3. Parametric effects

Oscillations in multilayer media

Neutrino-neutrino scattering and collective transformations

Parametric effects

Propagation on the Earth

Parametric enhancement

V. Ermilova ,V. Tsarev, V. Chechin,
Krat. Soob. Fiz. # 5, 26, (1986)

Strong transition if there is a harmonic modulation of density profile

$$n(x) = \langle n \rangle + n_1 \cos \omega_d x$$

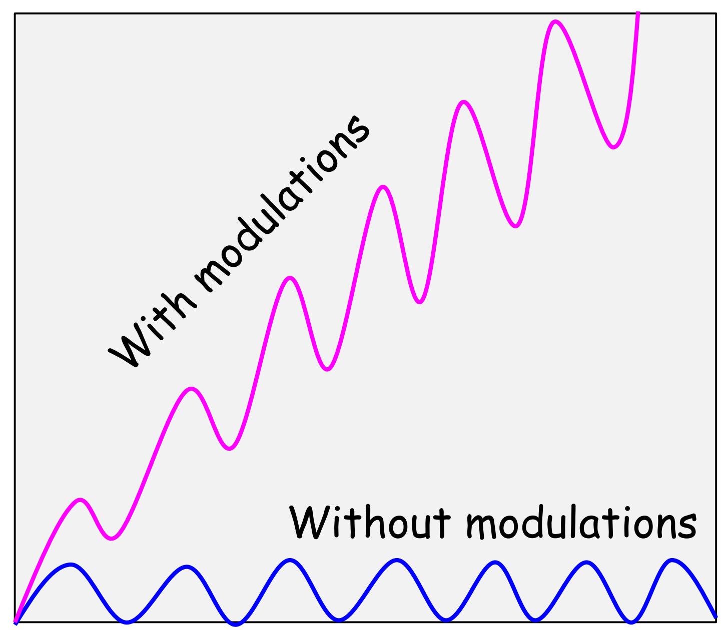
Parametric resonance condition:

$$k\, \omega_d = \Delta_m(\langle n \rangle)\ , \quad k = 1, 2 \ \ldots$$

$$\Delta_m(\langle n \rangle) = \frac{\Delta m^2}{2E} \left[(\cos 2\theta - 2VE/\Delta m^2)^2 + \sin^2 2\theta \right]^{\frac{1}{2}}$$

is frequency of oscillations for the average density

P



x

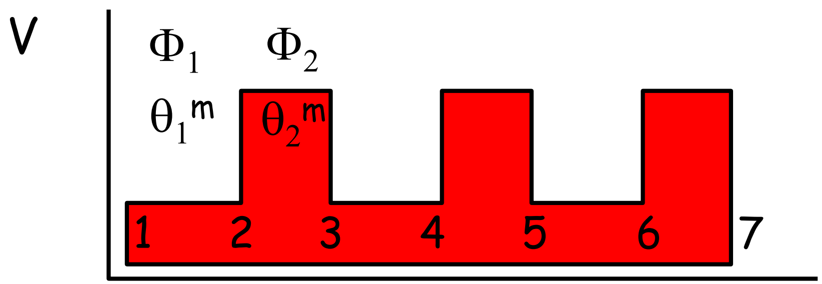
Realized in astrophysical objects?

Parametric enhancement of oscillations

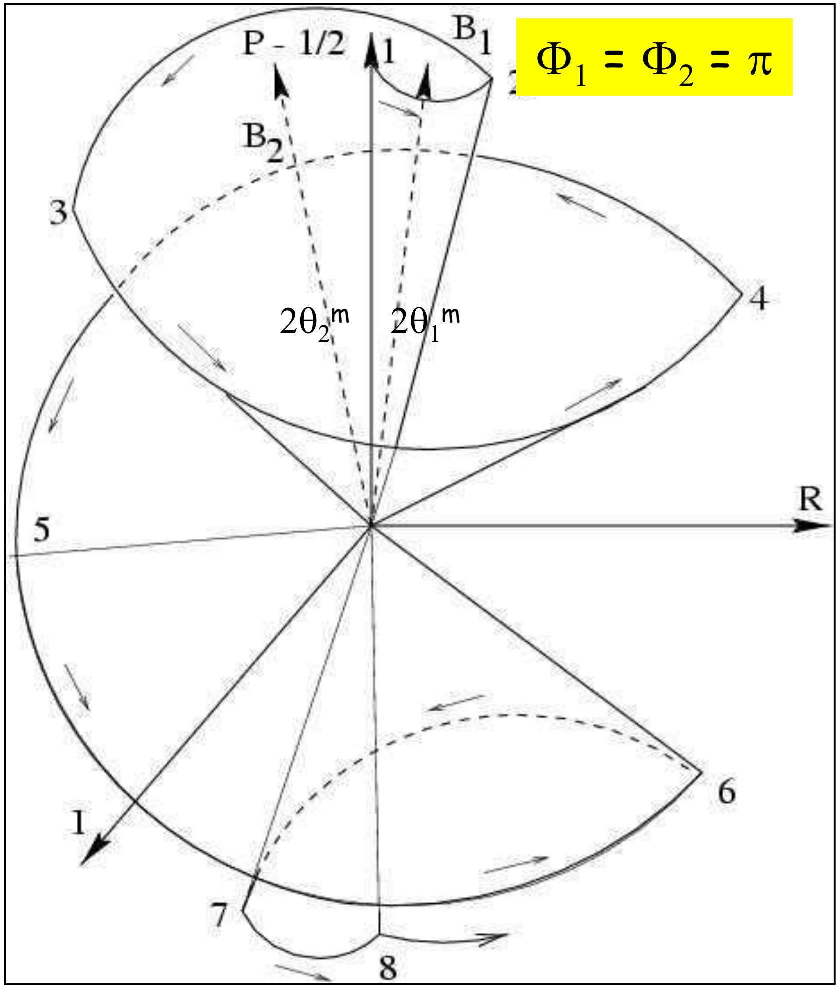
Parametric oscillations

E. Kh. Akhmedov , 1988

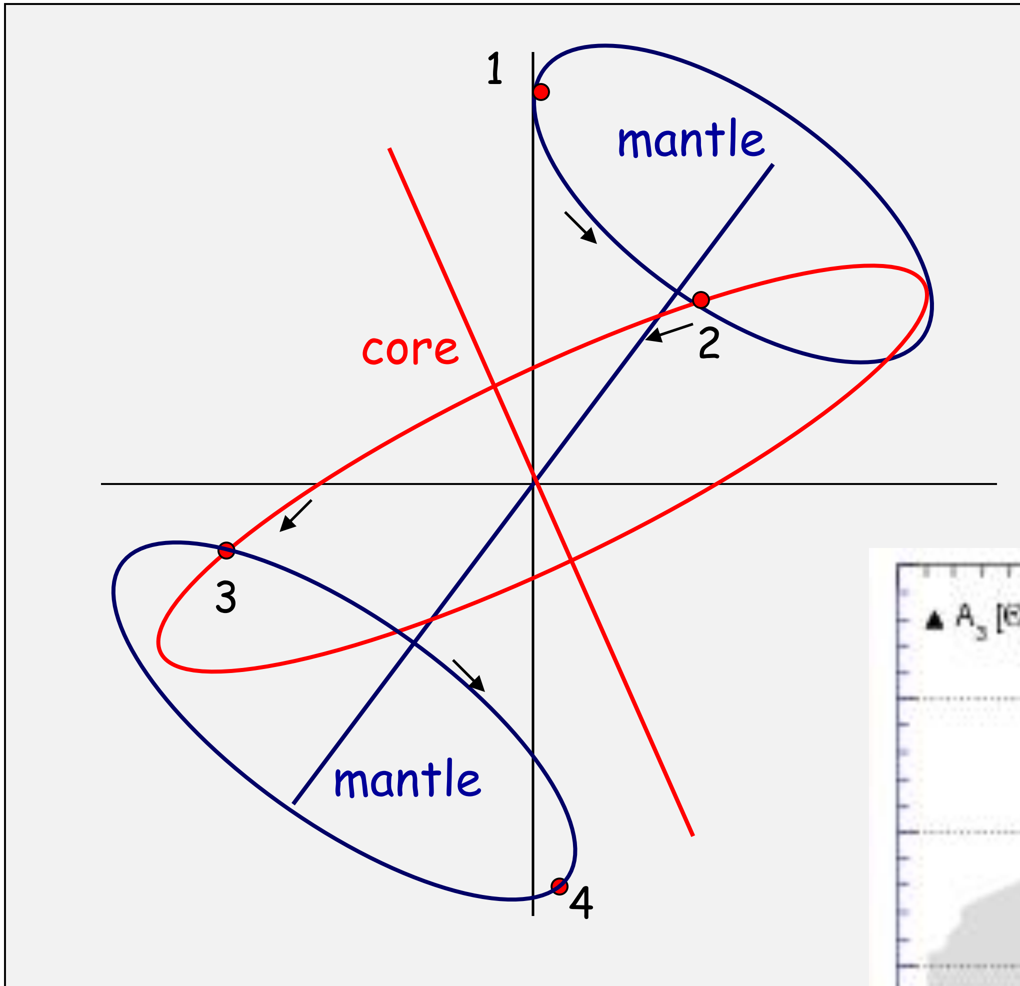
``Castle wall profile"



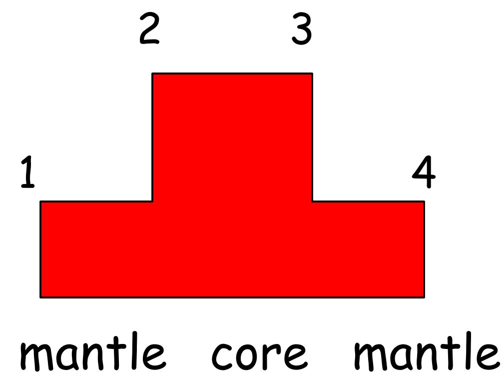
``Castle wall profile"

