Branes and the Swampland

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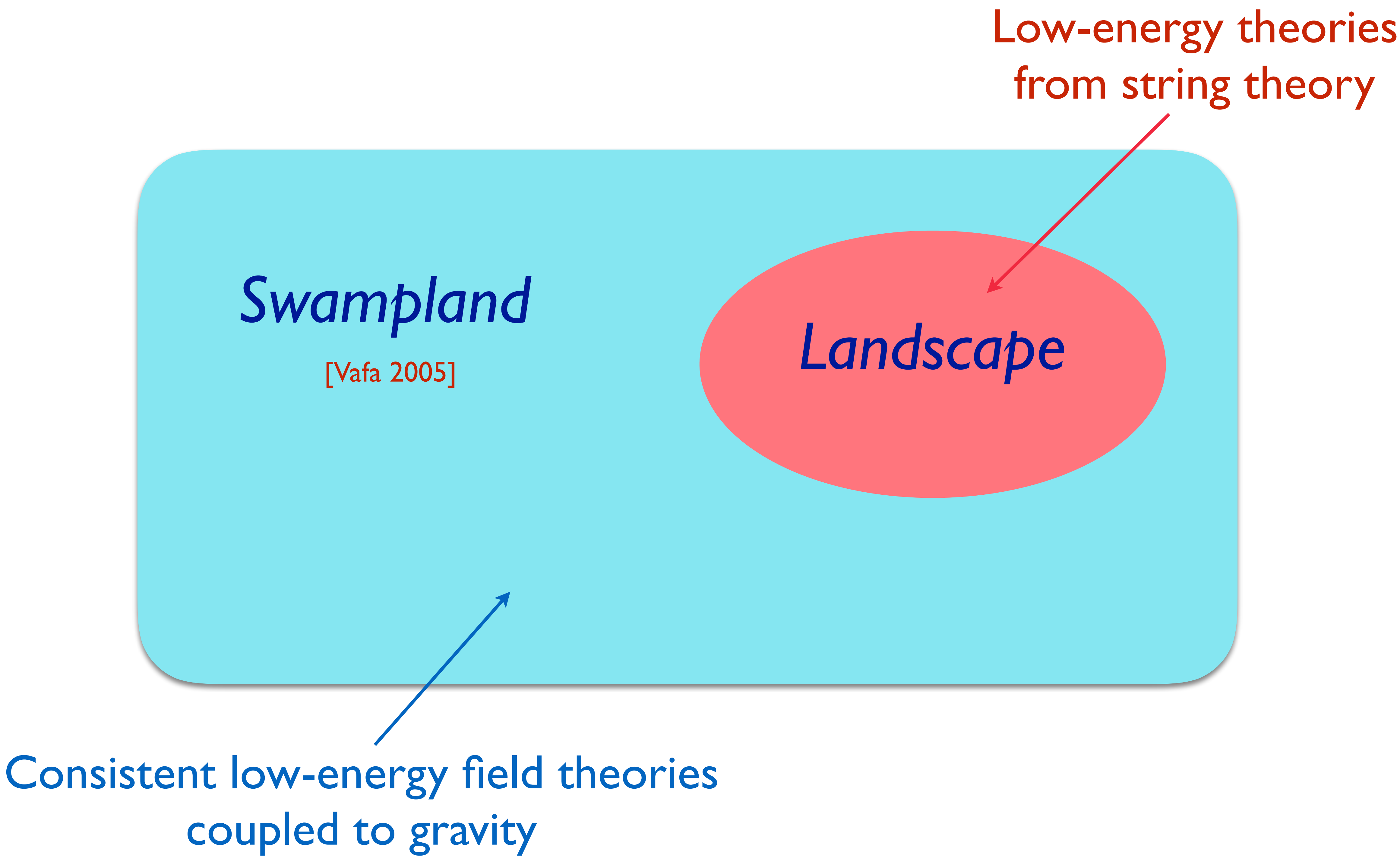
POSTECH

