Fluctuation theorem for quantum correlated systems

Jungjun Park

Collaborators: Vlatko Vedral, Hyunchul Nha, Sang Wook Kim

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- Introduction(literature review)
- Formalism and main results
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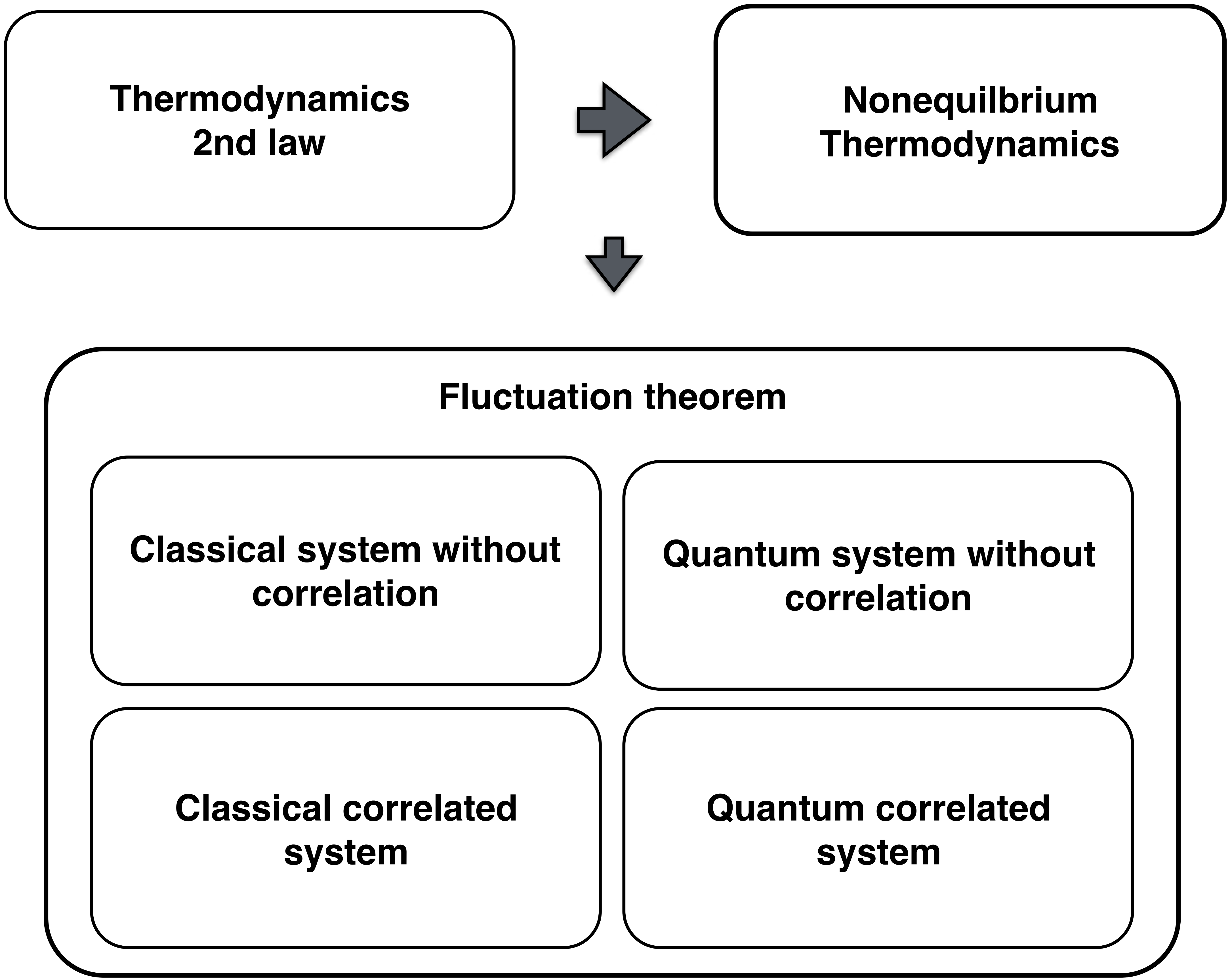
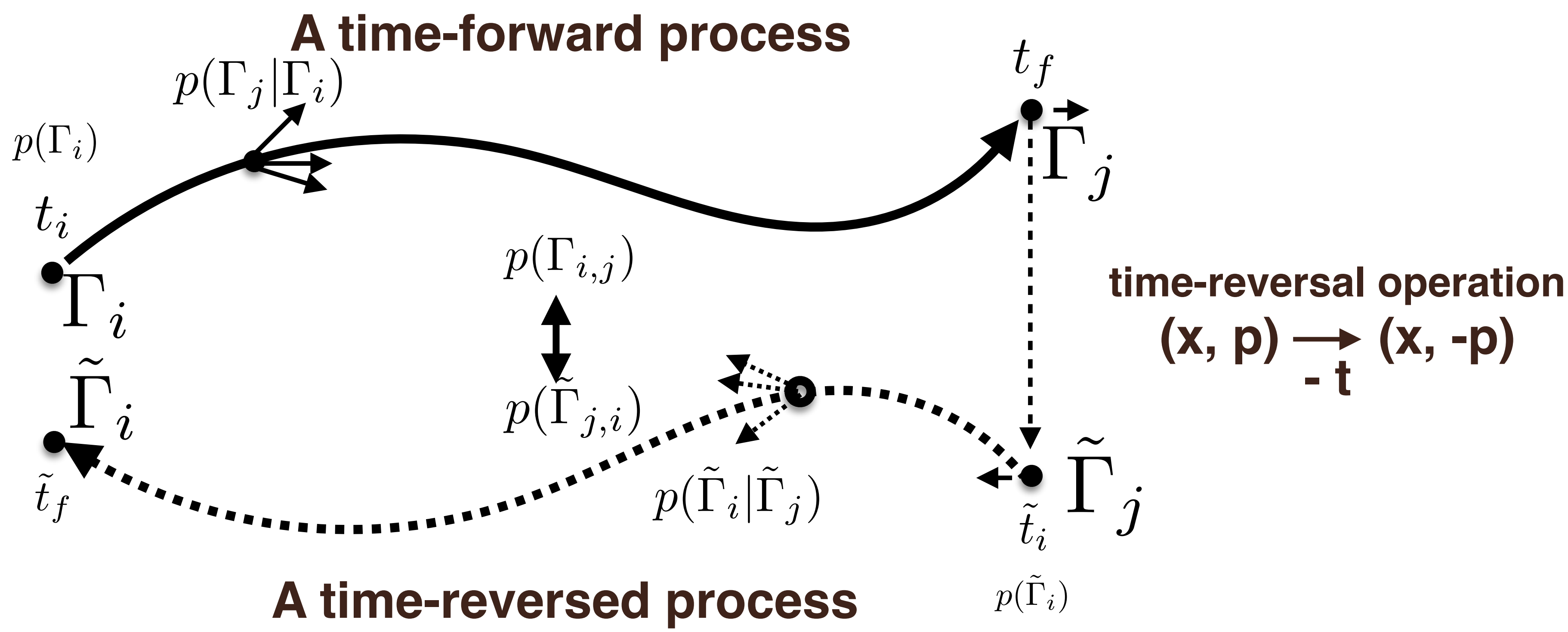
Overview


Thermodynamics 2nd law
Nonequilbrium Thermodynamics
Fluctuation theorem
Classical system without correlation
Quantum system without correlation
Classical correlated system
Quantum correlated system

Classical fluctuation theorem for a single particle regime

Example: Stochastic process



$p(\Gamma_{i,j})$: probability of a trajectory from i to j

$p(\tilde{\Gamma}_{j,i})$: probability of its time-reversed trajectory from j to i

Fluctuation theorems

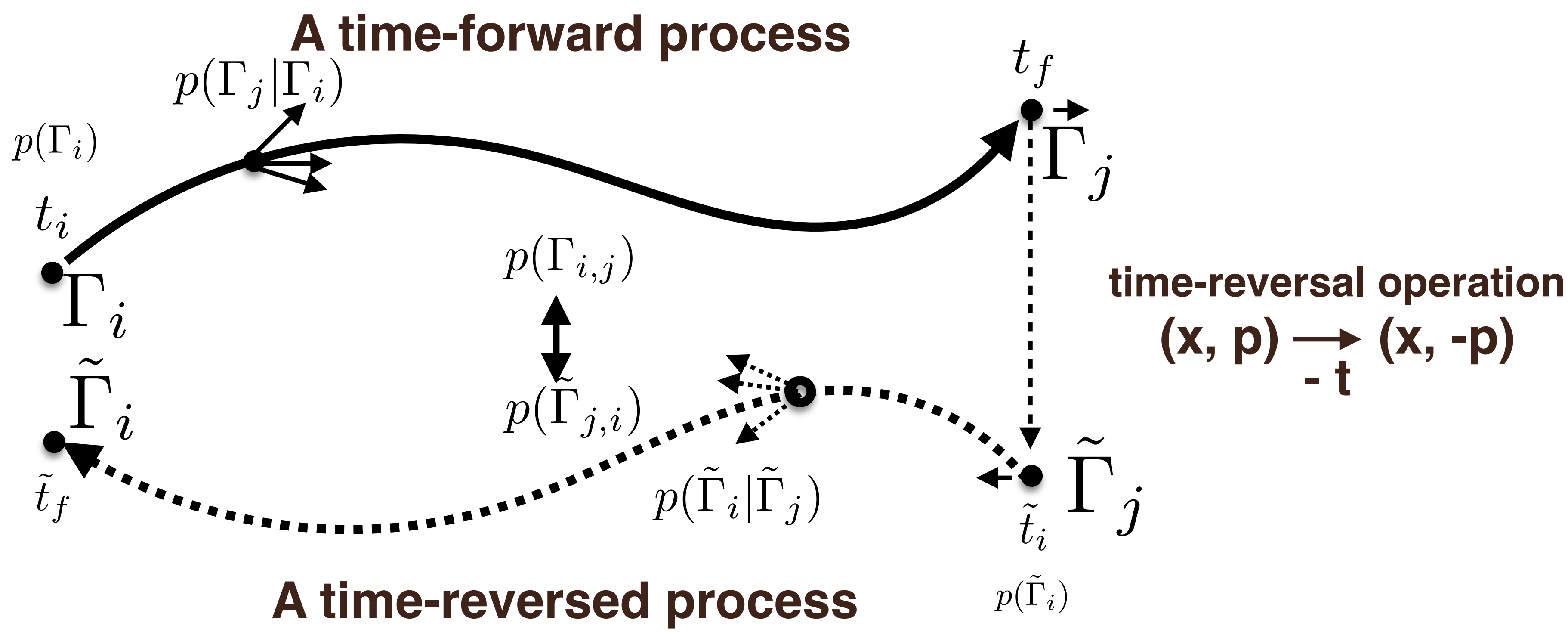
$$\frac{p(\tilde{\Gamma}_{j,i})}{p(\Gamma_{i,j})} = e^{-\sigma_{i,j}} \qquad \langle e^{-\sigma} \rangle = 1$$

U. Seifert, PRL (2005)

D. J. Evans, et al, PRL (1993) G. Gallavotti et al, PRL (1995) G. E. Crooks, PRE (1999) C. Jarzynski, PRL (1997)

Classical fluctuation theorem for a single particle regime

Example: Stochastic process



$p(\Gamma_{i,j})$: probability of a trajectory from i to j

$p(\tilde{\Gamma}_{j,i})$: probability of its time-reversed trajectory from j to i

Fluctuation theorems

$$\langle \Delta S \rangle - \beta \langle Q \rangle \geq 0 \longleftarrow \langle \sigma \rangle \geq 0 \longleftarrow \langle e^{-\sigma} \rangle = 1$$

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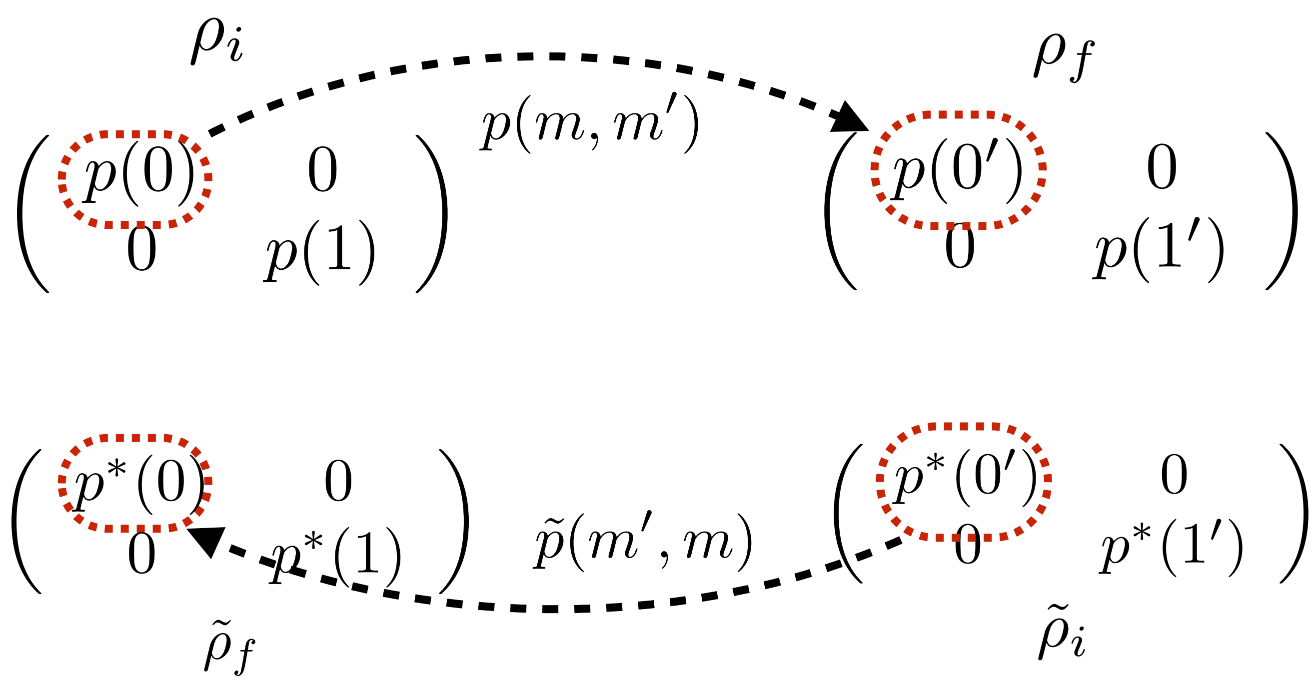
Quantum fluctuation theorem for a single particle regime

Quantum process

$|m\rangle \rightleftarrows |m'\rangle$

A time-forward process



A time-reversed process

$p(m, m')$: time-forward joint probability between m and m'

$\tilde{p}(m', m)$: time-reversed joint probability between m' and m

Quantum Fluctuation theorems

$$\frac{\tilde{p}(m', m)}{p(m, m')} := e^{-\sigma_{m,m'}} \qquad \langle e^{-\sigma} \rangle = 1$$

