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Thermodynamic resources in continuous-variable quantum systems

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Bosonic continuous-variable systems

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- Frequency modes of electromagnetic fields
- Vibration of trapped ions
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Motivation: How to understand thermodynamic aspect of resources used in these applications?

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- Canonical commutation relation

$$[\hat{q}, \hat{p}] = i\hbar$$

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- m-mode "vector of quadratures"

$$\hat{\mathbf{x}} \equiv (\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2 \cdots, \hat{q}_m, \hat{p}_m)$$

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- Phase-space moments of quantum states
  - First moments: $\langle \hat{\mathbf{x}} \rangle_\rho \in \mathbb{R}^{2m}$
  - Second moments: covariance matrix V_ρ

$$\left(V_\rho\right)_{jk} = \frac{1}{2}\left\langle\left\{\hat{x}_j - \langle \hat{x}_j \rangle_\rho, \hat{x}_k - \langle \hat{x}_k \rangle_\rho\right\}\right\rangle_\rho$$

Gaussian states

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- k-mode thermal state γ_k
  - First moments: $\langle\hat{\mathbf{x}}\rangle_{\gamma_k} = 0$
  - Covariance matrix $V_{\gamma_k} = \eta\mathbb{I}_{2k}$, where $\eta = \coth\left(\frac{\hbar\omega}{k_\mathrm{B}T}\right)$

Symplectic geometry of phase space

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- "Uncertainty principle" in terms of covariance matrix:

  $V_\rho + i\Omega \geq 0$, where

$$\Omega = \bigoplus_{k=1}^{m} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

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- Passive linear operations: $\mathbf{d} = 0$ & $S \in \mathrm{O}(2m) \cap \mathrm{Sp}(2m,\mathbb{R})$

Operational thermodynamic framework

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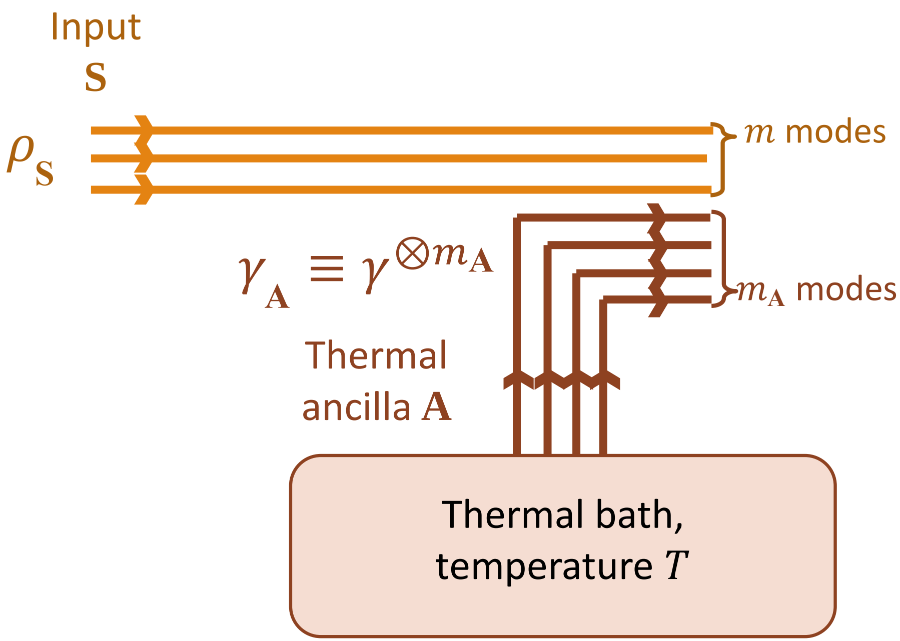
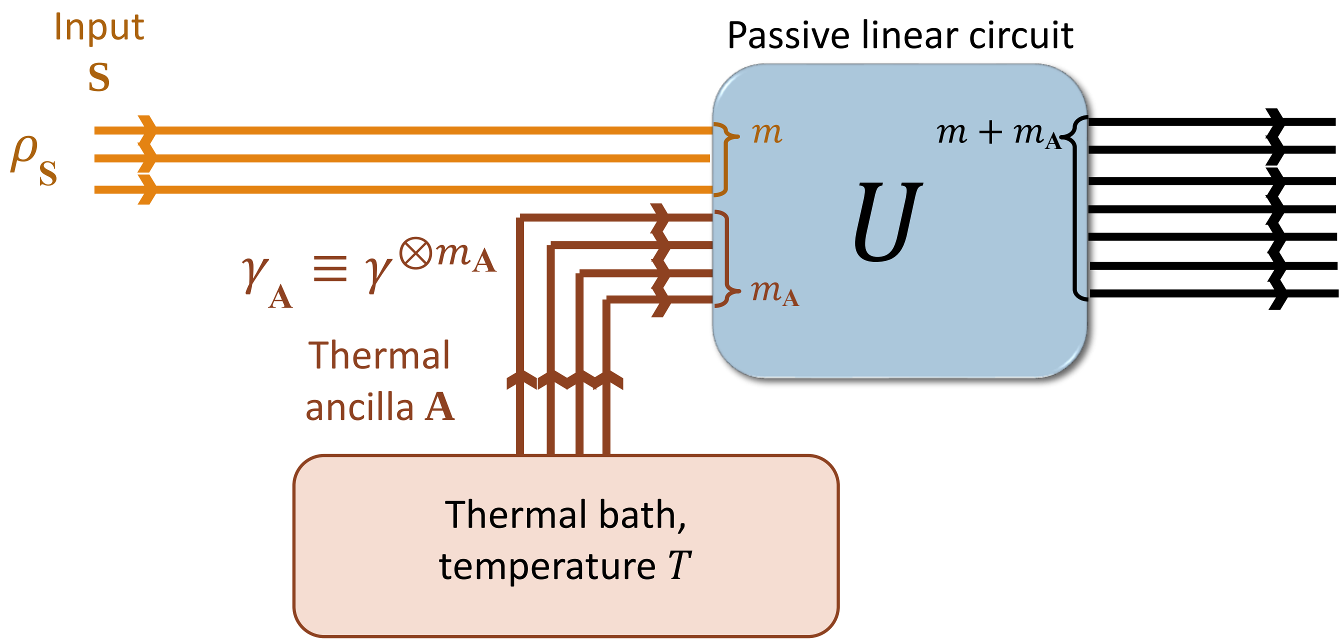