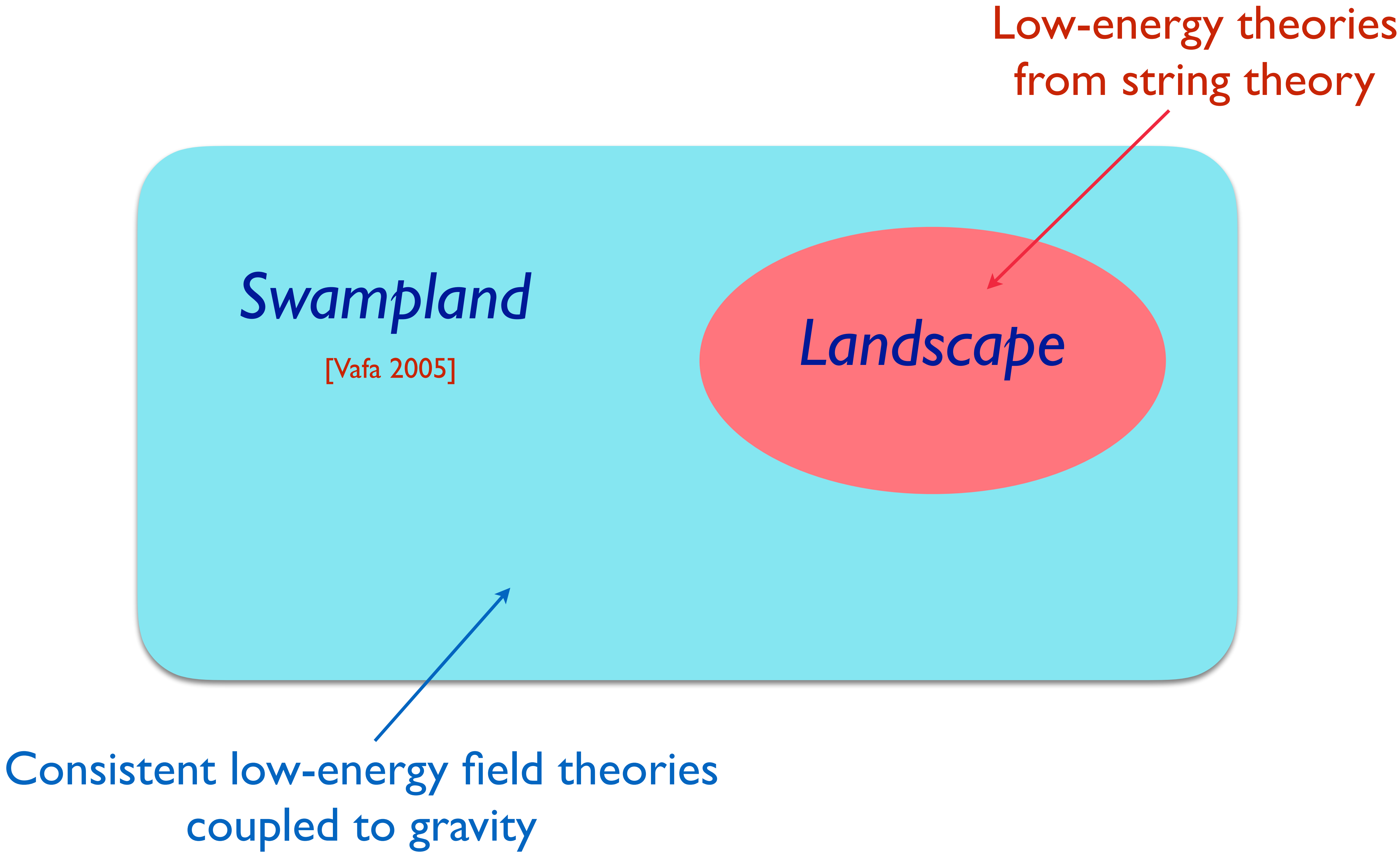
Based on

arXiv : 1905.08261 with Gary Shiu and Cumrun Vafa

String landscape and Swampland

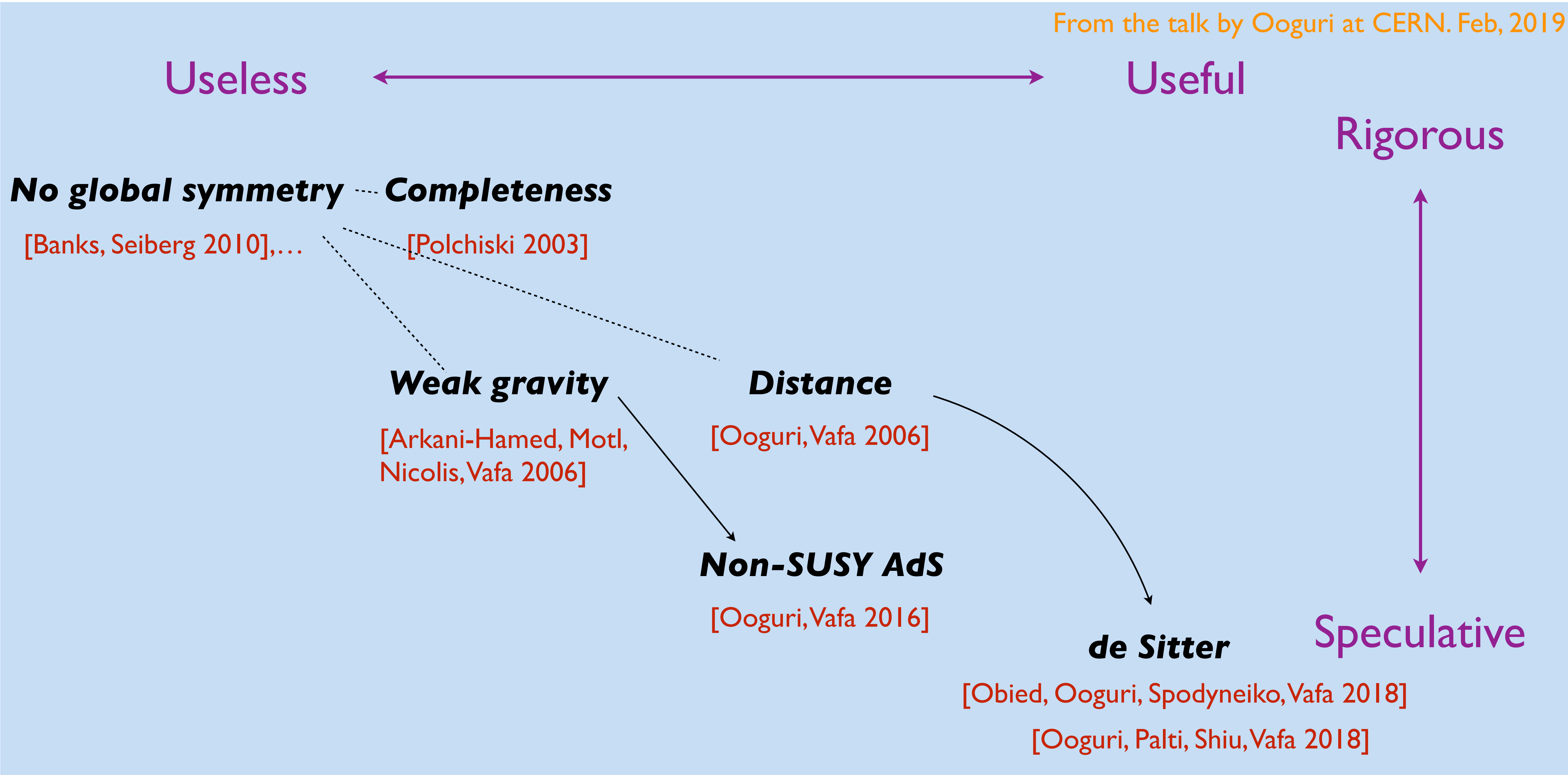


String landscape and Swampland

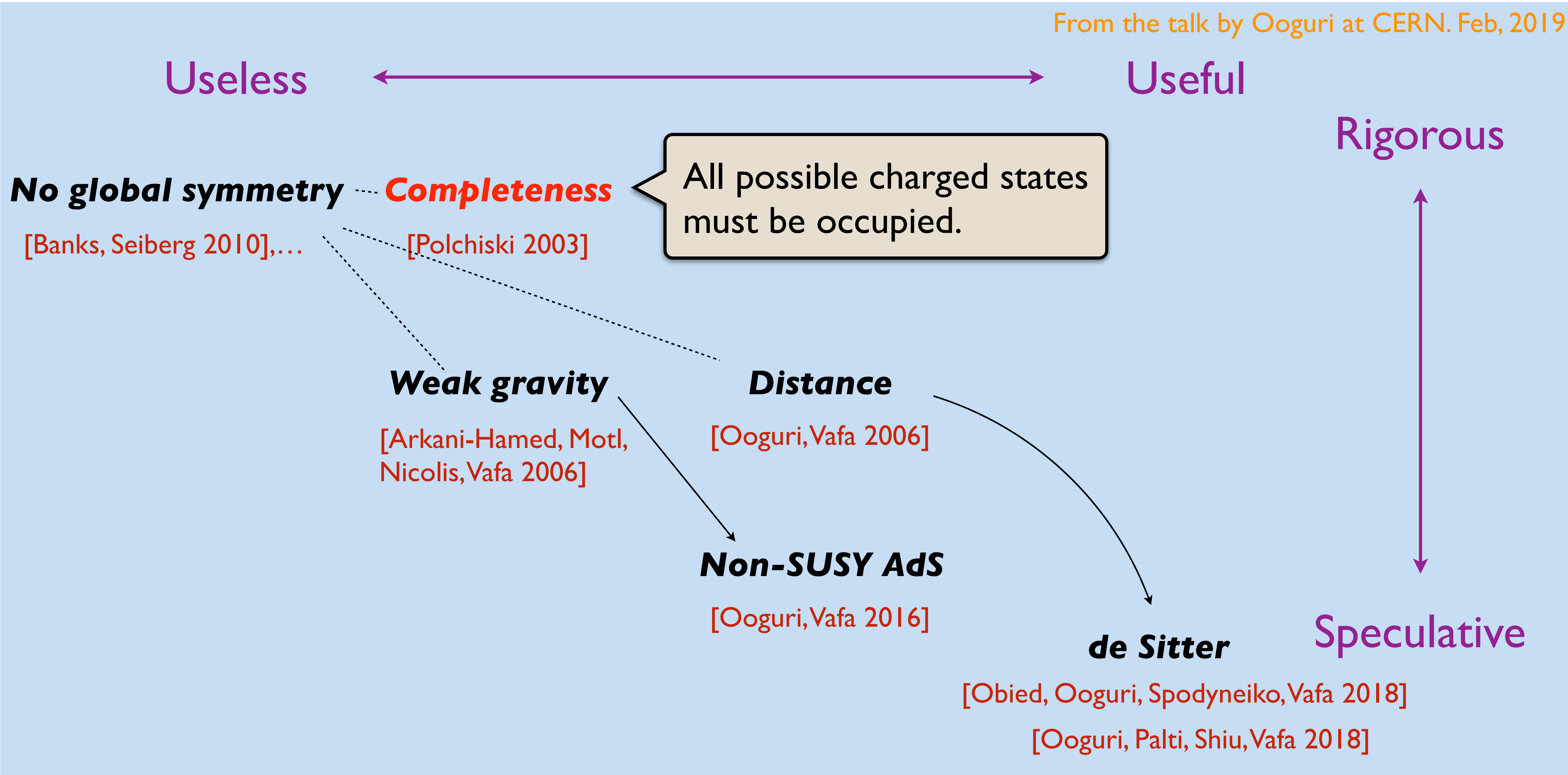


How can we distinguish between low-energy theories in the landscape from those in the Swampland?

Swampland conjectures



Swampland conjectures



In this talk, I will use completeness of charged strings/branes to discuss consistency of 10d & 6d low-energy field theories coupled to gravity.

10d (1,0) Supergravity

10d N=(1,0) Supergravity

- 1-loop gauge and gravitational anomalies can be cancelled by the Green-Schwarz mechanism which allows only 4-choices of gauge groups.

[Green, Schwarz 1984]

$$SO(32)\,,\ \ E_8\times E_8\,,\ \ E_8\times U(1)^{248}\,,\ \ U(1)^{496}$$

- First two, $SO(32),\ E_8\times E_8$, are realized as low energy limits of Type I and Heterotic string theories.

- Letter two including abelian gauge groups are conjectured to belong to the Swampland. [Vafa 2005]

- Indeed, cancellation of abelian anomalies in $E_8\times U(1)^{248},\ U(1)^{496}$ cannot be made compatible with SUSY and abelian gauge invariance!

[Adams, DeWolfe, Taylor 2010]

- We can also show by coupling string probes that two theories with abelian factors are inconsistent and thus are in the Swampland.

Strings in 10d (1,0) Supergravity

Consider 1/2 BPS strings in 10d (1,0) supergravity.