H. Tasaki, arXiv (2000)
J. Kurchan, arXiv (2000)
M. Campisi, et al, PRL (2010)

Quantum fluctuation theorem for a single particle regime

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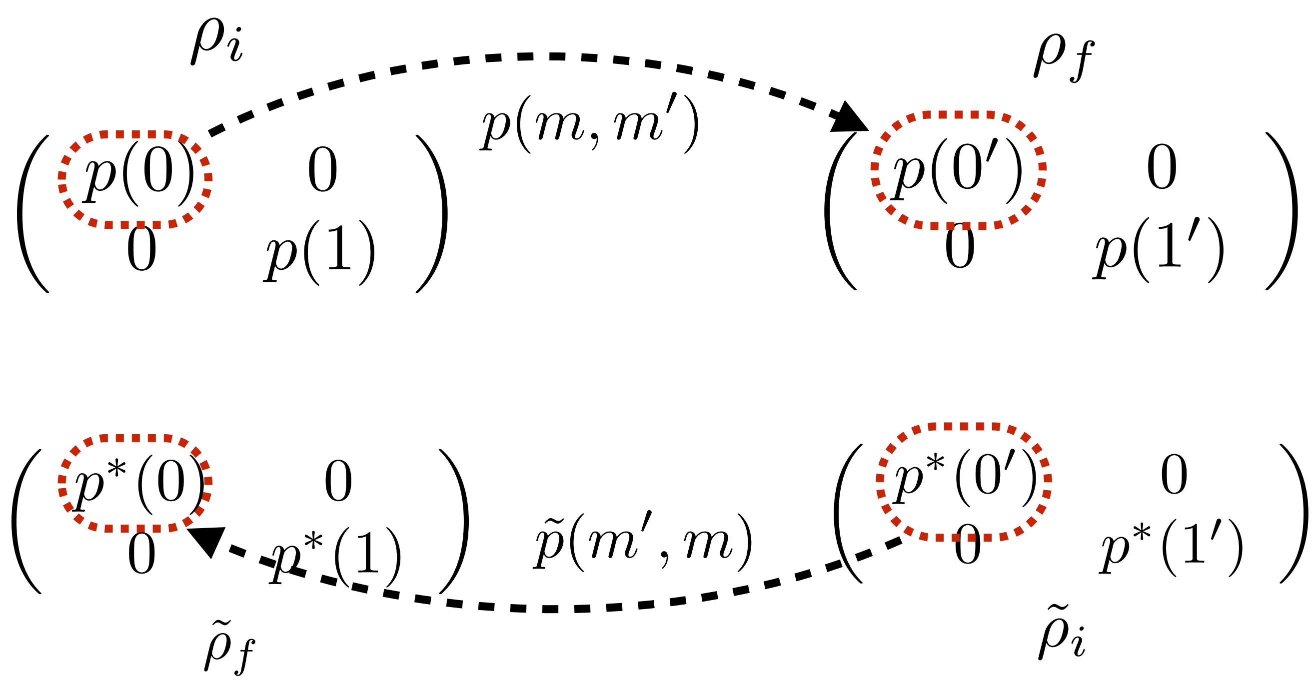

A time-reversed process

$$p(m, m') = p(m'|m)p(m) = |\langle m'|\Phi|m\rangle|^2 p(m)$$

$$\tilde{p}(m', m)$$

Quantum Fluctuation theorems

$$\frac{\tilde{p}(m', m)}{p(m, m')} := e^{-\sigma_{m,m'}} \qquad \langle e^{-\sigma}\rangle = 1$$

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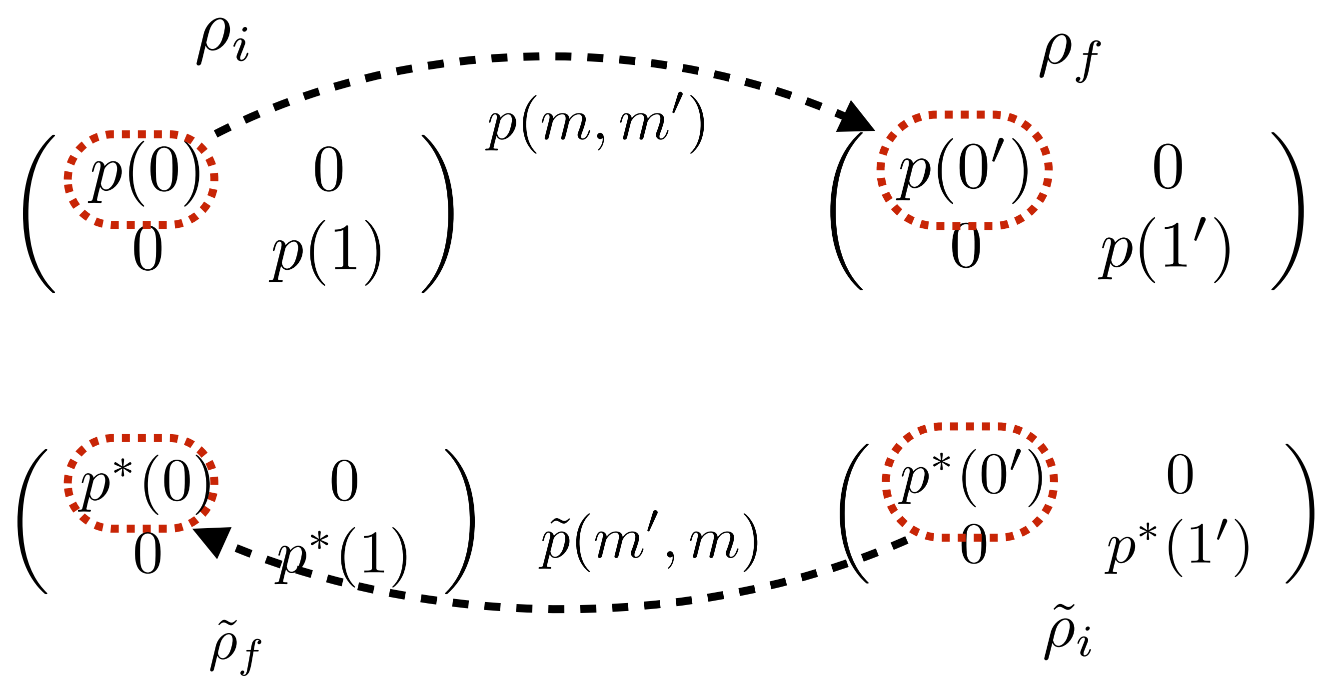
Quantum fluctuation theorem for a single particle regime

Quantum process

$$|m\rangle \rightleftarrows |m'\rangle$$

A time-forward process



A time-reversed process

$$p(m,m') = p(m'|m)p(m) = |\langle m'|\Phi|m\rangle|^2 p(m)$$

$$\tilde{p}(m',m) = \tilde{p}(m|m')p(m') = |\langle m|\tilde{\Phi}|m'\rangle|^2 p(m)$$

Quantum Fluctuation theorems

$$\frac{\tilde{p}(m',m)}{p(m,m')} := e^{-\sigma_{m,m'}} \qquad \langle e^{-\sigma}\rangle = 1$$

H. Tasaki, arXiv (2000)
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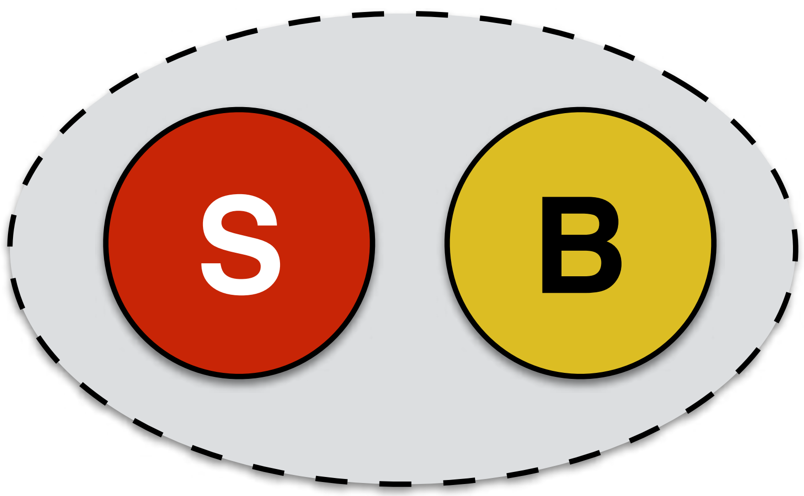
Generalisations of fluctuation theorems

- **Early generalisations without correlation**
- **Generalisations with correlation**
  - **System-environment correlation**
  - **System-system correlation in classical nonequilibrium regime**
  - **System-system correlation in quantum nonequilibrium regime**

Early generalisations without correlation

1. Weak coupling case

- System S & heat bath B in the weak coupling regime
- No correlation between S and B



C. Jarzynski, PRL (1997)
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Early generalisations without correlation

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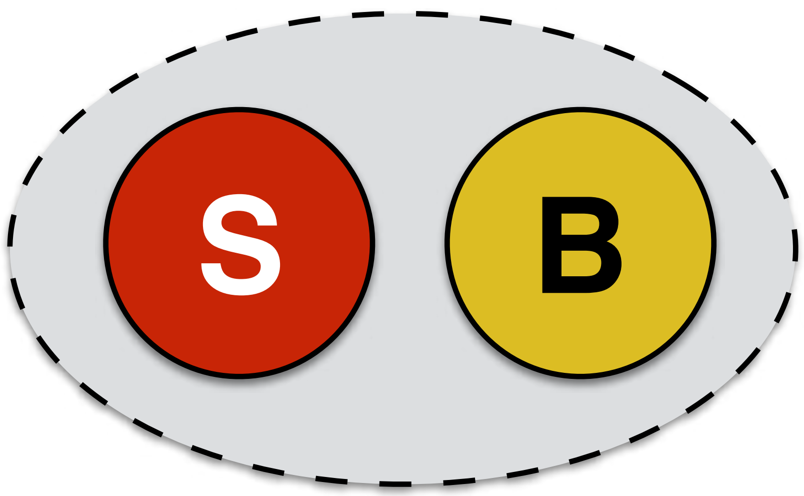
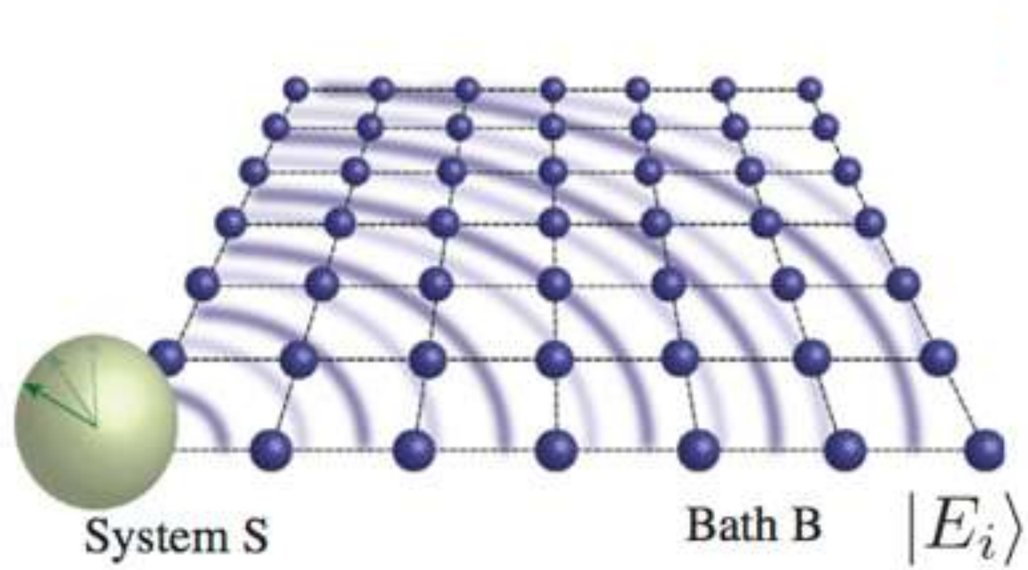


2. Strong coupling case

- Arbitrary strength of interaction Hamiltonian
- Heat bath + hyper heat bath
- No explicit consideration of correlation (S+B)

C. Jarzynski, PRL (1997)
G. E. Crooks, PRE (1999)
H. Tasaki, arXiv (2000)
J. Kurchan, arXiv (2000)
M. Campisi, et al.PRL (2010)
C. Jarzynski, J. Stat. Phys. (2000)

System-environment correlation

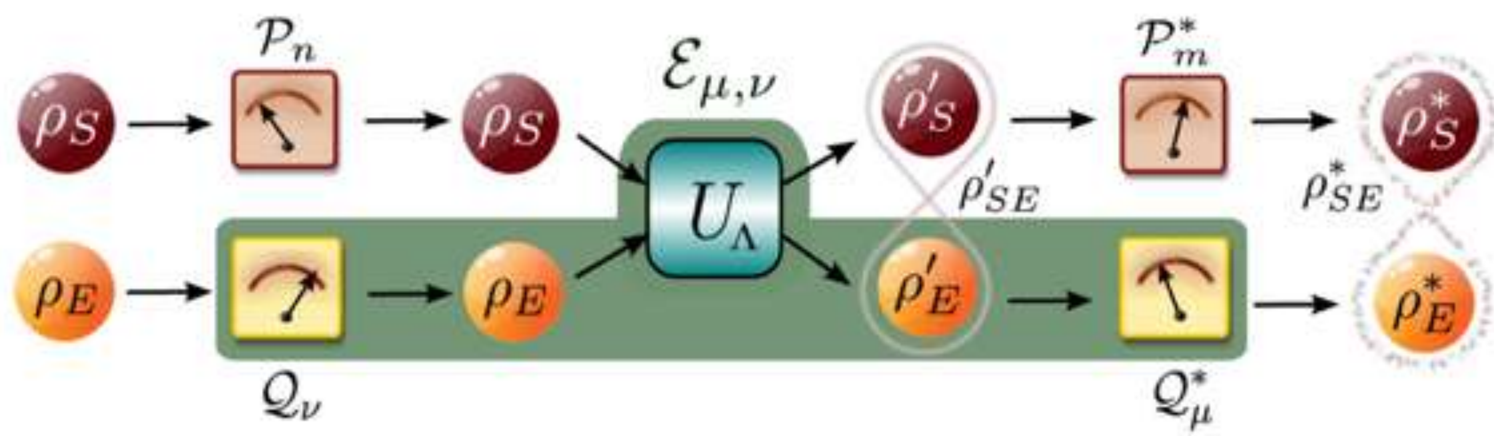


Many-body fluctuation theorem

- System & many-body particle reservoir
- Initial bath is assumed to be a pure state
- Entanglement between S and B is a central concept

Eiki Iyoda, et al. PRL (2017)