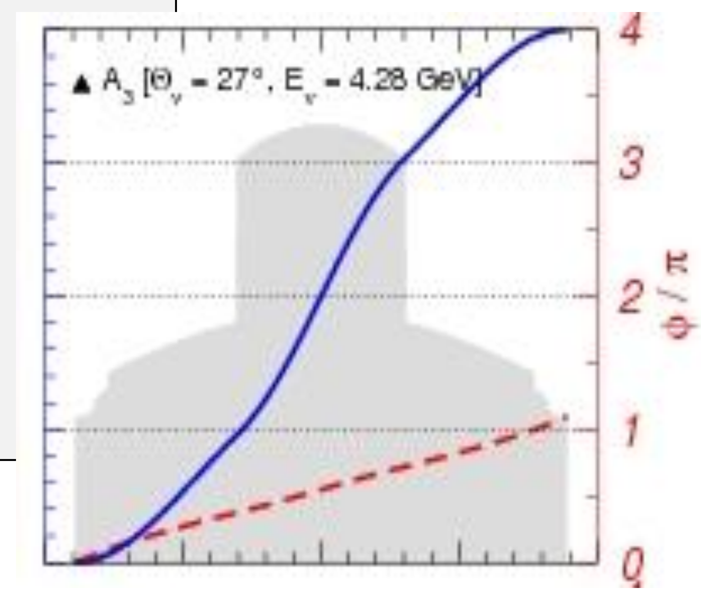


Parametric enhancement in the Earth, 1-3 mode

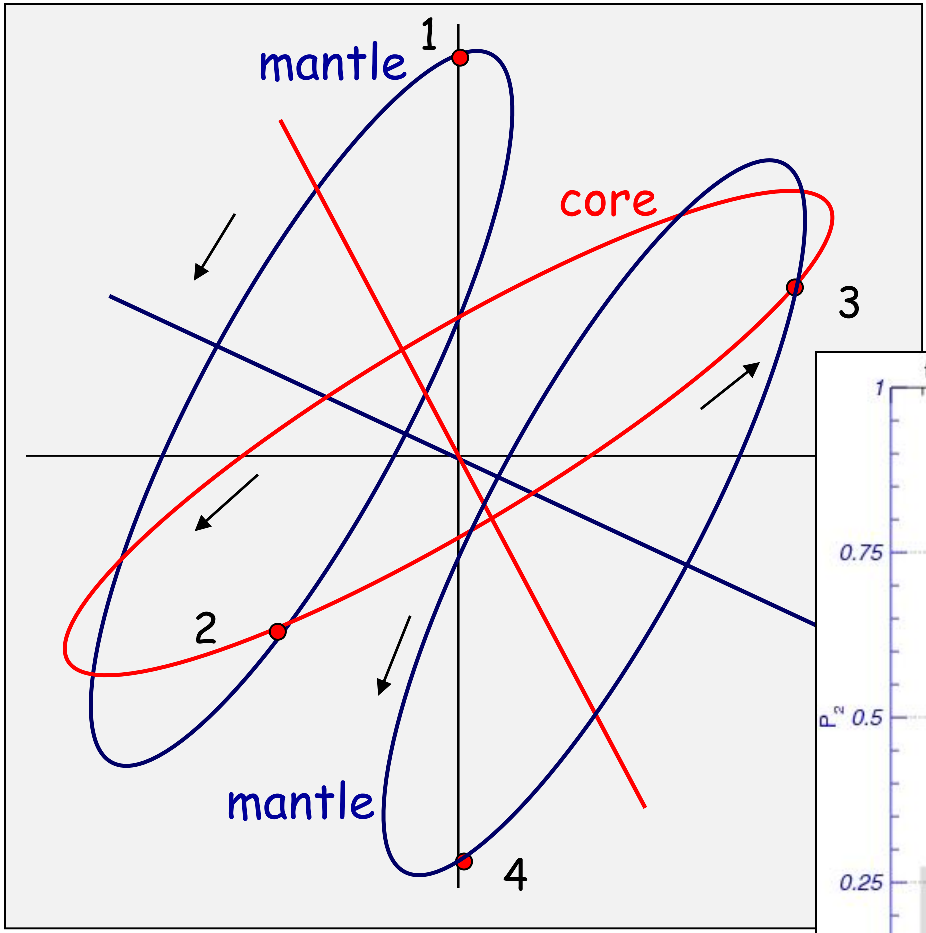


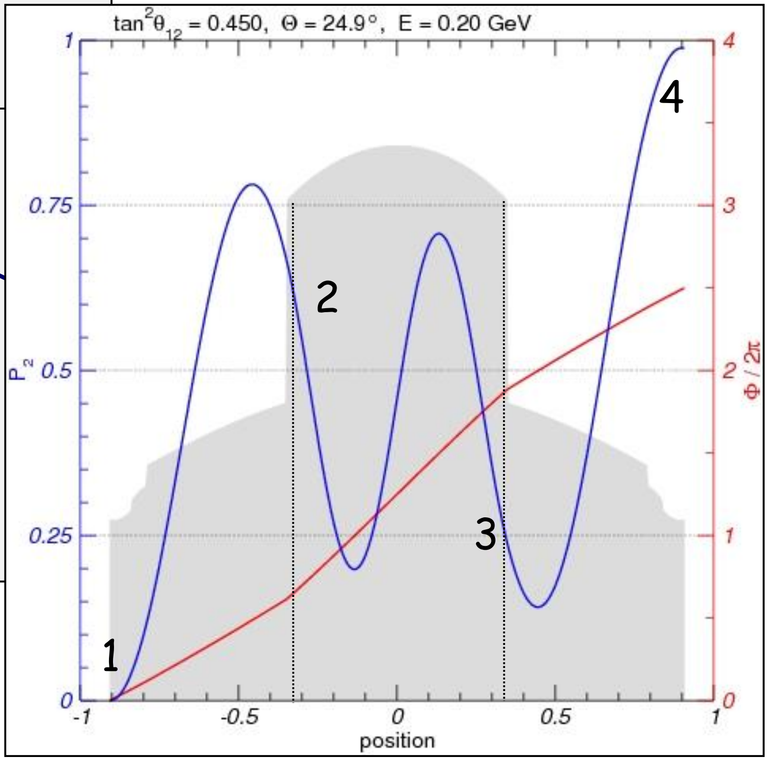
In the Earth





Parametric enhancement in the Earth,1-2 mode

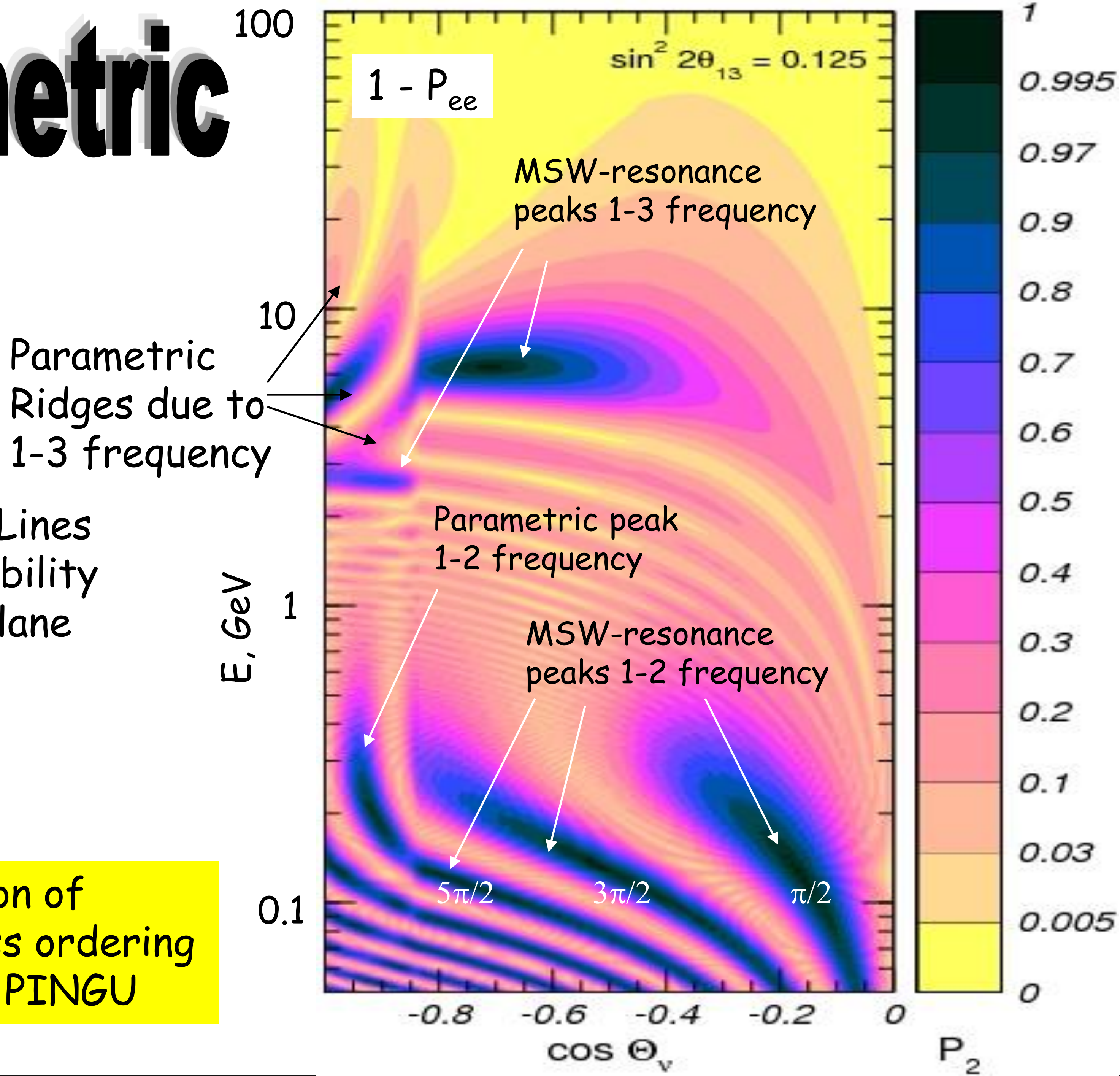




Parametric ridges

Oscillograms: Lines of equal probability in the $E - \theta_z$ plane

$\nu_e \rightarrow \nu_\mu + \nu_\tau$

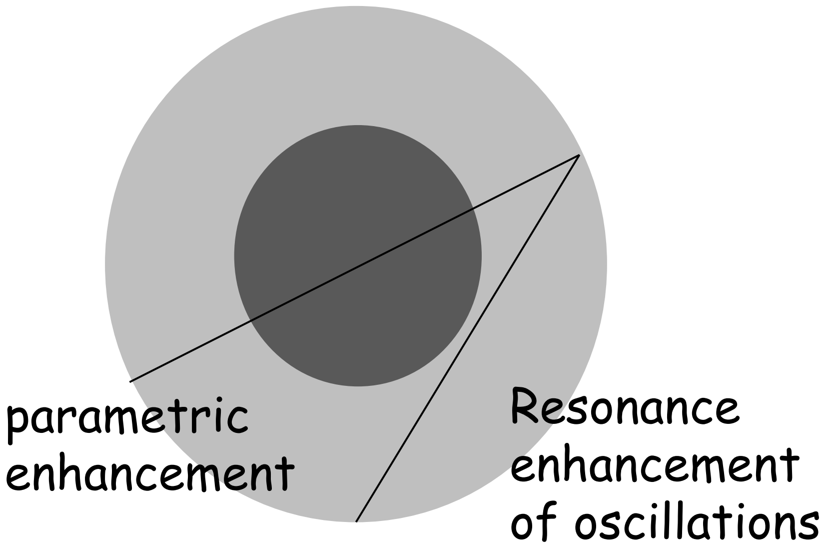


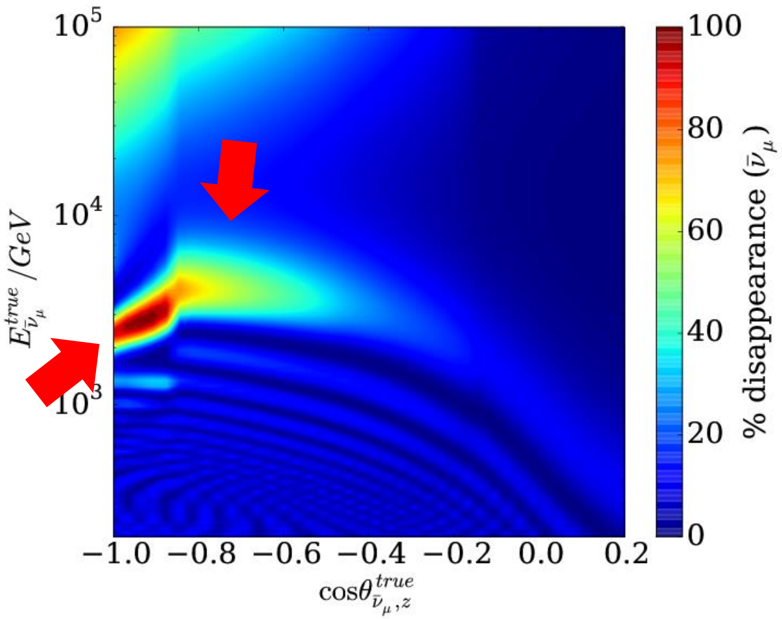
Determination of neutrino mass ordering With ORCA, PINGU

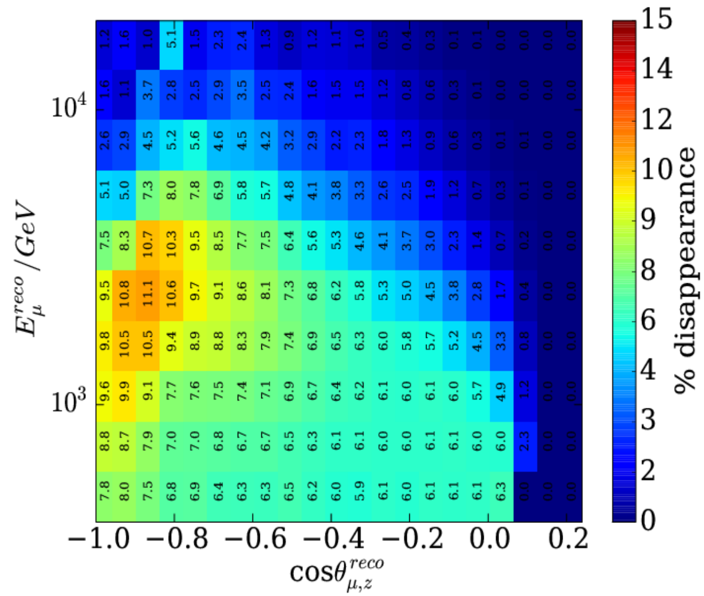
IceCube searches for sterile neutrinos

M.G. Aartsen et al, (IceCube Collaboration) 1605.01990 (hep-ex)

IC86, 2011 – 2012, 343,7 days, 20,145 muon events (reconstructed tracks) with E = 320 GeV – 20 TeV



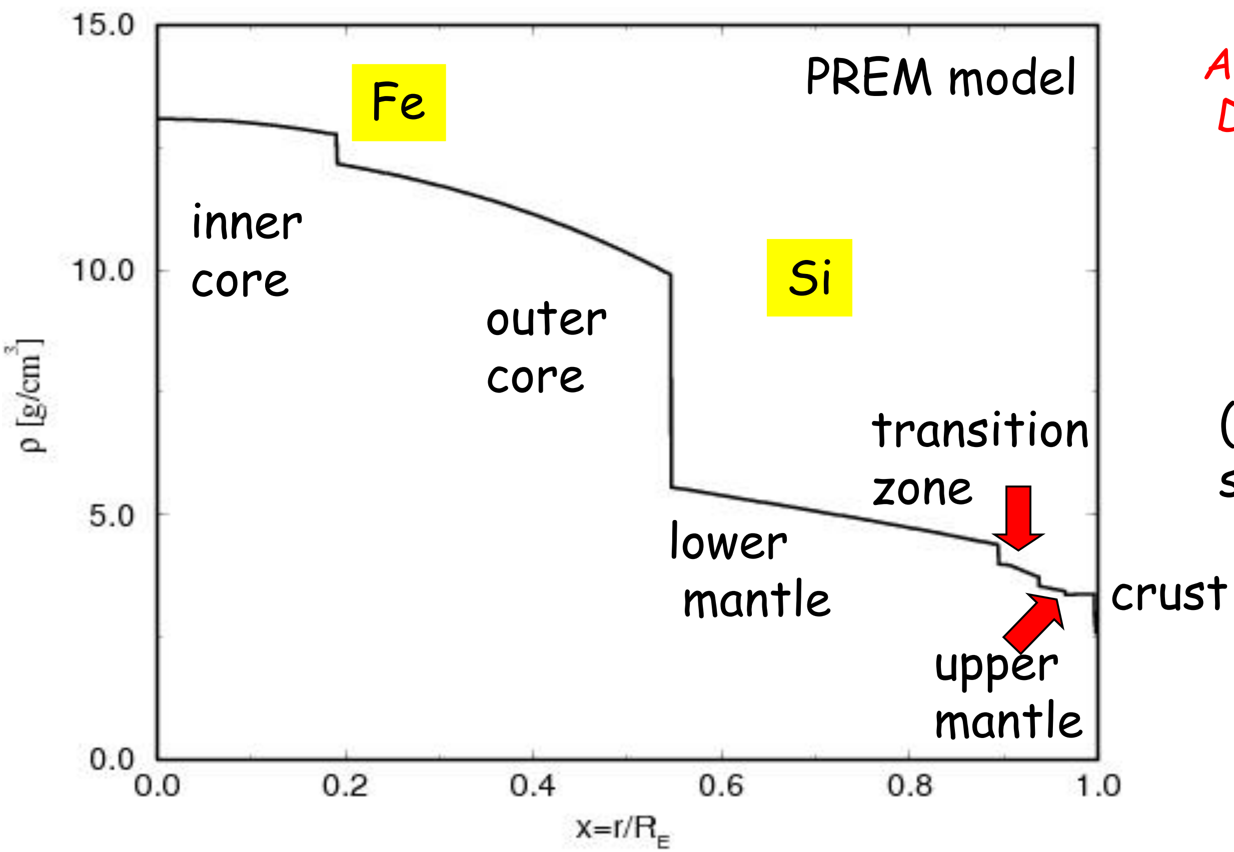




Oscillations in multi-layer media

Low energies
higher resolution

The earth density profile



A.M. Dziewonski D.L Anderson 1981

(phase transitions in silicate minerals)

R_e = 6371 km

solid liquid

Oscillations in the Earth

of low energy neutrinos

Incoherent fluxes of mass states arrive at the Earth from the Sun or SN. They split (decompose) into eigenstates in matter and oscillate.

