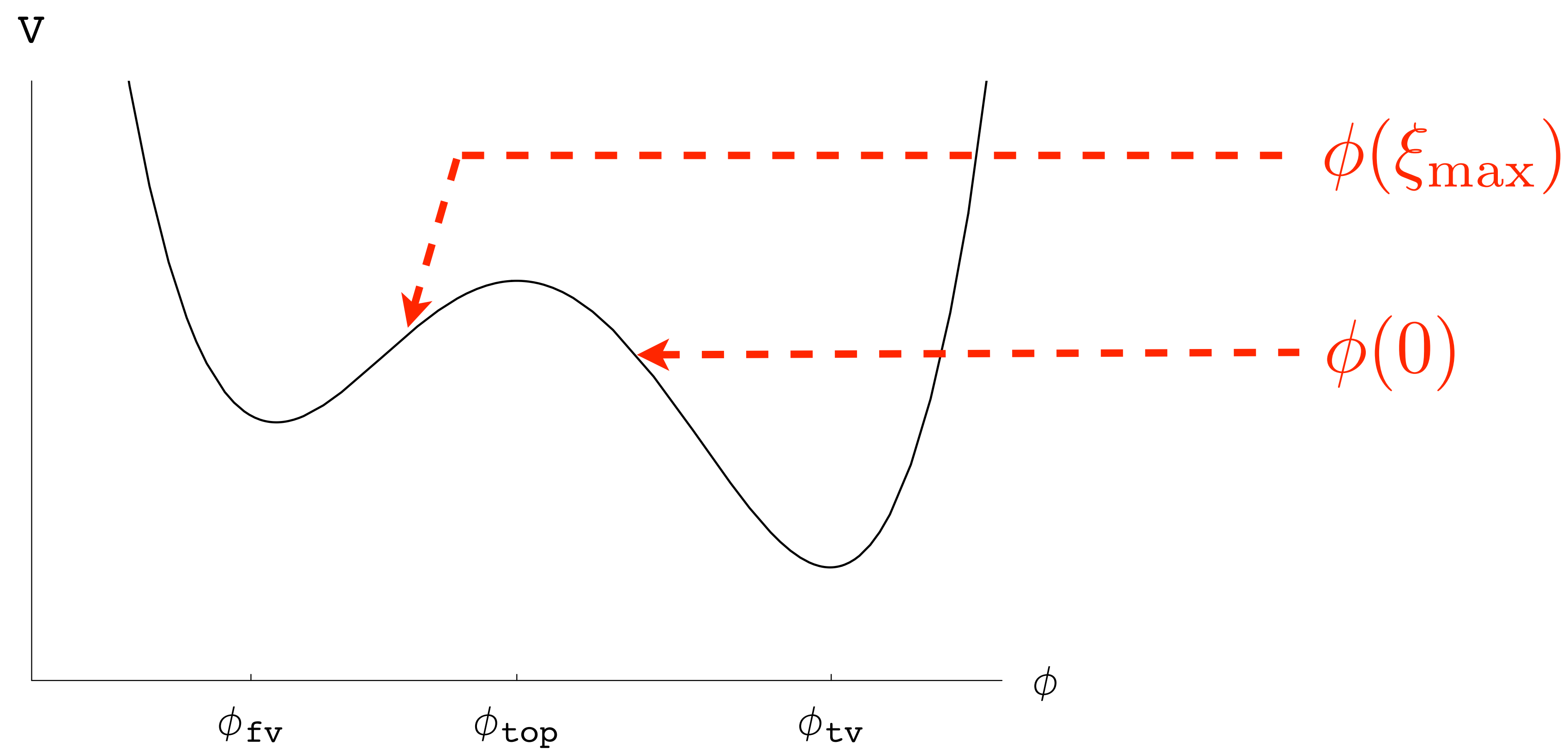
$$\tilde{\sigma} = \sqrt{\frac{12}{\kappa}} \int_0^{\ln(\rho_2/\rho_1)} dG \, e^{3G} \left[\frac{U(\phi_b)}{\sqrt{\frac{1}{2}\phi'^2_b - U(\phi_b)}} + \sqrt{-U_{\rm fv}} \right]$$

No Assumptions :

$$\int_0^{\infty} d\xi \, \phi_b'^2 < \frac{2}{\sqrt{3\kappa}} \left(\sqrt{|U_{\rm tv}|} - \sqrt{|U_{\rm fv}|} \right)$$

Tunneling from de Sitter





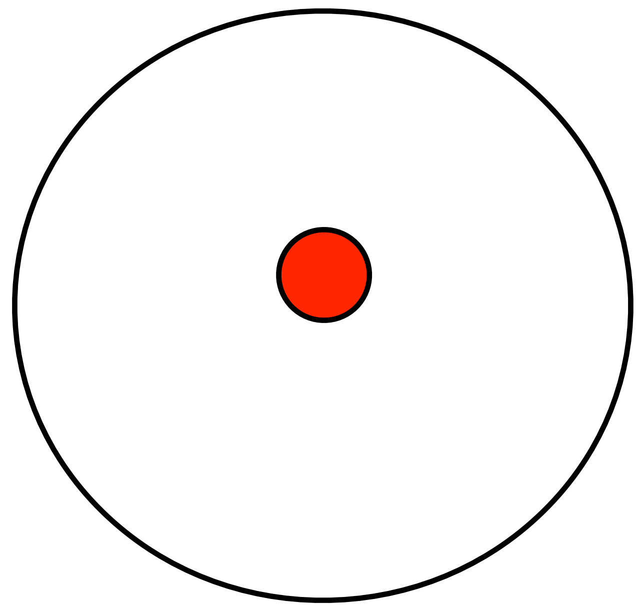
Issues/questions:

Bounce has finite four-volume

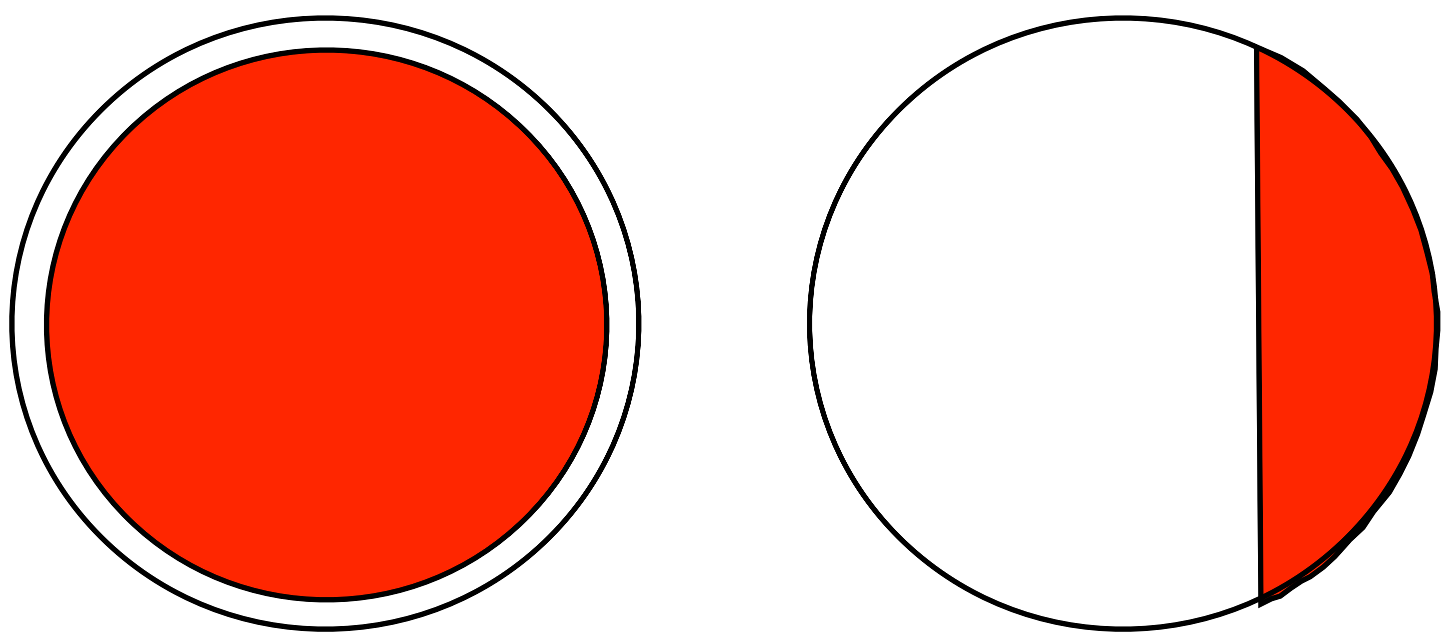
--- details of U near minimum unimportant

Can tunnel up as well as down K.Lee, EW

Path through configuration space? Configuration after tunneling?



Initial state?
Configuration after tunneling?



Answer — CDL bounce
Question — ?????

Strategy: Treat horizon volume as a finite volume system at a temperature

Brown, EW

$$T_{\rm dS} = \frac{H}{2\pi} = \frac{1}{2\pi\Lambda}$$

Periodic bounce solution

Fixed background approximation:

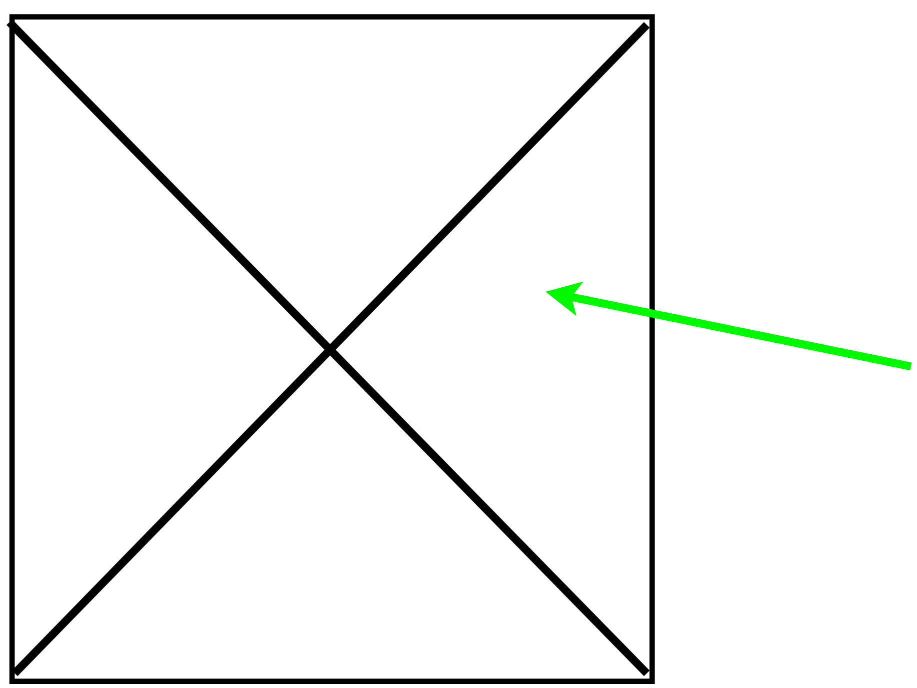
$$U(\phi_{\rm top}) - U(\phi_{\rm tv}) \ll U(\phi_{\rm tv})$$

To leading order, variations in Φ don't affect metric; can treat spacetime as de Sitter spacetime with fixed and uniform temperature.