Operational thermodynamic framework



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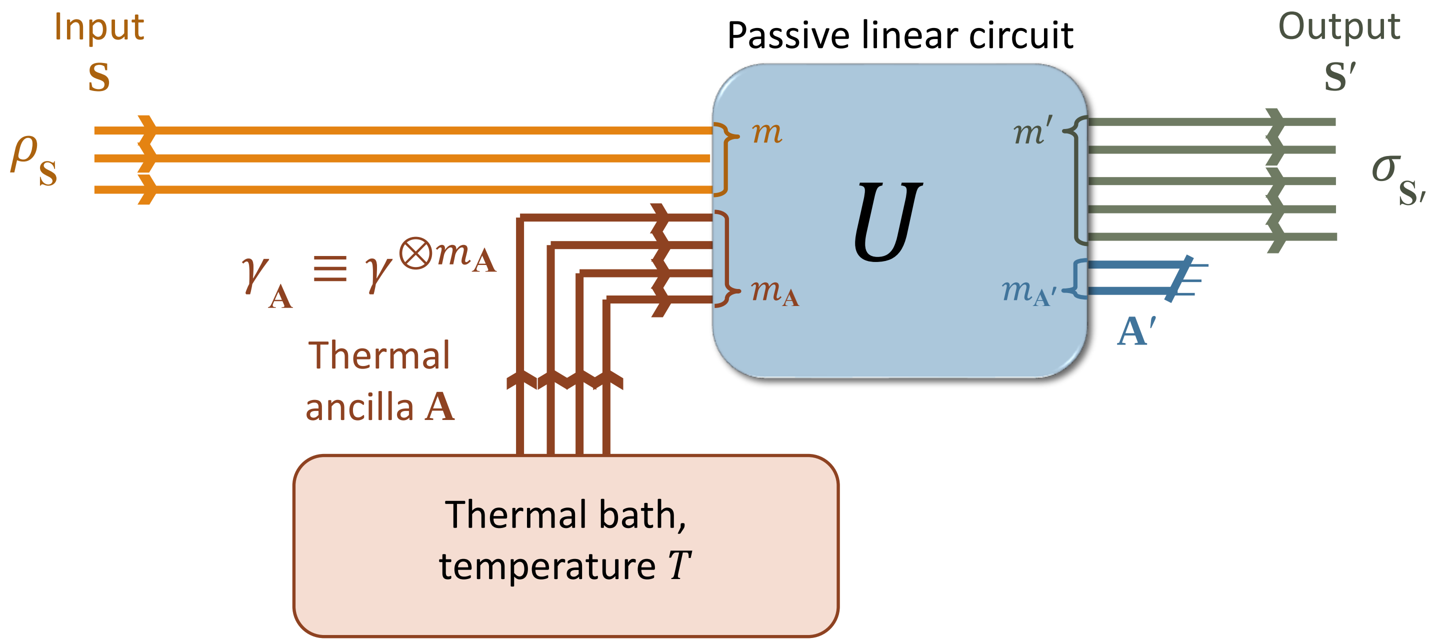

