Non-Lorentzian M5-branes from Holography

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Challenges and Advances in Theoretical Physics

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Outline

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Introduction and Motivation

The M5-Branes remains everyone's favourite Mystery:

- Mother of all field theories?
- Dual to M-theory on $AdS_7 \times S^4$
- M5 on $\mathcal{M}_5 \times S^1 \sim$ 5D SYM on $\mathcal{M}_5$, $g_{YM}^2 = 4\pi^2 R_{S^1}$
- M5 compactified on $\mathbb{T}^2$ = 4D SYM

It epitomizes our ignorance of strongly coupled QFT.

We don't expect a Lagrangian but we'd like some kind of working definition without appealing to String/M-theory.

Maybe a family of Lagrangians can be 'patched' together

We might hope to learn some lessons from M2-Branes. For example weak coupling arises from an orbifold.

[Kim,Lee] wrote a beautiful paper where the M5 on $\mathbb{R}^{1,5}$ is conformally mapped to $\mathbb{R} \times S^5$ and S^5 is realised as an S^1 Hopf fibration over $\mathbb{CP}^2$

- 5D SYM on $\mathbb{R} \times \mathbb{CP}^2$ with a Chern-Simons term
- Level k induces a $\mathbb{Z}_k$ orbifold of S^1.
- Dimensional parameter g_{YM}^2 is the radius of $\mathbb{CP}^2$, k is the coupling.

Here we exploit AdS$_7$ as a Hopf fibration over $\tilde{\mathbb{CP}}^3$.

Talk to [Kim, Mukhi, Tomassiello] about unpublished work

Timelike Fibration of AdS_7

[Pope,Sadrzadeh,Scuro] showed that AdS_7 can be written as a Hopf fibration

$$ds^2_{AdS_7} = -\frac{1}{4}\left(dx^+ + e^{\phi}\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right)\right)^2 + d\tilde{s}^2_{\tilde{\mathbb{CP}}^3}$$

$$ds^2_{\tilde{CP}^3} = \frac{R_+^2}{4}d\phi^2 + \frac{1}{4}e^{\phi}dx^i dx^i + \frac{e^{2\phi}}{4}\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right)^2$$

where $x^+ \cong x^+ + 4\pi R_+$ and Ω_{ij} is an anti-self-dual 2-form:

$$(\Omega^2)_{ij} = -R_+^2\delta_{ij}$$

We want to place M5-branes at constant ϕ:

$$ds^2_\phi = \frac{e^\phi}{4}\left[-e^{-\phi}dx^+dx^+ - 2dx^+\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right) + dx^i dx^i\right]$$

For finite ϕ, x^+ is timelike and isometric so we can reduce on it to obtain a deformed euclidean 5D SYM *c.f.* [Hull,NL].

Note that ϕ is related to the more usual AdS radius ρ by

$$e^{\phi/2} = \frac{e^\rho}{\cos(x^+/2R_+)}$$

but at fixed ρ

$$ds^2_\rho = \frac{e^{2\rho}}{4\cos^2(x^+/2R_+)}\Big[-\cos(x^+/R_+)e^{-2\rho}dx^+dx^+ \\ -2dx^+\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right) + dx^i dx^i\Big]$$

we now longer have an isometry along x^+.

Ultimately we want to go the boundary (UV):

$$ds^2_\infty = -2dx^+\left(dx^- - \frac{1}{2}\Omega_{ij}x^i dx^j\right) + dx^i dx^i$$

Looks like a null Omega-deformation of 6D Minkowski space.

In fact this metric is conformal to 6D Minkowski space:

$$ds^2_{Minkowski} = \frac{1}{\cos^2(x^+/2R_+)} ds^2_\infty$$

but with $x^+ \in (-\pi R_+, \pi R_+)$

Recall that for AdS_7 we had $x^+ \sim x^+ + 4\pi R_+$. So we can extend to periodic physics in $x^+ \in [-2\pi R_+, 2\pi R_+]$ by imposing reflecting boundary conditions.