System-environment correlation

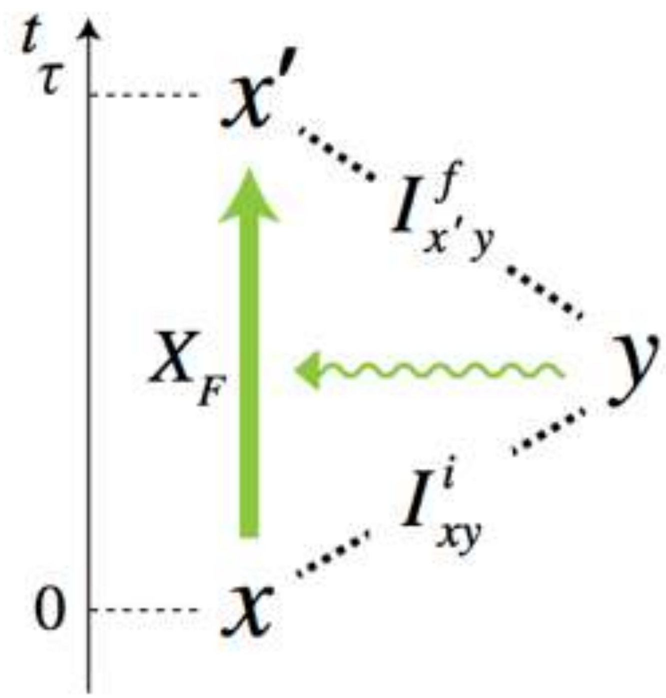


Heat-bath correlation fluctuation theorem

- System & squeezed boson heat reservoir
- Correlations between S and E are involved
- Description of correlation is classical between S and E

Gonzalo Manzano, et al. PRX (2018)

Classically correlated systems interacting with heat bath



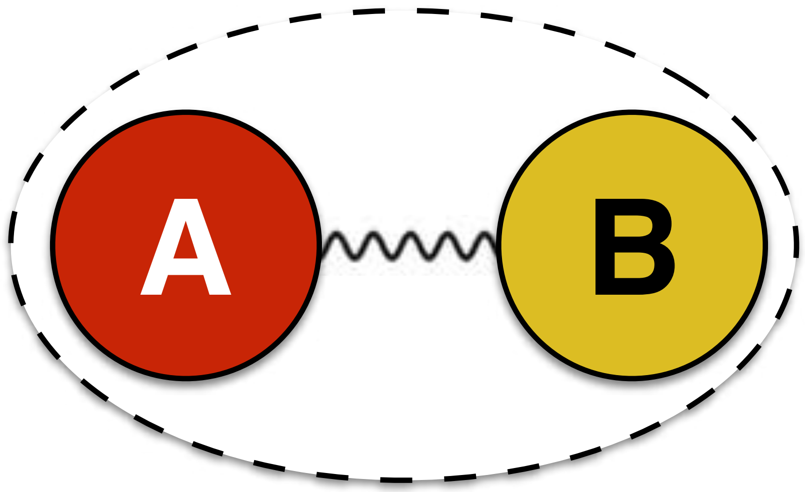
Classically correlated stochastic system

- Stochastic system + heat reservoir & external degree of freedom
- External system Y is invariant with time.
- Consider classically correlated system at t_i and t_f
- The formula of the fluctuation theorem has been changed

$$\langle e^{-\sigma} \rangle = 1 \quad \Rightarrow \quad \langle e^{-\sigma + \Delta J} \rangle = 1$$

T. Sagawa and M. Ueda, PRL. (2012)

Quantum correlated systems interacting with heat bath



<u>Fluctuation theorem for quantum correlated systems</u>

- System + system: quantum correlated bipartite systems
- Quantum phenomena of work and heat fluctuations due to correlation
- Correlation measure is still classical (Classical mutual information)

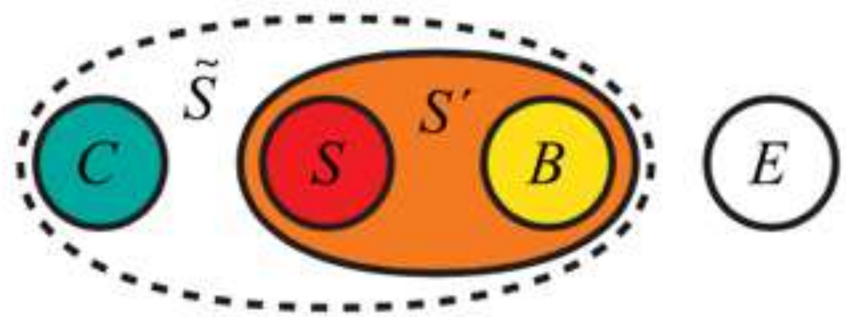
K. Funo, et al, PRE (2013)

S. S. Jevtic, et al. PRE (2015)

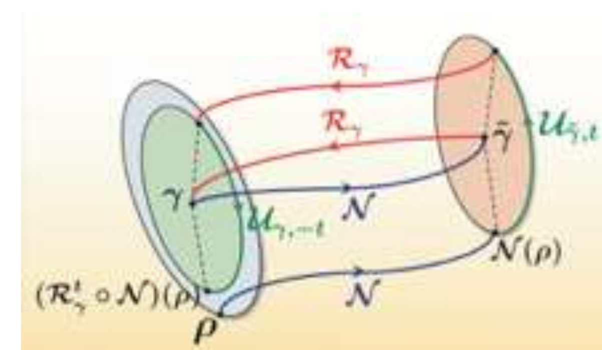
Quantum generalisations with quantum information approaches

<u>Coherence fluctuation theorem</u>

- Energy reservoir: measure of work without decoherence
- Method: Probability → Quantum channel descriptions
- Results: Initially arbitrary state, coherence effects



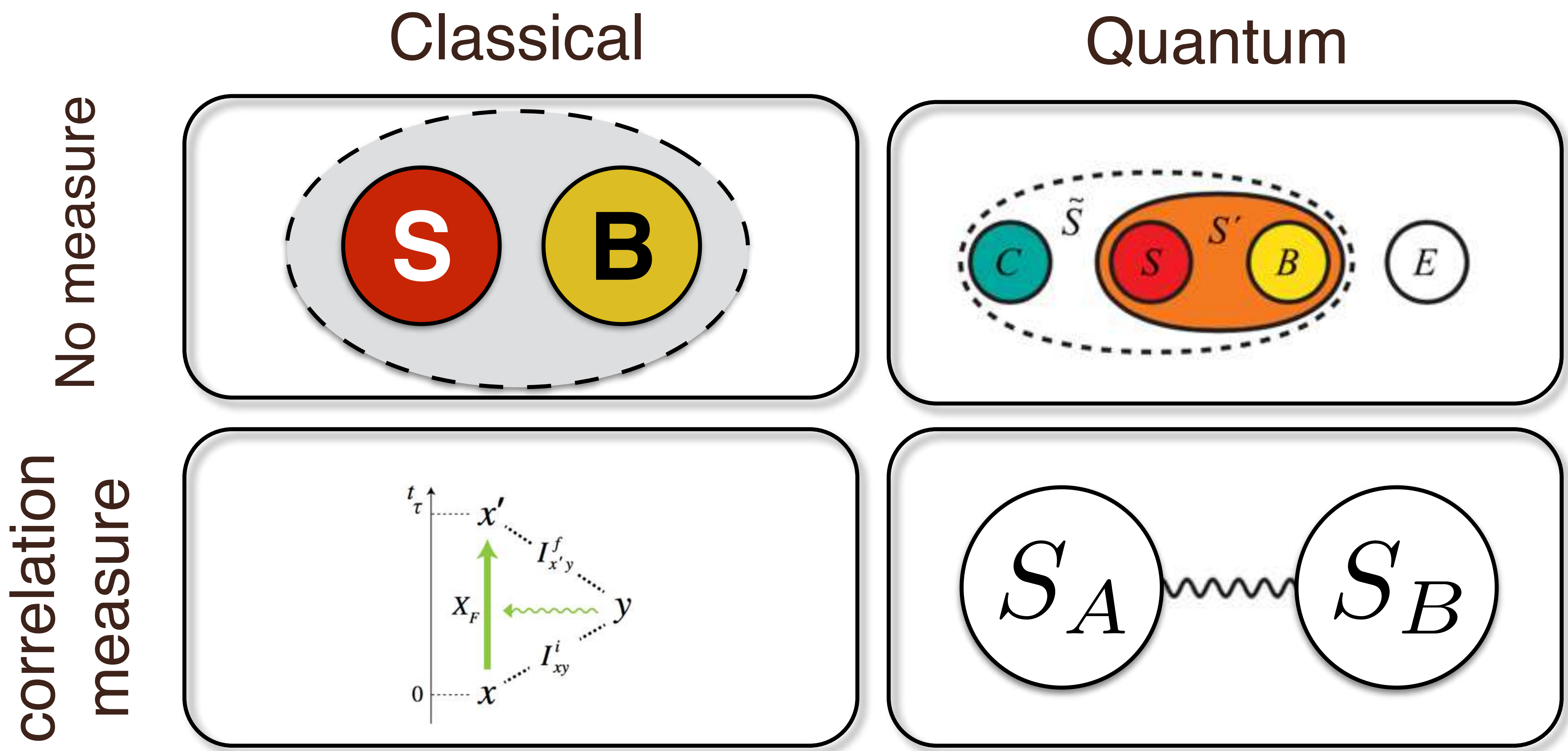
- Time-reversed process of quantum channels
  - From Hamiltonian approach to recovery map
  - Enable approaches to the resource theory



Johan Åberg, PRX (2018)

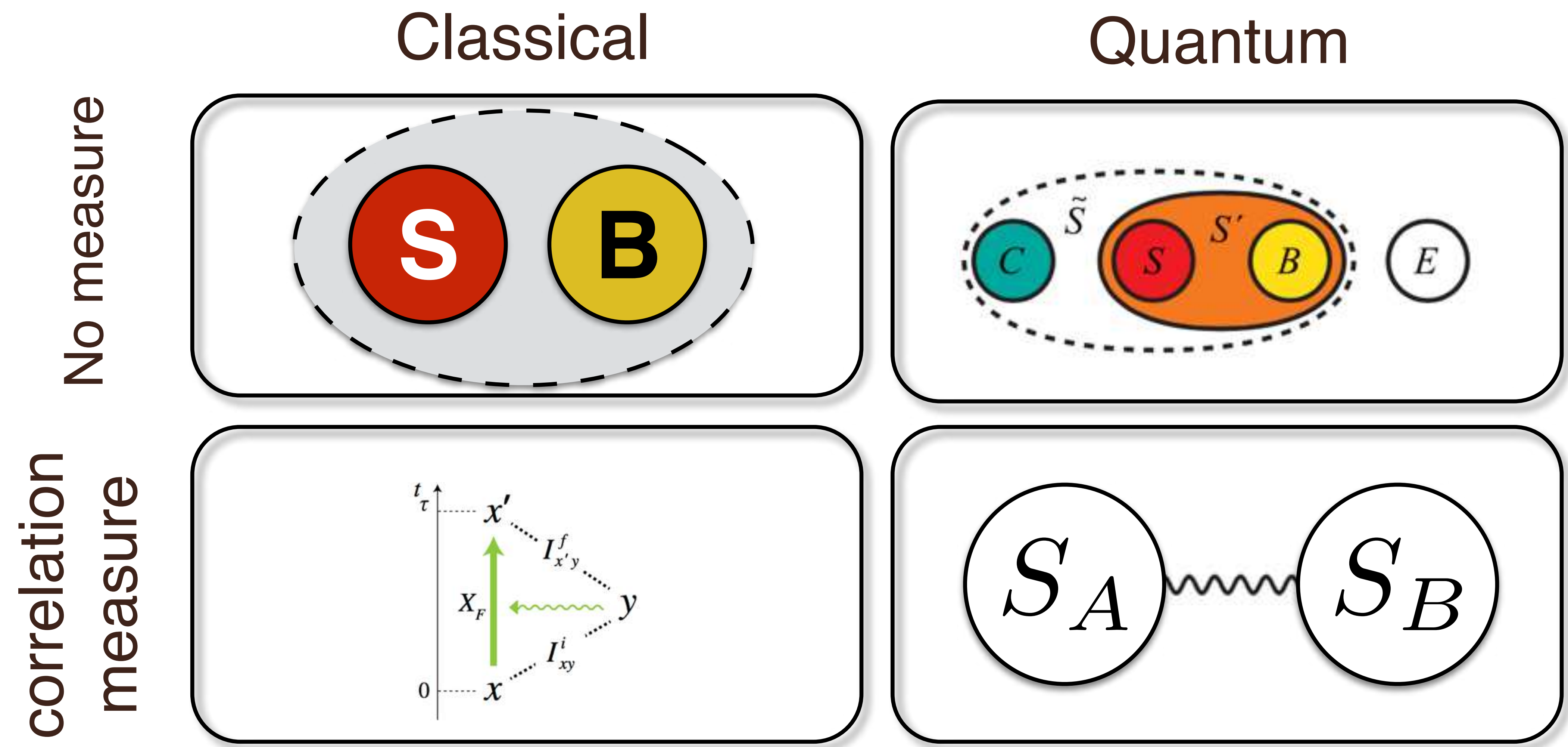
Hyukjoon Kwon, M. S. Kim PRX (2019)

Fluctuation theorem for correlated systems



- Absence of quantum correlation measure in nonequilibrium approaches
- Missing of irreversible characteristics of quantum correlation related to thermodynamics

Fluctuation theorem for correlated systems

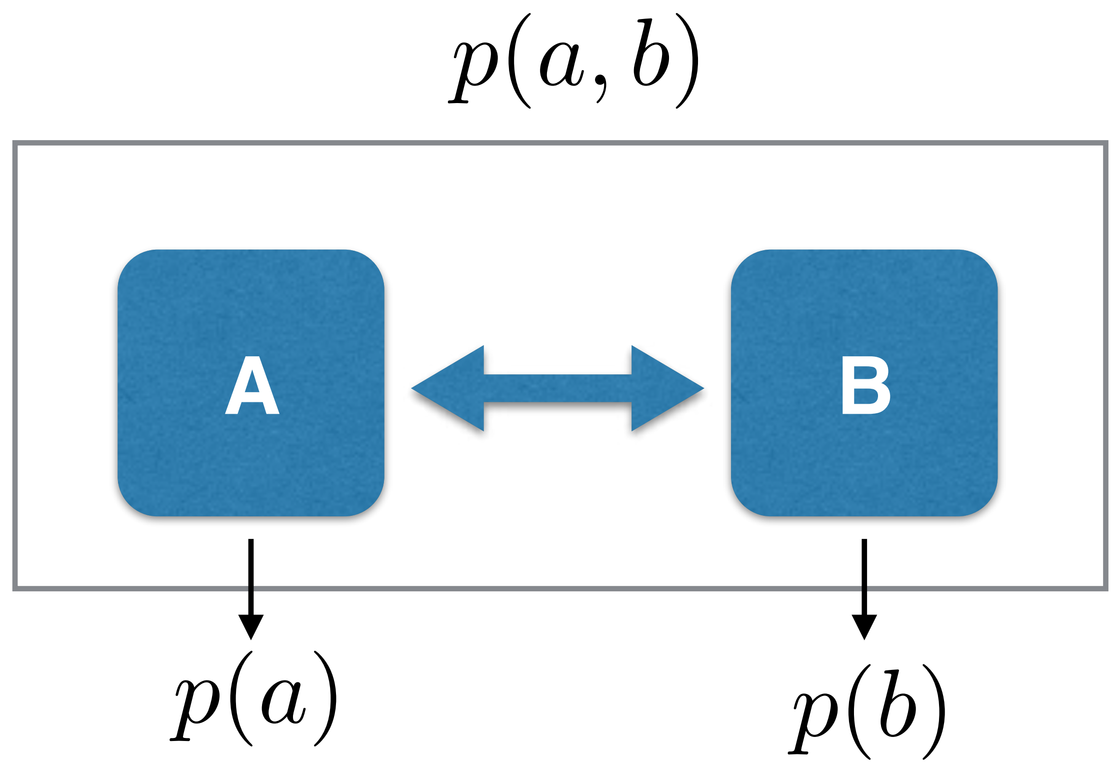


- **Fluctuation of quantum correlation is not measurable due to incompatibility**
- **Quantum approaches are required**

