The Resource Theoretic Paradigm of Quantum Thermodynamics with Control

POSTECH, 13/11/2019

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Joint work with

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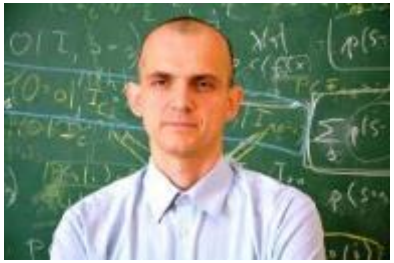
University of Gdańsk, Poland



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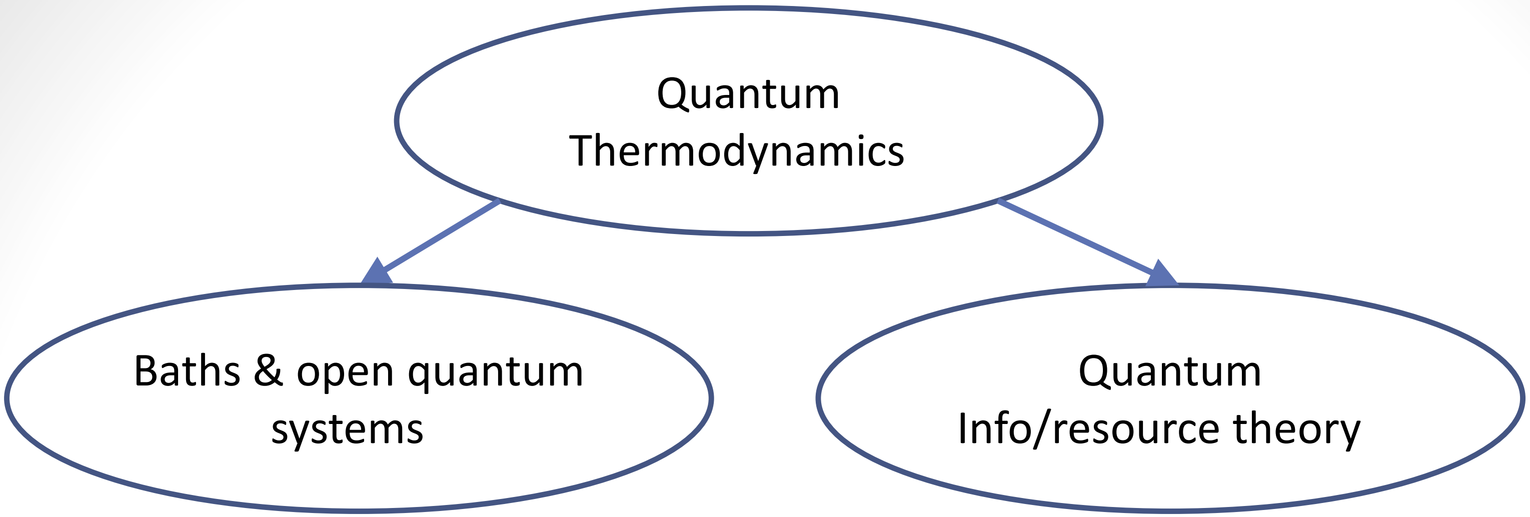
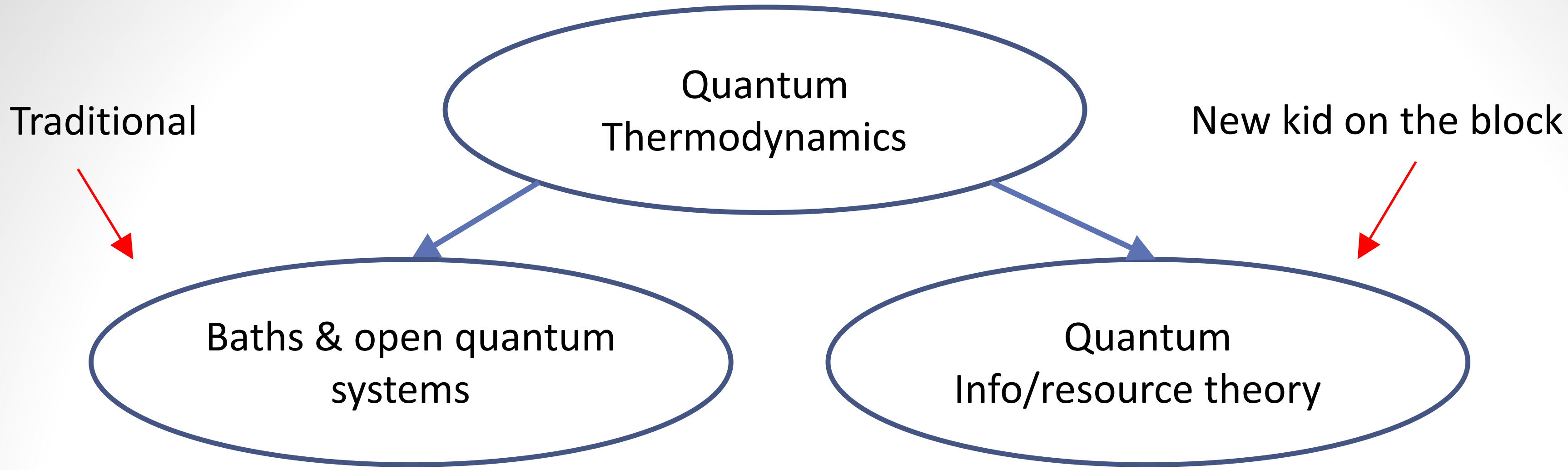

Quantum
Thermodynamics
Baths & open quantum
systems
Quantum
Info/resource theory


Quantum
Thermodynamics
Traditional
New kid on the block
Baths & open quantum systems
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Traditional