Static de Sitter coordinates

$$ds^2 = -\underbrace{\left(1 - \frac{r^2}{\Lambda^2}\right)}_{A(r)} dt^2 + \underbrace{\left(1 - \frac{r^2}{\Lambda^2}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)}_{h_{ij}}$$

$r = \Lambda$ → Horizon of r=0



Causal diamond

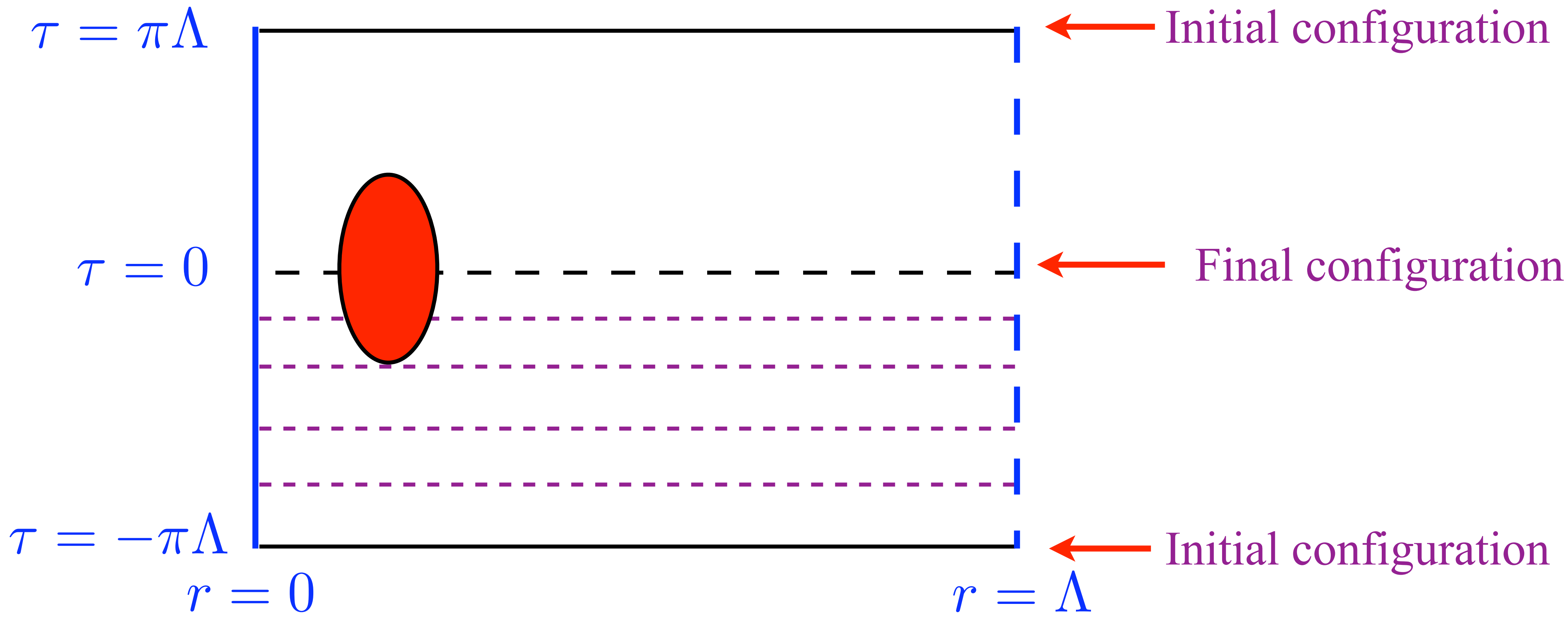
Field theory on thermal static patch

$$S = \int_{r \le \Lambda} d^4x \sqrt{-\det g} \left[-\frac{1}{2} g^{\mu\nu} \, \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

$$S \; = \; \int dt \int_{r \le \Lambda} d^3x \sqrt{\det h} \left[\frac{1}{2\sqrt{A(r)}} \left(\frac{d\phi}{dt} \right)^2 - \frac{1}{2} \sqrt{A(r)} \, h^{ij} \, \partial_i \phi \, \partial_j \phi - \sqrt{A(r)} \, V(\phi) \right]$$

$$E = \int_{r \le \Lambda} d^3x \sqrt{\det h} \left[\frac{1}{2\sqrt{A(r)}} \left(\frac{d\phi}{dt} \right)^2 + \frac{1}{2} \sqrt{A(r)} \, h^{ij} \, \partial_i \phi \, \partial_j \phi + \sqrt{A(r)} \, V(\phi) \right]$$

$$S_E = \int_{-\pi\Lambda}^{\pi\Lambda} d\tau \int_{r \le \Lambda} d^3x \sqrt{\det h} \left[\frac{1}{2\sqrt{A(r)}} \left(\frac{d\phi}{d\tau} \right)^2 + \frac{1}{2} \sqrt{A(r)} \, h^{ij} \, \partial_i \phi \, \partial_j \phi + \sqrt{A(r)} \, V(\phi) \right]$$



$\tau = \pi\Lambda$
Initial configuration
$\tau = 0$
Final configuration
$\tau = -\pi\Lambda$
Initial configuration
$r = 0$
$r = \Lambda$

$$S_E = \int_{-\pi\Lambda}^{\pi\Lambda} d\tau \int_{r \le \Lambda} d^3x \sqrt{\det h} \left[\frac{1}{2\sqrt{A(r)}} \left(\frac{d\phi}{d\tau} \right)^2 + \frac{1}{2} \sqrt{A(r)}\, h^{ij}\, \partial_i \phi\, \partial_j \phi + \sqrt{A(r)}\, V(\phi) \right]$$

Define:

$$\begin{aligned} \tilde{g}_{ab} dx^a dx^b &= A\, d\tau^2 + h_{ij}\, dx^i dx^j \\ &= \left(1 - \frac{r^2}{\Lambda^2} \right) d\tau^2 + \left(1 - \frac{r^2}{\Lambda^2} \right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right) \end{aligned}$$