Fluctuation theorem for quantum correlated systems

Fluctuations of correlation in non-equilibrium quantum bipartite systems

Classical fluctuation of correlation in nonequilibrium regime



- Fluctuation of correlation

$$J(a,b) := \ln \frac{p(a,b)}{p(a)p(b)}$$

- Averaged classical correlation fluctuation

$$\langle J \rangle = \sum p(a,b) \ln \frac{p(a,b)}{p(a)p(b)} = H(A:B)$$

Fluctuations of correlation in non-equilibrium quantum bipartite systems

$$\rho_{AB} = \sum p(m)|m\rangle\langle m|$$

$$\rho_A = \sum p(a)|a\rangle\langle a| \qquad \rho_B = \sum p(b)|b\rangle\langle b|$$

noncommutativity $[|m\rangle\langle m|_{AB}, |a\rangle\langle a|_A \otimes |b\rangle\langle b|_B] \neq 0$

<u>Quantum fluctuation of correlation</u>

- Measure of total correlation

$$I(m,a,b) = \ln \frac{p(m)}{p(a)p(b)}$$

Fluctuations of correlation in non-equilibrium quantum bipartite systems

$$\rho_{AB} = \sum p(m)|m\rangle\langle m|$$

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<u>Quantum fluctuation of correlation</u>

- Fluctuation of quantum correlation

$$\delta = \ln p(m) - \ln p(a,b)$$

$$p(a,b) = \langle a,b|\rho_{AB}|a,b\rangle$$

Fluctuations of correlation in non-equilibrium quantum bipartite systems

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<u>Quantum fluctuation of correlation</u>

- Fluctuation of quantum correlation

$$\delta = \ln p(m) - \ln p(a,b)$$

$$p(a,b) = \langle a,b|\rho_{AB}|a,b\rangle$$

- Joint probability

$$p(m,a,b) = p(m)|\langle m|a,b\rangle|^2$$

P. A. M. Dirac, RMP (1945)

H. Margenau and R. N. Hill, Progr. Theor. Phys. (1961); A. O. Barut, Phys. Rev. (1957)

Fluctuations of correlation in non-equilibrium quantum bipartite systems

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noncommutativity $[|m\rangle\langle m|_{AB}, |a\rangle\langle a|_A \otimes |b\rangle\langle b|_B] \neq 0$

Quantum fluctuation of correlation

- Average of quantum fluctuation of correlation

$$\langle\delta\rangle = \sum_{m,a,b} p(m,a,b)[\ln p(m) - \ln p(a,b)] = S(\rho'_{AB}) - S(\rho_{AB})$$

Fluctuations of correlation in non-equilibrium quantum bipartite systems

$$\rho_{AB} = \sum p(m)|m\rangle\langle m|$$

$$\rho_A = \sum p(a)|a\rangle\langle a| \qquad \rho_B = \sum p(b)|b\rangle\langle b|$$

noncommutativity $[|m\rangle\langle m|_{AB}, |a\rangle\langle a|_A \otimes |b\rangle\langle b|_B] \neq 0$

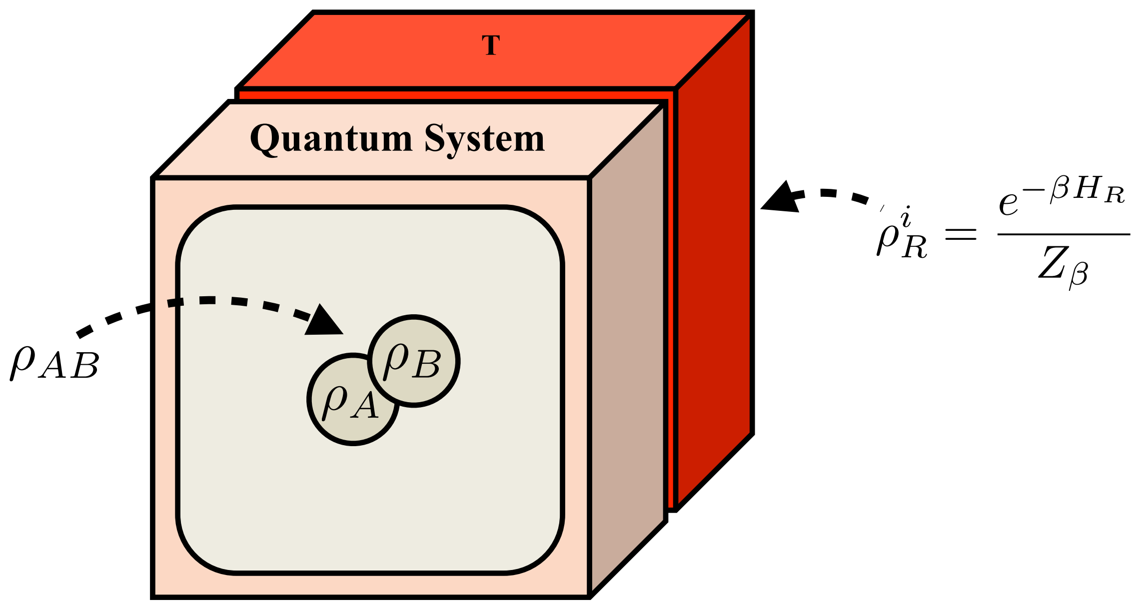
Quantum fluctuation of correlation

- δ is uniquely defined by a given composite system
- The non-classicality disappears when $[|m\rangle\langle m|_{AB}, |a\rangle\langle a|_A \otimes |b\rangle\langle b|_B] = 0$

P. A. M. Dirac, RMP (1945)

H. Margenau and R. N. Hill, Progr. Theor. Phys. (1961); A. O. Barut, Phys. Rev. (1957)

Joint probabilities in quantum non-equilibrium bipartite systems

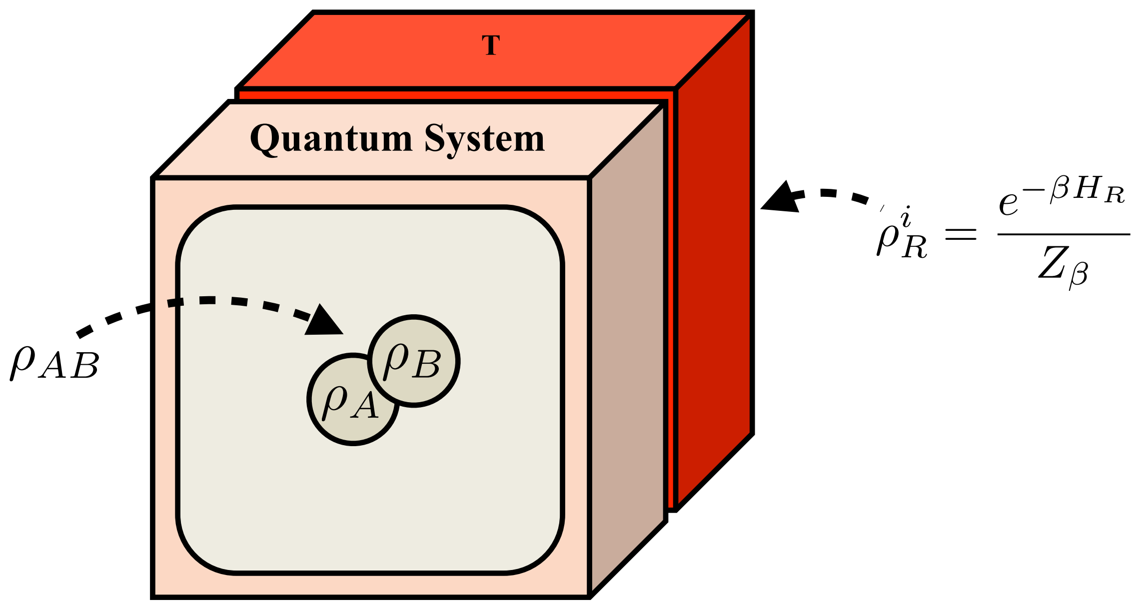


Joint probabilities(JPs) for joint systems

- The time-forward JP

$$p_{m,m',r,r'} = |\langle m', r'|U|m, r\rangle|^2 p_m p_r$$

Joint probabilities in quantum non-equilibrium bipartite systems



Joint probabilities(JPs) for joint systems

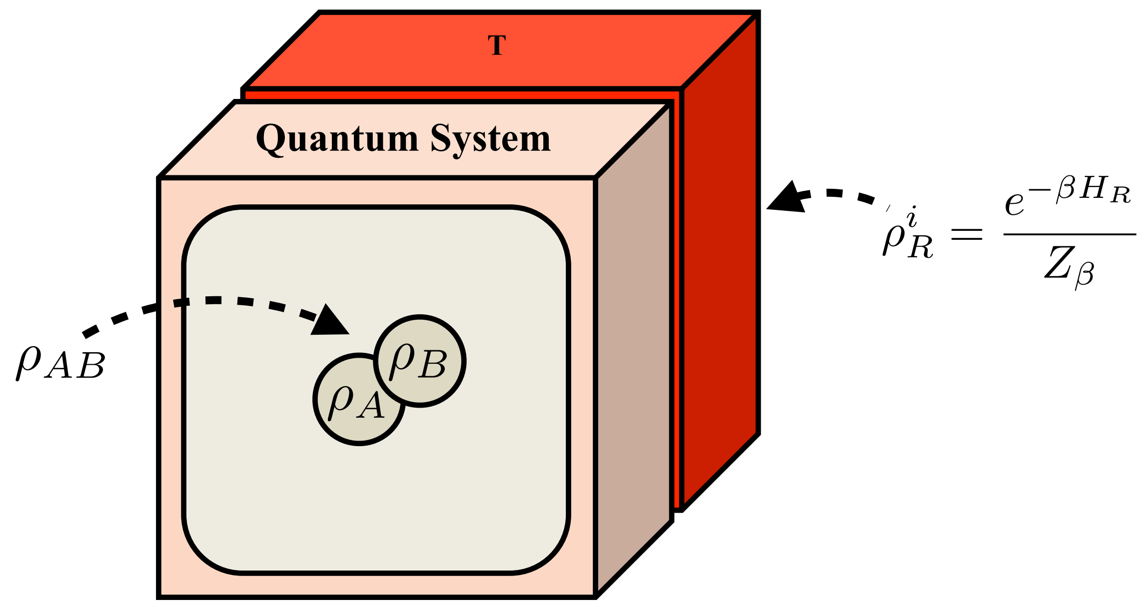
- The time-forward JP

$$p_{m,m',r,r'} = |\langle m', r'|U|m, r\rangle|^2 p_m p_r$$

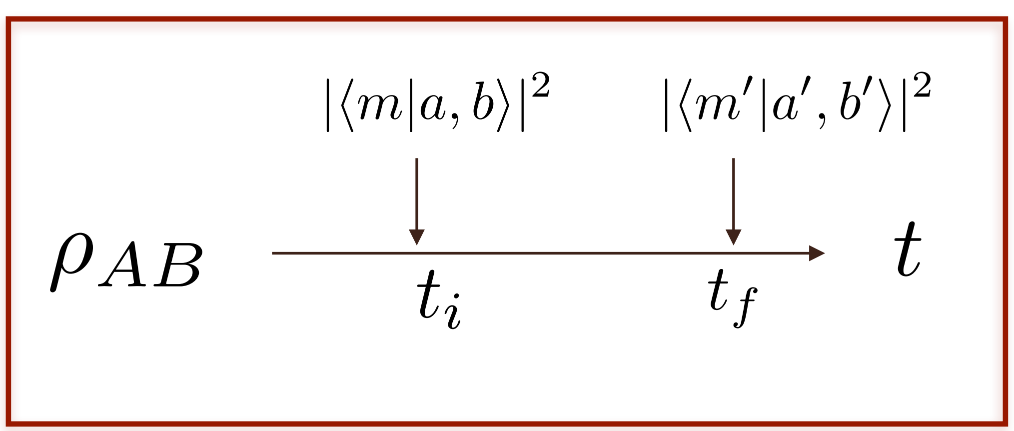
$$\downarrow \ \langle \Delta\delta \rangle = ?$$

$$p_{m,m',r,r';a,b,a',b'} = p_{m,m',r,r'} |\langle m|a, b\rangle|^2 |\langle m'|a', b'\rangle|^2$$

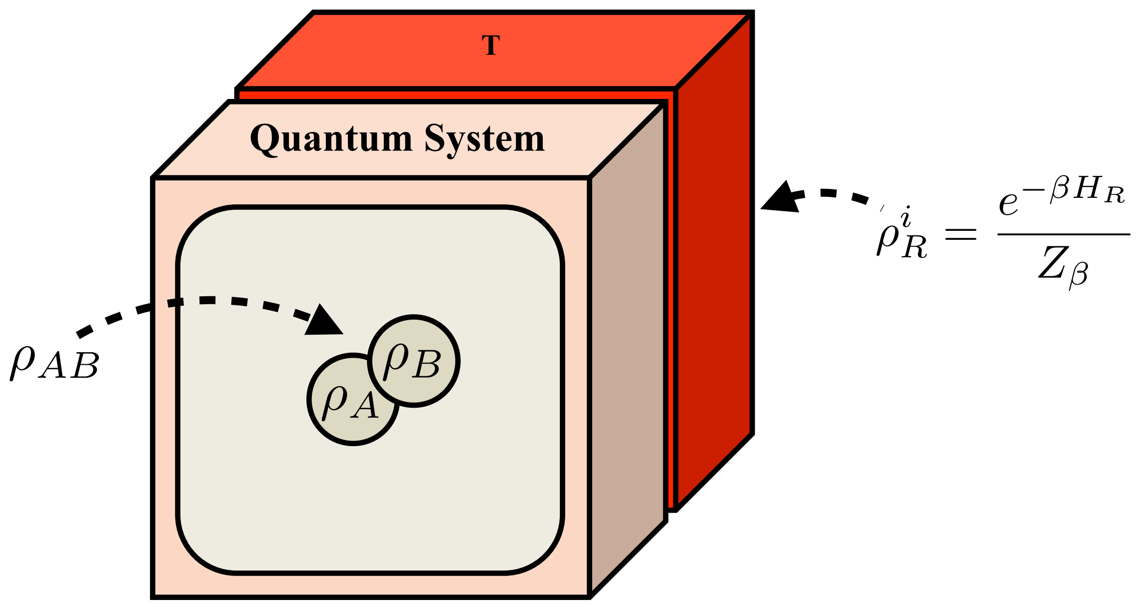
Joint probabilities in quantum non-equilibrium bipartite systems