Mixing of the mass states in matter

$$U^{mass} = U_{PMNS}^{+} U^{m}$$

flavor mixing matrix in matter

For the 1-2 mixing

$$\sin 2\theta' = \frac{\varepsilon \sin 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} - \varepsilon)^2 + \sin^2 2\theta_{12}}} = \varepsilon \sin 2\theta_{12}^{m}$$

$$\varepsilon = \frac{2VE}{\Delta m_{21}^2} = 0.03\, E_{10}\, \rho_{2.6}$$

MeV g/cm^3

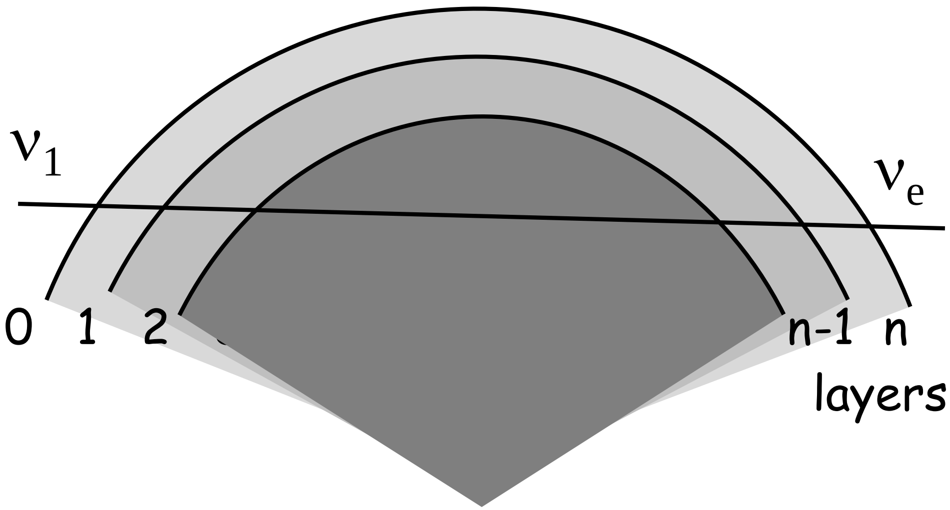
$c_{13} = \cos\theta_{13}$

In low density regime ε determines smallness (size) of effects

$\sin 2\theta' \sim V$

Regeneration

A. Ioannisian, A. Smirnov, D. Wyler, Phys.Rev. D96 (2017) no.3, 036005 arXiv:1702.06097 [hep-ph]



Layers with slowly changing density and density jump

Evolution matrix (matrix of transition amplitudes)

$$S = U^m{}_n \Pi_k D_k U_{k,k-1}$$

flavor mixing matrix, at the detector

D_k - describe the adiabatic evolution within layers:

$$D_k = \text{diag}\,(e^{-0.5i\phi_\kappa},\ e^{0.5i\phi_\kappa})$$

$$\phi_\kappa = \int dx (H_{2m} - H_{1m})$$

adiabatic phase acquired in k layer

$U_{k,k-1}$ - describes change of basis of eigenstates between k and k-1 layers

$$U_{k,k-1} = U(-\Delta\theta_{k-1})$$

$\Delta\theta_{k-1}$ -change of the mixing angle in matter after k-1 layer

Oscillation waves

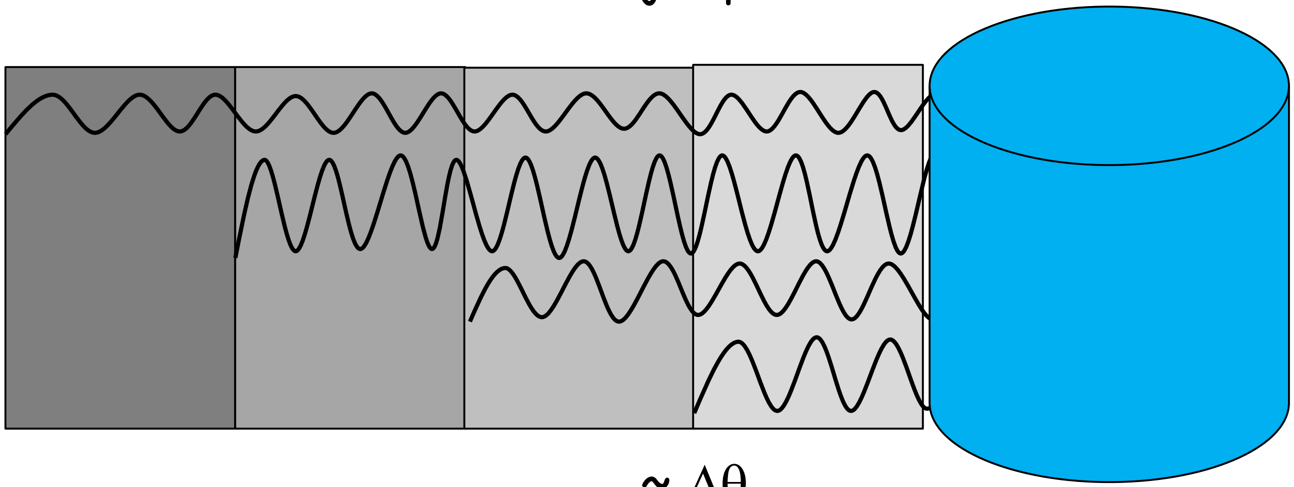
The lowest order plus waves emitted from different jumps

$$P_{1e} \sim c_{13}^2 \cos^2\theta_n^f + c_{13}^2 \sin 2\theta_n^f \; \Sigma_{j=0\,...n-1} \sin\Delta\theta_j \cos\phi_j^{after}$$

initial wave without density jumps

sum over density jumps

total phase acquired after jump j



$\sim \Delta\theta_j$

superposition of waves

the amplitude of wave: $c_{13}^2 \sin 2\theta_n^f \; \sin\Delta\theta_j$

$$\sin \Delta\theta_j = c_{13}^2 \sin 2\theta_{12} \, \Delta V_j \, \frac{E}{\Delta m_{21}^2}$$

Oscillation waves

**

Approximate (lowest order in ε) result

$$U_{k,k-1} = I - i\sigma_2 \sin \Delta\theta_{k-1}$$

Inserting this expression into formula for S and taking the lowest order terms in $\sin\Delta\theta_{k-1} \sim \varepsilon$

$$P_{1e} = |S_{e1}|^2 \sim \cos^2\theta_n^f + \sin 2\theta_n^f \, \Sigma_{j = 0 \ldots n-1} \sin\Delta\theta_j \cos \phi_j^{after}$$

the 1-2 angle in matter near detector

sum over density jumps

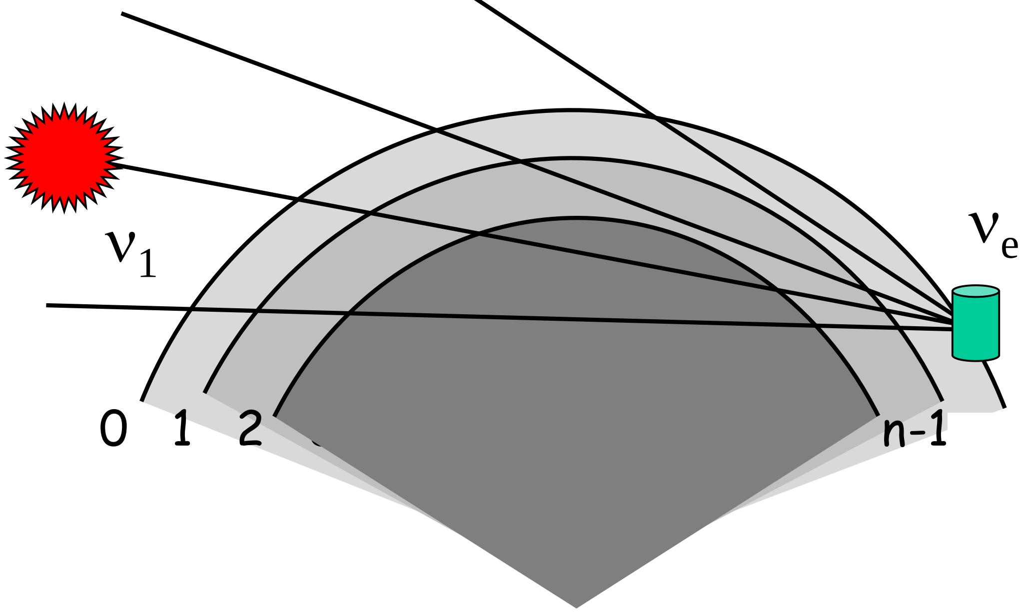
total phase acquired after jump j

for uniform sphere

$$\sin\Delta\theta_j \sim c_{13}^2 \sin 2\theta_{12} \Delta V_j \frac{E}{\Delta m_{21}^2}$$