- Strings are sources for 2-form field B_2.
- Action of Q strings is

$$S^{str} = Q \int_{\mathcal{M}_{10}} B_2 \wedge \prod_{a=1}^{8} \delta(x^a) dx^a = Q \int_{\mathcal{M}_2} B_2$$

- Note that B_2 transforms under local gauge and Lorentz symmetry as

[Bergshoeff, de Wit, Nieuwenhuizen 1982]

$$\delta_{\Lambda_i,\Theta} B_2 = -\frac{1}{4} \sum_i \mathrm{Tr}\Lambda_i F_i + \mathrm{tr}\Theta R$$

- Gauge and gravitational anomalies in the presence of 2d strings :

$$\delta_{\Lambda_i,\Theta} S^{str} = Q \int_{\mathcal{M}_2} \left[-\frac{1}{4} \sum_i \mathrm{Tr}\Lambda_i F_i + \mathrm{tr}\Theta R \right]$$

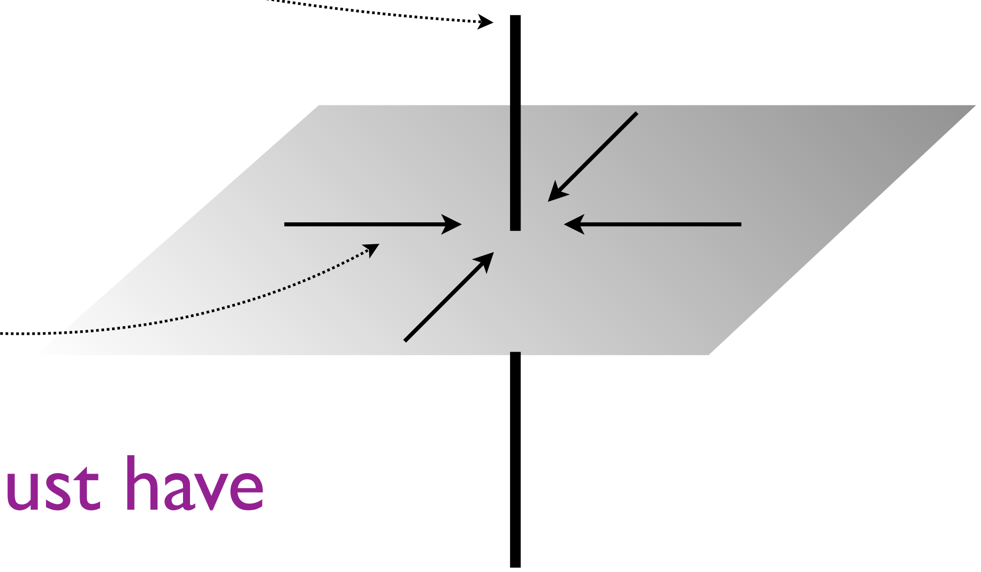
Anomaly inflow toward 2d strings

- Anomaly inflow toward 2d string worldsheet :

$$I_4^{\text{inflow}} = Q\left(-\frac{1}{4}\sum_i \mathrm{Tr}F_i^2 + \mathrm{tr}R^2\right)$$

- Anomaly inflow from bulk 10d theory must be cancelled by anomalies of the 2d CFT living on strings.

$$I_4^{\text{inflow}} + I_4^{WS} = 0$$



- Therefore, 2d world sheet CFT for Q-string must have

$$I_4^{WS} = Q\left(\underbrace{\frac{1}{2}p_1(T_2) - c_2(SO(8))}_{= -\mathrm{tr}R^2} + \frac{1}{4}\sum_i \mathrm{Tr}F_i^2\right)$$

[H-C.Kim, Shiu, Vafa 2019]

Anomalies in 2d worldsheet CFT

- Anomaly contributions from center-of-mass modes $(X_\mu, \lambda_+^I)\,,\ \mu, I = 1, \cdots, 8$

$$I_4^{\rm com} = -\frac{1}{6}p_1(T_2) - c_2(SO(8))$$

- Anomaly polynomial of interacting 2d worldsheet CFT (for Q=1) is

$$I_4 = I_4^{WS} - I_4^{com} = \frac{2}{3}p_1(T_2) + \frac{1}{4}\sum_i {\rm Tr}F_i^2$$

From this result, we can read off the left-moving and the right-moving central charges c_L, c_R and the level k_i's of gauge algebras in the 2d CFT.

$$I_4 = -\frac{c_R - c_L}{24}p_1(T_2) + \frac{c_R}{6}c_2(SO(8)) + \frac{1}{4}\sum_i k_i {\rm Tr}F_i^2$$

We therefore find :

$$c_L = 16\ , \quad c_R = 0\ , \quad k_i = 1$$

Unitary bounds and the Swampland

- Central charge contribution from level k Kac-Moody algebra of G is

$$c_G = \frac{k \cdot \mathrm{dim}G}{k + h^\vee} = \begin{cases} 8 & \text{for } E_8 \\ 16 & \text{for } SO(32) \\ 1 & \text{for } U(1) \end{cases}$$

- Current algebra for group G is on the left-moving sector and this means the left-moving central charge c_L of a unitary 2d CFT is bounded

$$\sum_i \frac{k_i \cdot \mathrm{dim}G_i}{k_i + h_i^\vee} \leq c_L$$

- The 2d worldsheet CFTs for $SO(32)$, $E_8 \times E_8$ saturate this bound.

- However, this unitary bound is violated for $E_8 \times U(1)^{248}$, $U(1)^{496}$.

$\cap$

Swampland

[H-C.Kim, Shiu, Vafa 2019]