Joint probabilities(JPs) for joint systems



Joint probabilities in quantum non-equilibrium bipartite systems



Joint probabilities(JPs) for joint systems

- The time-reversed JP

$$\tilde{p}_{m',m,r',r} = |\langle m, r|\tilde{U}|m', r'\rangle|^2 \tilde{p}_{m'}\tilde{p}_{r'}$$

$$\downarrow$$

$$\tilde{p}_{m',m,r',r;a',b',a,b} = \tilde{p}_{m',m,r',r}\boxed{|\langle m'|a', b'\rangle|^2|\langle m|a, b\rangle|^2}$$

The detailed fluctuation theorem for arbitrary bipartite systems

Detailed fluctuation theorem

$$\frac{\tilde{p}_{m',a',b',m,a,b;r',r}}{p_{m,a,b,m',a',b';r,r'}} = e^{-\sigma+\Delta I+\Delta\delta}$$

- Entropy production of individual systems and a heat bath

$$\sigma = \Delta s_A + \Delta s_B + \Delta s_R$$

- Changes in entropy of subsystems

$$\Delta s_{A(B)} := -\ln \tilde{p}_{a'(b')} - (-\ln p_{a(b)})$$

- Variations of fluctuations of correlations

$$\Delta I := I_f - I_i \qquad \Delta\delta := \delta_f - \delta_i$$

The detailed fluctuation theorem for arbitrary bipartite systems

Detailed fluctuation theorem

$$\frac{\tilde{p}_{m',a',b',m,a,b;r',r}}{p_{m,a,b,m',a',b';r,r'}} = e^{-\sigma+\Delta I+\Delta\delta}$$

- Classical version of DFT

$$\frac{\tilde{p}_{a',b',a,b;r',r}}{p_{a,b,a',b';r,r'}} = e^{-\sigma+\Delta I}$$

The detailed fluctuation theorem for arbitrary bipartite systems

Detailed fluctuation theorem

$$\frac{\tilde{p}_{m',a',b',m,a,b;r',r}}{p_{m,a,b,m',a',b';r,r'}} = e^{-\sigma+\Delta I+\Delta\delta}$$

- Integral fluctuation theorems

$$\langle e^{-\Delta s_A-\Delta s_B+\Delta I+\beta Q}\rangle = \boxed{\langle e^{-\Delta\delta}\rangle_{\tilde{R}}}$$

- The second law

$$\beta\langle Q\rangle \leq \langle\Delta s_A\rangle + \langle\Delta s_B\rangle - \langle\Delta I\rangle - \langle\Delta\delta\rangle$$

Known result

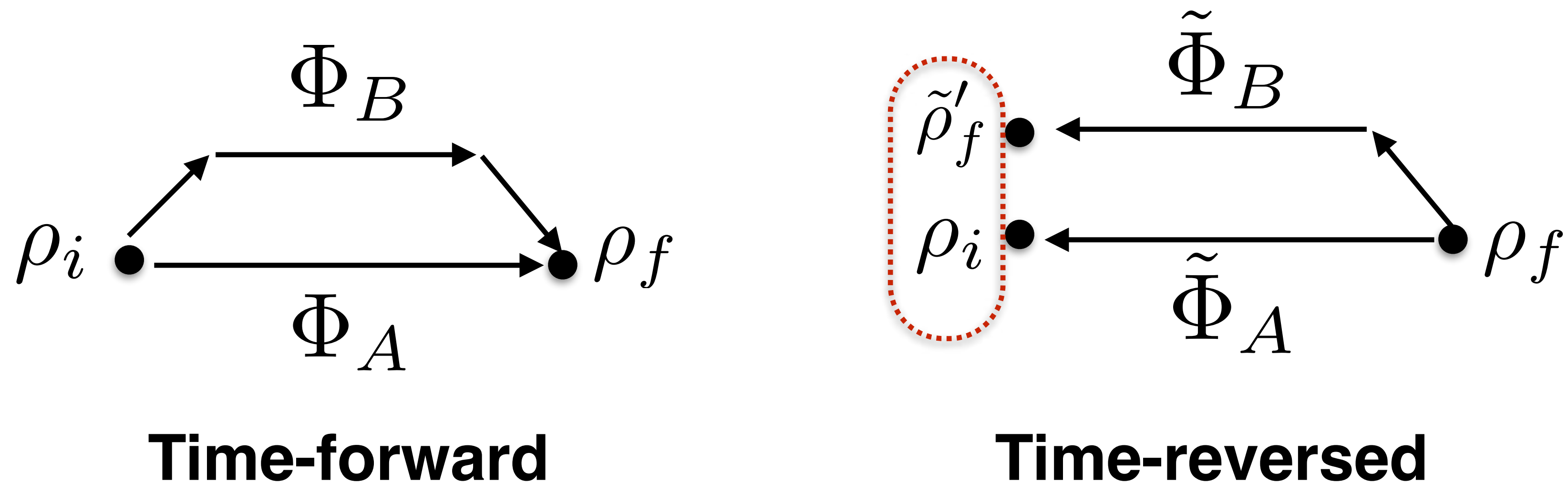
$$\beta\langle Q\rangle \leq \langle\Delta s_A\rangle + \langle\Delta s_B\rangle - \langle\Delta I\rangle + \boxed{\ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}}}$$

Illustrations

Two extreme cases of processes

A: Information conserving process

B: Information dissipative process



Process A

$$\beta\langle Q\rangle = \langle\Delta s_A\rangle + \langle\Delta s_B\rangle - \langle\Delta I\rangle + \ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

Two extreme cases of processes

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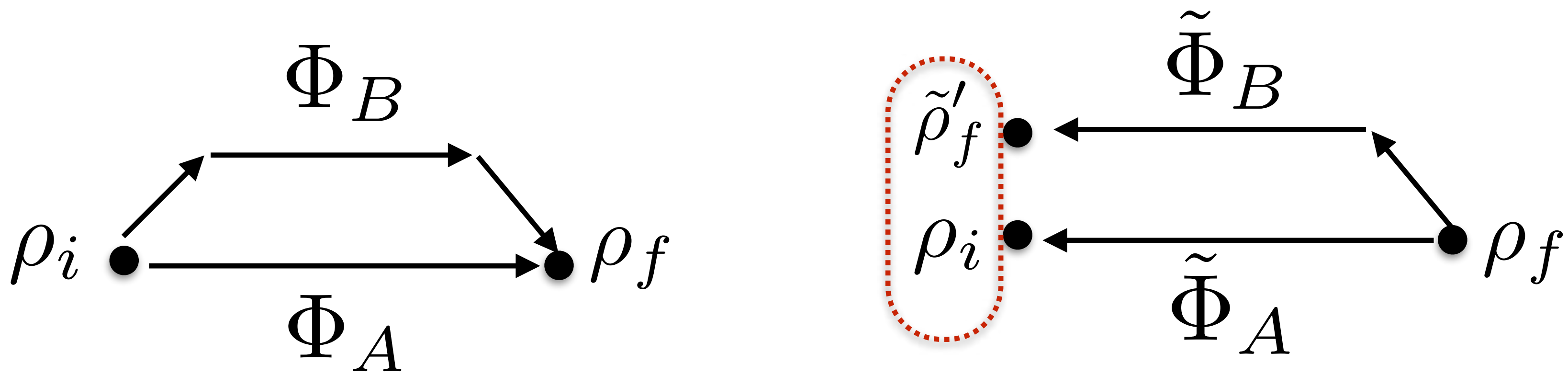
B: Information dissipative process



Time-forward **Time-reversed**

Process A

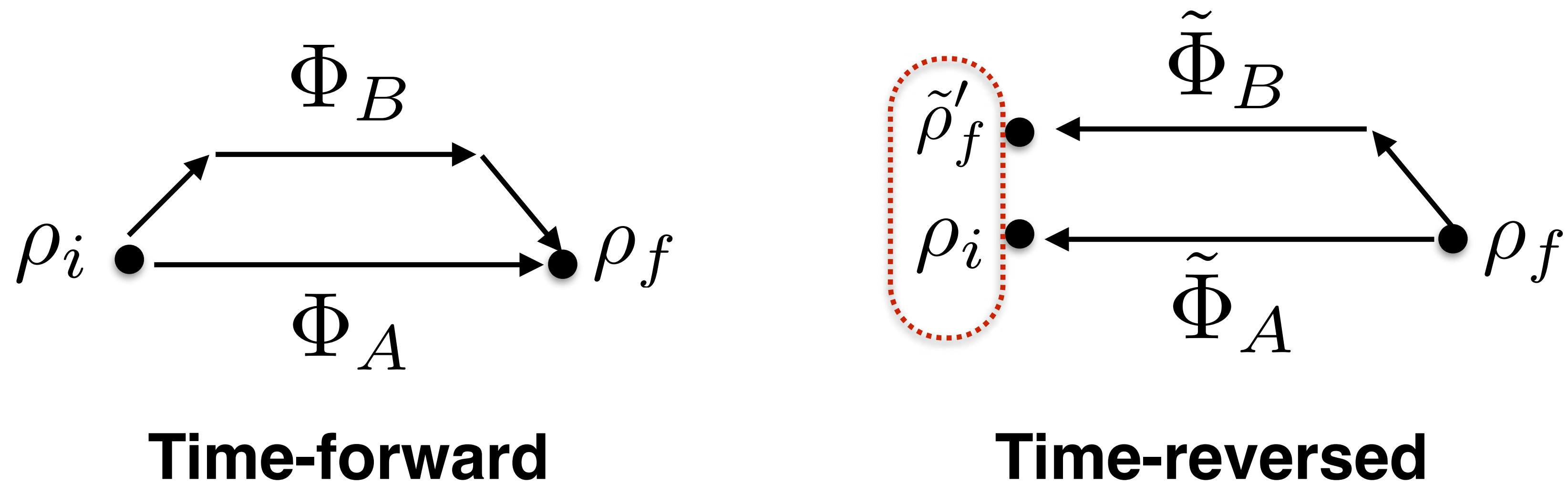
Maximum resource

$$\ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}} = -\langle\Delta\delta\rangle$$

Two extreme cases of processes

A: Information conserving process

B: Information dissipative process



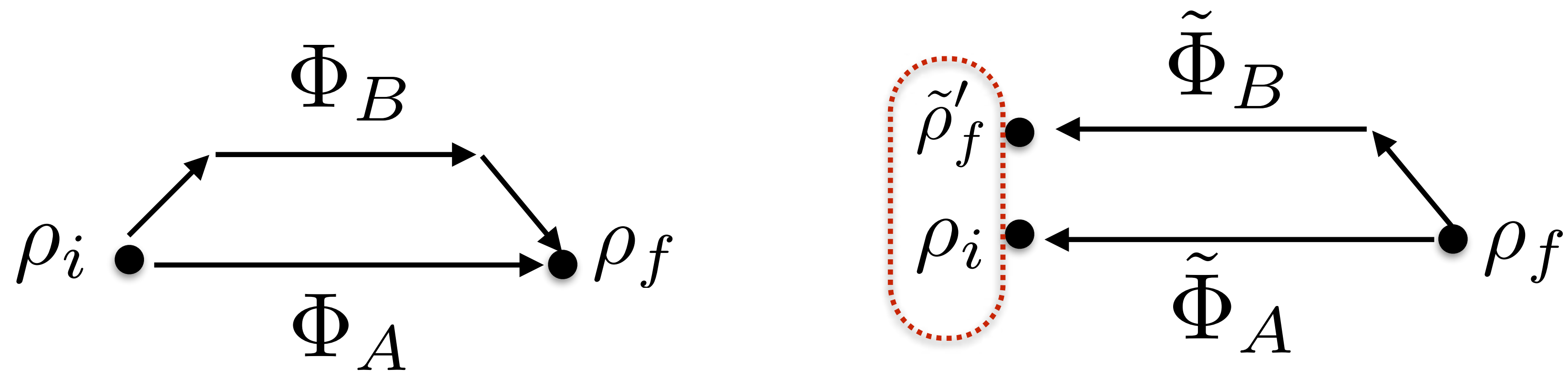
Process B

$$\beta\langle Q\rangle = \langle \Delta s_A\rangle + \langle \Delta s_B\rangle - \langle \Delta I\rangle + \ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

Two extreme cases of processes

A: Information conserving process

B: Information dissipative process



Time-forward **Time-reversed**

Process B

Minimum resource

$$\ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}} = 0$$

Information conserving process

- The time-forward process

- The time-reversed process

- The state changes
  - Forward $\rho^i_{AB} = |0\rangle\langle 0|_{AB} \rightarrow \rho^f_{AB} = |0'\rangle\langle 0'|_{AB} = |0'\rangle\langle 0'|_A \otimes |0'\rangle\langle 0'|_B$
  - Reversed $\tilde{\rho}^i_{AB} = |0'\rangle\langle 0'|_{AB} \rightarrow \tilde{\rho}^f_{AB} = |0\rangle\langle 0|_{AB}$

$$|0\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) = |\Psi^+\rangle \rightleftarrows |0'\rangle_{AB} = |0'\rangle_A \otimes |0'\rangle_B$$

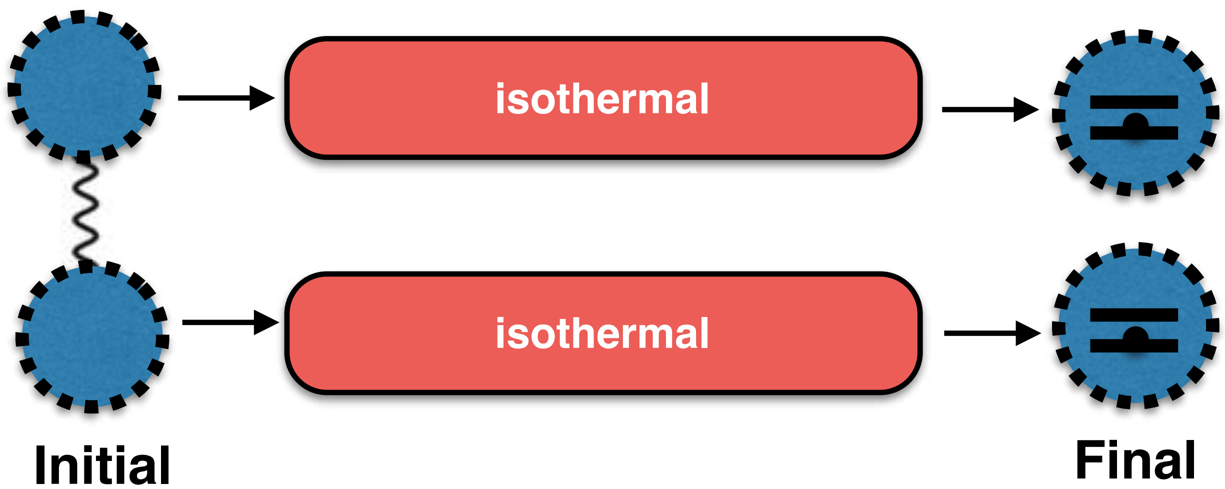
$$|1\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B) = |\Psi^-\rangle \qquad |1'\rangle_{AB} = |0'\rangle_A \otimes |1'\rangle_B$$

$$|2\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B) = |\Phi^+\rangle \qquad |2'\rangle_{AB} = |1'\rangle_A \otimes |0'\rangle_B$$