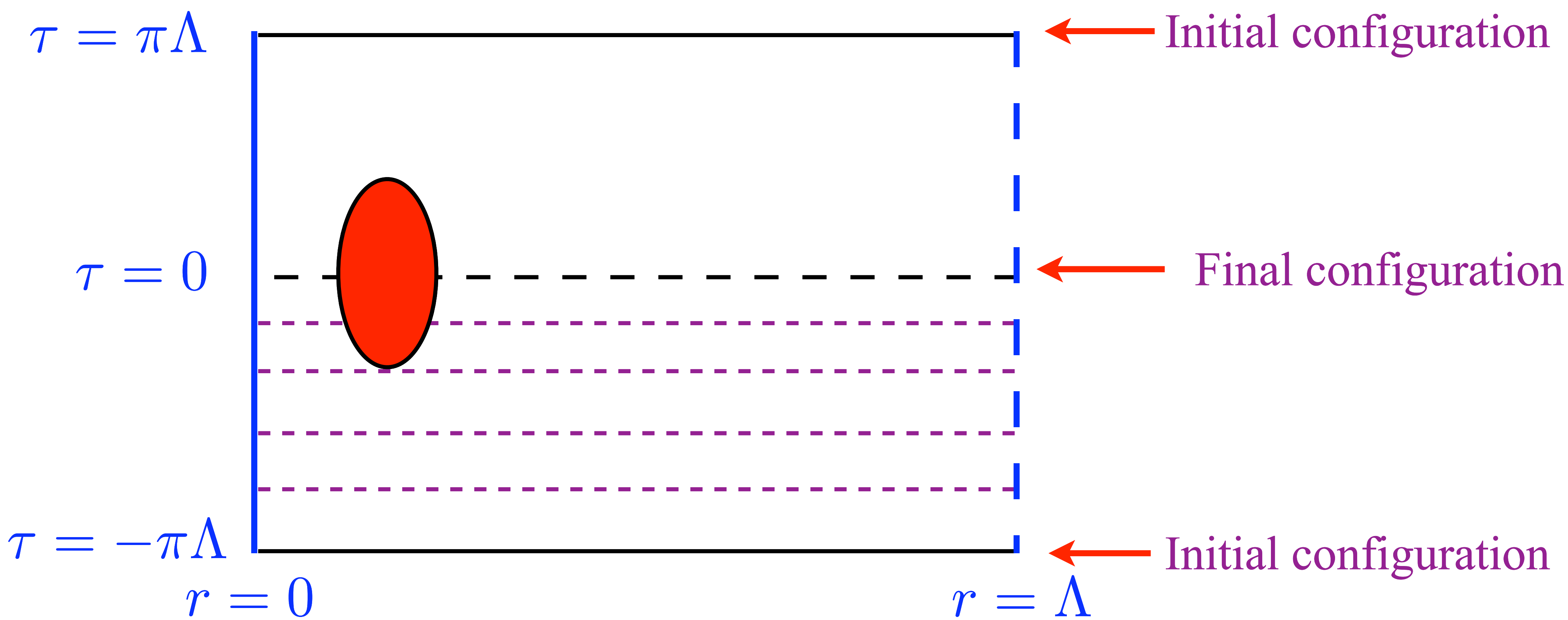
$$S_E = \int d^4x \sqrt{\det \tilde{g}} \left[\frac{1}{2} \tilde{g}^{ab}\, \partial_a \phi\, \partial_b \phi + V(\phi) \right]$$

$\tilde{g}$ is the round metric on a four-sphere :

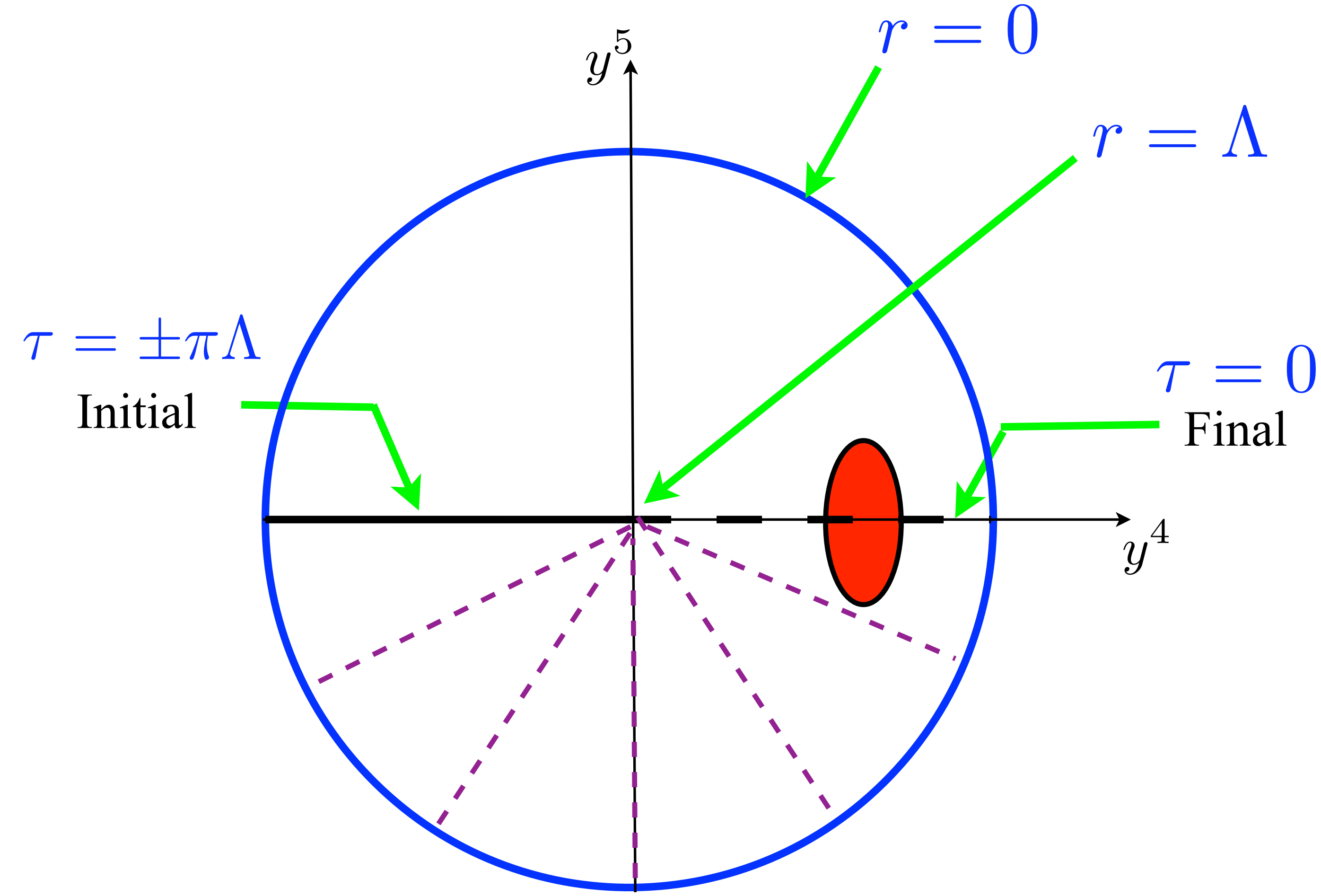
$$\begin{aligned} y^1 &= r\, \sin\theta \cos\phi \\ y^2 &= r\, \sin\theta \sin\phi \\ y^3 &= r\, \cos\theta \\ y^4 &= \sqrt{\Lambda^2 - r^2}\, \cos(\tau/\Lambda) \\ y^5 &= \sqrt{\Lambda^2 - r^2}\, \sin(\tau/\Lambda) \end{aligned}$$