M5-branes in AdS$_7$

We follow the prescription of [Linander,Ohlsson]: reduce the abelian equations on x^+, find a suitable action and generalise it to be non-abelian and supersymmetric.

For example the condition $dH = 0$ along with $\partial_+ = 0$ implies that in 5D

$$dF = 0$$

where $F_{\mu\nu} = H_{\mu\nu+}$. This means that we can introduce a one-form A_μ for $F_{\mu\nu}$, $\mu, \nu \neq x^+$.

$$S_\phi \;=\; \frac{1}{g_{YM}^2}\mathrm{tr}\int d^4x\,dx^-\Big\{ -\frac{1}{2}\nabla_i X^I\nabla_i X^I - \frac{1}{2}e^{-\phi}D_-X^I D_-X^I$$

$$-\frac{1}{4}e^{-\phi}\left[X^I,X^J\right]^2 + \frac{1}{2}F_{i-}F_{i-} + \frac{1}{4}e^{\phi}\left(\mathcal{F}_{ij}^2 + \epsilon_{ijkl}\Omega_{mi}x^m F_{jk}F_{-l}\right)$$

$$-\frac{i}{2}\bar{\Psi}\Gamma_+D_-\Psi + \frac{i}{2}\bar{\Psi}\Gamma_i\nabla_i\Psi + \frac{i}{4}e^{-\phi}\bar{\Psi}\Gamma_-D_-\Psi$$

$$-\frac{1}{2}\bar{\Psi}\Gamma_+\Gamma^I\left[X^I,\Psi\right] + \frac{1}{4}e^{-\phi}\bar{\Psi}\Gamma_-\Gamma^I\left[X^I,\Psi\right]\Big\}$$

where

$$\nabla_i \;=\; D_i - \frac{1}{2}\Omega_{ij}x^j D_-$$

$$\mathcal{F}_{ij} = F_{ij} - \Omega_{k[i}x^k F_{j]-}$$

This is an Omega-deformed version of euclidean 5D SYM and has 8 supersymmetries.

$$\begin{aligned}
\delta X^I &= i\bar{\epsilon}_-^{(0)}\Gamma^I\Psi\,, \\
\delta A_i &= i\bar{\epsilon}_-^{(0)}\Gamma_i\Gamma_+\Psi + \frac{i}{2}\Omega_{ij}x^j\bar{\epsilon}_-^{(0)}\Gamma_{-+}\Psi\,, \\
\delta A_- &= i\bar{\epsilon}_-^{(0)}\Gamma_{-+}\Psi\,, \\
\delta\Psi &= -\Gamma_+\Gamma^I D_- X^I\epsilon_-^{(0)} + \Gamma_i\Gamma^I\nabla_i X^I\epsilon_-^{(0)} \\
&\quad -\Gamma_i\Gamma_{+-}F_{-i}\epsilon_-^{(0)} - \frac{i}{2}\Gamma_+\Gamma^{IJ}[X^I,X^J]\epsilon_-^{(0)} - \frac{e^\phi}{2}\Gamma_{ij}\Gamma_+\mathcal{F}_{ij}\epsilon_-^{(0)}
\end{aligned}$$

where $\Gamma_{05}\epsilon_-^{(0)} = -\epsilon_-^{(0)}$ which is indeed a constant Killing spinor of the metric ds^2_ϕ.

The Boundary Theory

Next we want to construct the action for M5-branes reduced on x^+ in the limit $\phi \to \infty$

There is an almost scaling symmetry:

$$X^I \to \lambda^{-2} X^I , \qquad A_- \to \lambda^{-2} A_- , \qquad A_i \to \lambda^{-1} A_i ,$$

$$x^- \to \lambda^2 x^- , \qquad x^i \to \lambda x^i \qquad \Psi_+ \to \lambda^{-3} \Psi_+ , \qquad \Psi_- \to \lambda^{-2} \Psi_-$$

but we must shift $\phi \to \phi - 2 \ln \lambda$. We want to construct the fixed point theory