6d (1,0) Supergravity

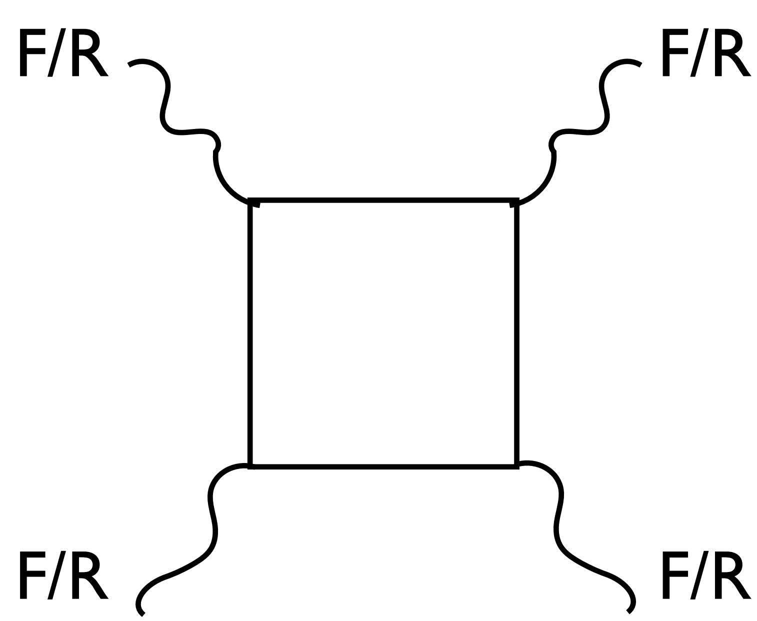
6d N=(1,0) Supergravity

- 6d N=(1,0) supermultiplets

Gravity	$(g_{\mu\nu}, B^+_{\mu\nu}, \psi^-_\mu)$
Tensor (T)	$(B^-_{\mu\nu}, \phi, \chi^+)$
Vector (V)	(A_μ, λ^-)
Hyper (H)	$(\varphi, \tilde{\varphi}, \psi^+)$

- Non-vanishing 1-loop gauge and gravitational anomalies from self-dual 2-form fields and chiral fermions.

1-loop box diagram



$\neq 0$

Green-Schwarz-Sagnotti mechanism

- The 1-loop anomalies can be cancelled by Green-Schwarz-Sagnotti mechanism if the 1-loop anomaly polynomial factorizes as

[Green, Schwarz 1984], [Sagnotti 1992]

$$I_8^{1-loop} = \frac{1}{2}\Omega_{\alpha\beta}X_4^\alpha X_4^\beta \ ,$$
$$X_4^\alpha = \frac{1}{2}a^\alpha \mathrm{tr}R^2 + \frac{1}{4}\sum_i b_i^\alpha \frac{2}{\lambda_i}\mathrm{tr}F_i^2$$

$\Omega_{\alpha\beta}$: symmetric bilinear form

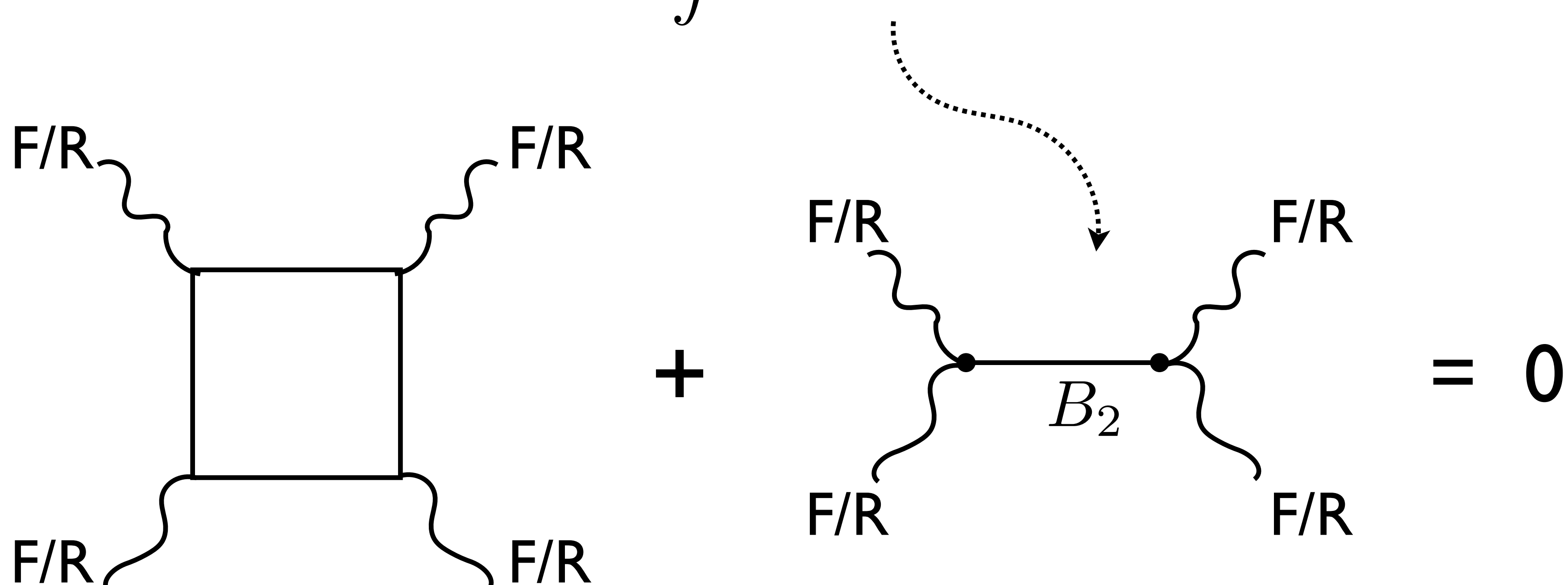
a^α, b_i^α : vectors in $\mathbb{R}^{1,T}$

$\alpha, \beta = 1, 2, \cdots, T+1$

$\lambda_{SU(N)} = 1, \lambda_{E_8} = 60, \cdots$

- This 1-loop anomaly can be cancelled by adding the Green-Schwarz term

$$S_{GS} = \int \Omega_{\alpha\beta}B_2^\alpha \wedge X_4^\beta$$



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$\alpha,\beta = 1,2,\cdots,T+1$