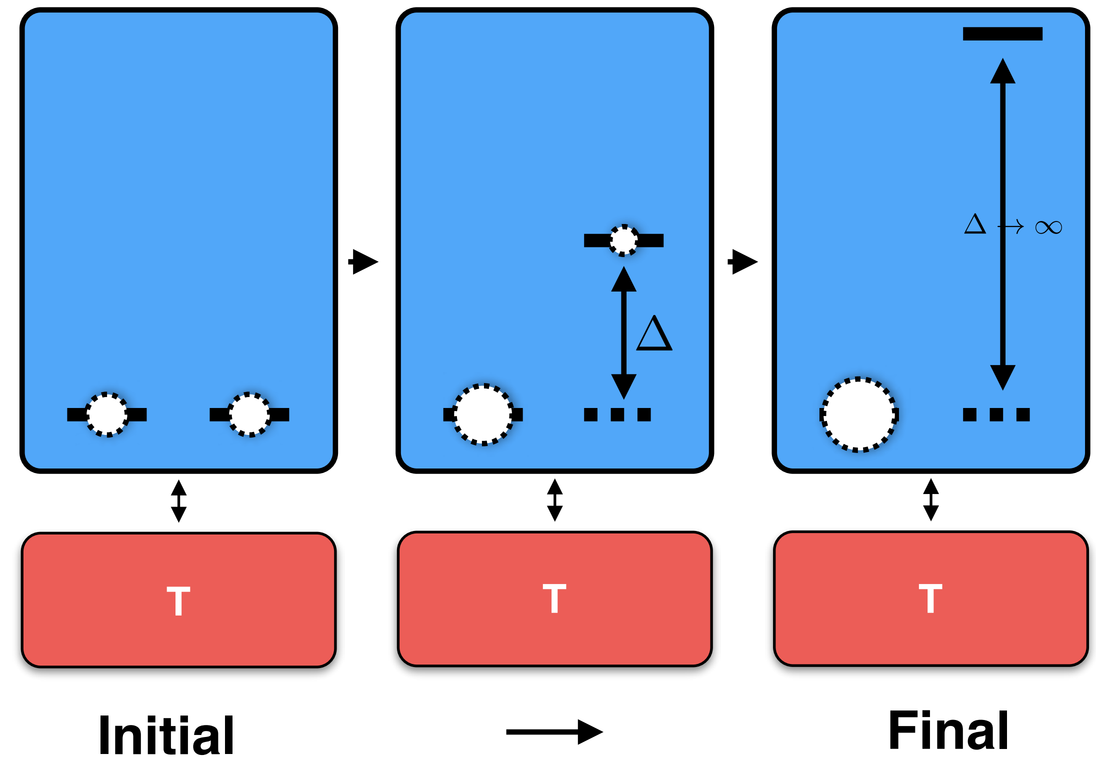
$$|3\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |1\rangle_B) = |\Phi^-\rangle \qquad |3'\rangle_{AB} = |1'\rangle_A \otimes |1'\rangle_B$$

Information dissipative process

- Time-forward process



- isothermal process



- state changes $\rho^i_{AB} = |0\rangle\langle 0|_{AB} \longrightarrow \rho^f_{AB} = |0'\rangle\langle 0'|_{AB} = |0'\rangle\langle 0'|_A \otimes |0'\rangle\langle 0'|_B$

- basis

$$|0\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) = |\Psi^+\rangle \longrightarrow |0'\rangle_{AB} = |0'\rangle_A \otimes |0'\rangle_B$$

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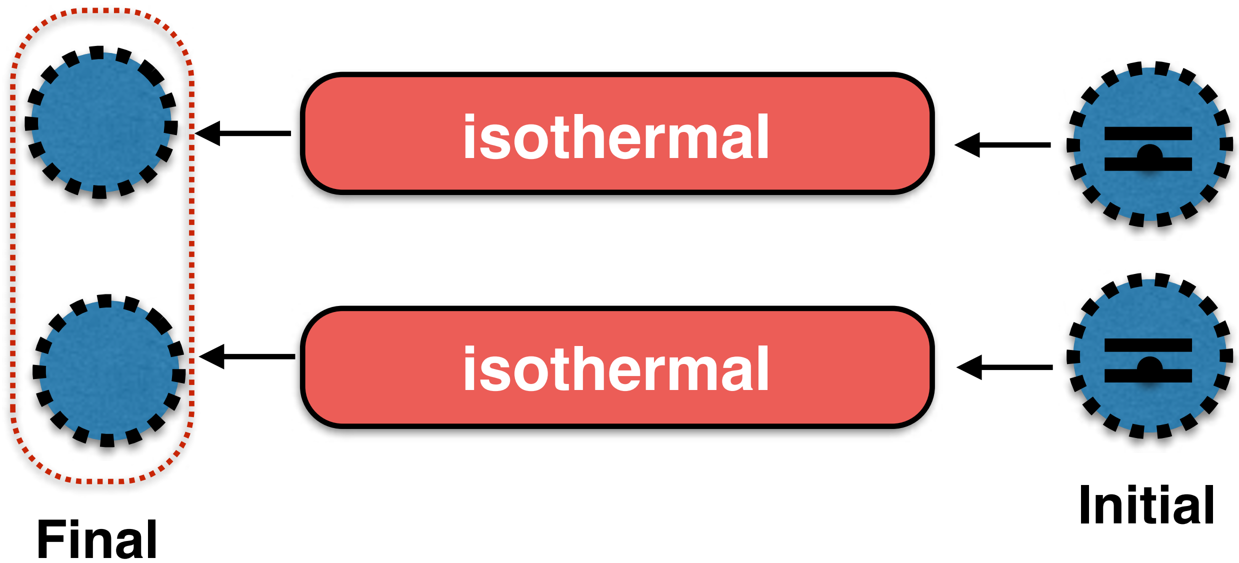
$$|2\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B) = |\Phi^+\rangle \qquad |2'\rangle_{AB} = |1'\rangle_A \otimes |0'\rangle_B$$

$$|3\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |1\rangle_B) = |\Phi^-\rangle \qquad |3'\rangle_{AB} = |1'\rangle_A \otimes |1'\rangle_B$$

Information dissipative process

- The time-reversed process



- Change in the states

$$\tilde{\rho}^{i}_{AB} = |0'\rangle\langle 0'|^{AB}$$

$$\rightarrow \tilde{\rho}^{f}_{AB} = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)_A \otimes \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)_B = \frac{1}{4}\left(|0\rangle\langle 0|_{AB} + |1\rangle\langle 1|_{AB} + |2\rangle\langle 2|_{AB} + |3\rangle\langle 3|_{AB}\right)$$

- The basis
  - The initial basis

$$|0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B\right) = |\Psi^+\rangle$$

$$|1\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B\right) = |\Psi^-\rangle$$

$$|2\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B\right) = |\Phi^+\rangle$$

$$|3\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |1\rangle_B\right) = |\Phi^-\rangle$$

  - The final basis

$$|0'\rangle_{AB} = |0'\rangle_A \otimes |0'\rangle_B$$

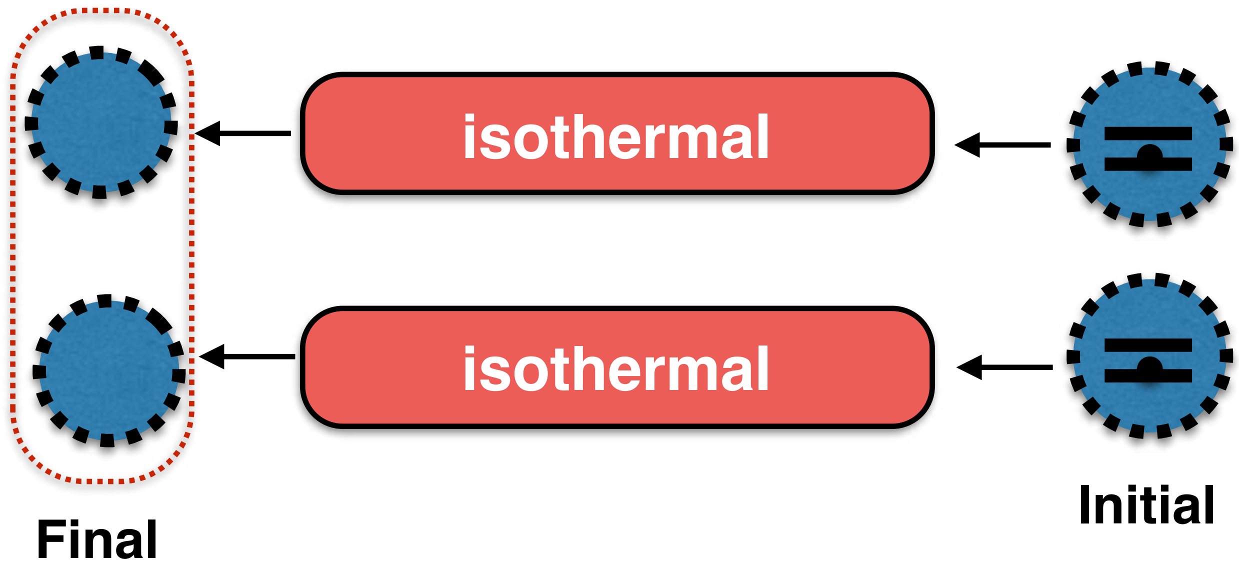
$$|1'\rangle_{AB} = |0'\rangle_A \otimes |1'\rangle_B$$

$$|2'\rangle_{AB} = |1'\rangle_A \otimes |0'\rangle_B$$

$$|3'\rangle_{AB} = |1'\rangle_A \otimes |1'\rangle_B$$

Information dissipative process

- The time-reversed process



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	$\langle Q\rangle$	$\langle -\Delta I\rangle$	$\langle \Delta s_A\rangle$	$\langle \Delta s_B\rangle$	$\ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}}$	$\langle -\Delta\delta\rangle$
(A)	0	ln2	-ln2	-ln2	ln2	ln2
(B)	-2ln2	ln2	-ln2	-ln2	-ln2	ln2

Summary: Information conserving process

- The time-forward process

- The time-reversed process

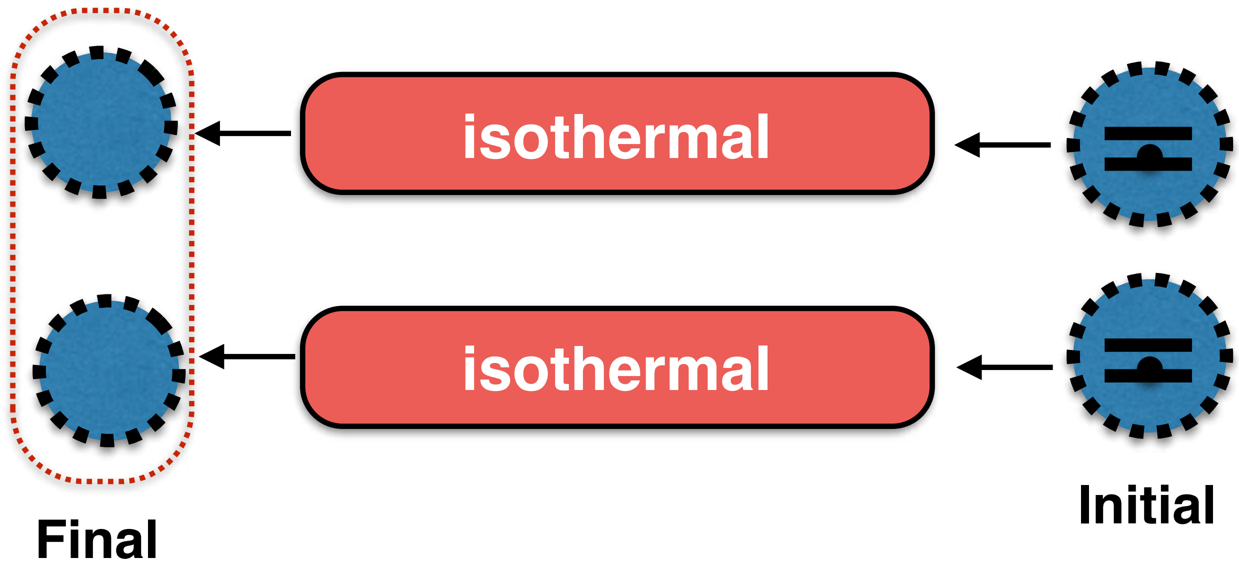
- The state changes
  - Forward $\rho^i_{AB} = |0\rangle\langle 0|_{AB} \longrightarrow \rho^f_{AB} = |0'\rangle\langle 0'|_{AB} = |0'\rangle\langle 0'|_A \otimes |0'\rangle\langle 0'|_B$
  - Reversed $\tilde{\rho}^i_{AB} = |0'\rangle\langle 0'|_{AB} \longrightarrow \tilde{\rho}^f_{AB} = |0\rangle\langle 0|_{AB}$

Maximum resource

$$\ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}} = -\langle\Delta\delta\rangle$$

Summary: Information dissipative process

- The time-reversed process



- Change in the states

$$\tilde{\rho}^{i}_{AB} = |0'\rangle\langle 0'|^{AB}$$

$$\longrightarrow \tilde{\rho}^{f}_{AB} = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)_A \otimes \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)_B = \frac{1}{4}\left(|0\rangle\langle 0|_{AB} + |1\rangle\langle 1|_{AB} + |2\rangle\langle 2|_{AB} + |3\rangle\langle 3|_{AB}\right)$$

$$\beta\langle Q\rangle \le \langle \Delta s_A\rangle + \langle \Delta s_B\rangle - \langle \Delta I\rangle - \langle \Delta\delta\rangle$$