New kid on the block

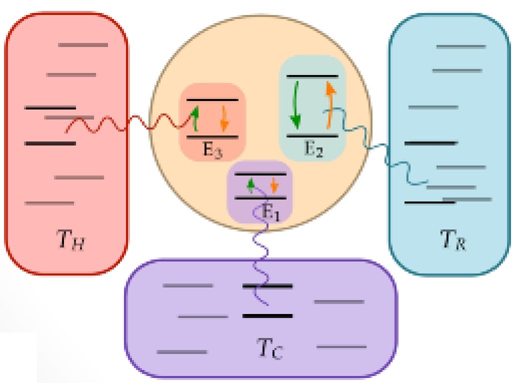
Quantum Thermodynamics

- Baths & open quantum systems
- Quantum Info/resource theory

- Continuous time dynamical models
  (often t-independent Hamiltonian)

  - $e^{-it\hat{H}_{\mathrm{SB}}}(\rho_{\mathrm{S}} \otimes \tau_{\mathrm{B}})e^{it\hat{H}_{\mathrm{SB}}}$

    $e^{-it\mathcal{L}_{\mathrm{S}}}(\rho_{\mathrm{S}})$



M. Mitchison, Contemp. Phys. (2019)
Book: Thermodynamics in the Quantum Regime (2019)

Traditional

New kid on the block

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Quantum Info/resource theory

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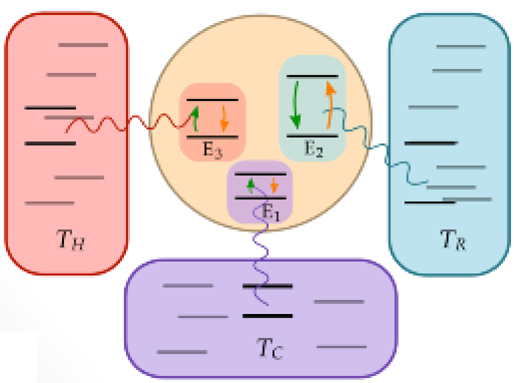
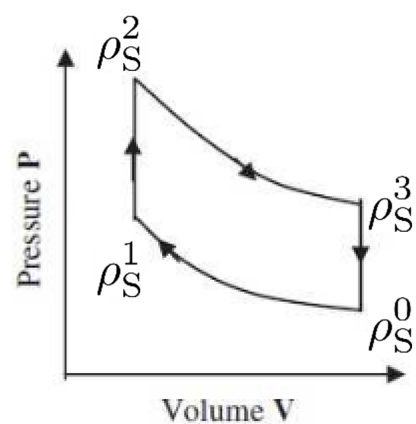


- Discrete time steps model (uses circuit model)

  ➢ $\rho_{\mathrm{S}}^0 \longrightarrow \rho_{\mathrm{S}}^1 \longrightarrow \rho_{\mathrm{S}}^2 \longrightarrow \dots$



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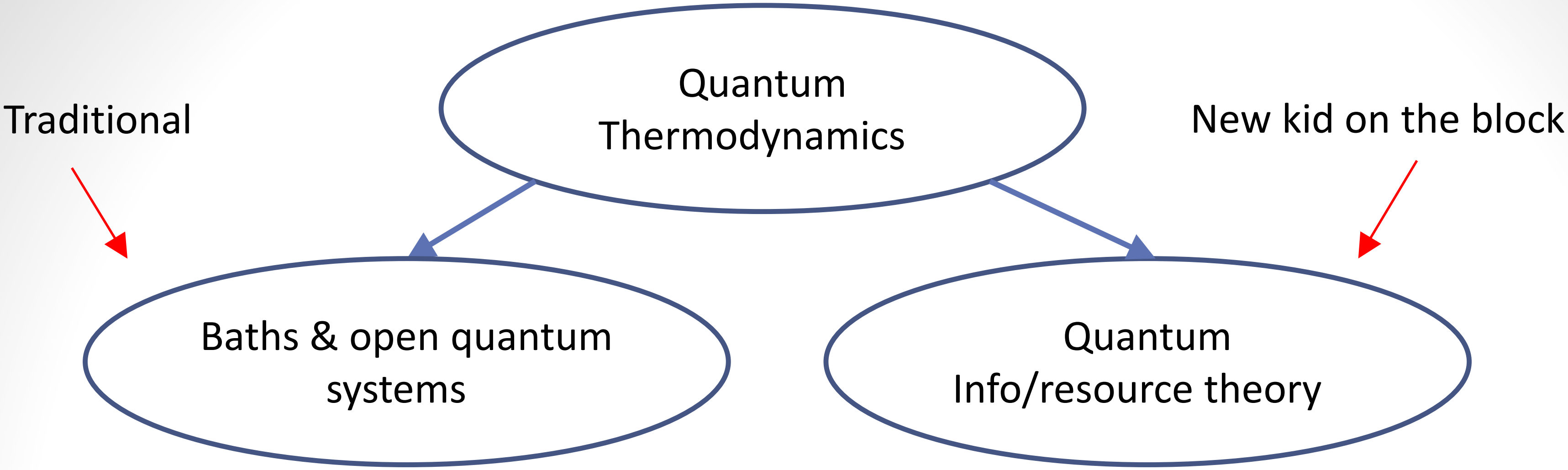

- ➢ Can we achieve a dynamical model without hidden thermodynamic costs?