$$\Gamma \sim e^{-[S_E(\mathrm{bounce}) - S_E(\mathrm{fv})]}$$

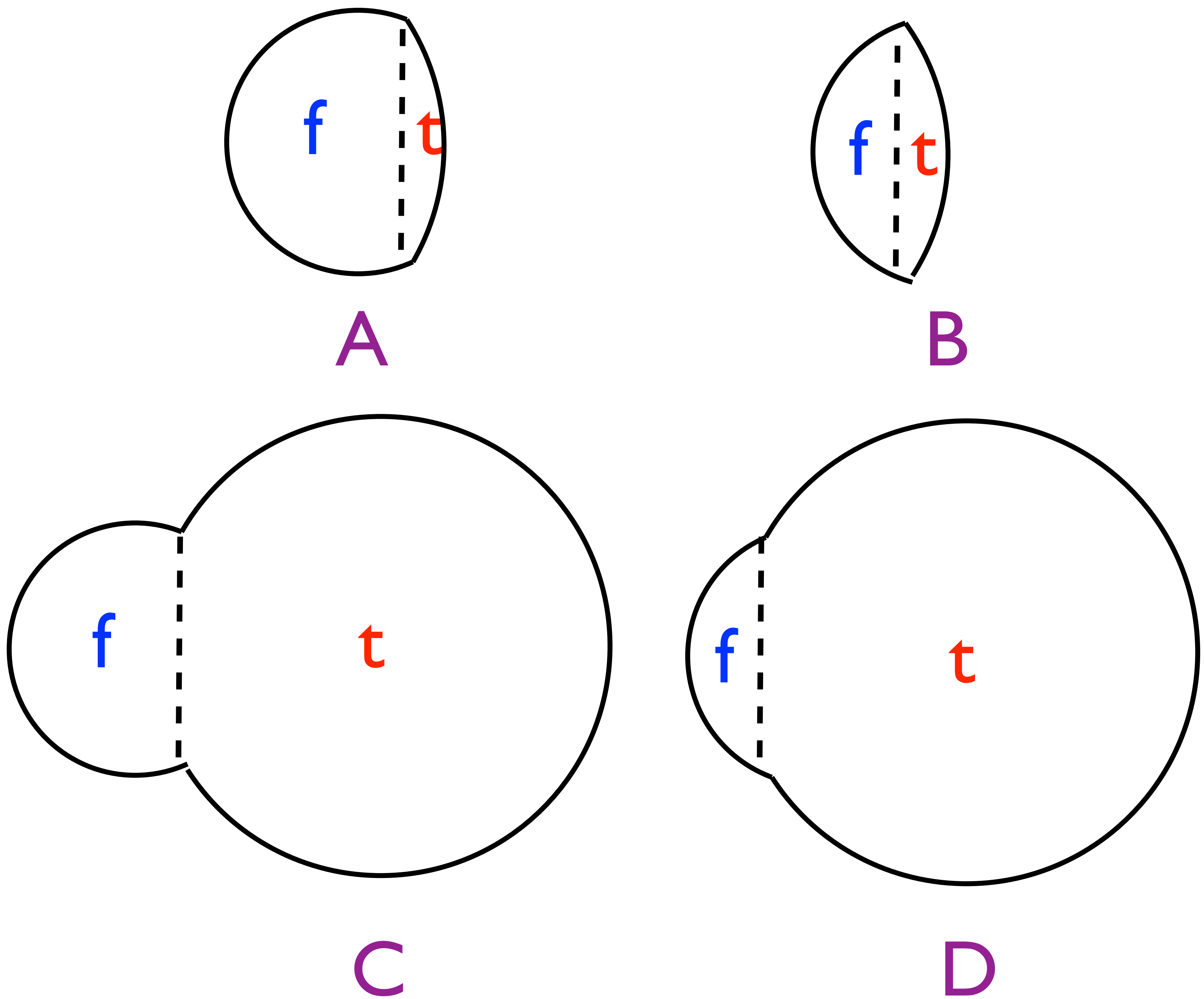
→ Coleman-De Luccia prescription for rate



$$
\begin{aligned}
y^1 &= r\ \sin\theta\cos\phi \\
y^2 &= r\ \sin\theta\sin\phi \\
y^3 &= r\ \cos\theta \\
y^4 &= \sqrt{\Lambda^2 - r^2}\ \cos(\tau/\Lambda) \\
y^5 &= \sqrt{\Lambda^2 - r^2}\ \sin(\tau/\Lambda)
\end{aligned}
$$