$\lambda_{SU(N)} = 1, \lambda_{E_8} = 60, \cdots$

- Factorization conditions :

$$H - V = 273 - 29T \ , \quad a\cdot a = 9 - T \ ,$$
$$0 = B^i_{\mathbf{adj}} - \sum_{\mathbf{R}} n^i_{\mathbf{R}} B^i_{\mathbf{R}} \ ,$$
$$a\cdot b_i = \frac{\lambda_i}{6}\left(A^i_{\mathbf{adj}} - \sum_{\mathbf{R}} n^i_{\mathbf{R}} A^i_{\mathbf{R}}\right) \ ,$$
$$b_i\cdot b_i = \frac{\lambda_i^2}{3}\left(\sum_{\mathbf{R}} n^i_{\mathbf{R}} C^i_R - C^i_{\mathbf{adj}}\right) \ ,$$
$$b_i\cdot b_j = 2\lambda_i\lambda_j \sum_{\mathbf{R},\mathbf{S}} n^{ij}_{\mathbf{R},\mathbf{S}} A^i_{\mathbf{R}} A^j_{\mathbf{S}} \quad (i\neq j)$$

$$\mathrm{tr}_{\mathbf{R}}F^2 = A_{\mathbf{R}}\mathrm{Tr}F^2$$
$$\mathrm{tr}_{\mathbf{R}}F^4 = B_{\mathbf{R}}\mathrm{Tr}F^4 + C_{\mathbf{R}}(\mathrm{Tr}F^2)^2$$

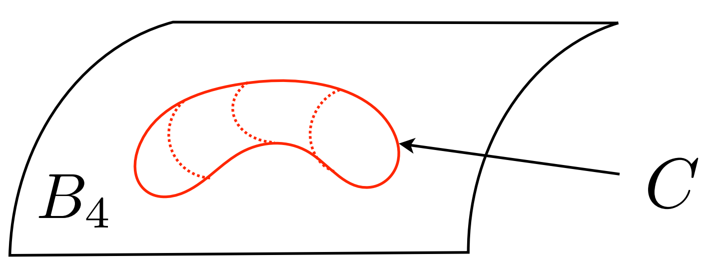
6d Supergravity from F-theory compactification

- F-theory on compact elliptic Calabi-Yau threefold (CY3)

$$\begin{array}{ccc} T^2 & \longrightarrow & X \\ & & \downarrow \\ & & B_4 \end{array} \qquad X \ : \ \text{elliptic } CY_3$$

or equivalently Type IIB on Kahler base B_4 leads to 6d (1,0) supergravity theory at low energy.

- Kahler parameters of 2-cycles $C \subset B_4$ are the V.E.Vs of scalar fields in 6d tensor multiplets, and 7-branes wrapped on C provide gauge symmetries.



T	$\rightarrow$	$h^{1,1}(B_4) - 1$
a	$\rightarrow$	K : Canonical class of B_4
b_i	$\rightarrow$	C_i : 2-cycles in B_4
G_i	$\rightarrow$	Type of singularity on C_i

A large class of 6d N=(1,0) supergravity theories have UV completion in F-theory or string theory.

Q) Are anomaly free 6d (1,0) supergravity theories all have UV completions?

Ex1) $T=9\,,\;G=SU(N)\times SU(N)$ with 2 bi-fundamental hypers is anomaly free for arbitrary N, while UV completion is known for $N\leq 8$.

[Dabholkar, Park 1996]

Ex2) $T=8k+9\,,\;G=(E_8)^k$ theory is anomaly free for arbitrary k , while UV completion is known only for $k=1,2$.

[Seiberg, Witten 1996]

We shall use "string probes" to see if these anomaly free effective theories are consistent.

Strings in 6d (1,0) Supergravity

6d tensor moduli space is parametrized by a vector $J \in \mathbb{R}^{1,T}$ satisfying

$$\mathrm{vol}(\mathcal{M}_T) \sim J \cdot J > 0 \ , \quad \frac{1}{g_i^2} \sim J \cdot b_i > 0 \ , \quad -J \cdot a > 0$$

In F-theory, a Kahler form $J \in H^{1,1}(B)$ satisfying these conditions defines positive-definite Kahler cone in the base.

Consider 1/2 BPS strings in 6d (1,0) supergravity which couple to T+1 2-form tensor fields B_2 .

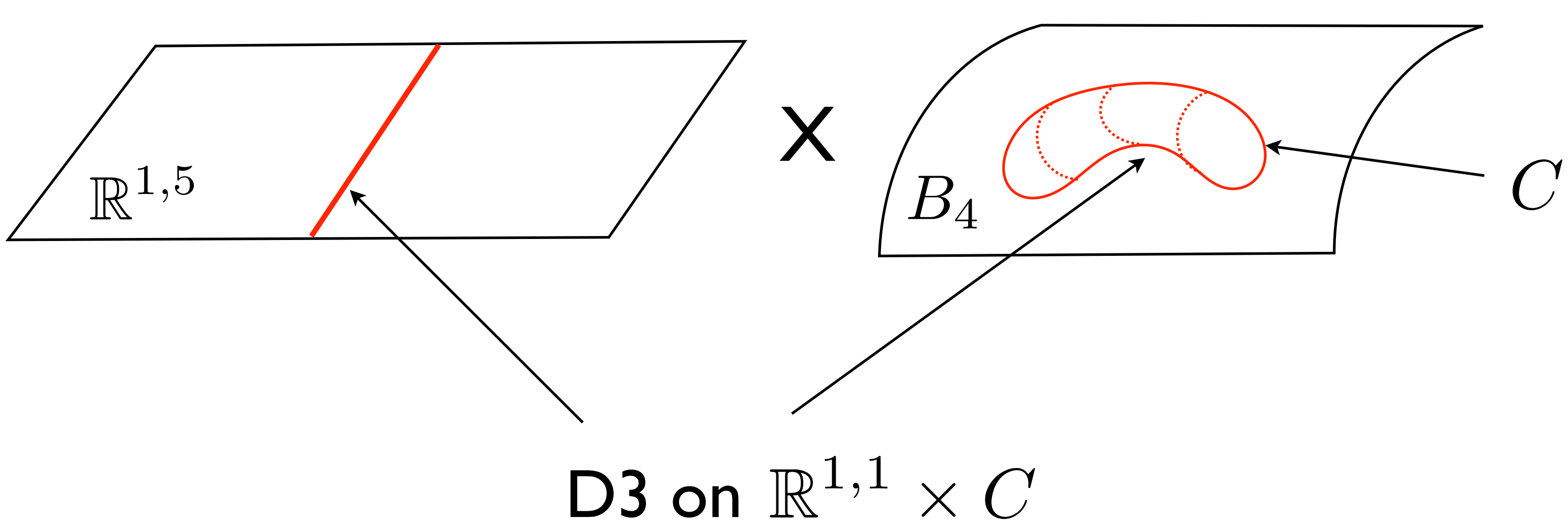
- String with charge Q has positive tension if $Q \cdot J > 0$.
- Such BPS string should exist by completeness of spectrum in gravity theory.