– Talk overview:

  - ✓ Yes: derive a continuous time dynamical model for circuit model
  - – But: difficult due to “catalytic embezzling”
  - – New physics: — 3rd law without light cone
    — laws only apply in equilibrium

Quantum Thermodynamics → Quantum Info/resource theory

Transformations (basics)

- T.O.s ρ_{S}^{0} **+**  **+** 

$$\rho_{\mathrm{S}}^{0} \mapsto \rho_{\mathrm{S}}^{1} \quad \text{if} \quad \exists \quad \tau_{\mathrm{B}} \quad \text{and} \quad [U_1, \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{B}}] = 0$$

$$\text{s.t.} \qquad \rho_{\mathrm{S}}^{1} = \mathrm{tr}_{\mathrm{B}}[U_1(\rho_{\mathrm{S}}^{0} \otimes \tau_{\mathrm{B}})U_1^{\dagger}]$$

M. Horodecki, J. Oppenhem, Nat. Commun (2013). F. Brandão et. al. (2015)

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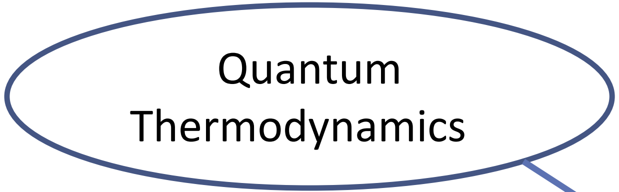
- C.T.O.s ρ_S^0 +  +  + 

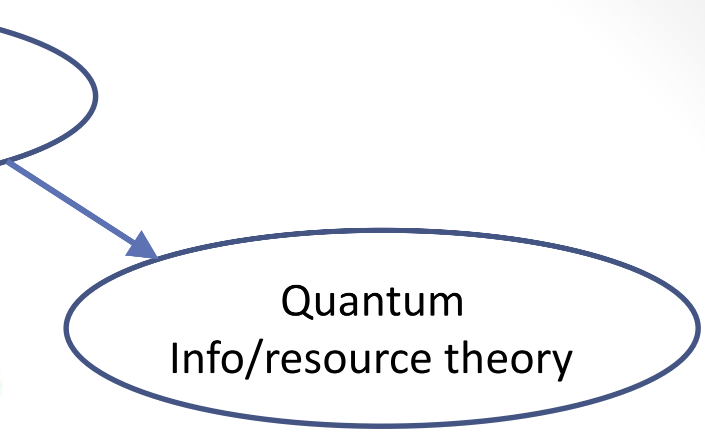
$$\rho_S^0 \mapsto \rho_S^1 \quad \exists \quad \tau_B, \; \rho_{Cat}^0 \; \text{and} \quad [U_1, \hat{H}_S + \hat{H}_{Cat} + \hat{H}_B] = 0$$

$$\text{s.t.} \quad \rho_S^1 \otimes \rho_{Cat}^0 = \mathrm{tr}_G[U_1(\rho_S^0 \otimes \rho_{Cat}^0 \otimes \tau_B)U_1^\dagger]$$

- If $\tau_B \propto \mathbb{1}_B$ we say C.N.O not C.T.O.

M. Horodecki, J. Oppenhem, Nat. Commun (2013). F. Brandão et. al. (2015)





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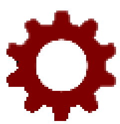
- C.T.O.s ρ_S^0 +  +  + 

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- David Jennings, 3rd KIAS workshop on Q. Info & and Thermo (2017).

Hamiltonian formulation of transitions

Assume transition occurs during time interval $t \in (t_1, t_2)$

$$\underset{t \leq t_1}{\rho_{\mathrm{S}}^{0}} \xrightarrow[\mathrm{TO}]{} \underset{t_2 \leq t}{\rho_{\mathrm{S}}^{1}}$$

$$\rho_{\mathrm{SCatB}}(t) = e^{-i\hat{H}_{\mathrm{SCatB}}\delta(t)}(\rho_{\mathrm{S}} \otimes \rho_{\mathrm{Cat}} \otimes \tau_{\mathrm{B}})e^{i\hat{H}_{\mathrm{SCatB}}\delta(t)},$$

$$\delta(t) = \begin{cases} 0 \text{ if } t \leq t_1 \\ 1 \text{ if } t \geq t_2 \end{cases}$$

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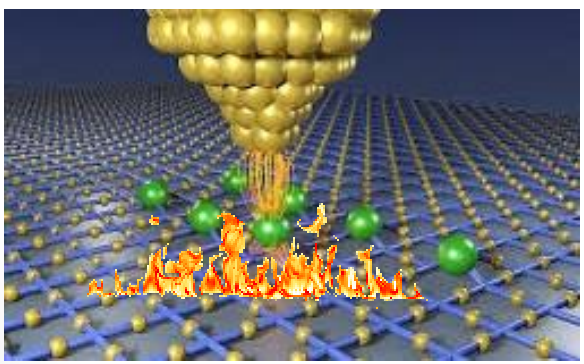
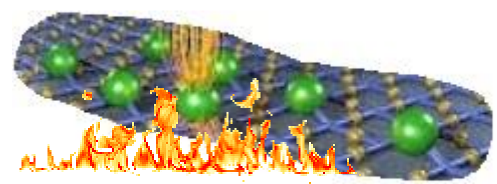
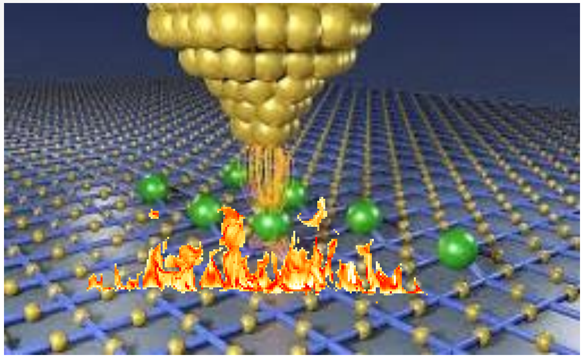