CDL thin-wall bounces:



$$\ddot{\rho} = -\frac{\kappa\rho}{3}\left(\dot{\phi}^2 + V\right) \Rightarrow$$

Types C and D impossible if V>0

For type A/B:

Wall tension

$$B[\bar{\rho}] = 2\pi^2\bar{\rho}^3 S_1 + \frac{4\pi^2}{\kappa}\Lambda_f^2\left[1 \mp \left(1 - \frac{\bar{\rho}^2}{\Lambda_f^2}\right)^{3/2}\right] - \frac{4\pi^2}{\kappa}\Lambda_t^2\left[1 - \left(1 - \frac{\bar{\rho}^2}{\Lambda_t^2}\right)^{3/2}\right]$$

$V_f - V_t$

$$\frac{\partial B}{\partial \bar{\rho}} = 0 \quad \Rightarrow \quad \frac{1}{\bar{\rho}^2} = \frac{1}{\Lambda_f^2} + \left(\frac{\epsilon}{3S_1} - \frac{\kappa S_1}{4}\right)^2$$

$\frac{\epsilon}{3S_1} > \frac{\kappa S_1}{4}$ $\Rightarrow$ Solution only for upper sign $\Rightarrow$ Type A

$\frac{\epsilon}{3S_1} < \frac{\kappa S_1}{4}$ $\Rightarrow$ Solution only for lower sign $\Rightarrow$ Type B

For type B, $\bar{\rho}$ decreases as ϵ decreases.

CDL prefactor? Negative modes?

For type A $$\frac{\partial^2 S_E}{\partial \bar{\rho}^2} < 0$$ Negative mode

For type B $$\frac{\partial^2 S_E}{\partial \bar{\rho}^2} > 0$$ No negative mode?

No tunneling for type B? — Implausible

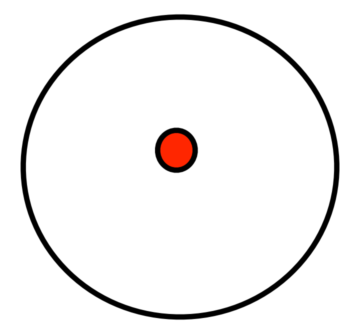
Negative modes of full $\frac{\delta^2 S_E}{\delta \chi^2}$

"Standard", slowly varying

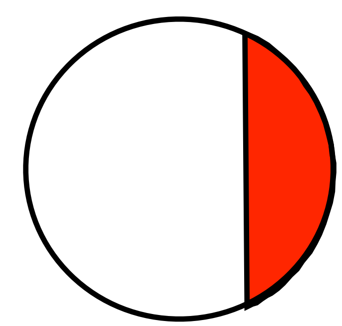
Rapidly oscillating — in wall
around $\rho_{\max}$

Bounces

Small type A



Large type A ($\rho_{\max}$ after wall)

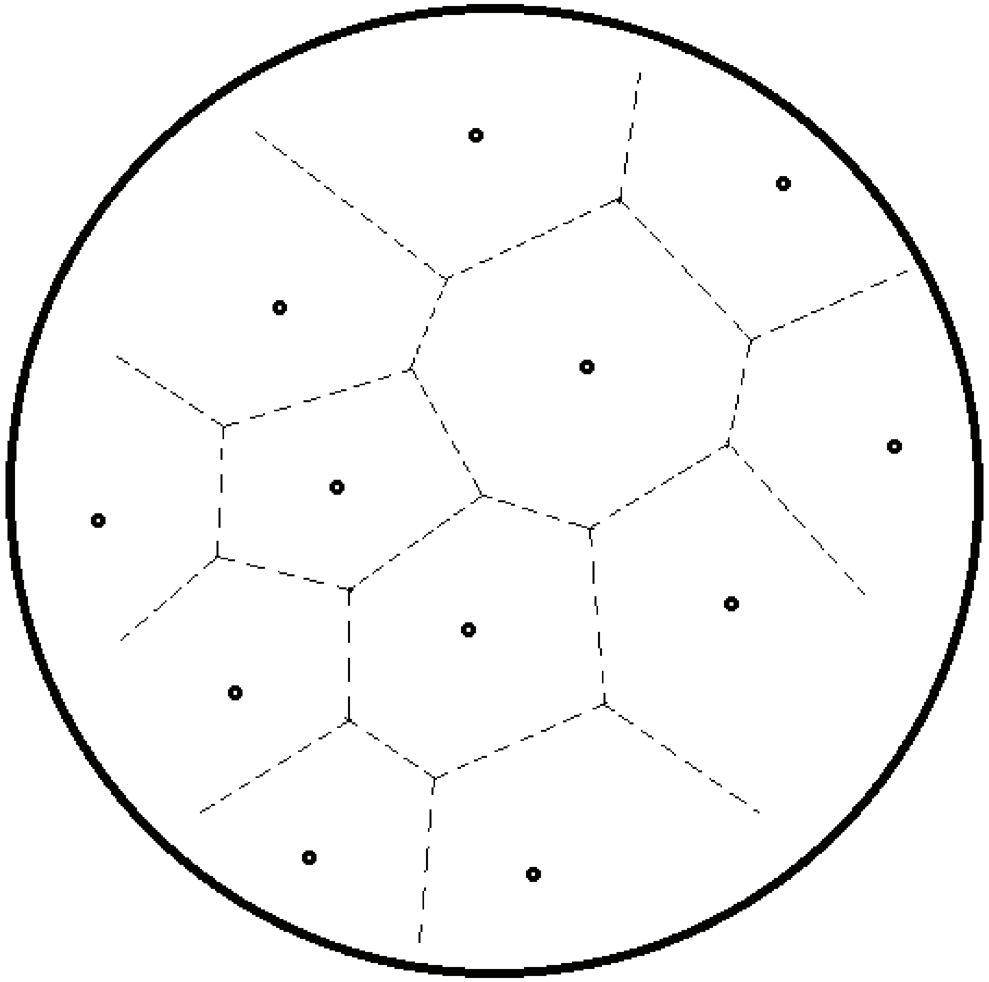


Type B ($\rho_{\max}$ in wall)

Negative modes present?