Strings in 6d (1,0) Supergravity

The 2d worldsheet theory is expected to flow N=(0,4) SCFT at low energy.

- Conformal R-symmetry is $SU(2)_R \subset SO(4)$ of transverse $\mathbb{R}^4$.
- Can host current algebra of G_i and $SU(2)_l \subset SO(4)$.

In Type IIB (or F-theory), strings come from D3-branes wrapped on a holomorphic 2-cycle $C \subset B_4$.



Worldsheet Anomalies

Anomalies of string worldsheet theory in 6d SCFTs can be computed by anomaly inflow computation.

[Bergman, Harvey 2004], [H-C. Kim, S. Kim, J. Park 2016], [Shimizu, Tachikawa 2016]

Anomaly polynomial of the worldsheet CFT in 6d supergravity theory:

[Haghighat, Murthy, Vafa, Vandoren 2015]
[Couzens, Lwarie, Martelli, Schafer-Nameki, Wong 2017]
[H-C. Kim, Shiu, Vafa 2019]

$$I_4 = -\frac{c_R - c_L}{24} p_1(T_2) + \frac{1}{4} k_i \, \mathrm{Tr} F_i^2 - \frac{c_R}{6} c_2(R) + k_l c_2(l)$$

$$= -\frac{3Q\cdot a - 1}{12} p_1(T_2) + \frac{1}{4} \sum_i Q\cdot b_i \mathrm{Tr} F_i^2$$

$$- \frac{Q\cdot Q - Q\cdot a}{2} c_2(R) + \frac{Q\cdot Q + Q\cdot a + 2}{2} c_2(l)$$

$SU(2)_R \times SU(2)_l = SO(4)$

- Central charges : $c_L = 3Q\cdot Q - 9Q\cdot a + 2 \; , \quad c_R = 3Q\cdot Q - 3Q\cdot a$
- Levels of $SU(2)_l \times \prod_i G_i$: $k_l = \frac{1}{2}(Q\cdot Q + Q\cdot a + 2) \; , \quad k_i = Q\cdot b_i$

Gravity string vs Instanton string

Worldsheet SCFT of a single string in 6d supergravity theories must have

$$c_L\ ,\ c_R \geq 0 \quad \& \quad k_l\ ,\ k_i \geq 0$$

- $k_l,\, k_i \geq 0$ means current algebra must be in left-moving sector.
- In F-theory, this means the curve C wrapped by D3-branes must be an irreducible effective curve in the Mori cone of the base.

Otherwise, the string degenerates to instanton string of gauge group G in local 6d SCFTs or little string theories (LSTs) embedded in supergravity.

- Instanton string in local 6d SCFTs or LSTs have accidental $SU(2)_I$ symmetry in IR and it becomes R-symmetry of another $(0,4)$ superconformal algebra which differs from $(0,4)$ algebra of supergravity strings we are interested in.

Unitary conditions

So we are interested in a single gravity string with $J \cdot Q > 0$ satisfying

$$1.\ c_L = 3Q \cdot Q - 9Q \cdot a + 2 \geq 0$$
$$2.\ c_R = 3Q \cdot Q - 3Q \cdot a \geq 0$$
$$3.\ k_i = Q \cdot b_i \geq 0$$
$$4.\ k_l = \frac{1}{2}(Q \cdot Q + Q \cdot a + 2) \geq 0$$

- Note that, as we saw in 10d strings, the level k current algebra of group G contributes to the left-moving central charge as

$$c_G = \frac{k \cdot \mathrm{dim} G}{k + h^\vee}$$

- A unitary SCFT on a 6d gravity string is subject to the bound on levels:

$$\sum_i \frac{k_i \cdot \mathrm{dim} G_i}{k_i + h_i^\vee} \leq c_L$$

Example I

6d supergravity theory coupled to T=9 with SU(N)xSU(N) gauge group with two bi-fundamental hypermultiplets. [Kumar, Morrison, Taylor 2010]

- No anomaly for arbitrary N with

$$\Omega = \mathrm{diag}(+1, (-1)^9) \ , \quad a = (-3, (+1)^9) \ ,$$
$$b_1 = (1, -1, -1, -1, 0^6) \ , \quad b_2 = (2, 0, 0, 0, (-1)^6)$$

- String theory realization for N=8 by [Dabholkar, Park 1996] .
- But no F-theory realization at large enough N. [Kumar, Morrison, Taylor 2010]

Let us consider a tensor vacuum of a Kahler form $J = (1, 0^9)$ satisfying $J^2 > 0, J \cdot b_i > 0, J \cdot a < 0$ conditions.