Open quantum system

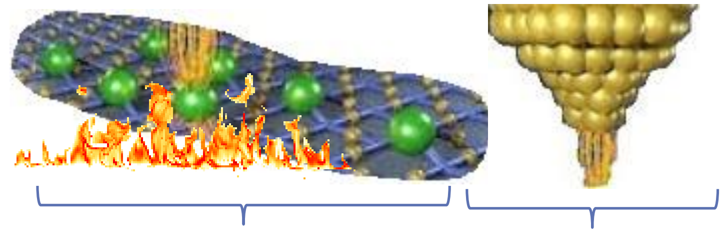


$\rho_{\mathrm{SCatB}}(t)$, $\hat{H}_{\mathrm{SCatB}}(t)$

Time independent Hamiltonian formulation of transitions



Closed quantum system $\hat{H}_{\mathrm{SCatBC}}$



$\rho_{\mathrm{SCatB}}(t)$, Control device $\rho_{\mathrm{C}}(t)$

$$\rho^{F}_{\mathrm{SCatBC}}(t) = e^{-it\hat{H}_{\mathrm{SCatBC}}}(\rho_{\mathrm{S}} \otimes \rho_{\mathrm{Cat}} \otimes \tau_{\mathrm{B}} \otimes \rho_{\mathrm{C}})e^{it\hat{H}_{\mathrm{SCatBC}}}$$

With free Hamiltonians $\hat{H}_{\mathrm{S}} = \hat{H}_{\mathrm{Cat}} = \mathbb{1},\ \hat{H}_{\mathrm{B}},\ \hat{H}_{\mathrm{C}}$

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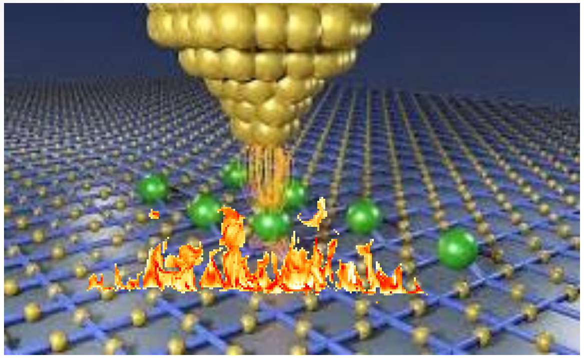

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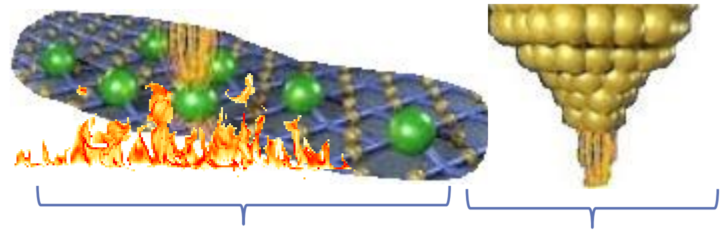

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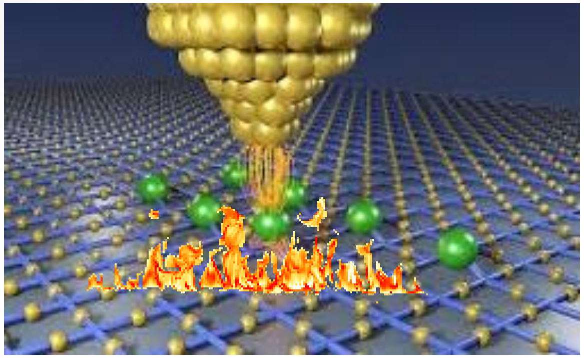
- Ideally need $\rho_{\mathrm{C}}(t)$ to be:
  - “free” resource, i.e. a “dynamic catalyst”:

    $$\rho_{\mathrm{C}}(t) = e^{-it\hat{H}_{\mathrm{C}}}\rho_{\mathrm{C}}\, e^{it\hat{H}_{\mathrm{C}}} \quad \text{for } t \in [0, t_1] \cup [t_2, t_3]$$

- Solution: yes, buy unphysical: $\hat{H}_{\mathrm{C}} = \hat{p}_{\mathrm{C}}$

Time independent Hamiltonian formulation of transitions



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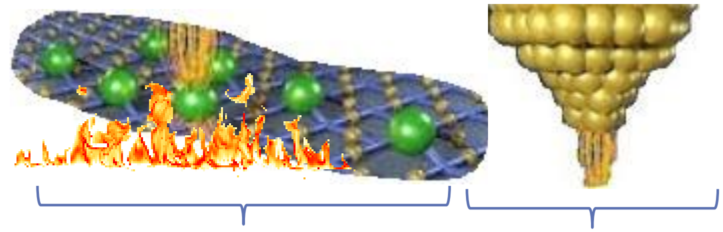

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Requires Infinite energy !

- Therefore need inexact catalysis

Why is there a potential issue? N. Ng et al NJP (2015)

- Consider Inexact Catalysis:

$$\rho_{\mathrm{S}}^{0} \otimes \rho_{\mathrm{Cat}}^{0} \xrightarrow[\mathrm{NO}]{} \rho_{\mathrm{S}}^{1} \otimes \rho_{\mathrm{Cat}}'$$

- Where

$$\|\rho_{\mathrm{Cat}}^{0} - \rho_{\mathrm{Cat}}'\|_1 = \frac{d_{\mathrm{S}}}{1 + (d_{\mathrm{S}} - 1)\log_{d_{\mathrm{S}}} d_{\mathrm{Cat}}} \to 0 \quad \text{as} \quad d_{\mathrm{Cat}} \to \infty$$

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$$\rho_S^0 \otimes \rho_{\text{Cat}}^0 \xrightarrow[\text{NO}]{} \rho_S^1 \otimes \rho_{\text{Cat}}^{\prime} \quad \forall\, \rho_S^0, \rho_S^1$$

Theory of thermodynamics becomes trivial!