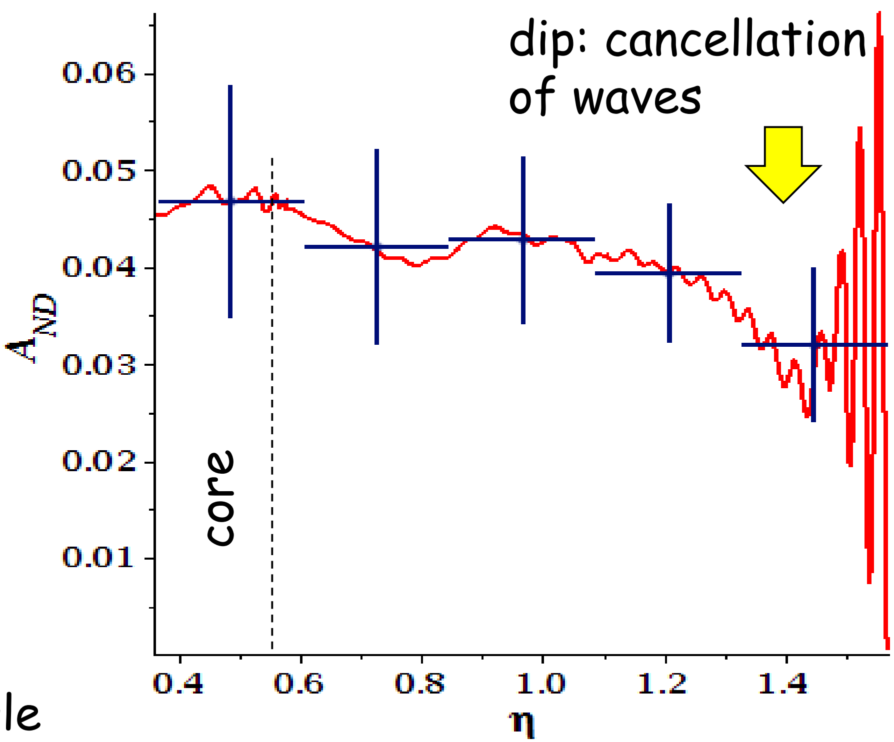
ΔV_j - jth density jump

Scanning the Earth



A. Ioannisian, A. Smirnov, D. Wyler, Phys.Rev. D96 (2017) no.3, 036005 arXiv:1702.06097 [hep-ph]

Integration above 11 MeV



Day –Night asymmetry

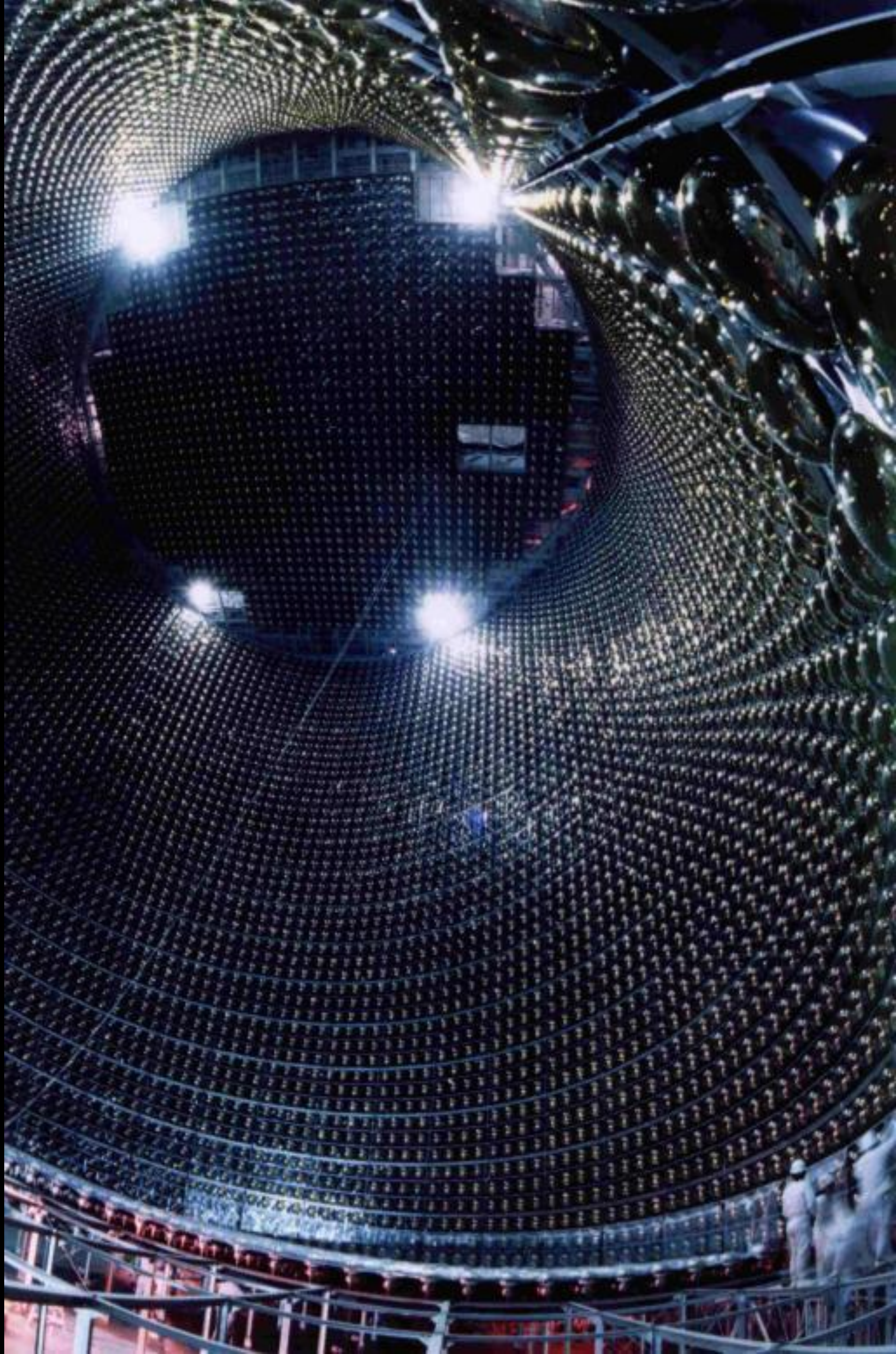
$$A_{DN} = \frac{N(\eta)}{D} - 1$$

Interference of waves

nadir angle

Attenuation effect

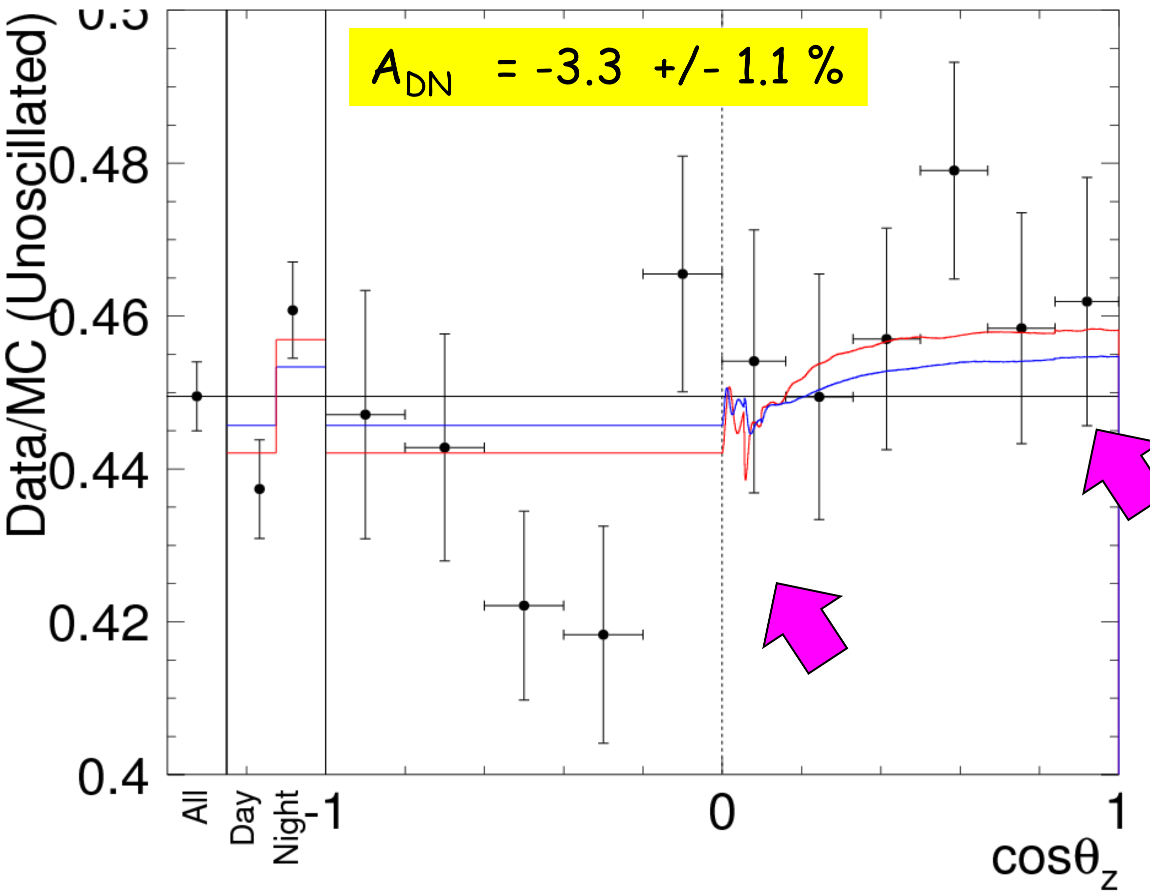
Consequence of finite energy resolution /reconstruction function



SK: Earth matter effect

SK Collaboration (Abe, K. et al.)
arXiv:1606.07538 [hep-ex]

SK-IV solar zenith angle dependence



no enhancement for core crossing trajectories

Explained by the attenuation effect

Generic features:

oscillatory pattern

dip

Integral formula

Matter effect (regeneration factor)

$$f_{reg} = P_{1e} - P_{1e}^{0} = P_{1e} - \cos^2\theta_{12}$$

determines the day-night asymmetry

A. Ioannisian and A Y S, PRL 93, 241801 (2004), hep-ph/0404060

In the integral form:

$$f_{reg} = -\frac{1}{2}\sin^2 2\theta_{12}\int_{x_0}^{x_f} dx\, V(x)\sin\phi^m(x\to x_f)$$

where the phase acquired from the point x to the final point of trajectory

$$\phi^m(x\to x_f)(E) = \int_{x}^{x_f} dx\, \Delta_{12}^{m}(x)$$

For potential with jumps explicit integration in f_{reg} reproduces the result of sum of waves emitted from the jumps

Attenuation effect

A. Ioannisian, A. Y. Smirnov, Phys.Rev.Lett., 93, 241801 (2004), 0404060 [hep-ph]

Integration with the energy resolution function R(E, E'):

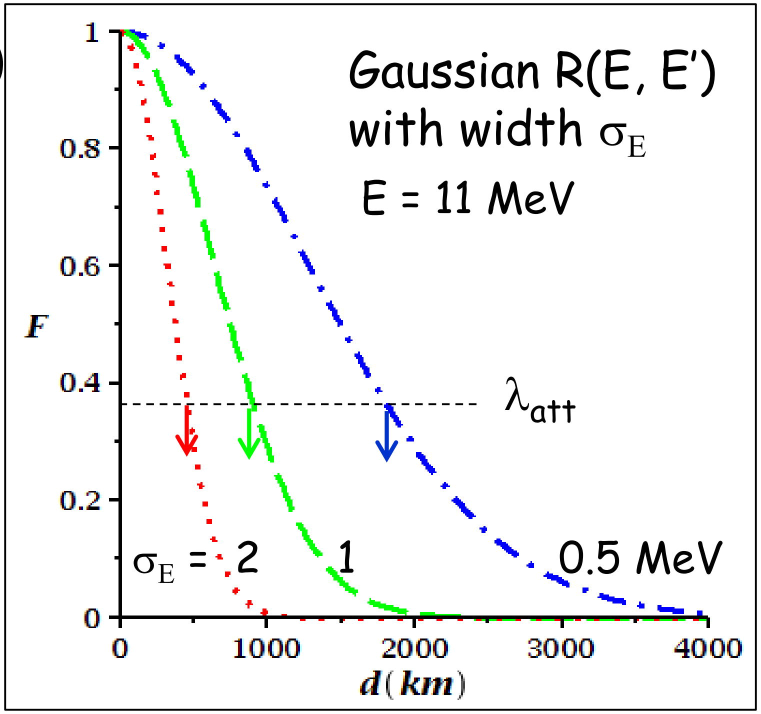
$$\langle f_{reg} \rangle = \int dE' \, R(E, E') \, f_{reg}(E')$$

changing order of integration

$$\langle f_{reg} \rangle = \frac{1}{2} \sin^2 2\theta \int_{x_0}^{x_f} dx \, F(x_f - x) \, V(x) \, \sin \phi^m(x \rightarrow x_f)$$

Attenuation factor

F(d)



Sensitivity to remote structures $d > \lambda_{att}$ is suppressed

Attenuation length

$$\lambda_{att} = l_\nu \frac{E}{\pi \sigma_E}$$

l_ν is the oscillation length

The better energy resolution, the deeper structures can be seen

Paradoxes of attenuation

A.N. Ioannisian, A. Yu. Smirnov
Phys.Rev. D96 (2017) no.8, 083009
arXiv:1705.04252 [hep-ph]

Effect of remote structures does not disappear completely.
Even for very large distances: it survives at the ε^2 level

Info about remote structure is still somehow stored.

$\nu_1 \rightarrow \nu_e$ - channel

Remote structures are attenuated by ε^2 ;
near structures are seen at ε

$\nu_e \rightarrow \nu_1$ - channel

Near structures are attenuated by ε^2 ;
remote structures are seen at ε

T-symmetry

$\nu_e \rightarrow \nu_e$ - channel

Three layer case: first layer prepare
incoherent state. Attenuation happens.
Applications for flavor – flavor transitions

Attenuation and decoherence

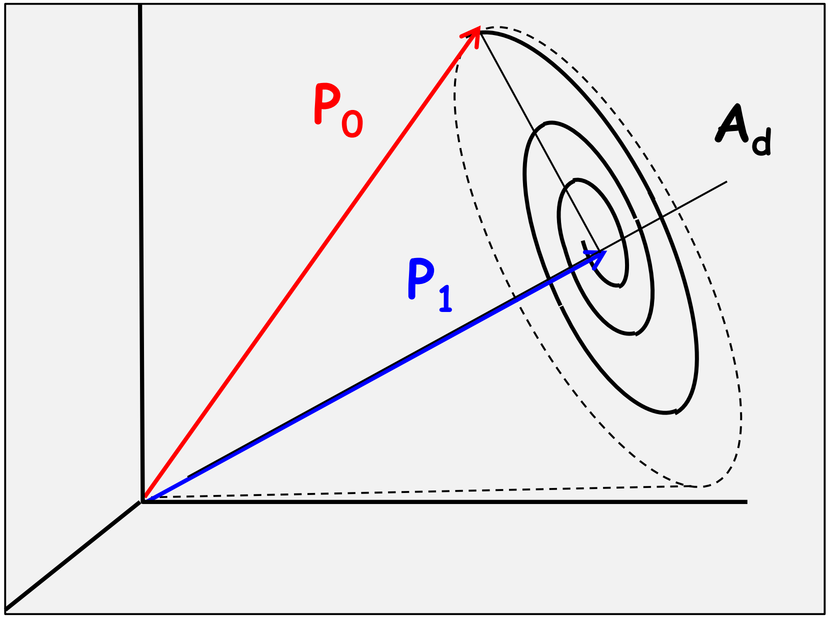
The oscillation phase acquired along the attenuation length:

$$\phi = 2\pi \frac{\lambda_{att}}{l_\nu} = 2\pi \frac{E}{\pi\sigma_E}$$

Difference of phases with ΔE

$$\Delta\phi = 2\pi \frac{\Delta E}{\pi\sigma_E}$$

For $\Delta E = \pi\sigma_E$ $\quad \Delta\phi = 2\pi$



Averaging – loss of coherence

$P_0 \rightarrow P_1$

converges to its projection onto axis of eigenstates A_d

- integration over the energy resolution interval leads to averaging of oscillations

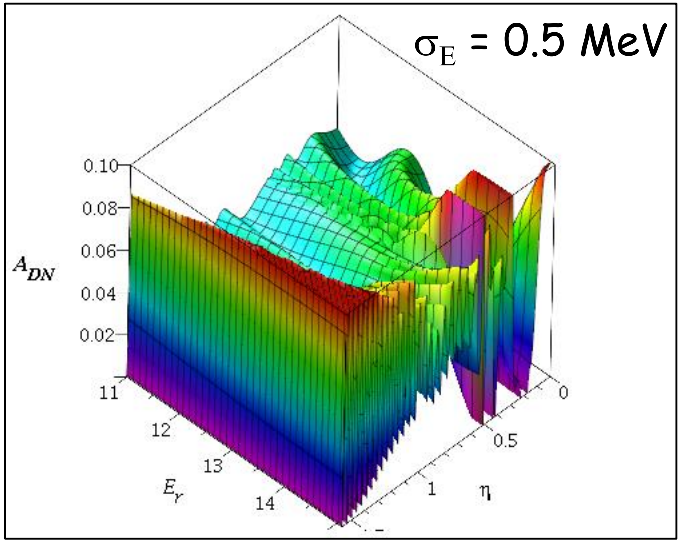
- λ_{att} is the distance over which oscillations observed with the energy resolution σ_E are averaged

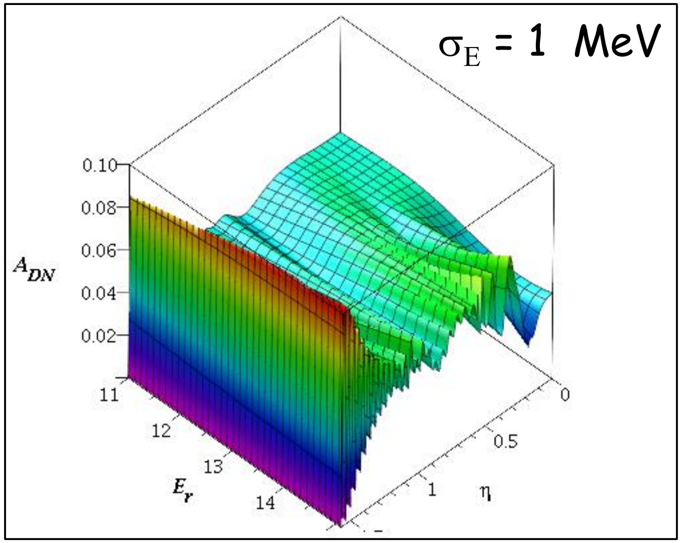
A.N. Ioannisian, A. Yu. S. Phys.Rev. D96 (2017) no.8, 083009, 1705.04252 [hep-ph]

