Signature of quantum information scrambling in operator entanglement in (1+1)d CFTs

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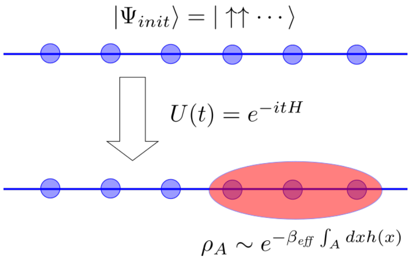
Introduction

- Today's topic: dynamics of many-body systems
- Far out of equilibrium properties of many-body systems; quantum quench, etc.
- Motivations: cold atom systems, quantum transport in strongly correlated systems, black hole physics, ...
- One universal behavior: *thermalization* even in isolated quantum systems



Introduction

- Memory of initial states is lost, or inaccessible locally. *Scrambling*: (fast) delocalization of quantum information. [Sekino-Susskind, Lashkari Stanford et al.]

- Can happen for sufficiently complex dynamics

- Out-of-time-order correlator (OTOC) [Kitaev, Stanford-Shenker, ...]

$$C(t) = \langle [W(t), V(0)]^2 \rangle_\beta \sim e^{\lambda t}$$

- Studied, for example, in the Sachdev-Ye-Kitaev models.

- Other probes: Spectral form factor, relative entropy, distance between states, etc.

- In this talk, I will be using *operator entanglement* defined for unitary time evolution operator of quantum systems

Operator entanglement

- Operators acting on $\mathcal{H}$ can be mapped to states in the doubled Hilbert space $\mathcal{H} \otimes \mathcal{H}$:

$$O = \sum_a O_{ab} |a\rangle\langle b| \longrightarrow |O\rangle = \sum_a O_{ab} |a\rangle |b\rangle$$

- Will call the two Hilbert spaces *input* and *output* Hilbert spaces.

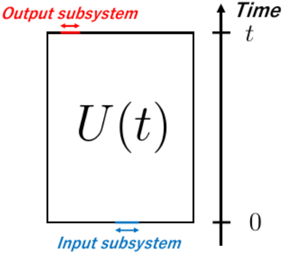
- For the unitary-time evolution operator in field theories or many-body systems

$$U_\epsilon(t) = e^{H(-\epsilon - it)} = \sum_a e^{E_a(-\epsilon - it)} |a\rangle\langle a|$$
$$\longrightarrow \quad |U_\epsilon(t)\rangle = \sum_a e^{E_a(-\epsilon - it)} |a\rangle |a\rangle \times \mathcal{N}$$

ϵ is the regulator.

Operator entanglement

- Once an operator is mapped to a state, we can discuss entanglement, in particular, entanglement between *input* and *output*.



- How efficiently $U(t)$ can be represented by Matrix product operator (MPO)

- Useful to diagnose scrambling and chaos.
  [Hosur-Qi-Roberts-Yoshida (16);Jonay-Huse-Nahum (18), ...]

The n-th Rényi entropy:

$$S_A^{(n)} = \frac{1}{1-n} \log \left[\mathrm{Tr}_A (\rho_A^n) \right]$$

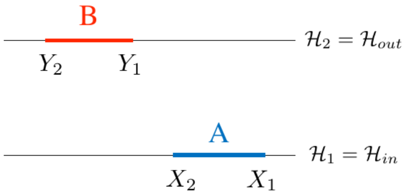
where $\rho_A = \mathrm{Tr}_{\bar{A}} |U_\epsilon(t)\rangle\langle U_\epsilon(t)|$ is the reduced density matrix for subsystem A. The limit $n \to 1$ gives the von-Neumann entropy:

$$S_A^{(n\to 1)} = S_A = -\mathrm{Tr}_A \left[\rho_A \log \rho_A \right]$$

The n-th *bi-partite operator mutual information* (BOMI)

$$I^{(n)}(A,B) = S_A^{(n)} + S_B^{(n)} - S_{A\cup B}^{(n)},$$

for two sub Hilbert spaces $\mathcal{H}_A, \mathcal{H}_B \subset \mathcal{H}_{tot}$,

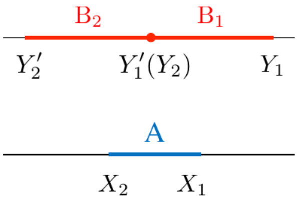


(1) BOMI

The n-th *tri-partite operator mutual information* (TOMI)

$$I^{(n)}(A, B_1, B_2) = I^{(n)}(A, B_1) + I^{(n)}(A, B_2) - I^{(n)}(A, B_1 \cup B_2),$$

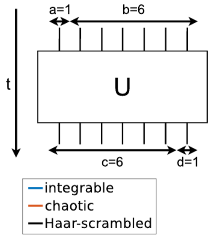
for three sub Hilbert spaces $\mathcal{H}_A, \mathcal{H}_{B_1}, \mathcal{H}_{B_2} \subset \mathcal{H}_{tot}$.