- A lot is still unknown about inexact catalysis

Conclusion:

- Consider implementation of CTO : $\rho_{\mathrm{S}}^{0} \xrightarrow[\mathrm{CTO}]{} \rho_{\mathrm{S}}^{1}$

$$\rho_{\mathrm{SCatBC}}^{F}(t) = e^{-it\hat{H}_{\mathrm{SCatBC}}}(\rho_{\mathrm{S}} \otimes \rho_{\mathrm{Cat}}^{0} \otimes \tau_{\mathrm{B}} \otimes \rho_{\mathrm{C}})e^{it\hat{H}_{\mathrm{SCatBC}}}$$

- Dynamics to be close to ``idealized" case:
  - ➢ After tracing out the bath, for all times $t \in [0, t_1] \cup [t_2, t_3]$

$$\|\rho_{\mathrm{SCatC}}^{F}(t) - \rho_{\mathrm{S}}(t) \otimes \rho_{\mathrm{Cat}}^{0} \otimes \rho_{\mathrm{C}}(t)\|_1 \rightarrow 0 \quad \text{as} \quad d_{\mathrm{C}} \rightarrow \infty$$

$$\rho_{\mathrm{C}}(t) = e^{-it\hat{H}_{\mathrm{C}}}\rho_{\mathrm{C}}\, e^{it\hat{H}_{\mathrm{C}}}$$

$$\rho_{\mathrm{S}}(t) = \begin{cases} \rho_{\mathrm{S}}^{0} \text{ for } t \in [0, t_1] \\ \rho_{\mathrm{S}}^{1} \text{ for } t \in [t_2, t_3] \end{cases}$$

- This is **not** sufficient to conclude CTO!

Thm 1: Consider $\tau_{\mathrm{B}} = \mathbb{1}/d_{\mathrm{B}}$

$$\rho^F_{\mathrm{SCatCB}}(t) := e^{-it\hat{H}_{\mathrm{SCatCB}}}(\rho^0_{\mathrm{S}} \otimes \rho^0_{\mathrm{Cat}} \otimes \rho^0_{\mathrm{C}} \otimes \tau_{\mathrm{B}})e^{it\hat{H}_{\mathrm{SCatCB}}},$$

$$\|\rho^F_{\mathrm{SCatC}}(t) - \rho^F_{\mathrm{S}}(t) \otimes \rho^0_{\mathrm{Cat}} \otimes \rho_{\mathrm{C}}(t)\|_1 \leq \varepsilon_{emb}(t; d_{\mathrm{S}}, d_{\mathrm{Cat}} d_{\mathrm{C}})$$

- There exists $\sigma_{\mathrm{S}}(t)$ which is ε_{res} close to $\rho^F_{\mathrm{S}}(t)$, i.e.

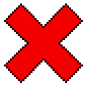
$$\|\sigma_{\mathrm{S}}(t) - \rho^F_{\mathrm{S}}(t)\|_1 \leq \varepsilon_{res}\left(d_{\mathrm{S}}, d_{\mathrm{Cat}} d_{\mathrm{C}}, \varepsilon_{emb}(t; d_{\mathrm{S}}, d_{\mathrm{Cat}} d_{\mathrm{C}})\right)$$

where $\rho^0_{\mathrm{S}} \otimes \tilde{\rho}^0_{\mathrm{Cat}} \otimes \rho^0_{\mathrm{C}} \xrightarrow[\mathrm{NO}]{} \sigma_{\mathrm{S}}(t) \otimes \tilde{\rho}^0_{\mathrm{Cat}} \otimes \rho_{\mathrm{C}}(t)$ for some $\tilde{\rho}^0_{\mathrm{Cat}}$.

- Explicitly:

$$\varepsilon_{res} = 5\sqrt{\frac{d_{\mathrm{S}}^{5/3} + 4(\ln d_{\mathrm{S}} d_{\mathrm{Cat}} d_{\mathrm{C}}) \ln d_{\mathrm{S}}}{-\ln \varepsilon_{emb}}}.$$

- Example 1: $\varepsilon_{emb} \propto d_{\mathrm{C}}^{-K}$,

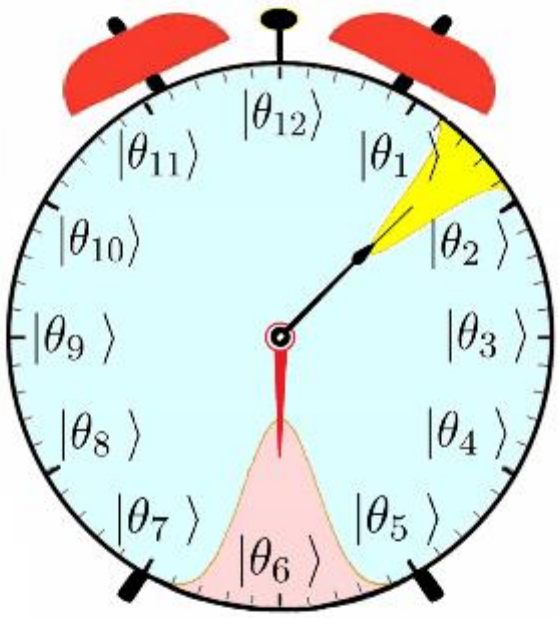


- Example 1: $\varepsilon_{emb} \propto e^{-d_{\mathrm{C}}}$

- Quasi-ideal clock

$$|\psi_{\mathrm{Cl}}\rangle\,, \qquad \hat{H}_{\mathrm{Cl}} + \hat{V}_{\mathrm{Cl}}$$