In this limit there are divergent terms in S_ϕ:

$$e^{\phi}\left(\mathcal{F}_{ij}^{2}+\epsilon_{ijkl}\Omega_{mi}x^{m}F_{jk}F_{-l}\right)=e^{\phi}\left(\mathcal{F}_{ij}^{-}\mathcal{F}_{ij}^{-}+\epsilon_{ijkl}F_{ij}F_{kl}\right)$$

The first term is positive definite and the second a total derivative

How to take the limit of such scalings were considered in general by [NL, Mouland] (and a related construction in supergravity by [Bershoeff,Rosseel, Zojer]):

We subtract $e^{\phi}\mathcal{F}^{-2}_{\ ij}$ and impose a Lagrange multiplier that sets it to zero:

$$e^{\phi}\mathcal{F}^{-}{}_{ij}\mathcal{F}^{-}{}_{ij}\to\mathcal{F}^{-}{}_{ij}G_{ij}$$

We then discard the divergent terms in the action and supersymmetry, take $\phi\to\infty$, and modify the supersymmetry.

Alternatively in our case we could also follow the route of [Linander,Ohlsson] but for null reductions of the M5-brane.

Recall that $F_{ij} = H_{ij+}$ but here the Lagrange multiplier arises from

$$G_{ij} = H_{ij-}$$

but no Bianchi identity for $G = \star G$.

Either way we arrive at

$$\begin{aligned} S \;=\; \frac{1}{g_{YM}^2}\mathrm{tr}\int d^4x\,dx^-\Big\{&\frac{1}{2}F_{-i}F_{-i} + \frac{1}{2}\mathcal{F}_{ij}G_{ij} - \frac{1}{2}\nabla_i X^I \nabla_i X^I \\ &- \frac{i}{2}\bar{\Psi}\Gamma_+ D_-\Psi + \frac{i}{2}\bar{\Psi}\Gamma_i\nabla_i\Psi - \frac{1}{2}\bar{\Psi}\Gamma_+\Gamma^I[X^I,\Psi]\Big\} \end{aligned}$$

This is an Omega-deformation of the maximally supersymmetric non-Lorentzian theory of [NL, Owen] (obtained at $\Omega_{ij} = 0$).

The on-shell condition imposed by G_{ij} sets $\mathcal{F}_{ij} = -(\star\mathcal{F})_{ij}$.

For $\partial_- = A_- = 0$ (or $\Omega_{ij} = 0$) this is just $F_{ij} = -(\star F)_{ij}$

Dynamics is restricted to motion on instanton moduli space

Generalizes the DLCQ description of M5-branes [Aharony, Berkooz, Kachru, Seiberg, Silverstein]

The general procedure of [NL, Mouland] preserves the original 8 supersymmetries.

But the boundary action has the Liftshitz scale symmetry

$$
\begin{aligned}
&x^- \ \to \ \lambda^2 x^- \,, \qquad && x^i \to \ \lambda x^i \,, \qquad && X^I \ \to \ \lambda^{-2} X^I \\
&A_- \ \to \ \lambda^{-2} A_- \,, \qquad && A_i \ \to \ \lambda^{-1} A_i \,, \qquad && G_{ij} \ \to \ \lambda^{-4} G_{ij} \\
&\Psi_+ \ \to \ \lambda^{-3} \Psi_+ \,, \qquad && \Psi_- \ \to \ \lambda^{-2} \Psi_- &&
\end{aligned}
$$

and an additional 16 superconformal supersymmetries:

$$
\begin{aligned}
\delta X^I &= i\bar{\epsilon}\Gamma^I\Psi\,,\\
\delta A_i &= i\bar{\epsilon}\Gamma_i\Gamma_+\Psi + \frac{i}{2}\Omega_{ij}x^j\bar{\epsilon}\Gamma_{-+}\Psi\,,\\
\delta A_- &= i\bar{\epsilon}\Gamma_{-+}\Psi\,,\\
\delta G_{ij} &= \frac{i}{2}\bar{\epsilon}\Gamma_k\Gamma_{ij}\Gamma_-\nabla_k\Psi - \frac{i}{2}\bar{\epsilon}\Gamma_+\Gamma_-\Gamma_{ij}D_-\Psi + \frac{1}{2}\bar{\epsilon}\Gamma_+\Gamma_-\Gamma_{ij}\Gamma^I[X^I,\Psi]\\
&\quad - 3i\bar{\eta}\Gamma_-\Gamma_{ij}\Psi\,,\\
\delta\Psi &= -\frac{1}{4}\Gamma_{ij}\Gamma_-\mathcal{F}_{ij}\epsilon - \Gamma_+\Gamma^I D_-X^I\epsilon + \Gamma_i\Gamma^I\nabla_iX^I\epsilon - \Gamma_i\Gamma_{+-}F_{-i}\epsilon\,,\\
&\quad -\frac{1}{4}\Gamma_{ij}\Gamma_+G_{ij}\epsilon - \frac{i}{2}\Gamma_+\Gamma^{IJ}[X^I,X^J]\epsilon - 4X^I\Gamma^I\eta
\end{aligned}
$$

where $D_\mu\epsilon = \Gamma_\mu\eta$ are the x^+-independent conformal Killing spinors of the boundary metric.

$$\text{type I}: \quad \epsilon_+ = e^{\frac{x^+}{4}\Omega\cdot\Gamma}\epsilon_+^{(0)} \qquad \epsilon_- = 0$$

$$\eta_+ = 0 \qquad \eta_- = -\frac{1}{16}e^{\frac{x^+}{4}}\Omega\cdot\Gamma(\Omega\cdot\Gamma)\Gamma_-\epsilon_+^{(0)}$$

$$\text{type II}: \quad \epsilon_+ = 0 \qquad \epsilon_- = \epsilon_-^{(0)}$$

$$\eta_+ = 0 \qquad \eta_- = 0$$

$$\text{type III}: \quad \epsilon_+ = \epsilon_+^{(0)} \qquad \epsilon_- = \frac{1}{2}x^i\Omega_{ij}\Gamma_j\Gamma_-\epsilon_+^{(0)}$$

$$\eta_+ = 0 \qquad \eta_- = \frac{1}{16}(\Omega\cdot\Gamma)\Gamma_-\epsilon_+^{(0)}$$

$$\text{type IV}: \quad \epsilon_+ = -\frac{1}{2}x^i\Gamma_i\Gamma_+\epsilon_-^{(0)} \qquad \epsilon_- = -\frac{1}{4}\Omega_{ik}\Gamma_{kj}x^ix^j\epsilon_-^{(0)} + x^-\epsilon_-^{(0)}$$

$$\eta_+ = -\frac{1}{2}\Gamma_+\epsilon_-^{(0)} \qquad \eta_- = -\frac{1}{16}(\Omega\cdot\Gamma)x^i\Gamma_i\epsilon_-^{(0)}$$

Conclusions/Comments

We have constructed non-Abelian theories in 5 dimensions for M5-branes embedded in AdS_7, reduced on a compact time direction.

Going to the boundary induces a classical RG flow leading to a novel boundary theory

- Lifshitz scale invariance, 8 supersymmetries and 16 superconformal supersymmetries
- Dynamics localizes on Omega-deformed quantum mechanics of instanton moduli space

There is no Chern-Simons term and so no natural integer k that leads to a $\mathbb{Z}_k$ orbifold

- reduction to type IIA is always strongly coupled
- can introduce an $\mathbb{Z}_k$ action by hand to make the string theory weakly coupled but this doesn't seem natural.

The Bosonic symmetries are also interesting: $SU(1,3)$ symmetry of $\tilde{\mathbb{CP}}^3$.

New class of field theories where the Manton approximation is exact on-shell.

The M5-brane continues to surprise with the richness of lagrangian field theories it begets.

Thank You

Thank You

and

Happy Birthday Kimyeong!