Then couple a string of charge $Q = (q_0, q_1, q_2, \cdots, q_9)$ having positive tension with respect to J:

$$\Longrightarrow \quad J \cdot Q = q_0 > 0$$

Example I

A string with charge $Q = (q_0, q_1, q_2, \cdots, q_9)$ is a supergravity string iff

$$c_R \geq 0,\ k_l \geq 0 \quad \rightarrow \quad q_0^2 - \sum_i^9 q_i^2 \geq -1\,,\ \ q_0^2 - \sum_i^9 q_i^2 - 3q_0 - q_{1:3} - q_{4:9} \geq -2$$

$$k_1 \geq 0,\ k_2 \geq 0 \quad \rightarrow \quad q_0 + q_{1:3} \geq 0\,,\ \ 2q_0 + q_{4:9} \geq 0$$

where $q_{1:3} = q_1 + q_2 + q_3$
$q_{4:9} = q_4 + \cdots + q_9$

and unitary bound on levels must be satisfied, i.e.

$$\underset{SU(2)_l}{\frac{3k_l}{k_l+2}} + \underset{SU(N)_1}{\frac{k_1(N^2-1)}{k_1+N}} + \underset{SU(N)_2}{\frac{k_2(N^2-1)}{k_2+N}} \leq c_L$$

where $c_L = 3(q_0^2 - \sum_i^9 q_i^2) + 9(3q_0 + \sum_i^9 q_i) + 2$

- The strongest bound is given by a string of $Q = (1, -1, 0, 0, -1, 0^5)$:

$$\frac{k_i \text{dim} G}{k_i + h^\vee} \leq c_L \ \rightarrow \ \frac{N^2-1}{N+1} \leq 8 \ \rightarrow \ N \leq 9$$

$$k_l = 0,\ \ k_1 = Q \cdot b_1 = 0,\ \ k_2 = Q \cdot b_2 = 1$$

Example I

We found a unitary bound for a supergravity string of $Q = (1, -1, 0, 0, -1, 0^5)$

$$\frac{k_i \mathrm{dim} G}{k_i + h^\vee} \leq c_L \;\rightarrow\; \frac{N^2 - 1}{N+1} \leq 8 \;\rightarrow\; N \leq 9$$

- The worldsheet theory on this string cannot be a unitary SCFT if $N > 9$.
- This shows that the bulk 6d supergravity theory, though it's anomaly free, belongs to ***the swampland*** if $N > 9$.
- Kodaira condition for elliptic CY3 in F-theory requires $N \leq 12$. [Kumar, Morrison, Taylor 2010]
- String theory realization is known for $N = 8$. [Dabholkar, Park 1996]

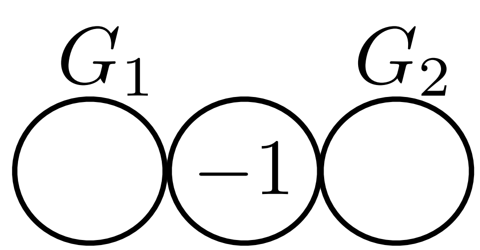
Q) Is there a string theory construction for the theory with $N = 9$?

Example I

- The worldsheet theory on a string with charge $Q=(1,-1,0,0,-1,0^5)$ is in fact a single E-string theory.

  - The curve Q is a rational curve ($k_l=g=0$) with $Q^2=-1$.
  - The theory on a single string in 6d E-string theory on -1 curve.
  - The interacting CFT consists of 8 left-moving fermions forming E8 current algebra.
  - No accidental $SU(2)_I$ symmetry and thus the theory can also be considered as a supergravity string.

- Adjacent curves can only support gauge symmetry of $\prod_i G_i \subset E_8$.



$$G_1 \times G_2 \subset E_8$$

[Heckman, Morrison, Vafa 2013]

- $Q\cdot b_1 = 0,\ Q\cdot b_2 = 1 \Longrightarrow SU(N)_2 \subset E_8 \Longrightarrow N \leq 9$

Example II

6d supergravity theory coupled to T=8k+9 with $(E_8)^k$ gauge group.

- No anomaly for arbitrary k with [Kumar, Morrison, Taylor 2010]

$$\Omega = \mathrm{diag}(1,(-1)^{8k+9}) \ , \quad a = (-3, 1^{8k+9}) \ ,$$
$$b_i = (-1,-1,0^{4(i-1)},(-1)^3,-3,0^{8k+8-4i})$$

- Kahler form can be chosen as

$$J = (-j_0, 0^{4k+1}, 1^{4k+8}) \ , \quad (4k+8)/3 > j_0 > \sqrt{4k+8}$$

- A string of charge $Q = (-q, 0^{8k+9})$ with $q \geq 9$ is a supergravity string with positive tension.

- The unitary bound on left-moving central charge :

$$\frac{3k_l}{k_l+2} + \sum_{i=1}^{k} \frac{248k_i}{k_i+30} \leq c_L$$

where

$$k_l = \frac{q^2+3q}{2}$$
$$k_i = Q.b_i = q$$
$$c_L = 3q(q-9)+2$$

Example II

- Unitary bound cannot be satisfied with charge $9 \leq q \leq 14$ for any $k \geq 3$.

$$\frac{3k_l}{k_l+2} + \sum_{i=1}^{k} \frac{248k_i}{k_i+30} \leq c_L \qquad \text{with} \quad 9 \leq q \leq 14$$

- 6d supergravity theory with $k \geq 3$ belongs to the swampland!

- However, when k=1,2 there exist another solutions of Ω, a, b_i :

k=1 : $\Omega = \mathrm{diag}(1, (-1)^{17})$, $a = (-3, 1^{17})$,
$b_1 = (0, 1, (-1)^{11}, 0^5)$

k=2 : $\Omega = \mathrm{diag}(1, (-1)^{25})$, $a = (-3, 1^{25})$,
$b_1 = (0, 1, (-1)^{11}, 0^{13})$, $b_2 = (0, 0^{13}, 1, (-1)^{11})$

- Therefore, the above string analysis doesn't apply. In fact, the 6d gravity theory for k=2 is realized by M-theory compactfication on $K3 \times (S^1/\mathbb{Z}_2)$ with 24 M5-branes on the interval. [Seiberg, Witten 1996]

Conclusion

Conclusion

- We initiated a new program by using string/brane probes for a deeper understanding of Swampland criteria.

- The unitarity of worldsheet CFTs associated to central charges give rise to new constraints on consistent 10d and 6d supergravity theories.

- This work using brane probes can be generalized in various directions:
    - 6d supergravity with abelian gauge groups. [S-J. Lee, Weigand 2019]
    - Other types of branes and defects.
    - Supergravity theories in other dimensions. [Katz, H-C. Kim, Tarazi, Vafa in progress]