M. Woods, R. Silva, J. Oppenheim,
J. Annales Henri Poincaré (2018)



Thm 2:

- The Quasi-Ideal Clock can achieve $\forall \; t \in [0, t_1] \cup [t_2, t_3]$

$$\varepsilon_{emb}(t; d_{\mathrm{S}}, d_{\mathrm{Cat}} d_{\mathrm{C}}) = \mathcal{O}\left(poly(d_{\mathrm{C}}) \exp\left[-c_0 d_{\mathrm{C}}^{1/4} \right] \right), \quad \text{as } d_{\mathrm{C}} \to \infty$$

- Quasi-ideal clock

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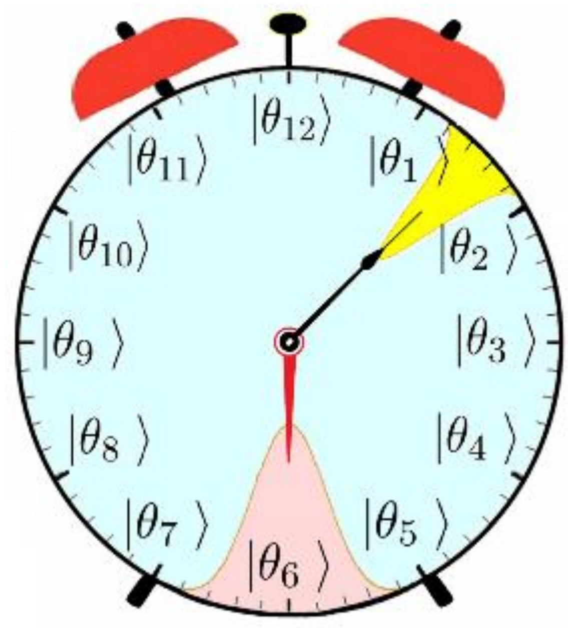

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- Therefore $\forall\ t\in[0,t_1]\cup[t_2,t_3],\quad \varepsilon_{res}\to 0,\quad \text{as } d_{\rm C}\to\infty$

- Definition: The embezzlement-free Control model

Let $\hat{H}_{\mathrm{SCatCB}} = \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{Cat}} + \hat{H}_{\mathrm{B}} + \hat{H}^{\mathrm{int}}_{\mathrm{SCatB}} \otimes \hat{H}^{\mathrm{int}}_{\mathrm{C}} + \hat{H}_{\mathrm{C}}$

and $\hat{H}^{\mathrm{int}}_{\mathrm{SCatB}}$ satisfy:

1) $[\hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{Cat}} + \hat{H}_{\mathrm{B}}, \hat{H}^{\mathrm{int}}_{\mathrm{SCatB}}] = 0$

2) $\mathrm{tr}_{\mathrm{B}}[e^{-i\hat{H}^{\mathrm{int}}_{\mathrm{SCatB}}}(\rho^0_{\mathrm{S}} \otimes \rho^0_{\mathrm{Cat}} \otimes \tau_{\mathrm{B}})e^{i\hat{H}^{\mathrm{int}}_{\mathrm{SCatB}}}] = \rho^1_{\mathrm{S}} \otimes \rho^0_{\mathrm{Cat}}$

for some $\rho^1_{\mathrm{S}}, \rho^0_{\mathrm{Cat}}, \tau_{\mathrm{B}}$

$$\rho^{\mathrm{target}}_{\mathrm{SCatB}}(t) = e^{-i\delta(t)\hat{H}^{\mathrm{int}}_{\mathrm{SCatB}}}\left(\rho^0_{\mathrm{S}}(t) \otimes \rho^0_{\mathrm{Cat}} \otimes \tau_{\mathrm{B}}\right) e^{i\delta(t)\hat{H}^{\mathrm{int}}_{\mathrm{SCatB}}}$$

$$\delta(t) = \begin{cases} 0 & \text{for } t \in [0, t_1] \\ 1 & \text{for } t \in [t_2, t_3]. \end{cases}$$

Thm 3:

- The deviation from the idealised dynamics is bounded by

$$\|\rho^F_{\mathrm{SCatC}}(t) - \rho^F_{\mathrm{S}}(t) \otimes \rho^0_{\mathrm{Cat}}(t) \otimes \rho^0_{\mathrm{C}}(t)\|^4_1 \leq d_{\mathrm{S}} d_{\mathrm{Cat}} d_{\mathrm{B}} \mathrm{tr}\left[\tau^2_{\mathrm{B}}\right] \Delta_{\mathrm{C}}(t)$$