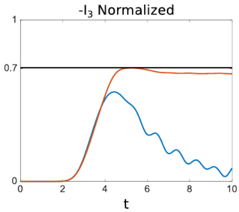


(2) TOMI

Operator entanglement and scrambling

- Negativity of tripartite information indicates scrambling [Hosur-Qi-Roberts-Yoshida]



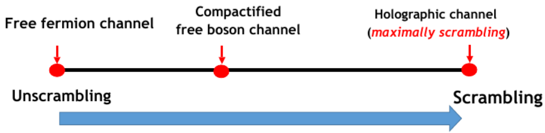


- Chaos and scrambling in CFT? Can we use operator entanglement?

Systems of interest

- We will consider operator mutual information of the time-evolution operator of 2d CFTs.

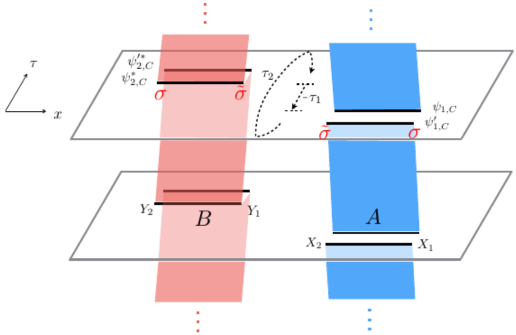
  - Free fermion CFT with $c = 1$
  - Compactified free boson CFT with radius R with $c = 2$ (Tomonaga-Luttinger liquid)
    Realizing both rational and irrational CFT
  - Holographic CFTs with large c



Methods

- Replica method $\to$ Twist operator formalism [Calabrese-Cardy]

$$\mathrm{Tr}\,\rho^n \sim \langle \sigma_n(X_1)\tilde{\sigma}_n(X_2)\tilde{\sigma}_n(Y_1,\tau_1)\sigma_n(Y_2,\tau_1)\rangle$$

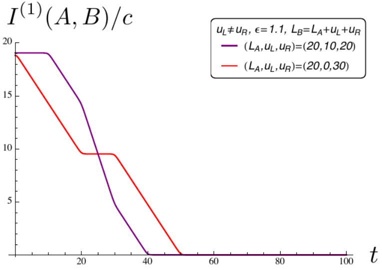


- τ_1 is analytically continued to the real time $\tau_1 \to it$.
- Depends on four point correlation function; depends on the operator content of the theory.

Results

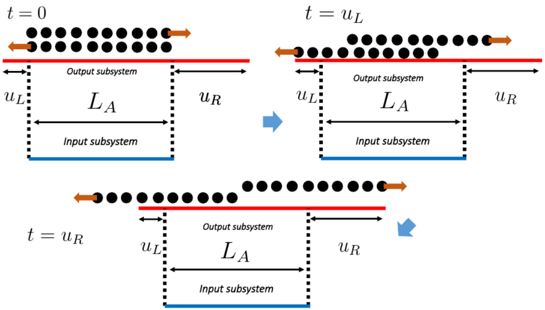
- Bipartite operator mutual information
  Free fermion, compactified free boson, holographic CFTs
- Tripartite operator mutual information
  Free fermion, compactified free boson, holographic CFTs

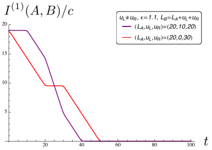
Bipartite, free fermion CFT



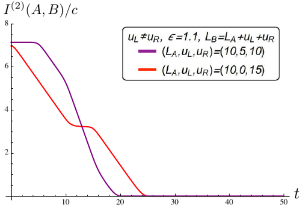
Bipartite, free fermion CFT

Can be explained by the quasi-particle picture [C.f. Calabrese-Cardy]:



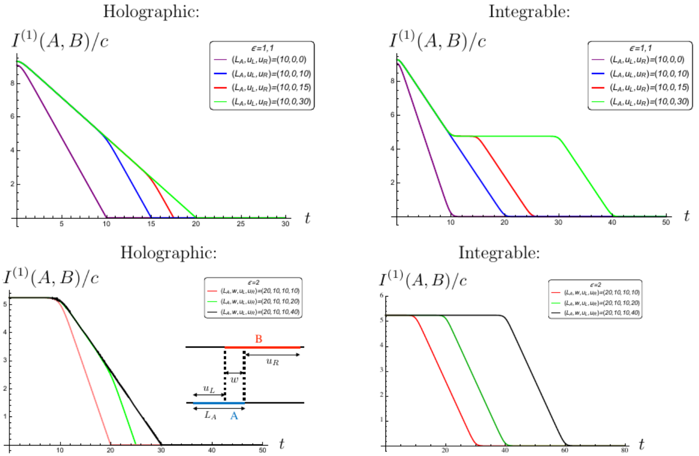


Bipartite, compactified free boson CFT



- Very similar to the free fermion case.
- Corners are slightly rounded.
- Tri-partite case is more interesting.

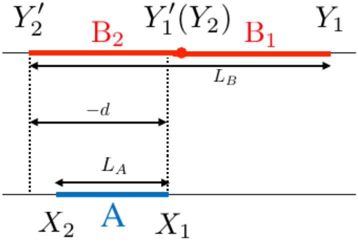
Bipartite, holographic

- 

- No plateau; strong violation of quasi-particle picture

Tri-partite

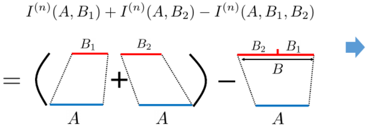
- Setting:

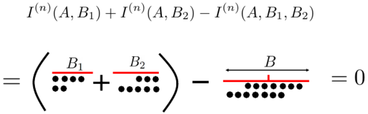


We consider the case where $B_{1,2}$ are both semi-infinite.

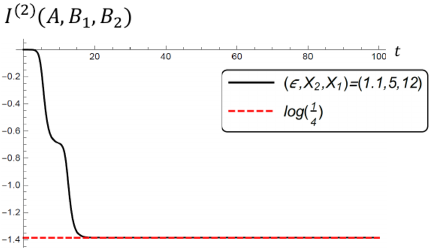
Tri-partite, free fermion CFT

- $I(A, B_1, B_2)$ is identically zero.





Tri-partite, compactified free boson



- Parameterize the radius as:

$$R = \sqrt{\eta}, \quad \eta = \frac{p}{p'}, \quad p, p' : \text{relatively coprime integers}$$

$\eta = 1$ is the self-dual radius.

- Rational case: $(\tilde{\eta} = max\{\eta, 1/\eta\})$

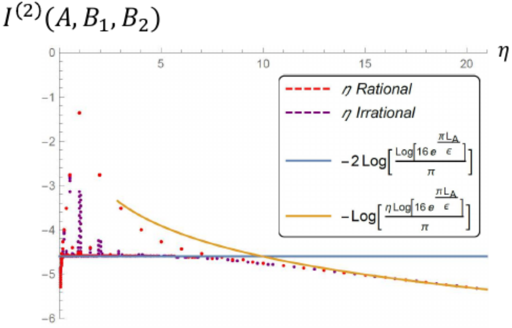
$$I^{(2)}(A, B_1, B_2) \xrightarrow[L_A \gg \epsilon]{t \to \infty} \begin{cases} -2 \log d_{\sigma_2} & 2pp', \tilde{\eta} \ll \frac{L_A}{\epsilon} \\ -2 \log \frac{L_A}{\epsilon} & \tilde{\eta} \ll \frac{L_A}{\epsilon} \ll 2pp' \\ -\log \left(\tilde{\eta} \frac{L_A}{\epsilon} \right) & \frac{L_A}{\epsilon} \ll 2pp', \tilde{\eta} \end{cases}$$

where d_{σ_2} is the quantum dimension of the twist operator.