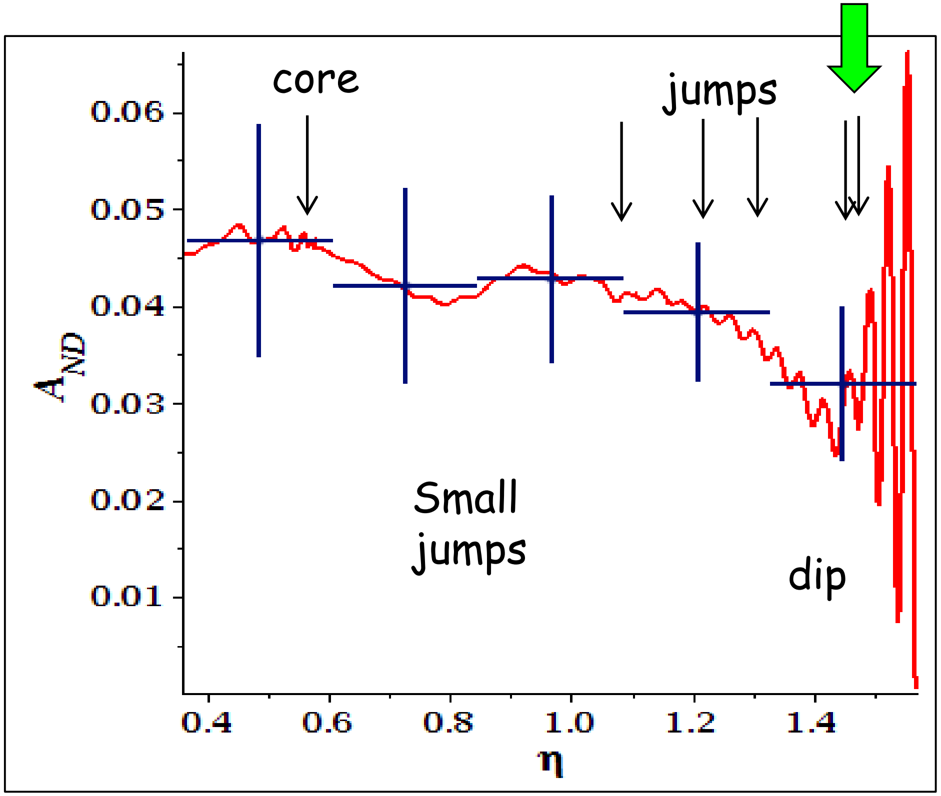
Relative D-N asymmetry

A. Ioannisian,
A.Y.S., D. Wyler
1702.06097 [hep-ph]

$$A_{DN} = \frac{N - D}{D}$$







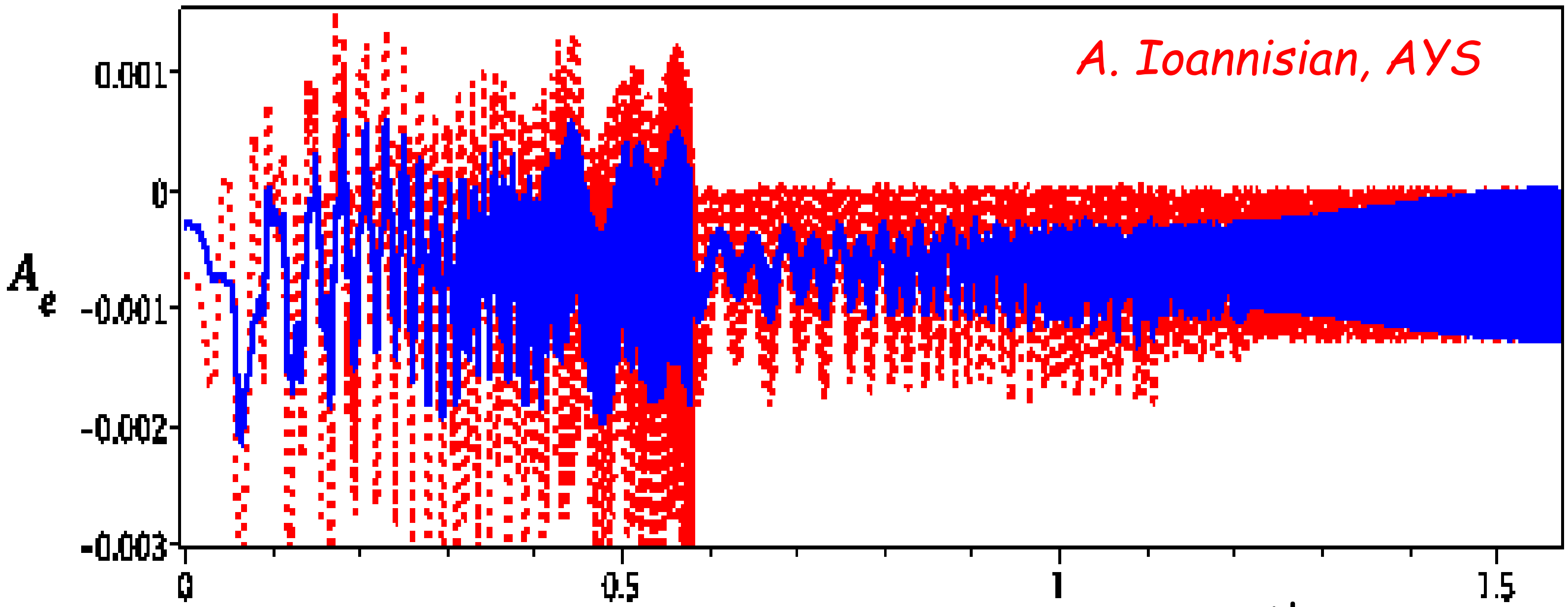
Relative excess of the night events integrated over E > 11 MeV
Sensitivity of DUNE experiment
40 kt, 5 years

ν_{Be} - neutrinos: no attenuation of the Earth core

σ_E determined by the width of the Be- line

σ_E /E ~ 0.2%

λ_{att} ~ 10^4 km

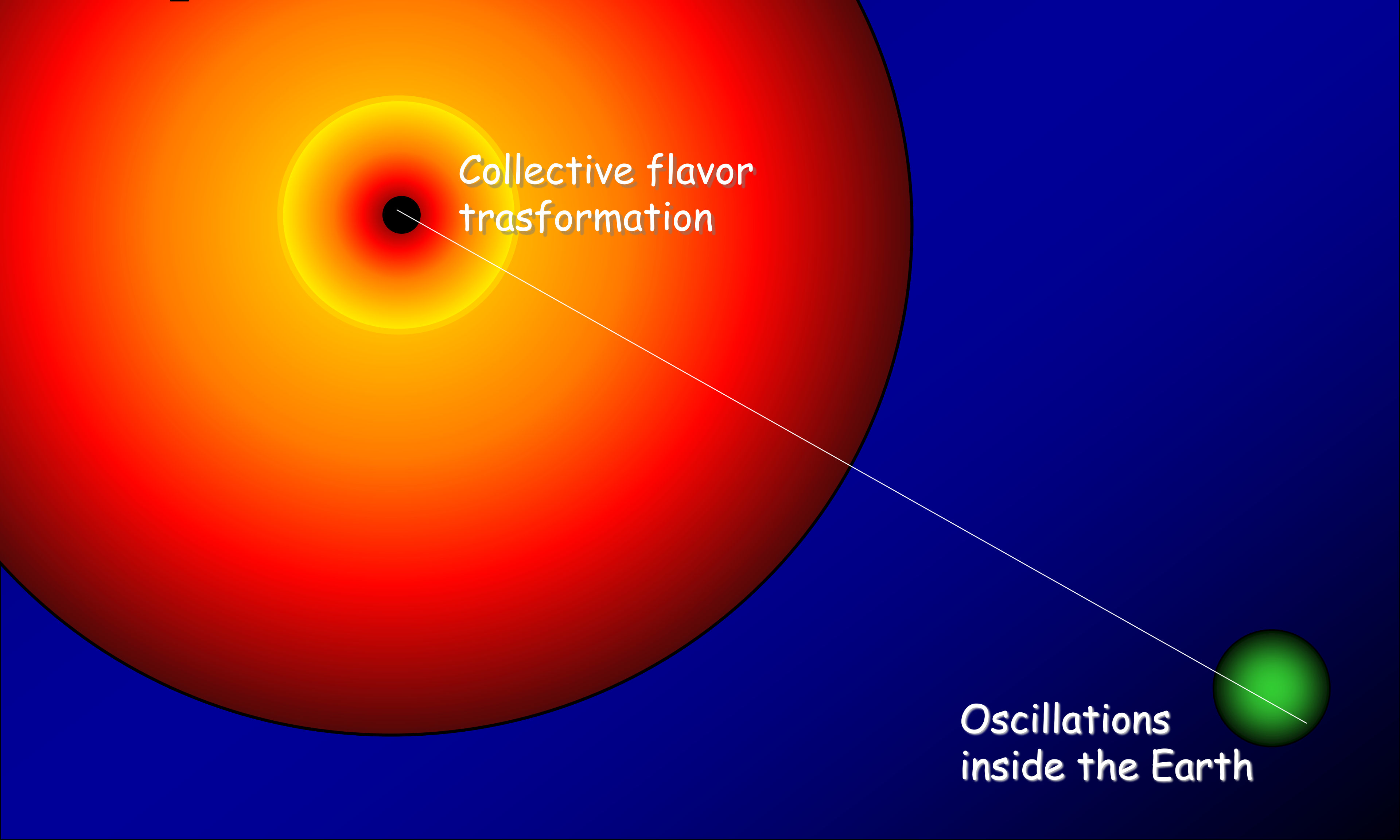


But 0.1% effect

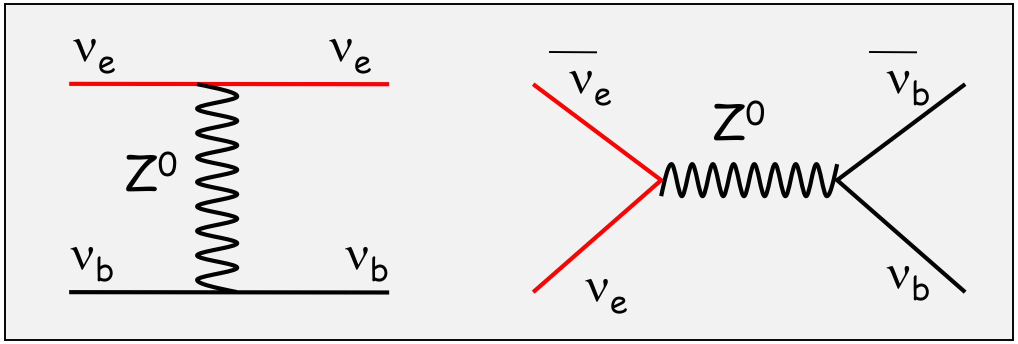
Another possibility: use high energy (GeV scale) neutrinos

Neutrino-neutrino scattering and collective transformations

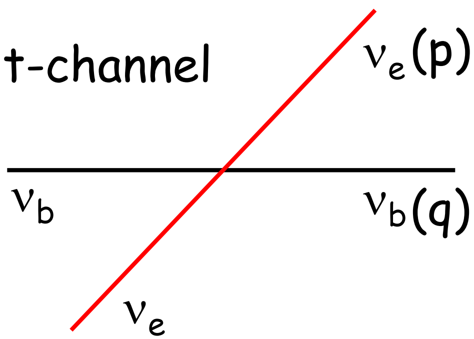
Supernova neutrinos



$\nu\nu$-scattering



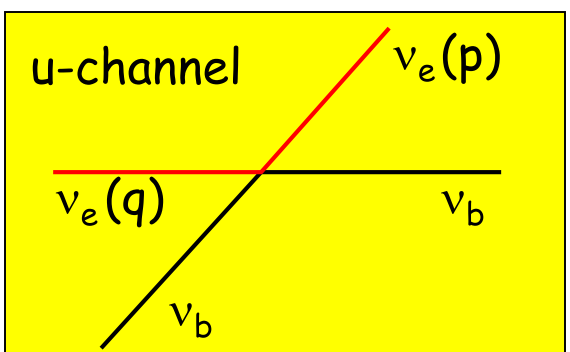
Refraction in neutrino gases



elastic forward scattering
→ coherent

$$V = \sqrt{2}\, G_F \,(1 - v_e\, v_b\,)\, n_b$$

velocities



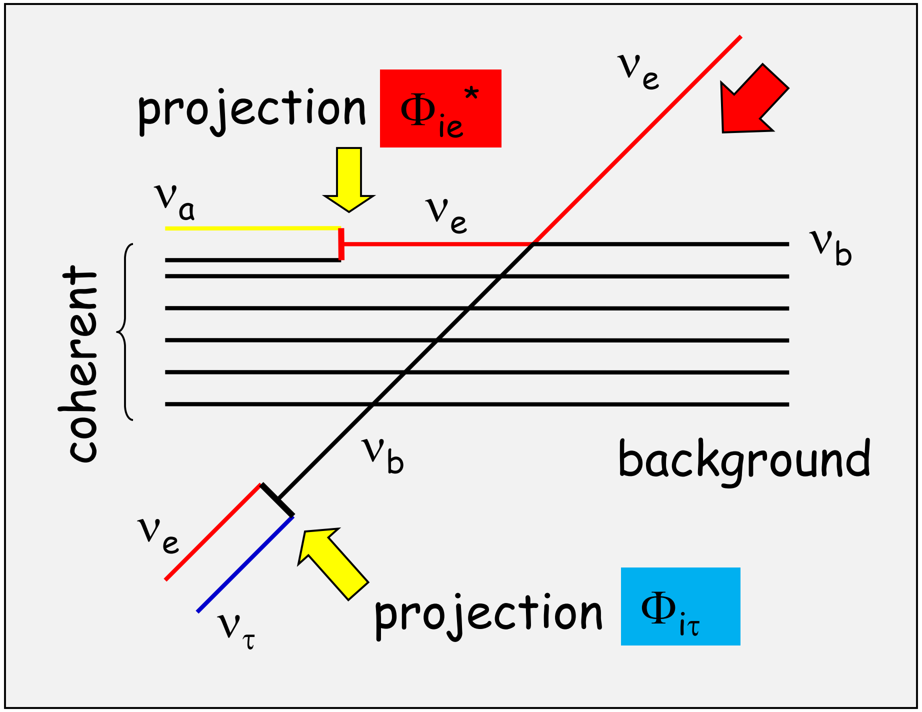
J. Pantaleone

Momentum exchange = flavor exchange
→ flavor mixing

Can it be coherent?