	Slowly varying	Wall oscillating	$\rho_{\max}$ oscillating
Small type A	Yes	No	Planckian
Large type A	Usually	Possible	Yes
Type B	No	—	Yes

Multibounce configurations?



No oscillating zero modes

Multibounce contributions:

$$I = I_0 \sum_{n=0} \frac{1}{n!} \left(i^? \mathcal{V}_4 K e^{-B} \right)^n$$

Dominated by $n_* \approx \mathcal{V}_4 K e^{-B} \approx \mathcal{V}_4 \Gamma$

Flat space: $\mathcal{V}_4 = \Omega T \to \infty$ ⇨ finite bounce density

de Sitter: $\mathcal{V}_4 = \frac{8\pi^2}{3} H^{-4}$ ⇨ $n_* \approx \frac{\Gamma}{H^4} \sim \left(\frac{M_{\rm Pl}}{\mu} \right)^4 e^{-B}$

$\frac{\Gamma}{H^4} \gg 1$ ⇨ no oscillating negative modes

$\frac{\Gamma}{H^4} \ll 1$ ⇨ oscillating mode issues

$\frac{\Gamma}{H^4} \gg 1$ ⇨ phase transition completes normally

Guth, EW

$\frac{\Gamma}{H^4} \ll 1$ ⇨ no percolation -- eternal inflation

Flat space limit: $\mu \ll M_{\rm Pl}$, $R \ll H^{-1}$ ⇨ Bounce $\approx$ flat space bounce

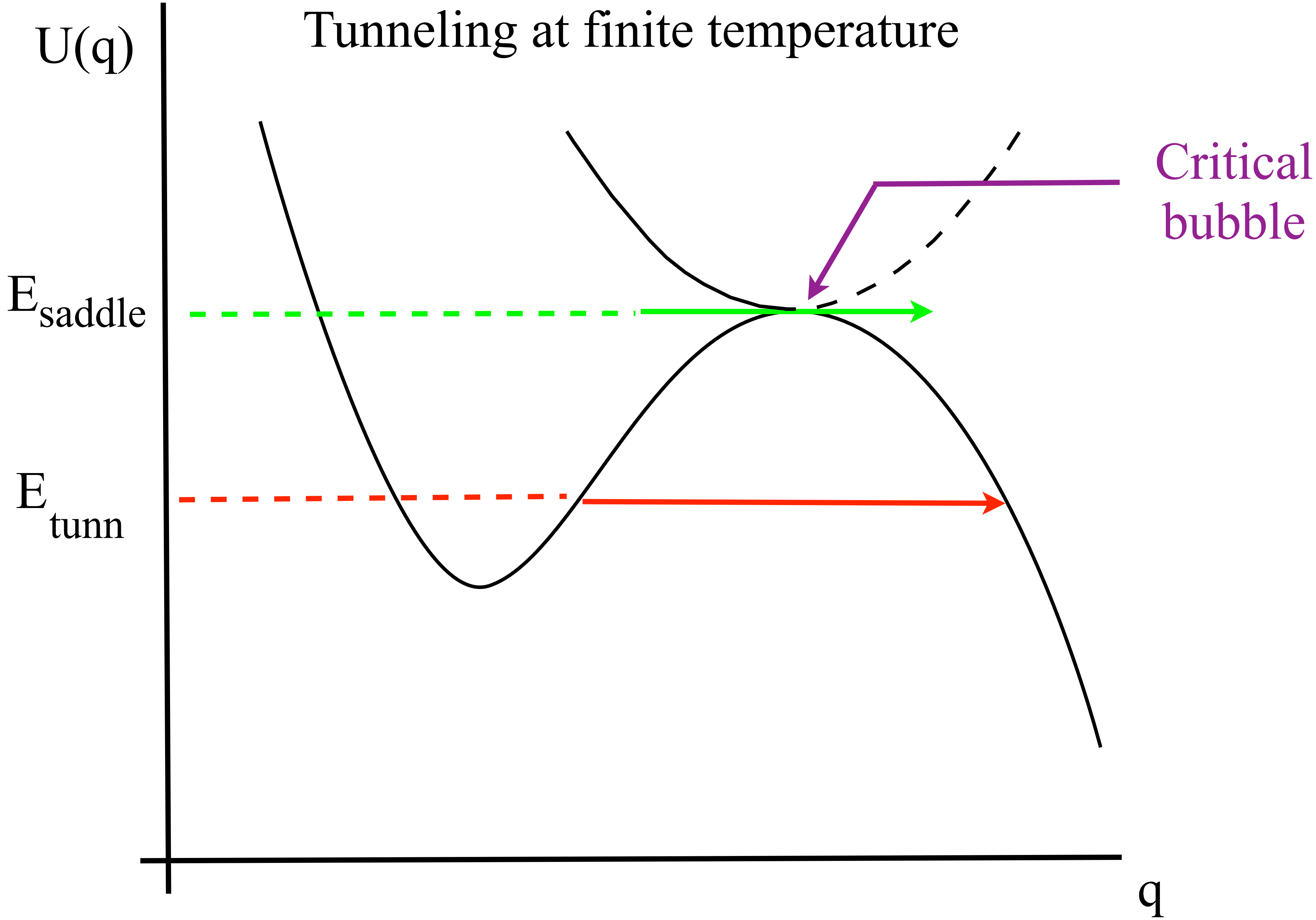
$\text{Time scale} \ll H^{-1}$ Transition $\approx$ flat space transition

Open Questions

Meaning of type B bounces?
Tunneling of entire horizon volume?
Analogous to tunneling in symmetric double well?