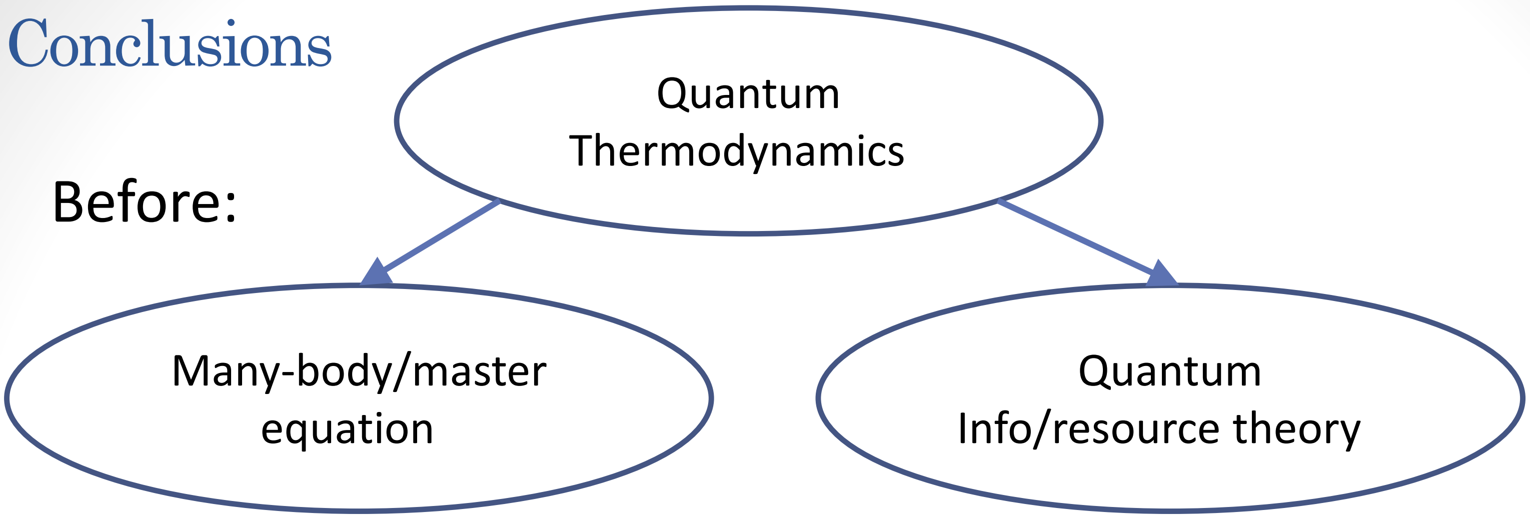
- The final state $\rho^F_{\mathrm{S}}(t)$ is

$$\|\rho^F_{\mathrm{S}}(t) - \rho^{\mathrm{target}}_{\mathrm{S}}(t)\|_1 \leq \sqrt{d_{\mathrm{S}} d_{\mathrm{Cat}} d_{\mathrm{B}} \mathrm{tr}\left[\tau^2_{\mathrm{B}}\right]} \Delta_{\mathrm{C}}(t)$$

close to one which can be reached via t-CTO: For all $t \in [0, t_1] \cup [t_2, t_3]$ the transition $\rho^0_{\mathrm{S}} \otimes \rho^0_{\mathrm{Cat}} \otimes \rho^0_{\mathrm{C}}$ to $\rho^{\mathrm{target}}_{\mathrm{S}}(t) \otimes \rho^0_{\mathrm{Cat}}(t) \otimes \rho^0_{\mathrm{C}}(t)$ is possible via a TO.

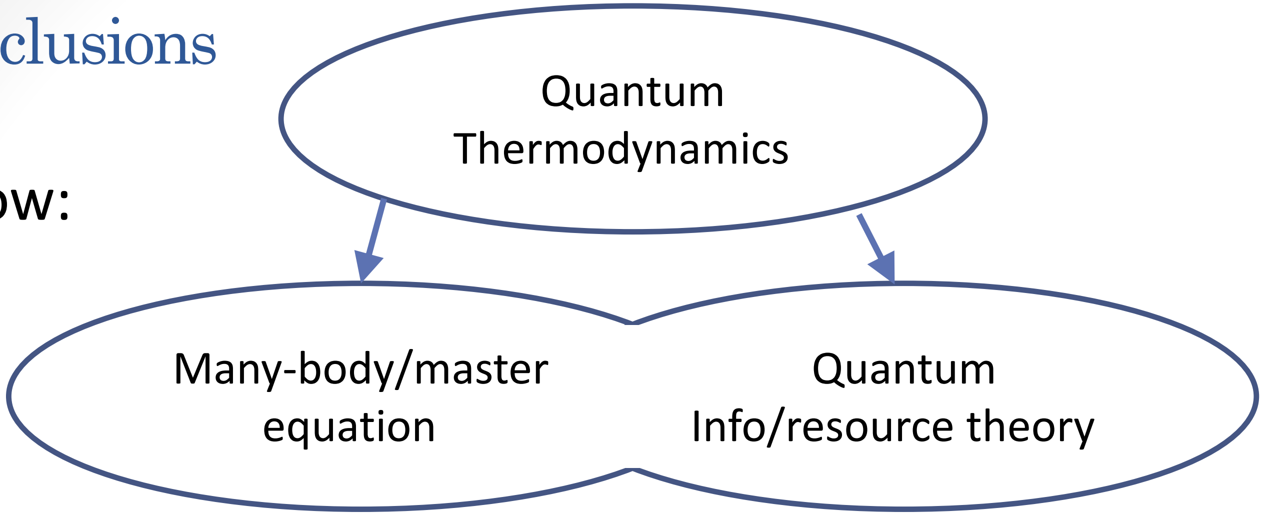
Conclusions

Before:



Conclusions

Now:



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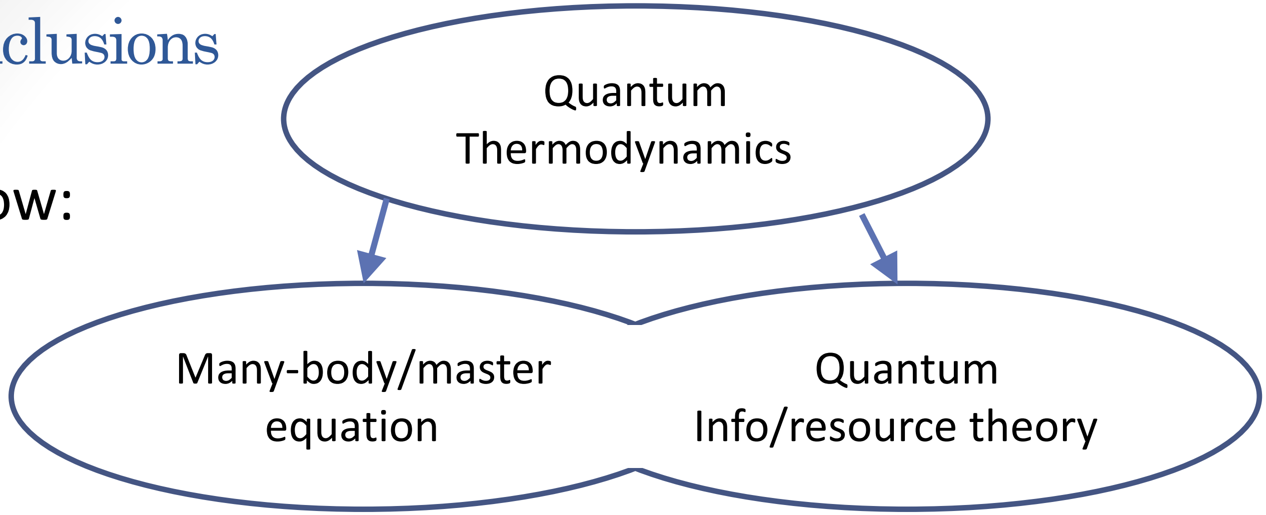