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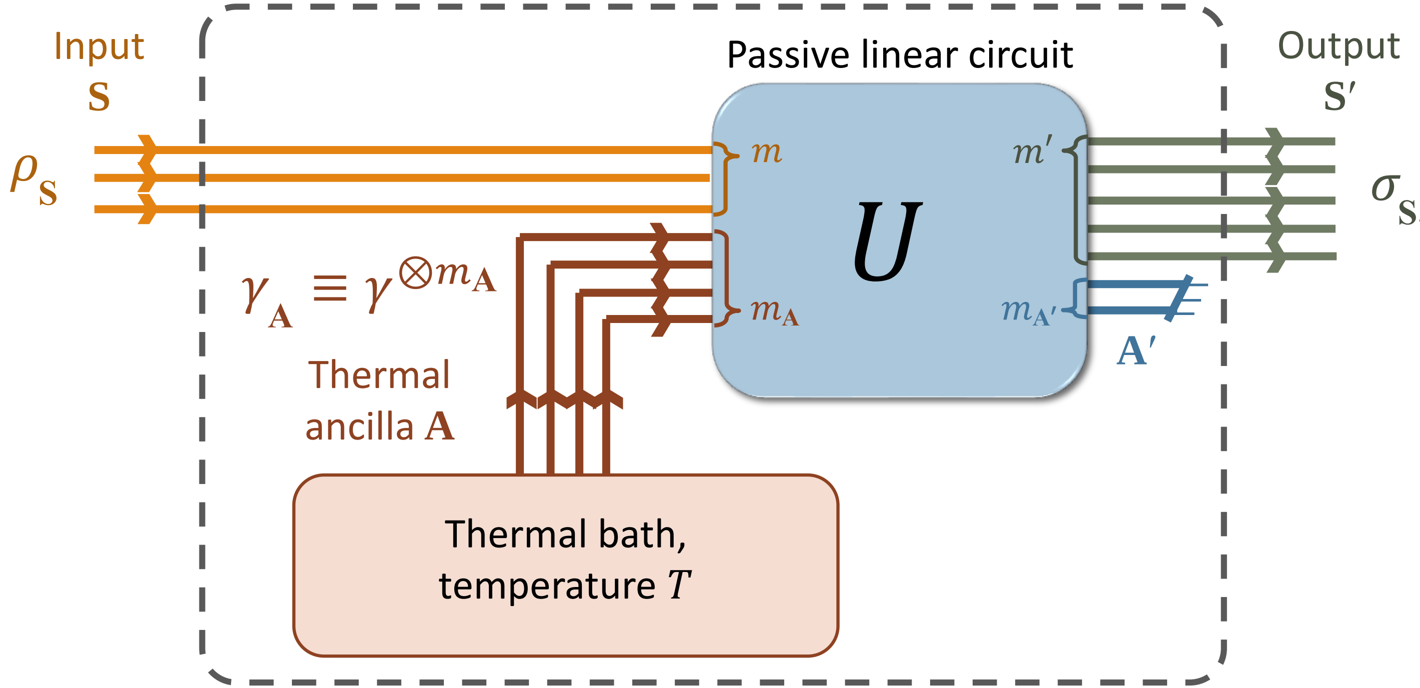