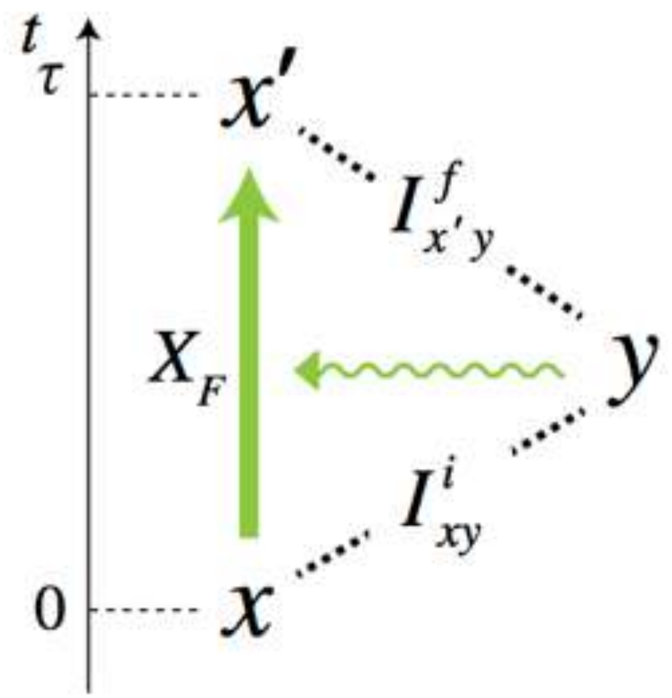
Known result

$$\beta\langle Q\rangle = \langle \Delta s_A\rangle + \langle \Delta s_B\rangle - \langle \Delta I\rangle + \ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

$$\downarrow \ \ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}} = 0$$

Tight bound

Classically correlated systems interacting with heat bath



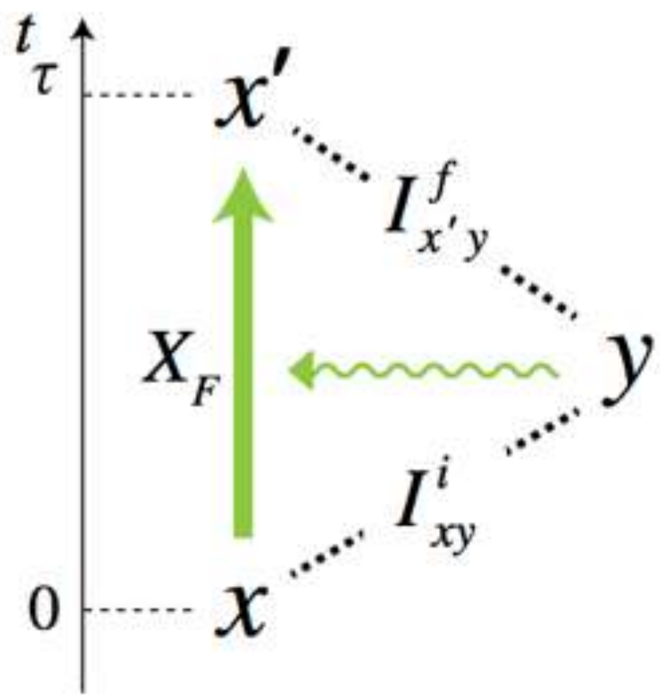
Classically correlated stochastic system

- Stochastic system + heat reservoir & external degree of freedom
- External system Y is invariant with time.
- Consider classically correlated system at t_i and t_f
- The fluctuation theorem has been changed.

$$\langle e^{-\sigma} \rangle = 1 \quad \Rightarrow \quad \langle e^{-\sigma + \Delta J} \rangle = 1$$

T. Sagawa and M. Ueda, PRL. (2012)

Classically correlated systems interacting with heat bath



Classically correlated stochastic system

- Time-forward

$$p(X_F, y)$$

- Time-reversed

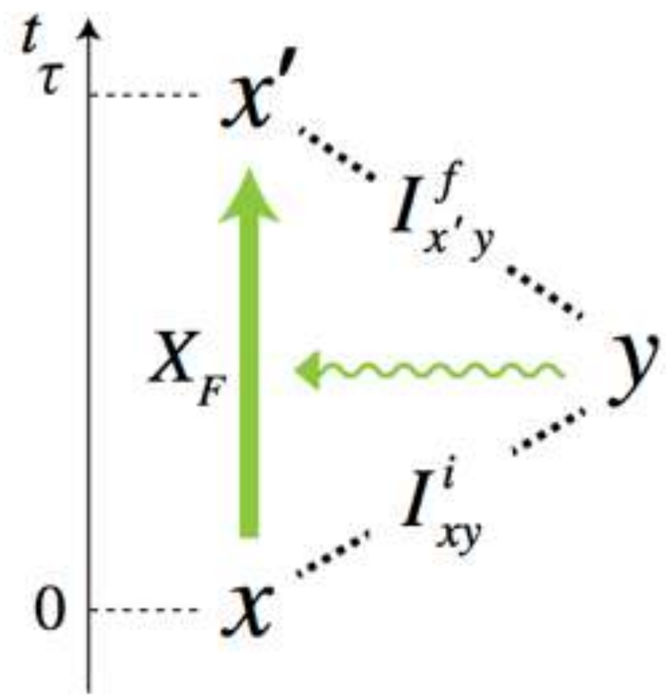
$$\tilde{p}(X_B, y)$$

- Mutual information

$$J_i(x, y) = \ln p_i(x, y) - \ln p_i(x) - \ln p_i(y)$$

$$J_f(x', y') = \ln p_f(x', y') - \ln p_f(x') - \ln p_f(y')$$

Classically correlated systems interacting with heat bath



Classically correlated stochastic system

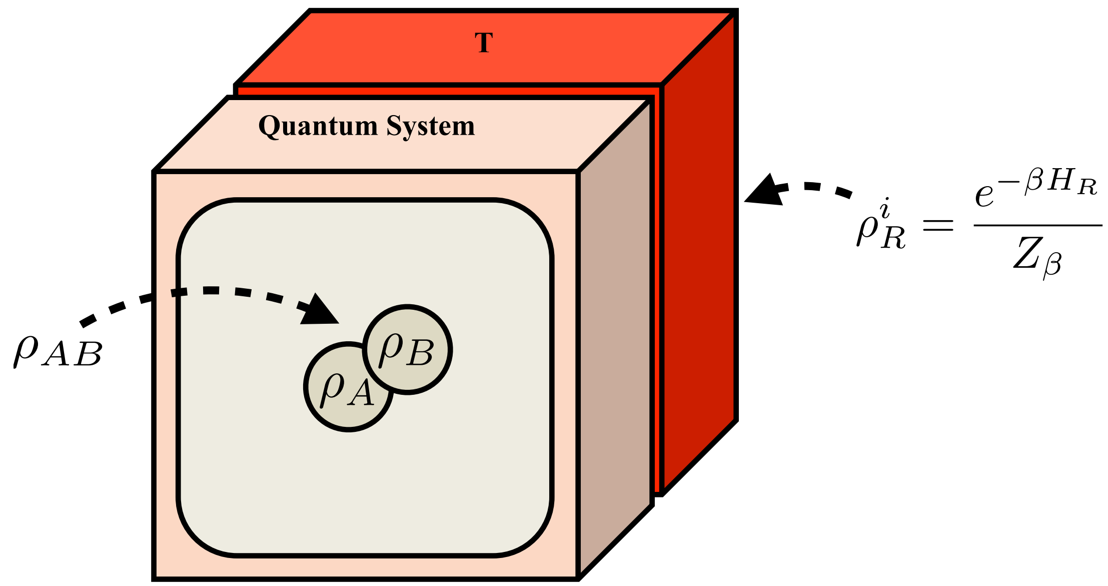
- The detailed fluctuation theorems

$$\frac{\tilde{p}(X_B, y)}{p(X_F, y)} = e^{-\sigma + \Delta J}$$

- The second law of thermodynamics

$$\langle \sigma \rangle \geq \langle \Delta J \rangle$$

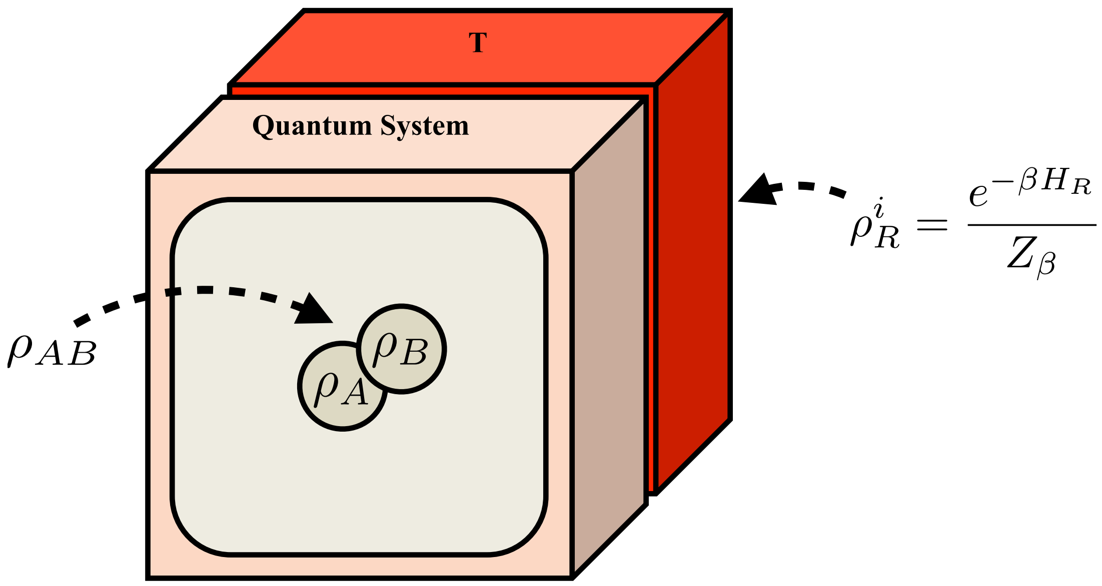
Fluctuation theorem for quantum correlated systems



Quantum correlated system

- Quantum system A + heat reservoir & external degree of freedom
- External system B is invariant with time.
- The systems are correlated during the process.
- Role of the initial correlation

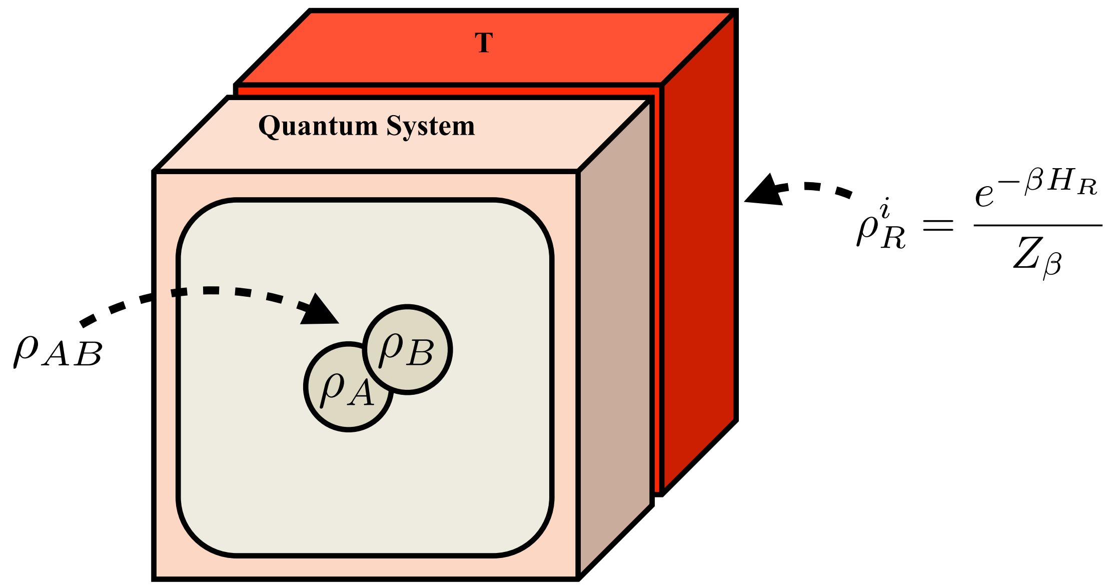
Fluctuation theorem for quantum correlated systems



Fluctuation theorems

$$\langle e^{-\sigma + \Delta I} \rangle_Q = \gamma \neq 1$$

Fluctuation theorem for quantum correlated systems



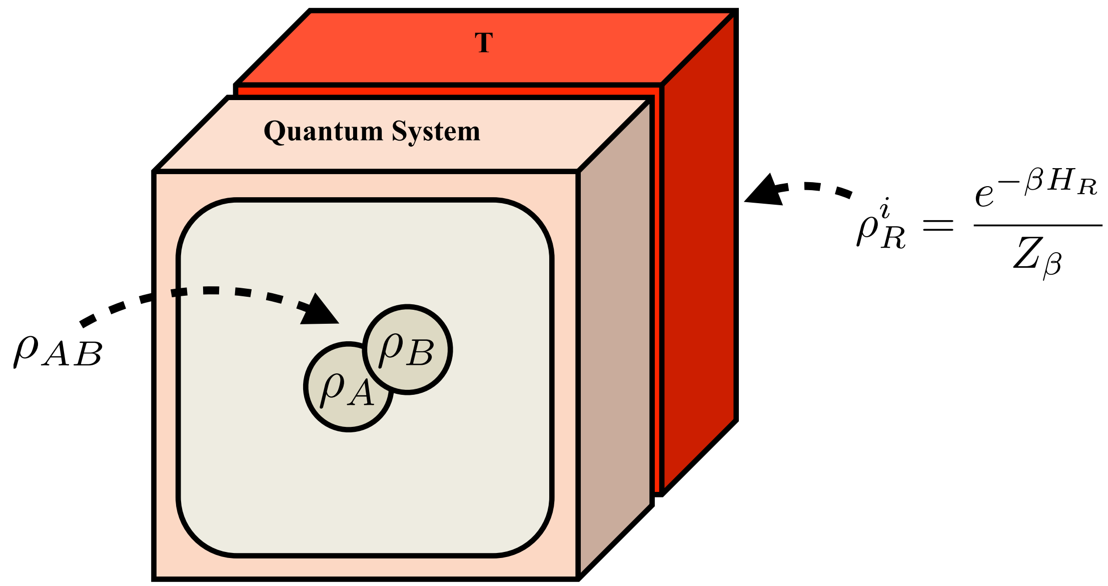
Fluctuation theorems

$$\langle e^{-\sigma+\Delta I}\rangle_Q = \langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

The second law of thermodynamics

$$\langle\sigma\rangle \geq \langle\Delta I\rangle - \ln\langle e^{-\sigma}\sum e^{-[\alpha+\Delta\delta]}\rangle$$

Fluctuation theorem for quantum correlated systems



Fluctuation theorems

$$\langle e^{-\sigma+\Delta I}\rangle_Q = \langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