Can (in principle) the control be thermodynamically free?

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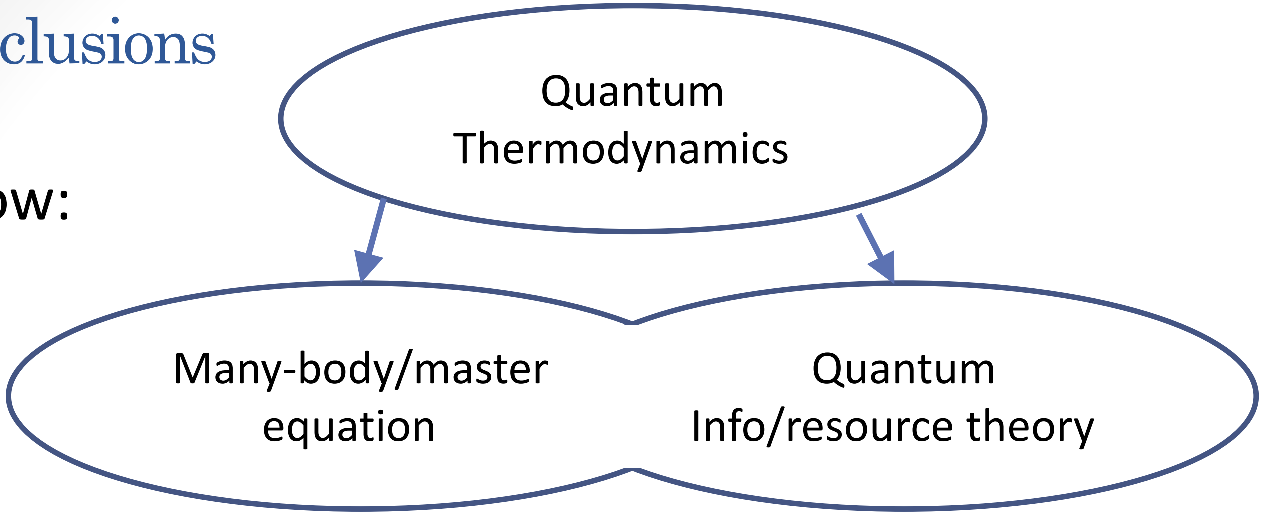

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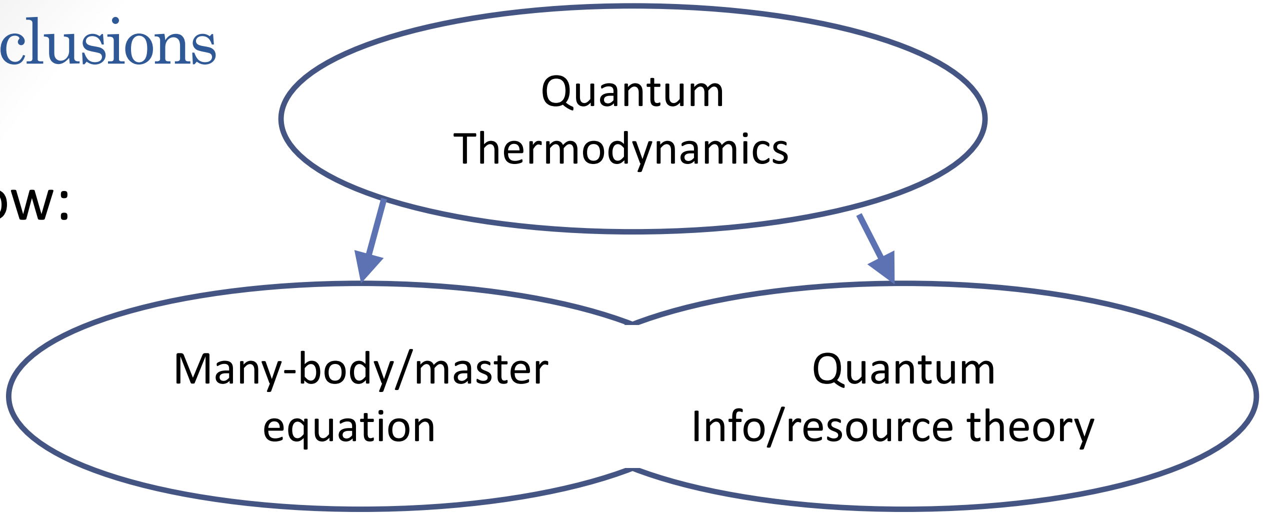

Can (in principle) the control be thermodynamically free?

Selection Open Questions

- Does the control need to be highly coherent?
- Characterization of 3rd law without light-cone bound
- Do the 2nd laws of quantum thermodynamics hold out of equilibrium?