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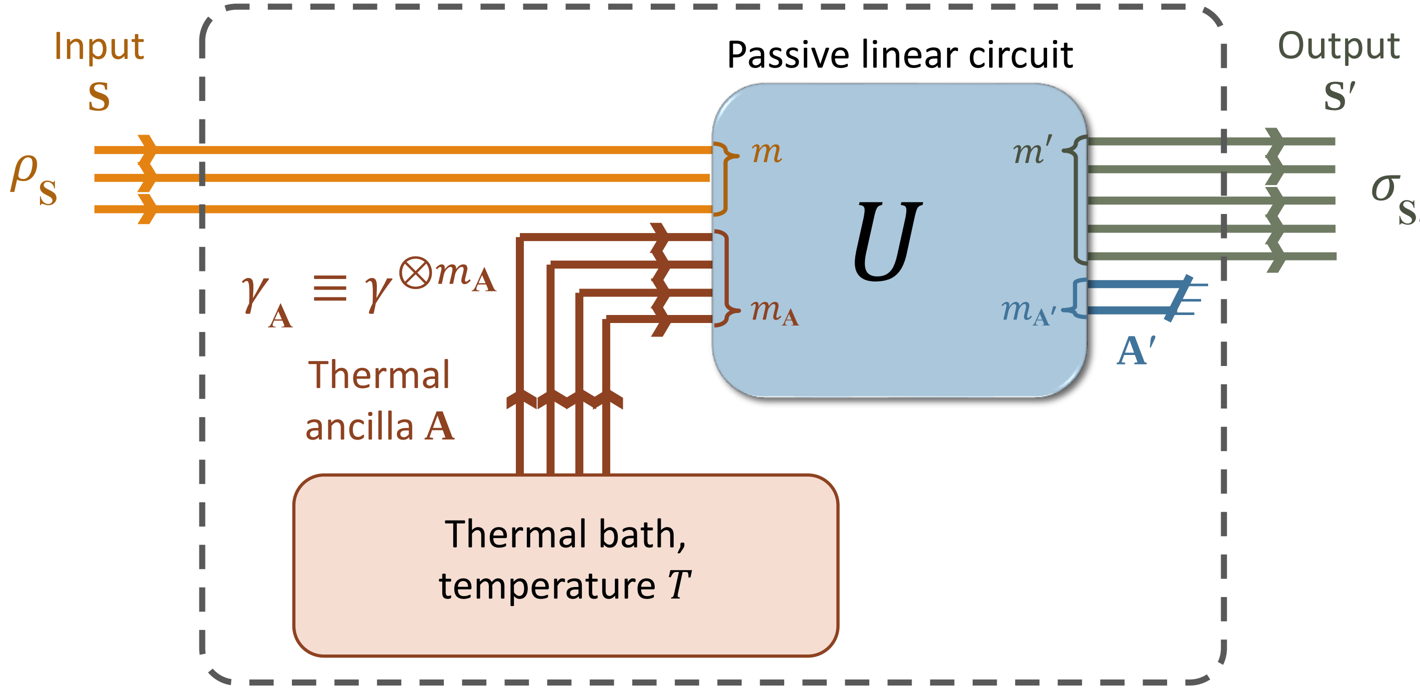

Bosonic linear thermal (BLT) operations

Thermodynamic “laws” under BLTO

I. Thermalization of generalized temperatures

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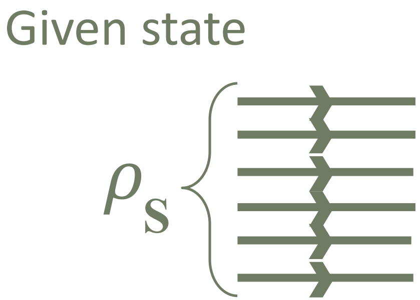
Define the *principal mode temperatures* as follows:



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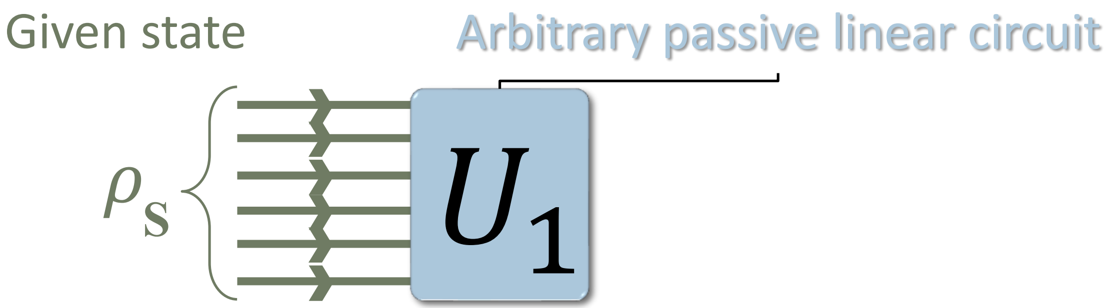

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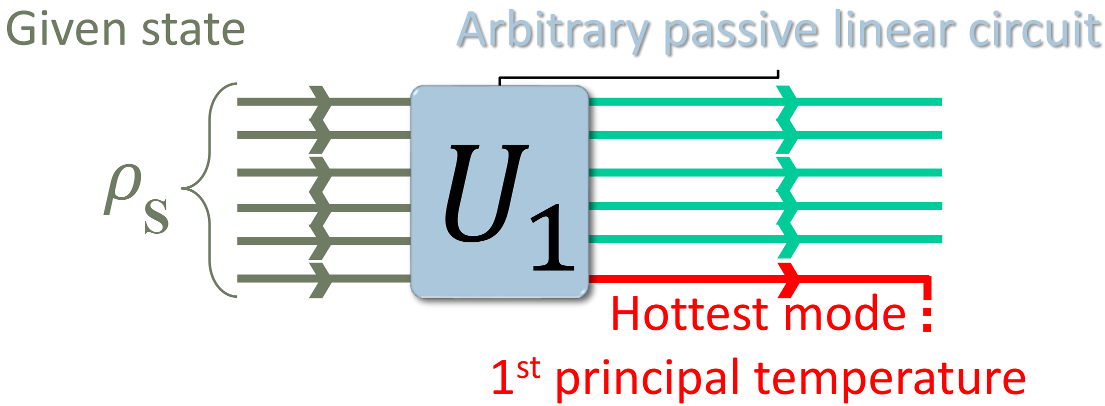