Coherence of flavor exchange

J. Pantaleone
S. Samuel
V.A. Kostelecky



Coherence if the background is a mixture of flavor states:

for ith particle of background

$$|\nu_{ib}\rangle = \Phi_{ie}\,|\nu_e\rangle + \Phi_{i\tau}\,|\nu_\tau\rangle$$

Φ_{ie} - ($\Phi_{i\tau}$ -) amplitude to find ν_e (ν_τ) in ith bkgr. neutrino

Inverting

$$|\nu_e\rangle = \Phi_{ie}^*\,|\nu_b\rangle + \Phi_{i\tau}^*\,|\nu_a\rangle$$

transition $\nu_e + \nu_b \rightarrow \nu_\tau + \nu_b$

with amplitude ~ $\Phi_{ie}^*\Phi_{i\tau}$ and unchanged background

→ summation over background neutrinos is coherent

This generates flavor non - diagonal potential $V_{e\tau} \sim \Sigma_i\, \Phi_{ie}^*\Phi_{i\tau}$

also diagonal

Neutrino term in the Hamiltonian

Contribution to the Hamiltonian in the flavor basis

$$H_{\nu\nu} = \sqrt{2}\, G_F \Sigma_i (1 - v_e v_b) \begin{pmatrix} |\Phi_{ie}|^2 & \Phi_{ie}^* \Phi_{i\tau} \\ \Phi_{ie}\Phi_{i\tau}^* & |\Phi_{i\tau}|^2 \end{pmatrix} \quad \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix}$$

where $\Phi_{ie} = \sqrt{P_{be}}$ $\quad \Phi_{i\tau} = \sqrt{P_{b\tau}}$

The Hamiltonian in symmetric form:

$$H_{\nu\nu} = \frac{1}{2} \begin{pmatrix} V_\nu & 2\overline{V}_\nu e^{i\phi} \\ 2\overline{V}_\nu e^{-i\phi} & - V_\nu \end{pmatrix}$$

where

$$V_\nu \sim \sqrt{2}\, G_F\, n(1 - v_e v_b)\, P_{be} \qquad \overline{V}_\nu \sim \sqrt{2}\, G_F\, n(1 - v_e v_b) \sqrt{P_{be} P_{b\tau}}$$

The effective coupling constants in V include probabilities

Total Hamiltonian

$$H = \frac{1}{2}\begin{pmatrix} -\cos 2\theta\ \omega_p + V_e + V_\nu & \sin 2\theta\ \omega_p + 2\bar{V}_\nu e^{i\phi} \\ \sin 2\theta\ \omega_p + 2\bar{V}_\nu e^{-i\phi} & \cos 2\theta\ \omega_p - V_e - V_\nu \end{pmatrix} \qquad \begin{bmatrix} \nu_e \\ \nu_\tau \end{bmatrix}$$

includes the vacuum contribution: $\omega_p = \Delta m^2/2E$ and usual matter potential V_e

Neutrino potentials

$$V_\nu \sim V_\nu^0 (1 - P^B_{e\tau})$$

$$\bar{V}_\nu \sim V_\nu^0 \sqrt{P^B_{e\tau}(1 - P^B_{e\tau})}$$

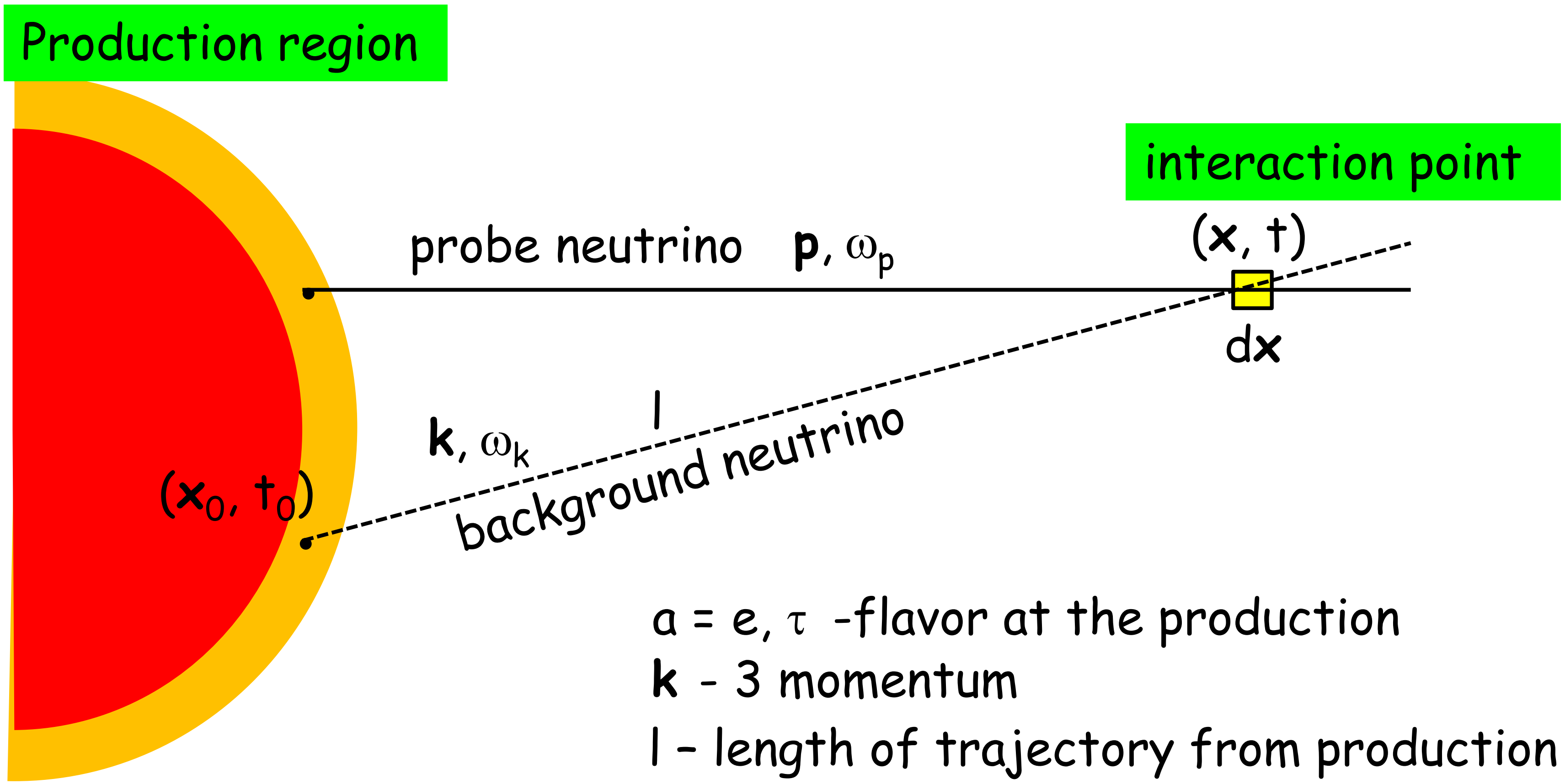
if ν_e is produced

$\phi = Arg\,[\Phi_e \Phi_\tau^*]$

$P^B_{e\tau}(x)$ – effective transition probability of the background neutrinos

In the central parts of collapsing star $V_e \gg V_\nu \gg \omega$

Summation over background neutrinos



Production point - the point of last inelastic collision

a = e, τ -flavor at the production

k - 3 momentum

l – length of trajectory from production ($\mathbf{x}_0$, t_0) to interaction point ($\mathbf{x}$, t)

→ (a, l, **k**) characterize a given mode

($\mathbf{x}$, t), **k** and l determine ($\mathbf{x}_0$, t_0)

Total Hamiltonian and potentials

After integration

$$H = \frac{1}{2}\begin{pmatrix} -\cos 2\theta\, \omega_p + V_e + V_\nu & \sin 2\theta\, \omega_p + 2\overline{V}_\nu e^{i\phi} \\ \sin 2\theta\, \omega_p + 2\overline{V}_\nu e^{-i\phi} & \cos 2\theta\, \omega_p - V_e - V_\nu \end{pmatrix} \quad \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix}$$

Potentials

$$V_\nu = \int d\mathbf{k} \int dl \; [V_\nu^e(\mathbf{k}, l) - V_\nu^\tau(\mathbf{k}, l)]\,(1 - P_{e\tau}(\mathbf{k}, l))$$

$$\overline{V}_\nu e^{i\phi} = \int d\mathbf{k} \int dl \; [V_\nu^e(\mathbf{k}, l) - V_\nu^\tau(\mathbf{k}, l)]\, e^{i\phi(\mathbf{k}, l)} \sqrt{P_{e\tau}(\mathbf{k}, l)\,(1 - P_{e\tau}(\mathbf{k}, l))}$$

where

$$V_\nu^a(\mathbf{k}, l) = \sqrt{2}\, G_F\, n_\nu^a(\mathbf{k}, l)\,(1 - \mathbf{v}_p \mathbf{v}_k) \qquad a = e, \tau$$

$n_\nu^a(\mathbf{k}, l)$ - number density of neutrinos emitted from $(\mathbf{x}_0, t_0)$ and arriving at the point $(\mathbf{x}, t)$

Removing the phase

Off-diagonal term: introduce real V' and the phase ϕ' as

$$\sin 2\theta\, \omega_p + 2\overline{V}_\nu e^{i\phi} = V' e^{-i\phi'} \qquad (*)$$

Transformation of the fields

$$\psi = U\psi' \qquad U = \mathrm{diag}\,(e^{0.5\, i\phi'},\ e^{-0.5 i\phi'})$$

it does not change flavor probabilities.

Hamiltonian for ψ'

$$H = \tfrac{1}{2}\begin{pmatrix} V^r(t) & V'(t) \\ V'(t) & -V^r(t) \end{pmatrix}$$

$$V^r(t) = V_e + V_\nu - \cos 2\theta\, \omega_p + d\phi'/dt$$

$V'(t)$ is def. in (*)

Probe neutrino propagation in external neutrino potentials
(as in the case of NSI but) with non-trivial time dependence

The problem

Evolution equation for the probe particle

$$i \frac{d\,\Phi_p}{d\,t} = H\,(\Phi_{ib}{}^{x})\,\Phi_p$$

$x = ct$ – point along the ν_p trajectory

Total Hamiltonian depends on the wave functions of all background neutrinos $\Phi_{ib}{}^{x}$ which cross the probe neutrino trajectory in a point **x**

To find $\Phi_{ib}{}^{x}$ one needs to solve the corresponding evolution equation for each ν_{ib}

$$i \frac{d\,\Phi_{ib}}{d\,t} = H\,(\Phi_{jb}{}^{y})\,\Phi_{ib}$$

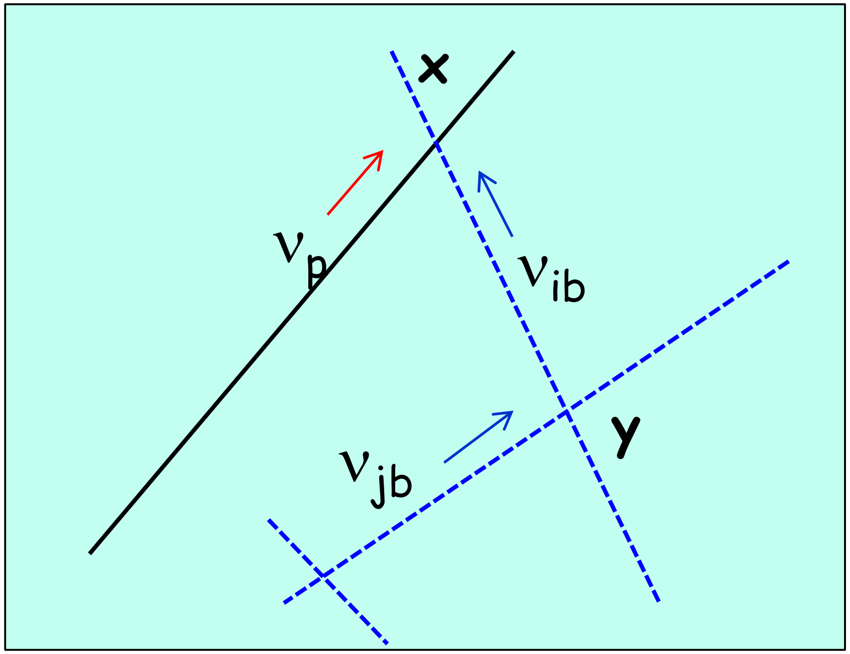
$y = ct$ along the ν_{ib} trajectory

Here H depends on the wave functions of all background neutrinos $\Phi_{jb}{}^{y}$ which cross the ν_{ib} trajectory in a point **y**

The equation should be integrated over **y** from the ν_{ib} production point to **x**

→ Huge number of coupled equations

The problem



Stationary case: pattern does not depend on time

Modes involved are characterized by ($\mathbf{x}_0$, $\mathbf{k}$, a)

a = e, τ -flavor at production

$\mathbf{x}_0$ - production point

$\mathbf{k}$ - momentum

Specific limits of integration for each point

Modes with the same set ($\mathbf{x}_0$, $\mathbf{k}$, a) evolve in the same way

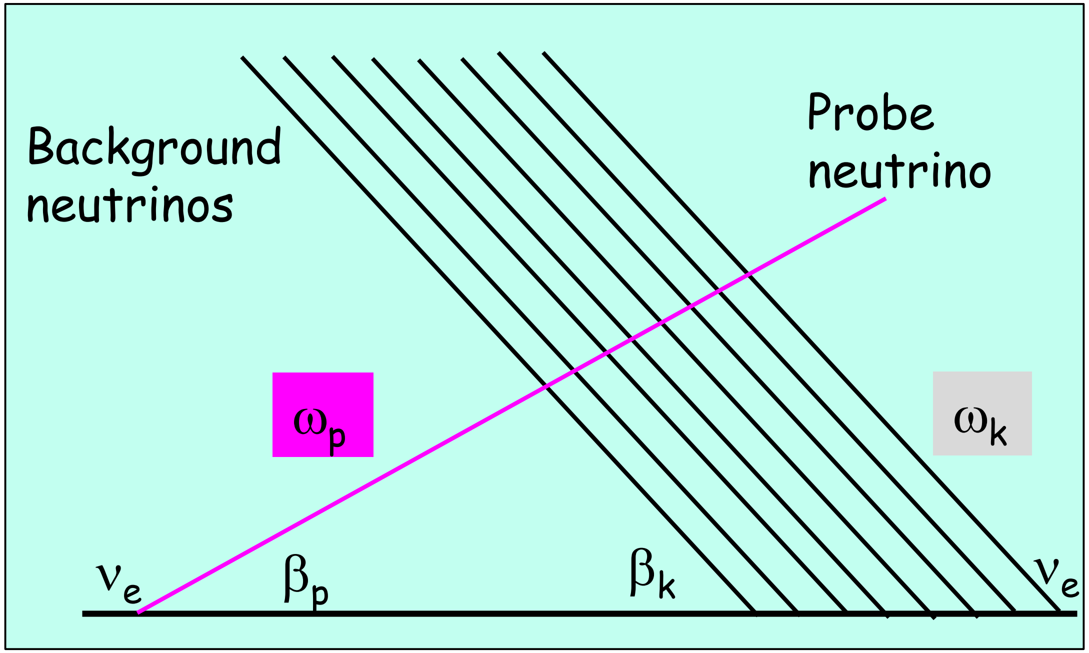
Discretize parameters – numerical solutions