The second law of thermodynamics

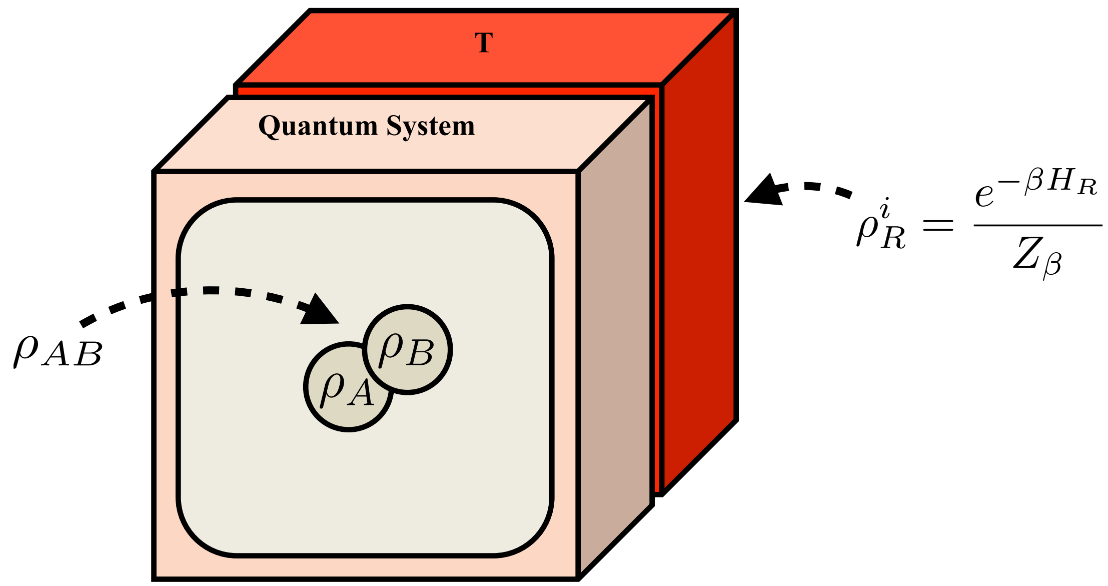
Classical thermodynamic bound

$$\langle\sigma\rangle \geq \langle\Delta I\rangle$$

Quantum extended bound

$$\rightarrow \langle\sigma\rangle \geq \langle\Delta I\rangle - \ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

Fluctuation theorem for quantum correlated systems



Fluctuation theorems

$$\langle e^{-\sigma+\Delta I}\rangle_Q = \langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

The second law of thermodynamics

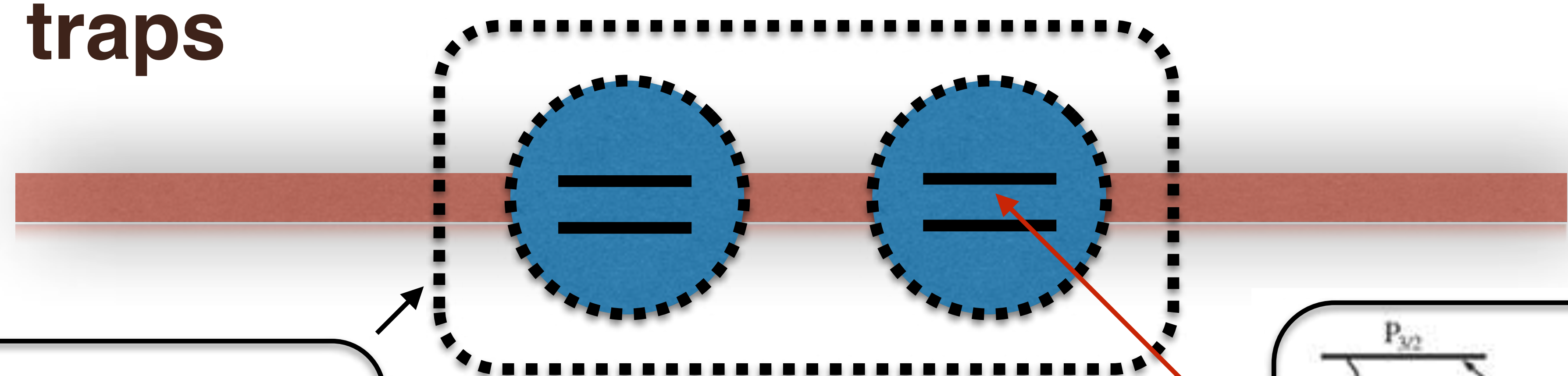
If the initial quantum correlation exists,

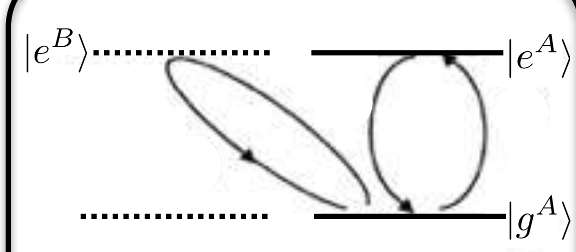
- Irreversibility factor is considered due to the non-commutativity
- The quantum correlation gain and the cost are in a trade-off relation

➡ The extra information term is a quantum measure of thermodynamic gain in nonequilibrium processes

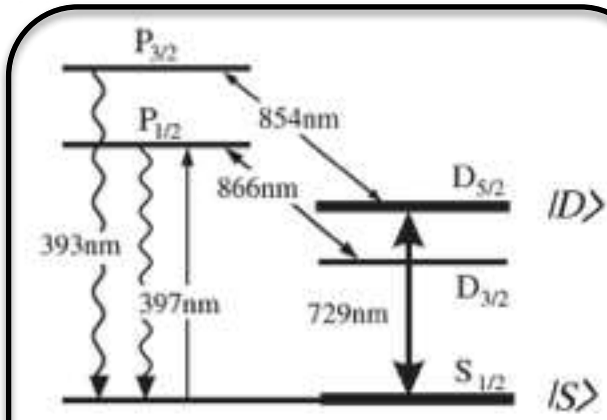
Future work: Experiments

Ion traps



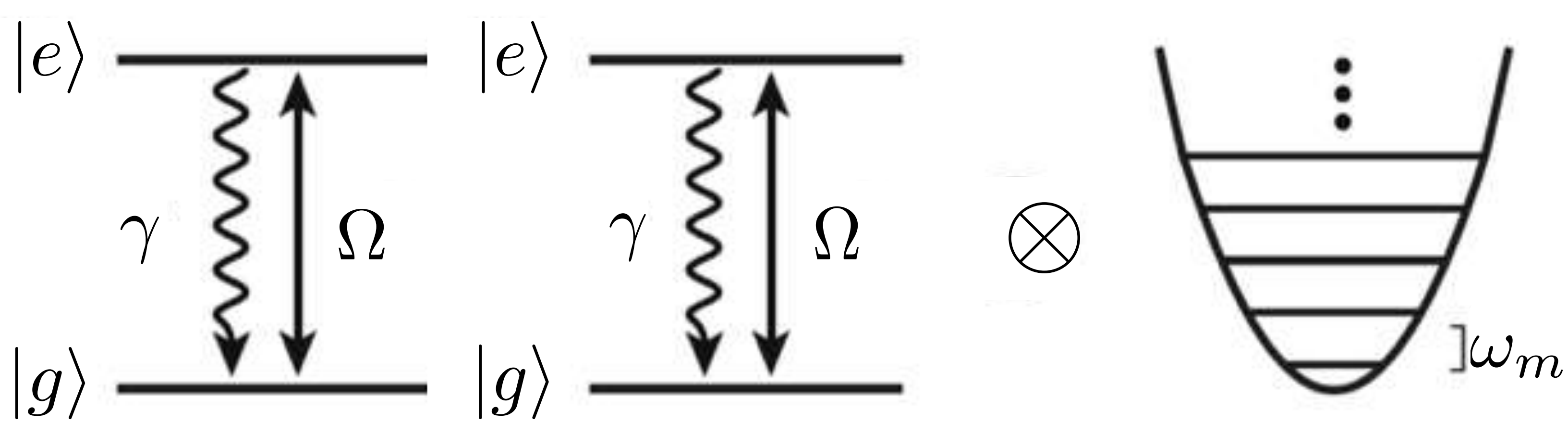


Joint control



Individual control

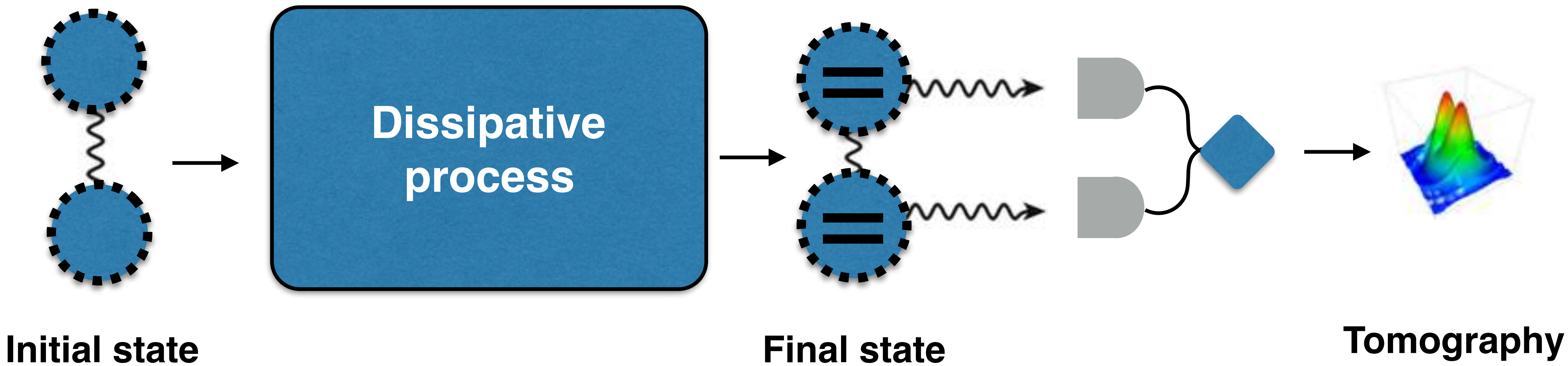
Entangled internal states and a heat bath



Two entangled spins

Single mode heat bath

Future work: Experiments



Verification of the thermodynamic inequality

$$\beta\langle Q\rangle \leq \langle \Delta s_A\rangle + \langle \Delta s_B\rangle - \langle \Delta I\rangle + \ln\langle e^{-\Delta\delta}\rangle_{\tilde{R}}$$

Future work: Squeezed Reservoir

Squeezed reservoir can replace equilibrium thermal bath

- Thermal bath(Boltzmann) ⟶ **idealised**
- Squeezed bath (nonequlibrium) ⟶ experimentally **realisable**

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REPORT

A single-atom heat engine

Johannes Roßnagel[1,*], Samuel T. Dawkins[1], Karl N. Tolazzi[2], Obinna Abah[3], Eric Lutz[3], Ferdinand Schmidt-Kaler[1], Kilian Singer[1]...

+ See all authors and affiliations

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Future work: Squeezed Reservoir

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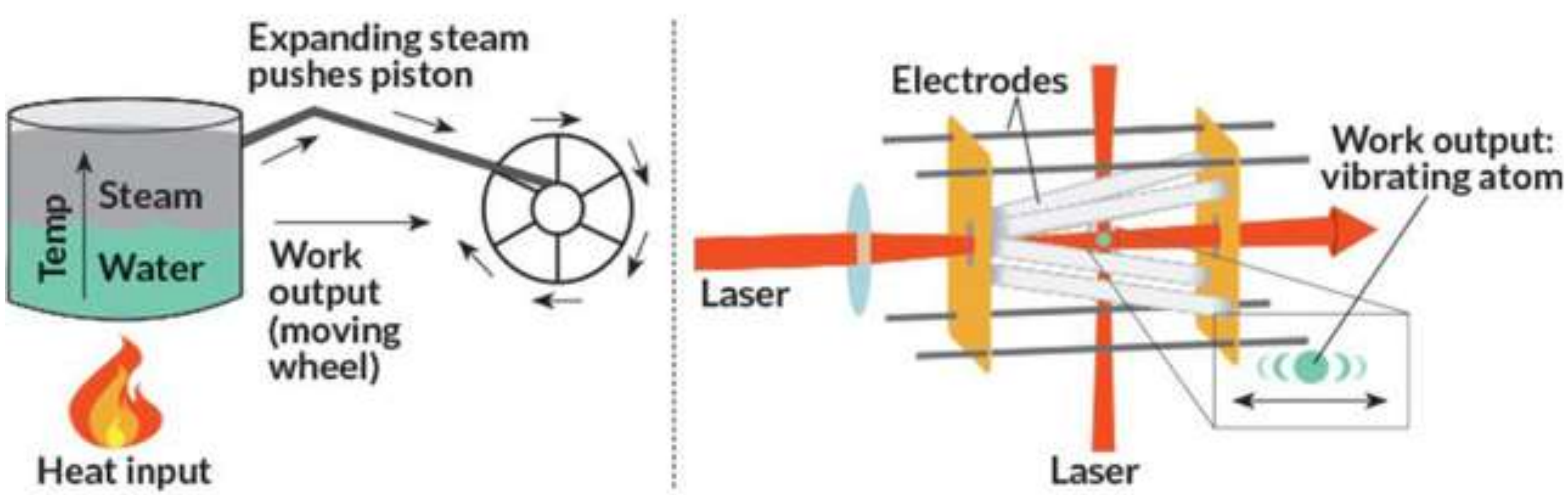

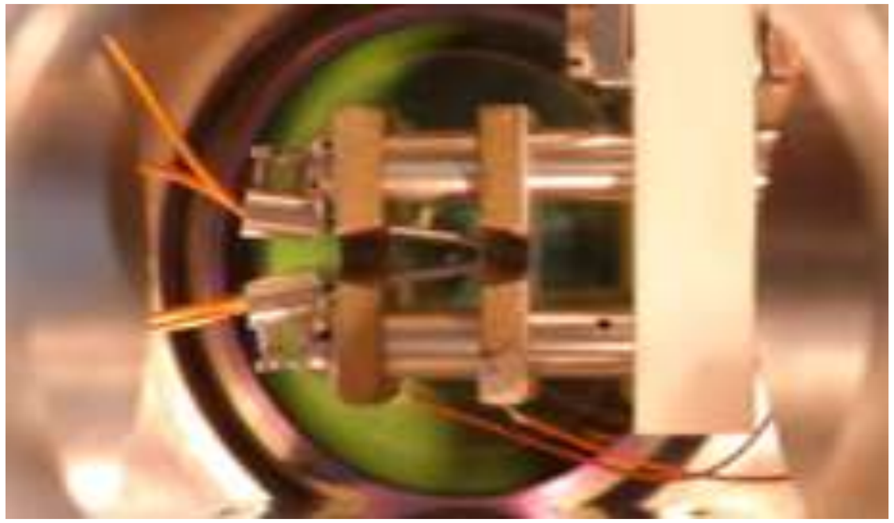

- **A single atom heat engine in an ion trap**

J. Robnagel, et al, Science (2016)

Future work: Squeezed Reservoir

Squeezed reservoir can replace equilibrium thermal bath

- Thermal bath(Boltzmann) ⟶ **idealised**
- Squeezed bath (nonequlibrium) ⟶ experimentally **realisable**

Future work

- **Tripartite correlated system including squeezed reservoir**
- **Quantum network thermodynamics**
- **Thermodynamic uncertainty relation**
- **Quantum cooling algorithm**

Summary

- Motivation: a study of the role of quantum correlation in non-equilibrium thermodynamics for **quantum correlated systems**.

- We introduce the multi-indexed **joint probabilities**, the new definition of the measure of **quantum fluctuations** for nonequilibrium systems, and so on.

- The applications show that the fluctuation theorems and the thermodynamic inequalities present **non-classical features** in terms of thermodynamic **gain** and **cost**.

- The resulting equations lead to the nonequilibrium **tight bound** and the benefits by obtaining **time-reversed** entropy production.

Thank you!