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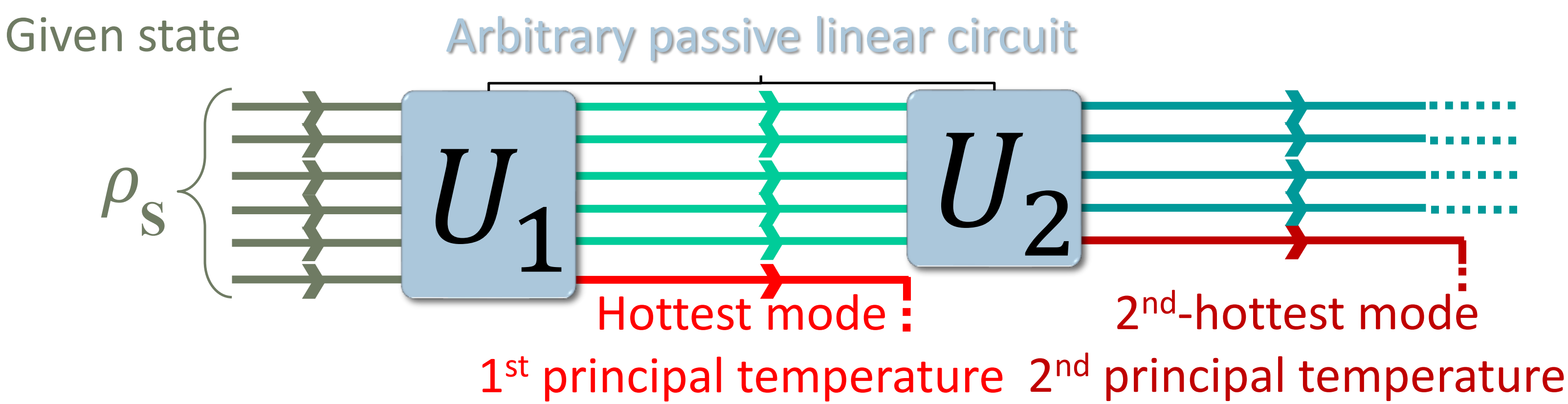

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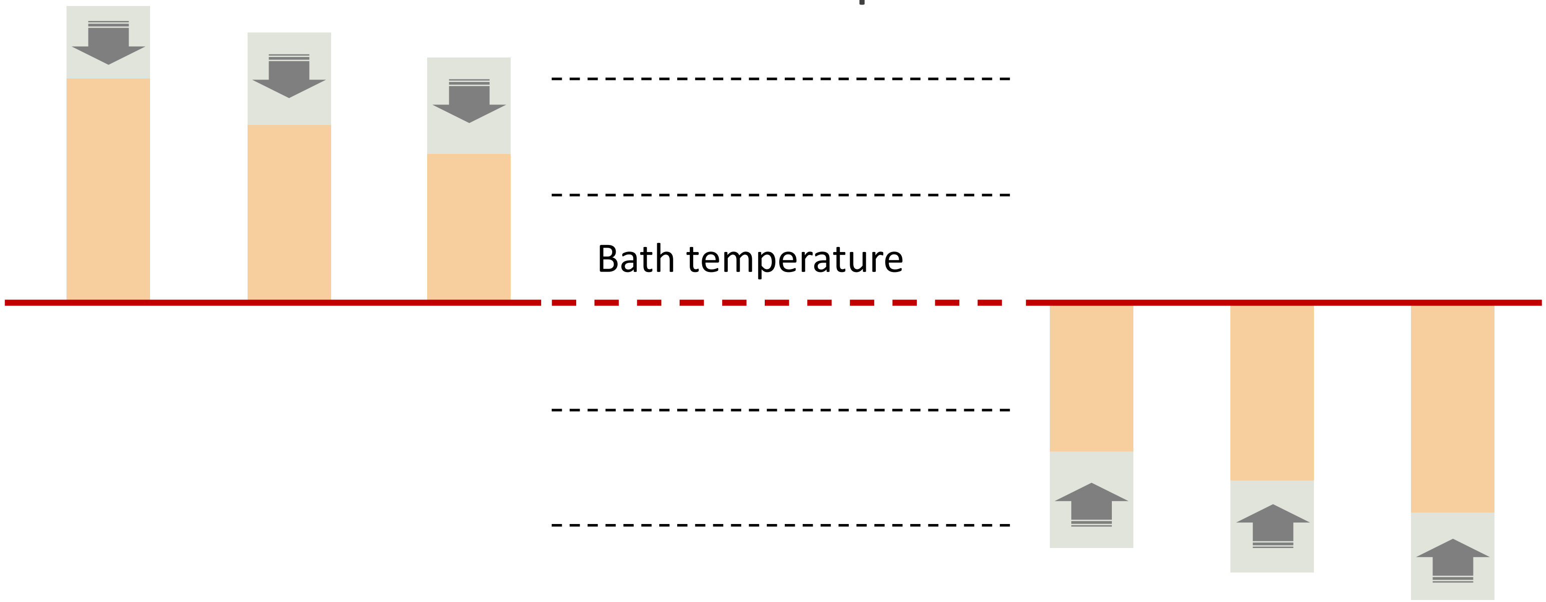
Law: Under BLT operations, every principal temperature thermalizes towards the bath temperature.