Solvable example

Neutrinos in neutrino flux

R. S. L. Hansen, A. Yu. S.
1801.09751 [hep-ph]

Flux of collinear background neutrinos



→ no $\nu\nu$– interactions in the flux

Still coherent flavor exchange

$$\omega = \frac{\Delta m^2}{2E}$$

$$V_\nu = V_\nu^0 (1 - 2 P_{e\tau}) \qquad \bar{V}_\nu = V_\nu^0 \sqrt{P_{e\tau}(1 - P_{e\tau})} \qquad V_\nu^0 = \sqrt{2}\, G_F n_\nu (1 - \mathbf{v}_p \mathbf{v}_k)$$

$$P_{e\tau} = \sin^2 2\theta_m \, \sin^2 \tfrac{1}{2} A_\beta \Delta_m t \qquad \phi_m = Arg\,[\Phi_e \Phi_\tau^*] = A_\beta \Delta_m t$$

$$A_\beta = \frac{\sin\beta_p}{\sin\beta_k}$$

level splitting

Potentials – periodic functions of time → parametric effects

Potentials: time dependence

Combining different terms in the Hamiltonian (in rotating frame)

$$H = \frac{1}{2}\begin{pmatrix} V^r(t) & V'(t) \\ V'(t) & -V^r(t) \end{pmatrix}$$

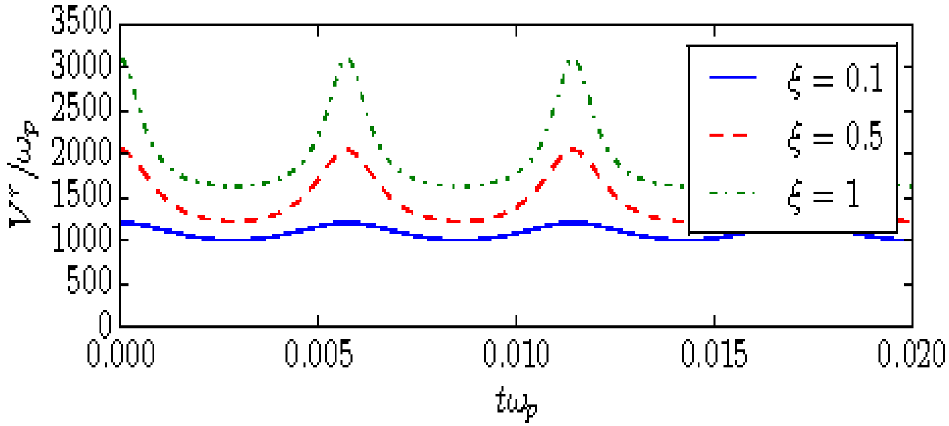
including usual matter potential V_e which dominates

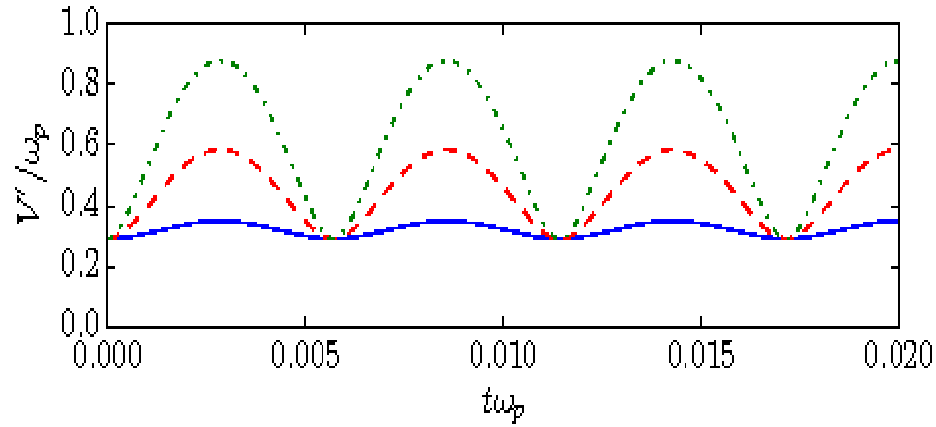
$$V_e = 10^3\ \omega_k$$

and vacuum terms

Neutrino to matter potential ratio

$$\xi = \frac{V_\nu^{\,0}}{V_e}$$





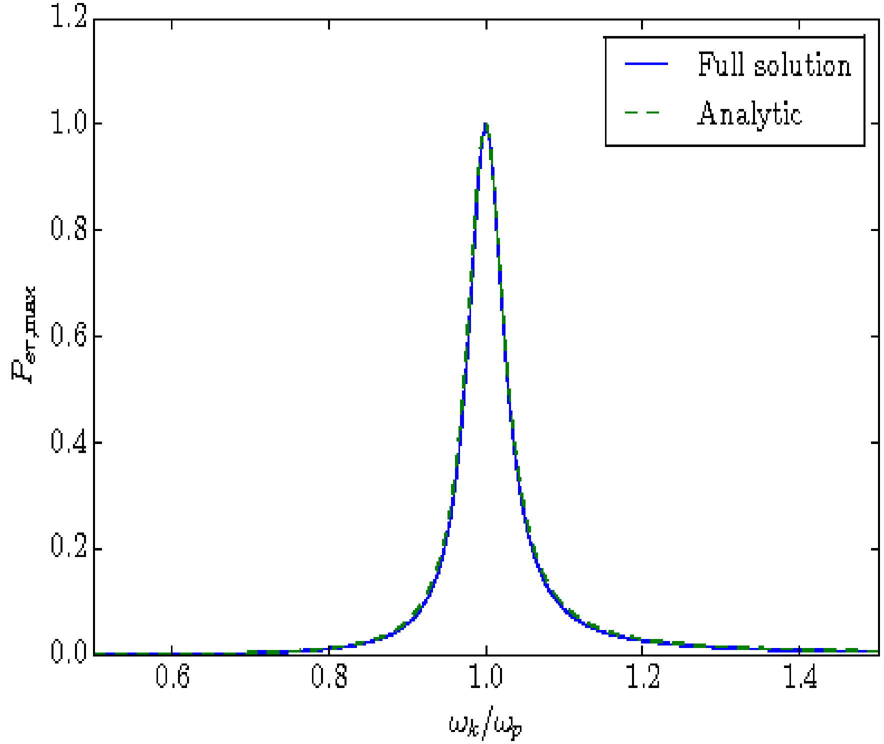
V_r and V' as functions of the time for different values of ξ. $A_\beta = 1.1001$, $\omega_p = \omega_k$

Parametric resonance

Dependence of the depth of parametric oscillations on ω_k / ω_p

$A_\beta = 1.1001$, $V_\nu^0 = 100\ \omega_p$

$V_\nu^0 = 100\ \omega_p$



Parametric resonance condition:

$$- \cos 2\theta\ \omega_p + V_e + V_\nu^0 = A_\beta\ \Delta_m$$

Frequency of oscillations of the probe neutrino determined by the averaged density

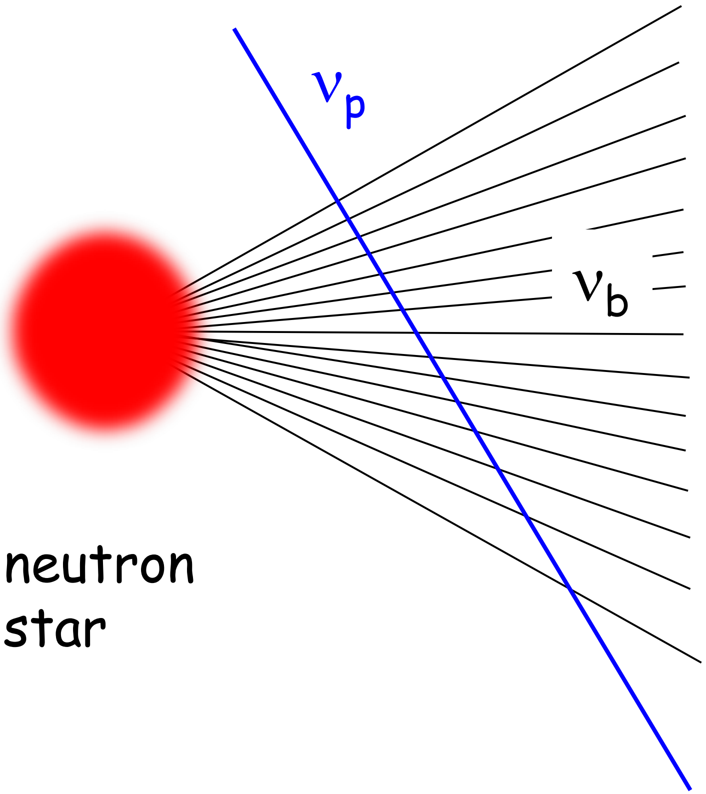
Frequency of modulations of the potentials

Width of the resonance

$$\Gamma / \omega_k = \sin 2\theta\ \xi\ (1 + A_\beta)$$

proportional to ξ

Diverging flux



neutron star

Far from source, diverging flux, $\nu\nu$ interactions in the flux are negligible.

External probe neutrino

The angle between the probe and flux neutrinos changes A_β and neutrino density change

Realization of simplified example

This case is reduced to the case of varying density

An effective theory

Effective theory of collective oscillations

Above the neutrinosphere collective effects can be completely described by evolution of individual neutrinos in external potentials produced by usual matter and other neutrinos.

Flavor diagonal $V_\nu(t)$ and flavor changing $\bar{V}_\nu(t)$ potentials are generated

All possible collective oscillations effects are consequences of particular time dependences of the potentials

The problem is reduced to determination of time dependences of the potentials

Consider effects of inelastic interactions in certain approximations

Effective theory approach

**

Since it is not feasible to perform explicit computations of these potentials, the approach is

Extract certain properties of the potentials from their general expressions

Some integrations in the potentials which correspond to averaging over energy or over neutrino production point can be done explicitly (approximations) using general form of potentials

Assumptions and conjectures on the time dependence of potentials can be introduced

Construct potentials using various limits, existing numerical results, simplified solvable examples

Properties of potentials

Potentials are integrals of oscillation amplitudes which have oscillatory dependence on time.

Therefore potentials are also expected to be oscillatory functions of time determined by intrinsic frequencies of the system

$$V_e\ ,\ V_\nu,\ \omega_p,\ \omega_k$$

Furthermore, there is the hierarchy of frequencies

$$V_e > V_\nu \gg \omega$$

Inverse problem:

Find potentials and their time dependences or general conditions for potentials which can lead to effects found in certain simplified models

Synchronized oscillations

Fast flavor transitions

As parametric effect with increasing amplitude of periodic modulations

Bi-polar oscillations

As parametric effect for negligible ω → transition is the same for neutrinos and antineutrinos

Spectral splits

Check how realistic are these conditions in realistic supernova

Two effects of enhancement

Phase velocity cancellation

Rotation of the fields that eliminates the phase from the off-diagonal terms leads to appearance of phase velocity in the diagonal terms

$$V^r(t) = V_e + V_\nu - \cos 2\theta\, \omega_p - d\phi/dt$$

if $d\phi/dt \sim V_e + V_\nu$ strong cancellation → matter suppression is removed

Oscillations with maximal depth and frequency $1/V_\nu$

Parametric enhancement

V_ν and $\overline{V}_\nu$ - periodic functions

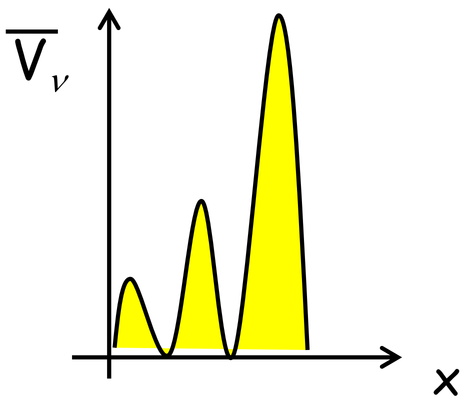
Parametric resonance if the frequency of modulations of potentials coincides with eigenfrequency of the probe neutrino

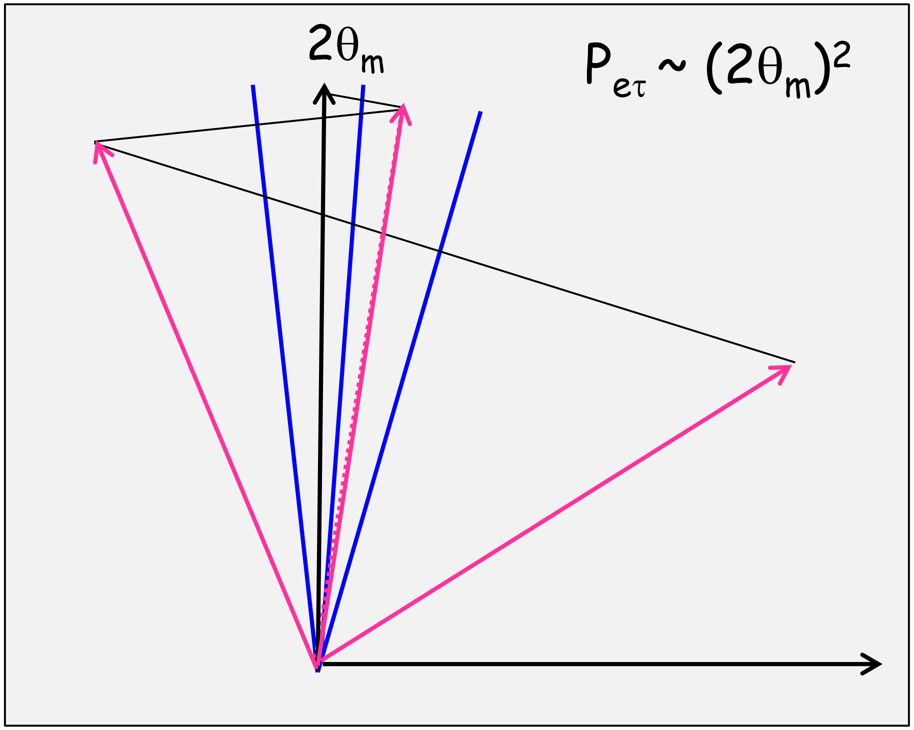
Instabilities and fast transition in the flavor field

Exponential grow of the transition probabilities

Two conditions:

1. Phase velocity cancellation
2. Parametric enhancement induced by modulations of the neutrino potentials with growing amplitude