- Irrational case

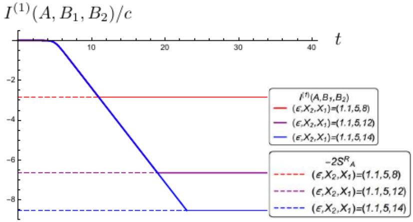
$$I^{(2)}(A, B_1, B_2) \xrightarrow[L_A \gg \epsilon]{t \to \infty} \begin{cases} -2 \log \frac{L_A}{\epsilon} & \tilde{\eta} \ll \frac{L_A}{\epsilon} \\ -\log \left(\tilde{\eta} \frac{L_A}{\epsilon} \right) & \tilde{\eta} \gg \frac{L_A}{\epsilon} \end{cases}$$

Tri-partite, compactified free boson



- Least negative at the self-dual radius (because of $SU(2)$ symmetry?)

Tri-partite, holographic CFTs

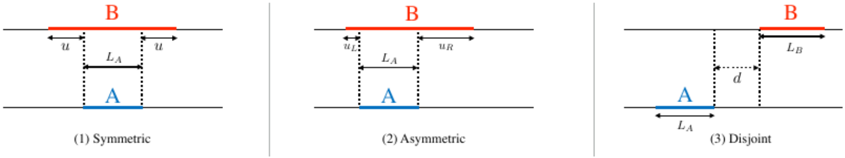


- Late-time behavior: $I^{(1)}(A, B_1, B_2) \to \frac{-\pi c}{3\epsilon} L_A =: -2S_A^R$
- S_A^R is regulated entanglement entropy:

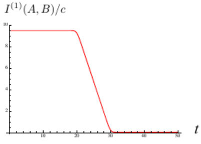
$$S_A^R = \lim_{n\to 1} \frac{1}{1-n} \ln \left[\frac{\langle \sigma_n(w_1, \bar{w}_1)\bar{\sigma}_n(w_2, \bar{w}_2)\rangle}{|dz/dw|_{w=0}^{4h_n}} \right]$$

- Saturating the bound $I(A, B_1, B_2) \geq -2Min[S_A, S_B]$

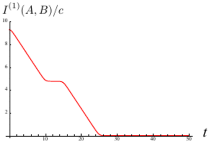
Summary: bipartite



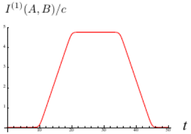
Summary: bipartite



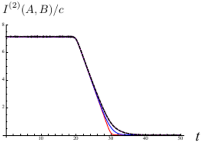
(1a) Free fermion



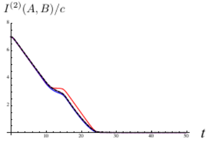
(2a) Free fermion



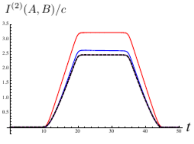
(3a) Free fermion



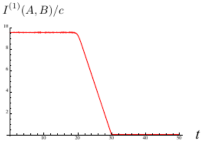
(1b) Compactified boson



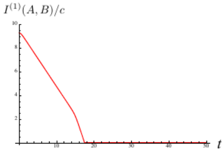
(2b) Compactified boson



(3b) Compactified boson



(1c) Holographic



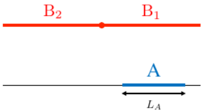
(2c) Holographic

$$I^{(1)}(A,B)/c \approx 0$$

(up to numerical error, in large c limit)