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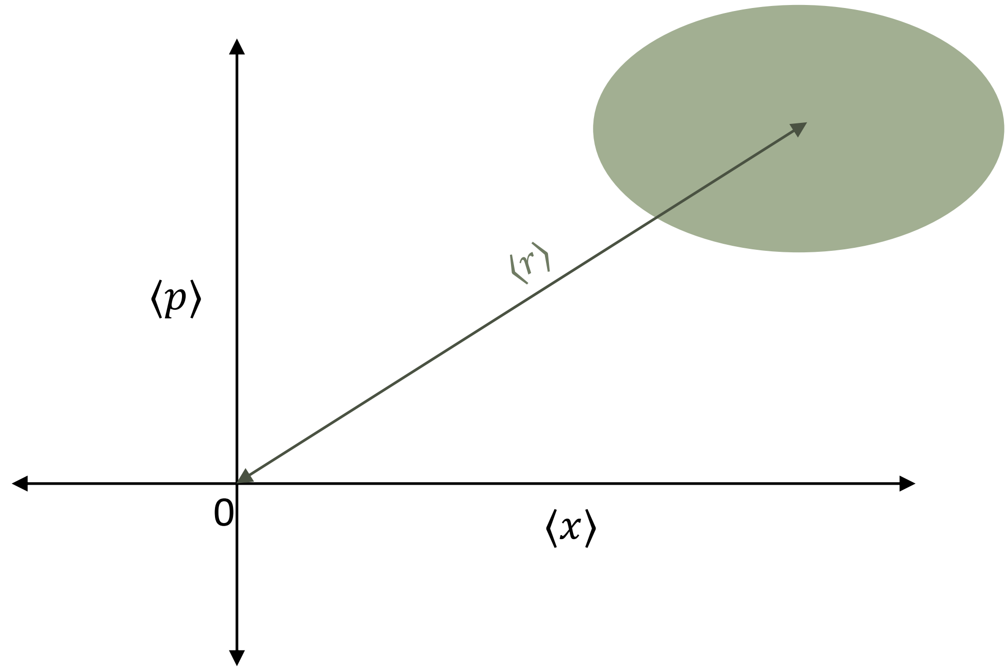
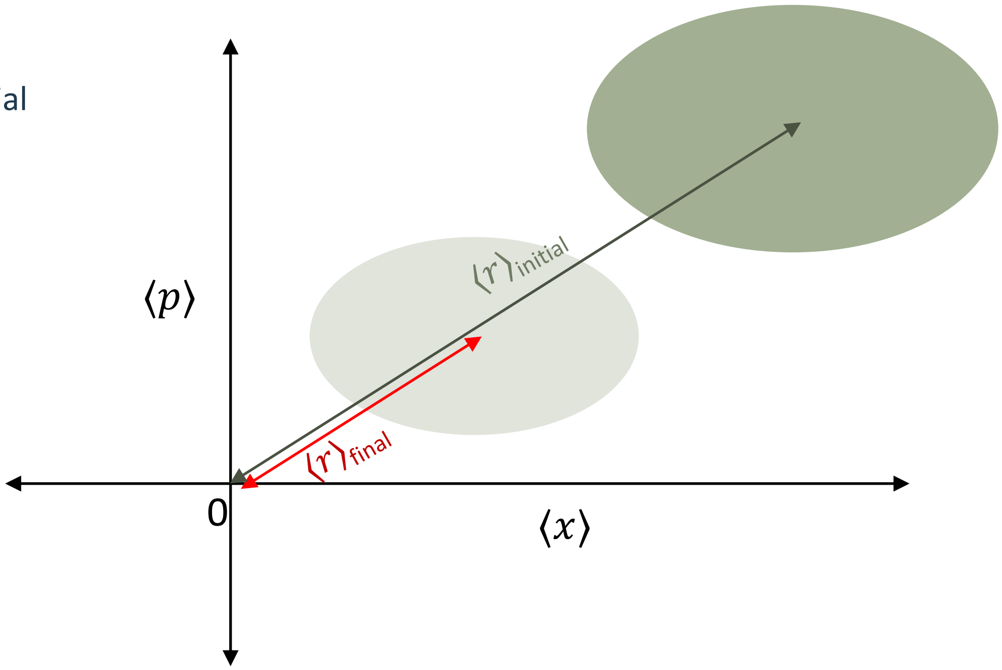