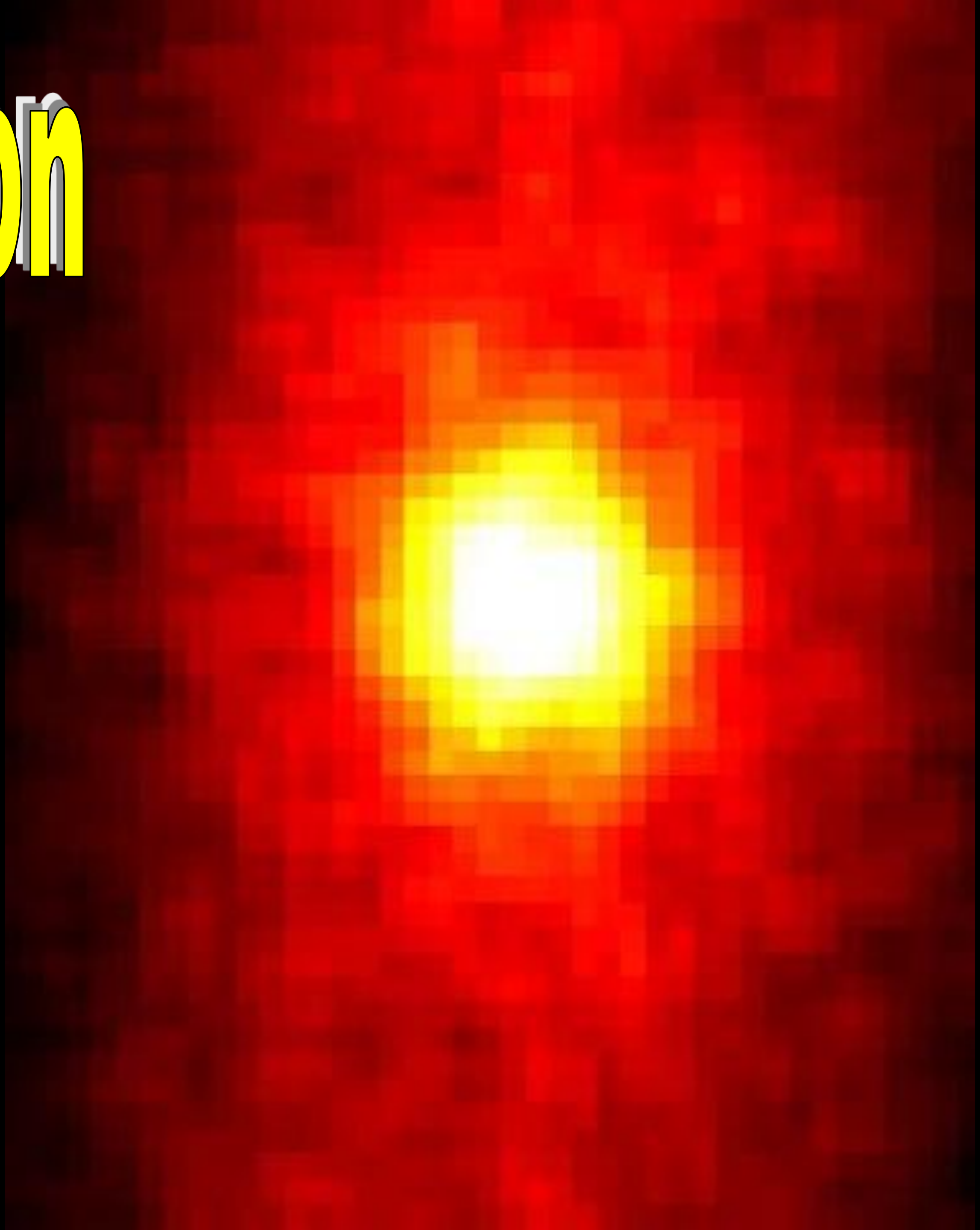




$\Delta\theta_m / \Delta t \sim \theta_m$

The cone angle and transition probability increase exponentially

In conclusion



Theory of the neutrino flavor transformations will play the key role in future developments in neutrino physics:

- establishing mass hierarchy,
- measurements of CP phase,
- searches for sterile neutrinos,
- oscillation tomography of the Earth,
- understanding supernova neutrinos
- searches for new physics

Theory of neutrino flavor transformation is to a large extent elaborated. Still theory of collective transformations in realistic supernovae is missing. Some subtle aspects – to be clarified. Some small effects become accessible and important with new experimental precision.

New effects of flavor transformations in the presence of new physics can emerge and should be explored.
Neutrino probes of the Dark Universe

Fuzzy dark matter

A. Berlin, 1608.01307 [hep-ph]

Modulating mass?

Ultra-light scalar DM

$$\phi(t, x) \sim \frac{\sqrt{2\rho(x)}}{m_\phi} \cos(m_\phi t)$$

Couples to neutrinos $g_\phi \, \phi \, \nu_i \, \nu_j + \ldots$ → mass states oscillate

give contribution to neutrino mass and modifies mixing

$$\delta m(t) = g_\phi \, \phi(t) \qquad \Delta\theta_m(t) = g_\phi \, \phi(t) / \Delta m_{ij}$$

Neutrinos propagating in this field will experience variation of mixing with frequency given by m_ϕ

For $m_\phi = 10^{-22}$ eV, the modulation length $l_{mod} = 2\pi/m_\phi = 10^{17}$ cm

Parametric resonance: $l_{mod} = l_\nu$

For solar mass splitting $E_{res} = 3 \times 10^3$ PeV $(10^{-22}\ \text{eV}/m_\phi)$

⇨ $E_{res} = 3$ PeV for $m_\phi = 10^{-19}$ eV

Evolution equation

Ensemble of neutrino polarization vectors $\mathbf{P}_\omega$

Negative frequencies for antineutrinos

$$d_t \mathbf{P}_\omega = (-\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{P}) \times \mathbf{P}_\omega$$

Vacuum mixing term

$\mathbf{B} = (\sin 2\theta\ ,\ 0,\ \cos 2\theta)$

$\omega = \Delta m^2 / 2E$

Usual matter potential

$\mathbf{L} = (0,\ 0,\ 1)$

$\lambda = V = \sqrt{2}\, G_F\, n_e$

Collective vector

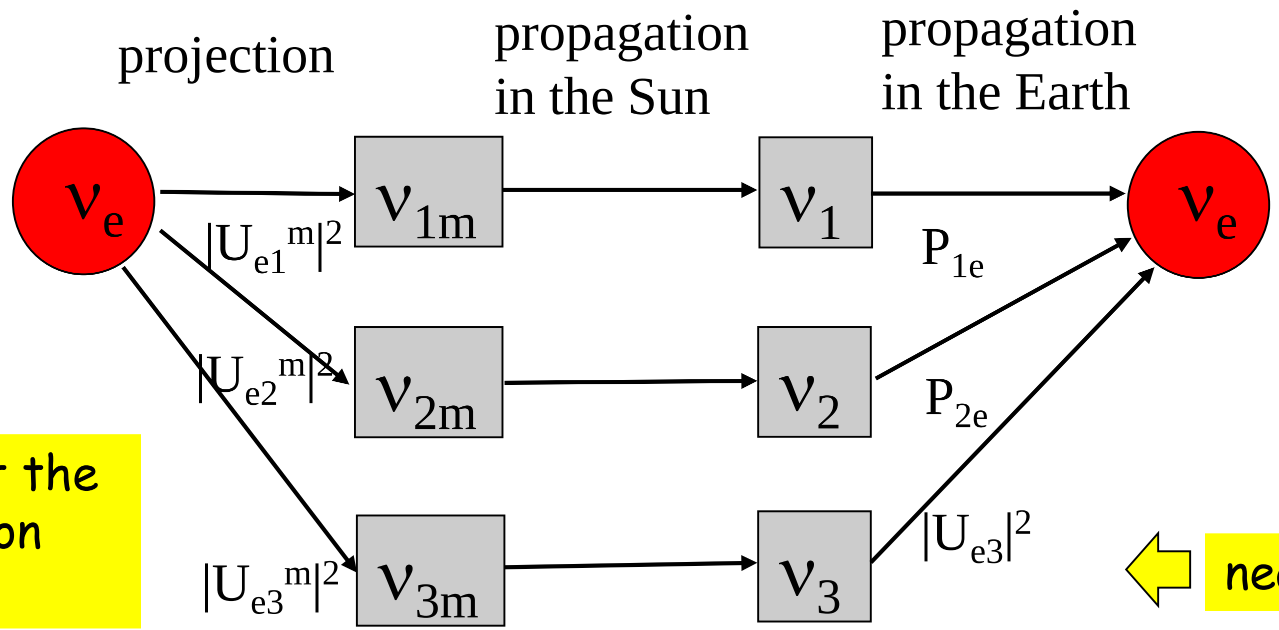
$$\mathbf{P} = \int_{-\inf}^{+\inf} d\omega\ \mathbf{P}_\omega$$

$\mu = \sqrt{2}\, G_F\, n_\nu\, (1 - \cos\theta_{\nu\nu})$

The term describes collective effects

Scheme of transitions

and between the Sun and the Earth



mixing at the production point n_0

nearly decouples

adiabatic conversion

oscillations in multi-layer medium

$$P_{ee} = \Sigma_i | U_{ei}^m(n_0)|^2 P_{ie}$$

during the day $P_{ie} = | U_{ei}|^2$ scale invariant

3ν – oscillations

$$\nu_f = U_{23} I_\delta U_{13} U_{12} \nu_{mass}$$

$U_{13} U_{12} \nu_{mass} = \tilde{\nu}$

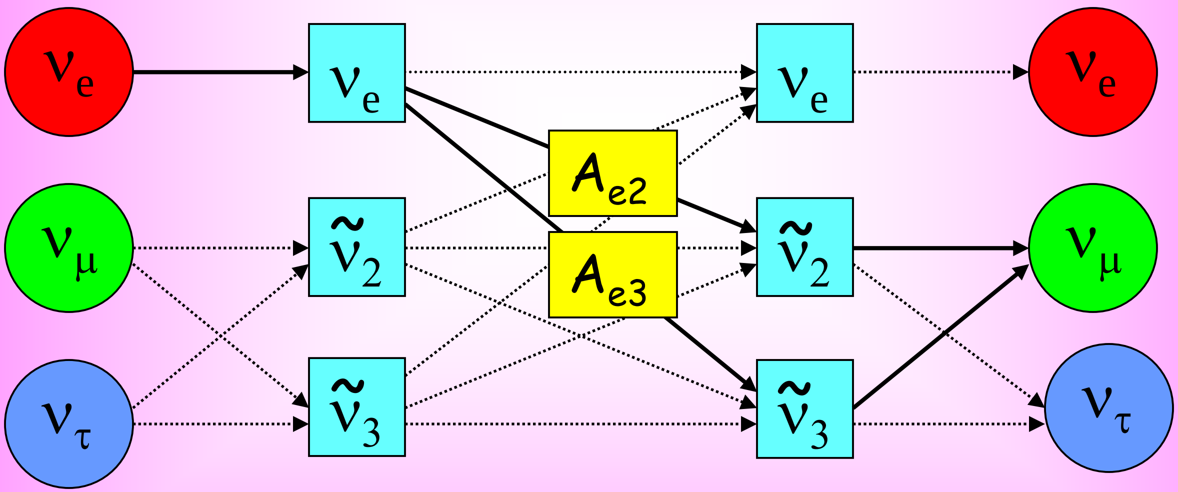
CP-violation and 2-3 mixing are excluded from dynamics of propagation

Propagation basis

$$\nu_f = U_{23} I_\delta \tilde{\nu}$$

$$I_\delta = \text{diag}(1, 1, e^{i\delta})$$

Matter potential is unchanged in new basis



projection propagation projection

A_{22} A_{33} A_{23}

CP appears in projection only

For instance: $A(\nu_e \rightarrow \nu_\mu) = \cos\theta_{23} A_{e2} e^{-i\delta} + \sin\theta_{23} A_{e3}$

Total Hamiltonian and potentials

$$H = \frac{1}{2}\begin{pmatrix} -\cos 2\theta\, \omega_p + V_e + V_\nu & \sin 2\theta\, \omega_p + 2\bar{V}_\nu e^{i\phi} \\ \sin 2\theta\, \omega_p + 2\bar{V}_\nu e^{-i\phi} & \cos 2\theta\, \omega_p - V_e - V_\nu \end{pmatrix} \quad \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix}$$

Potentials

$$V_\nu \sim V_\nu^0 (1 - P^B_{e\tau})$$

$$\bar{V}_\nu \sim V_\nu^0 \sqrt{P^B_{e\tau}(1 - P^B_{e\tau})}$$

$P^B_{e\tau} = P_{e\tau}$

non-linearity

$P^B_{e\tau}(x)$ – effective transition probability of the background neutrinos

$$V_e \gg V_\nu \gg \omega$$

$H^{diag} \sim V_e$ $\qquad$ $H^{non-diag} \sim V_\nu^0 \sqrt{P^b_{e\tau}} < V_\nu^0$ $\qquad$ $\phi \sim \int dt\, \Delta H$

$\Delta H \sim V_e$ $\qquad$ $d\phi/dt \sim V_e$

if $\omega \ll V_\nu$, H depends on potentials only – evolution of neutrinos and antineutrinos is the same → bi-polar oscillations

Conditions for strong transformations **

1. Resonance oscillations

$V^r \ll V'$ ⇒ $V_e + V_\nu + d\phi'/dt - \cos 2\theta\ \omega_p \sim 0$

or $d\phi'/dt \sim - V_e - V_\nu$

The system oscillate with maximal depth and frequency ~ V'

If there is no significant modulations of the non-diagonal element

2. Adiabatic conversion

Performing series of transformations of fields - exclude fast time variations in V^r and V'

In new frame $\tilde{V}^r$ and $\tilde{V}'$ may satisfy adiabatic condition → strong transition if V^r changes from $\tilde{V}^r \gg \tilde{V}'$ to $\tilde{V}^r \ll \tilde{V}'$

... continued

**

3. Parametric enhancement, resonance

Potentials are modulated by periodic functions, so that the mixing angle in medium $\tan 2\theta_m = - V^l / V^r$ varies with a period T_θ

Parametric enhancement if the frequency of modulations coincides with eigenfrequency of the system $1/T_p$

$$T_\theta = T_p$$

$$T_p = \frac{2\pi}{\sqrt{\langle V^r \rangle^2 + \langle V^l \rangle^2}}$$

$\langle V^r \rangle$, $\langle V^l \rangle$ - potentials averaged over modulations

Large transition probability develops over many periods

Solvable example

Consequence of finite energy resolution /reconstruction function