Meaning of oscillating negative modes?
Connection between completed phase transition
and “normal” negative mode structure?

Tunneling at finite temperature



$$\Gamma_{\rm tunn} \sim e^{-[S_E({\rm bounce}) - S_E({\rm fv})]}$$

Periodic bounce

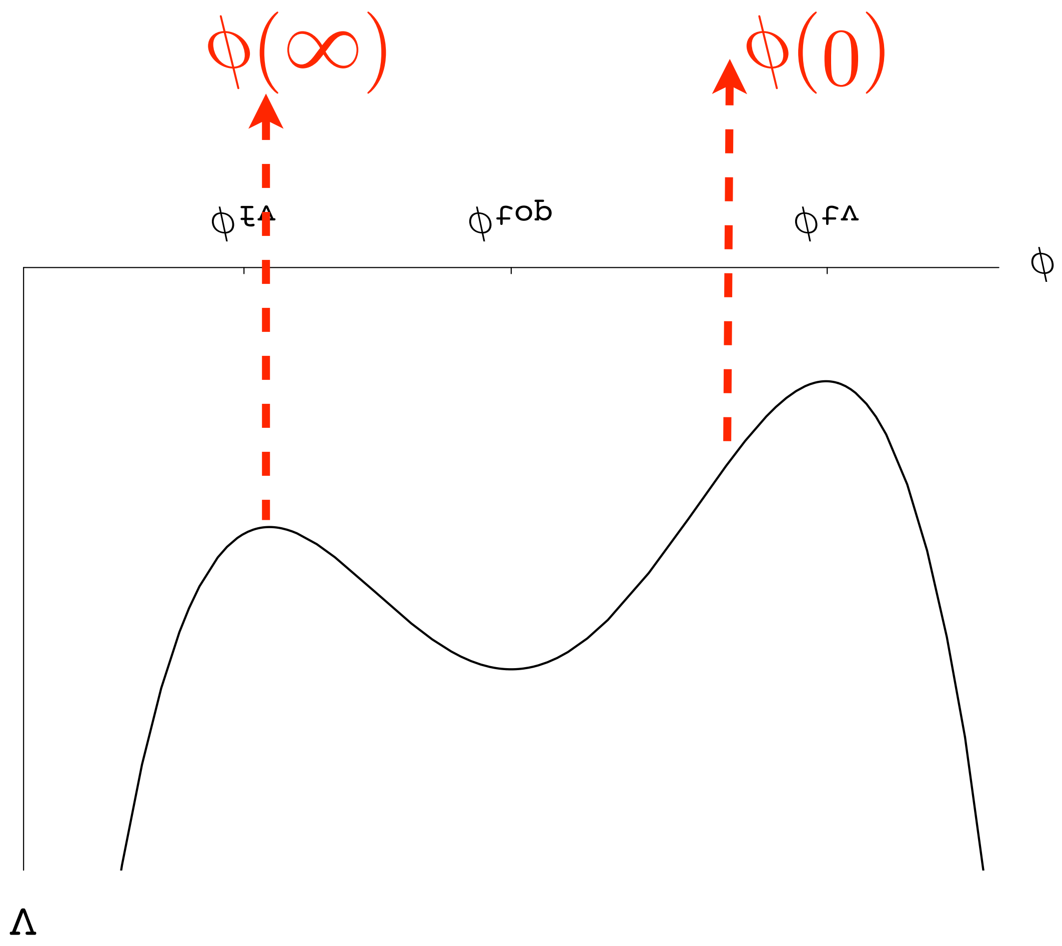
$$\Gamma_{\rm therm} \sim e^{-\beta(E_{\rm saddle} - E_{\rm fv})}$$

$$\sim e^{-[S({\rm saddle}) - S({\rm fv})]}$$

O(4)-symmetric bounces --- one field

$$\phi(\mathbf{x},\tau) = \phi(\sqrt{\mathbf{x}^2+\tau^2}) = \phi(s)$$

$$\frac{d^2\phi}{ds^2} + \frac{3}{s}\frac{d\phi}{ds} = \frac{dV}{d\phi} \qquad \dot\phi(0) = 0 \qquad \phi(\infty) = \phi_{\rm fv}$$



$$S = 4\pi^2 \int_0^\infty d\xi \, \left[\rho^3 U(\phi) - \frac{3}{\kappa}\rho\right]$$ Divergent

$$S = -2\pi^2 \int_0^\infty d\xi \, \rho^3 U(\phi) + \text{boundary terms}$$ Divergent

Minkowski false vacuum:

$$B = -2\pi^2 \int_0^\infty d\xi \, \rho^3 U(\phi)$$

Minkowski or AdS vacuum to AdS vacuum

$$\bar{\rho} = \frac{\bar{\rho}_0}{1 - (\bar{\rho}_0/2\ell)^2} \qquad B = \frac{B_0}{[1 - (\bar{\rho}_0/2\ell)^2]^2} \qquad \ell = (\kappa\epsilon/3)^{-1/2}$$

$$\bar{\rho} = \frac{\bar{\rho}_0}{(1 - \kappa/\kappa_{\rm cr})} \qquad B = \frac{B_0}{(1 - \kappa/\kappa_{\rm cr})^2} \qquad \kappa_{\rm cr} = \frac{4\epsilon}{3\sigma^2}$$

If $\bar{\rho}_0 \geq 2\ell$, $\epsilon \leq \frac{3}{4}\sigma^2\kappa$, no bounce → decay is quenched.