II. Signal deterioration

- Always: $\langle r \rangle_{\text{final}} \leq \langle r \rangle_{\text{initial}}$



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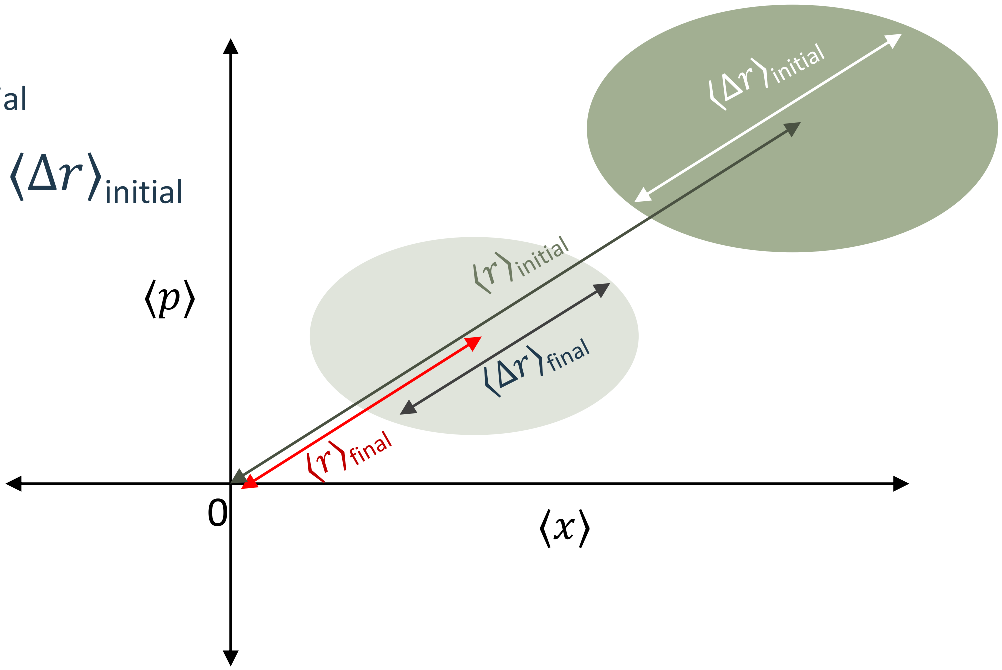

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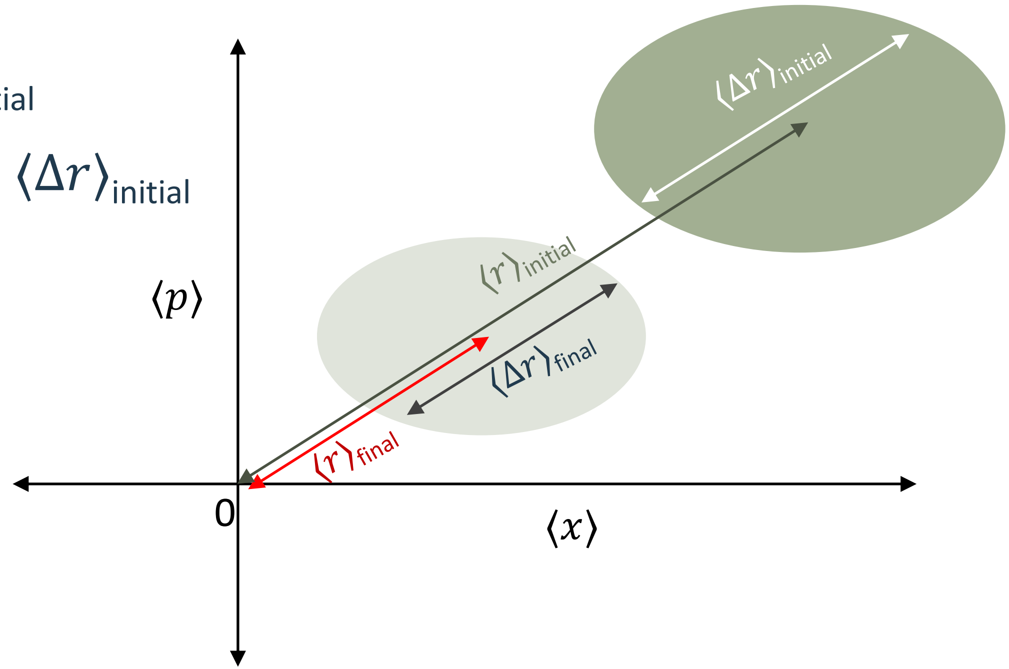