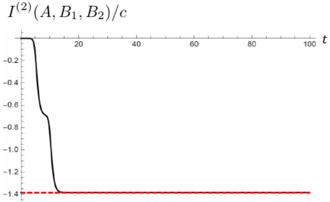
(3c) Holographic

Summary: tripartite

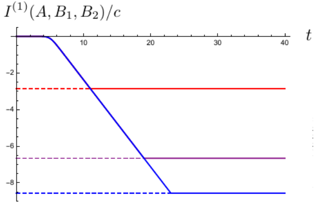


$$I^{(1)}(A, B_1, B_2) = 0$$

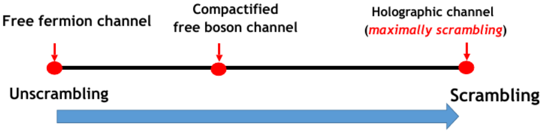
(1) Free fermion



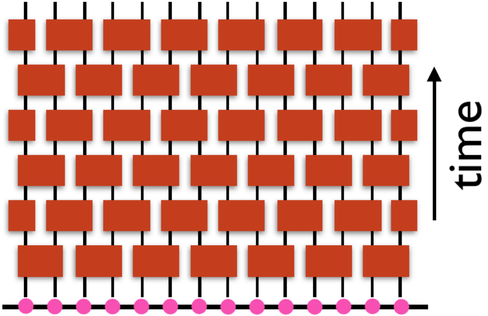
(2) Compactified boson



(3) Holographic



v.s. Random unitary circuit

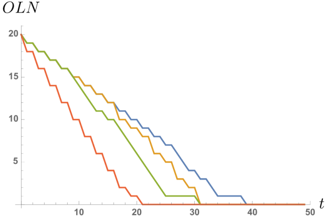
- 

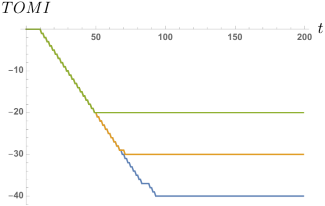
[Nahum-Ruhman-Vijay-Haah (16-18); Khemani-Vishwanath-Huse (18); Zhou-Nahum (18); Joney-Huse-Nahum (18), ...]

v.s. Random unitary circuit

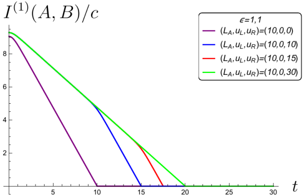
[Kudler-Flam, Tan, Nozazki, SR]

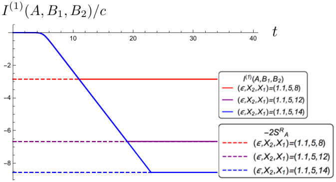
- Random unitary circuit:



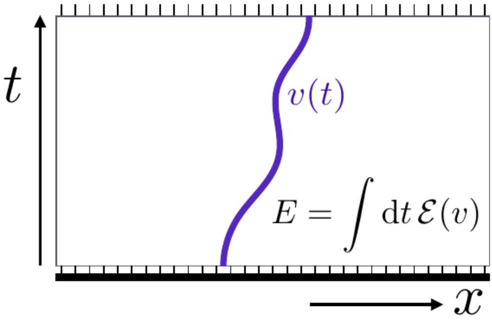


- Holographic CFTs:





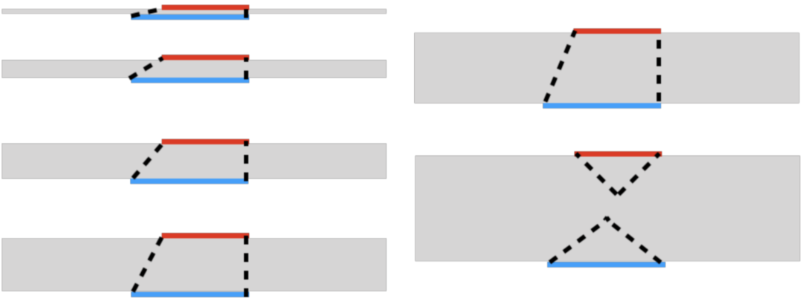
Line-tension picture



$$\mathcal{E}(v,x,t) = \begin{cases} \log q & v < 1 \\ v \log q & v > 1 \end{cases}$$

[Joney-Huse-Nahum (18),...]

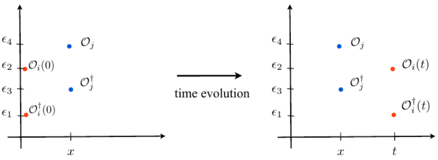
Line-tension picture

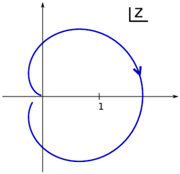


- Time v.s. radial coordinate in AdS
- Phase transition in minimal surface area
- Kink in the slope

v.s. OTOC

- [Hosur-Qi-Roberts-Yoshida] relates the averaged OTOC to the 2nd Renyi operator entanglement for qubit systems.
- For the compactified boson theory our analysis is similar to the calculations of the OTOC [Caputa-Kusuki-Takayanagi-Watanabe (17)].
- For 4-pt function relevant to OTOC, the cross ratio makes a round trip. Hence, for rational CFT, OTOC at late time is given by quantum dimension. [Roberts-Stanford(16), Gu-Qi(16), Caputa-Numasawa-Veliz-Osorio (16)]





For operator mutual information, the cross ratio does not make a round trip.

Summary

- The unitary time-evolution operators of CFTs with different quantum information scrambling capabilities show distinct late time behaviors in the operator entanglement.
- It would be interesting to link to hydrodynamic pictures of entanglement spreading, and operator hydrodynamics.
- Precise relation to other indicators of scrambling/chaos ?
- The precise definition of quantum chaos?