

# Functional Transcendence for Uniformizations of Finite-Volume Quotient Spaces of Bounded Symmetric Domains

*Mini-Course*

Ngaiming Mok  
The University of Hong Kong

**Abstract** Finite-volume quotients of bounded symmetric domains  $\Omega$ , which are naturally quasi-projective varieties, are objects of immense interest to Several Complex Variables, Algebraic Geometry, Arithmetic Geometry and Number Theory, and an important topic revolves around functional transcendence in relation to universal covering maps of such varieties (in analogy to the exponential map  $\exp : \mathbb{C} \rightarrow \mathbb{C}^*$ ). While a lot has already been achieved in the case of Shimura varieties (such as the moduli space  $\mathcal{A}_g$  of principally polarized Abelian varieties) by means of methods of Diophantine Geometry, Model Theory, Hodge Theory and Complex Differential Geometry, techniques for the general case of not necessarily arithmetic quotients  $\Omega/\Gamma =: X_\Gamma$  have just begun to be developed. For instance, uniformization problems for subvarieties of products of arbitrary compact Riemann surfaces of genus  $\geq 2$  have hitherto been intractable by existing methods. We will explain a differential-geometric approach leading to characterization results for totally geodesic subvarieties of  $X_\Gamma$  for the universal covering map  $\pi : \Omega \rightarrow X_\Gamma$ . Especially, we will explain how the study of holomorphic isometric embeddings of the Poincaré disk into  $\Omega$  can be further developed to derive uniformization theorems for bi-algebraic varieties, generalizing earlier results of Ullmo-Yafaev in 2011 in the case of arithmetic quotients. More generally, we will consider the Zariski closure of images of algebraic sets under the universal covering map  $\pi : \Omega \rightarrow X_\Gamma$ . In the arithmetic case, after partial results of Ullmo-Yafaev and Pila-Tsimerman it was proven by Klingler-Ullmo-Yafaev in 2016 that such Zariski closures are weakly special subsets of  $X_\Gamma$ , equivalently totally geodesic subsets of  $X_\Gamma$ , resolving in the affirmative the Hyperbolic Ax-Lindemann Conjecture (which is one of the two major components for the confirmation of the André-Oort Conjecture for Shimura varieties). I will explain how the arithmeticity condition can be dropped in the rank-1 case by a proof using foliation theory, Chow schemes and Kähler geometry.