Law: $\frac{\langle \Delta r \rangle_{\text{final}}}{\langle r \rangle_{\text{final}}} \geq \frac{\langle \Delta r \rangle_{\text{initial}}}{\langle r \rangle_{\text{initial}}}$ even when $\langle \Delta r \rangle_{\text{final}} \leq \langle \Delta r \rangle_{\text{initial}}$

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- Directionwise versions of generalized temperatures also thermalize
- Directionwise and modewise signal-to-noise ratios deteriorate
- Symplectic eigenvalues below bath level increase
- Squeezing of formation[1] decreases
- Fisher information relative to phase-space displacement and other nonclassicality measures found in earlier work[2, 3] decrease

1. M Idel, D Lercher, and MM Wolf. Journal of Physics A: Mathematical and Theoretical 49(44):445304, 2016.
2. H Kwon, KC Tan, T Volkoff, and H Jeong. Physical review letters, 122(4):040503, 2019.
3. B Yadin, FC Binder, J Thompson, VN, M Gu, and MS Kim. Physical Review X, 8(4):041038, 2018.

Illustrative example

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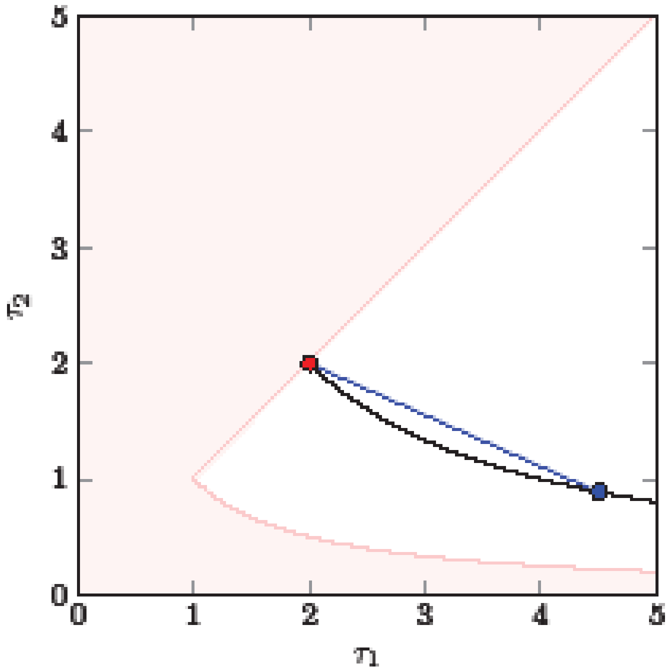

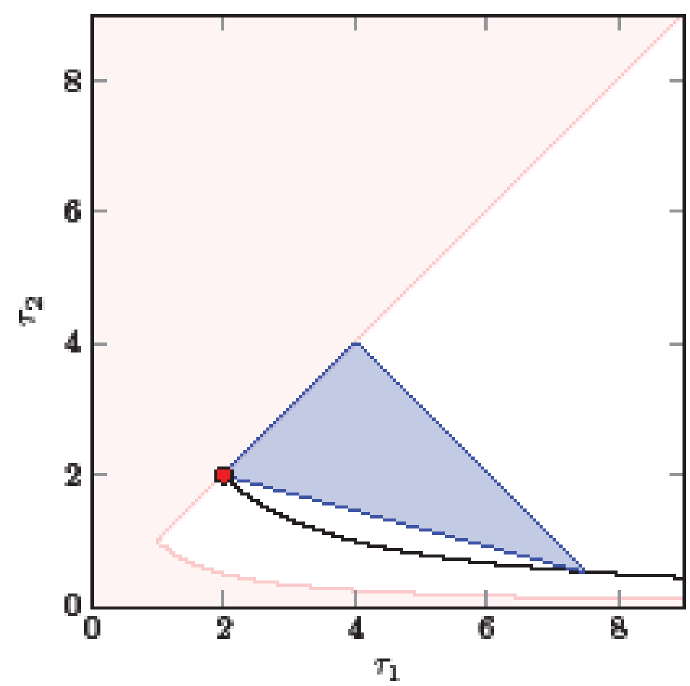

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- Operationally-motivated framework for thermodynamic processes on bosonic continuous-variable systems
- Families of "generalized temperatures" that equilibrate with bath
- Uncovering thermodynamic significance of signal-to-noise ratios, squeezing measures, Fisher information of displacement, etc.
- Outlook: connect with other approaches to thermodynamics, applications to engines, quantum control, etc.

Thank you!

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