Black hole thermodynamics & quantum gravity

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KIAS-SNU winter camp

Dec 30, 2019

Introduction

Ingredients of general relativity:

"causality ~ speed limit " (special relativity)

+

"mass = source" "local gravity = acceleration" (equivalence principle)

↓

Gravity determines the <u>causal structure</u> of spacetime.

→ how physical phenomena can propagate

Black holes are extreme examples: exists "event horizon"

Consequences:

- Information "loss" problem, thermodynamic behaviors
- Area law of entropy → "holographic" behaviors of gravity
- Makes quantum gravity very unique (demands drastic completion, e.g. to string theory)

Today's talk: Concrete relations between black holes physics & quantum gravity.

Naïve quantum gravity

Fundamental kinematic constants:

$$\left.\begin{aligned} &\text{quantum mechanics} \;:\; \hbar = \frac{h}{2\pi} = (\text{energy})(\text{time}) \equiv 1 \\ &\text{special relativity} \;:\; c = \frac{(\text{length})}{(\text{time})} \equiv 1 \end{aligned}\right\}$$

The only dimensionful quantity is:
length ~ time ~ $(\text{energy})^{-1}$ ~ $(\text{mass})^{-1}$ ...

Remaining constants are about dynamics/interactions: "coupling constants"

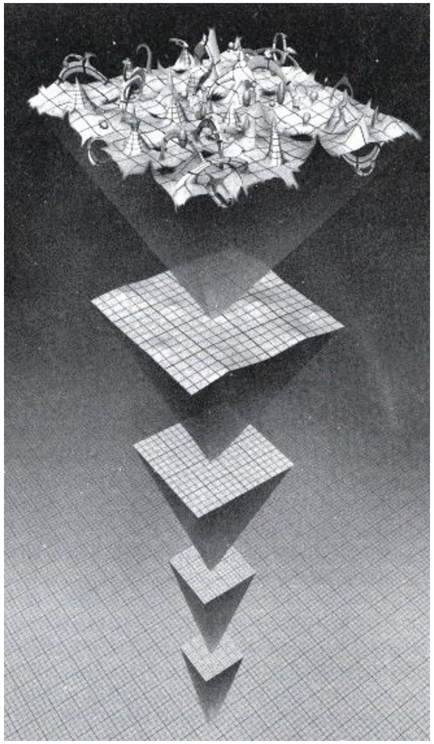
- EM: dimensionless coupling/charge.

$$F \sim e_1 e_2 / r^2 \;,\; [F] = [mLT^{-2}] \;= L^{-2} \quad \rightarrow \quad [e] = L^0$$

Forces have "universal strengths" at all scales (w/ quantum caveat)

- Gravity: Equivalence principle demands "inertia = charge"

$$F \sim G\, m_1 m_2 / r^2. \quad \rightarrow \quad [G] = L^2$$



Strong gravity at short distance ~ high E : $GE^2 \rightarrow \infty$

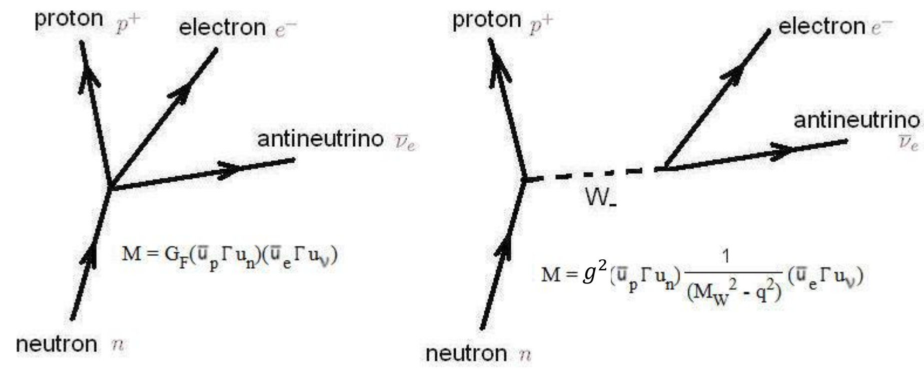
- Naïve gravity inconsistent in the quantum regime
- Interactions "diverge" for short distance quantum fluctuations
- Leaves essential challenges, such as early universe, ...

Naïve attempts of improvement

Strong-coupling problems & their solutions are common in physics.

- Fermi's weak interaction: $G_F \propto L^2$
- Electroweak theory:

  vector bosons at short distance. $g \sim L^0$



Can gravity emerge? Exists a no-go theorem [Weinberg, Witten] (1980):

- Cannot form massless spin 2 bound states from $s \leq 1$.

Steven Weinberg

Edward Witten

Exist other attempts to fit GR into particle physics (local quantum field theory, QFT)

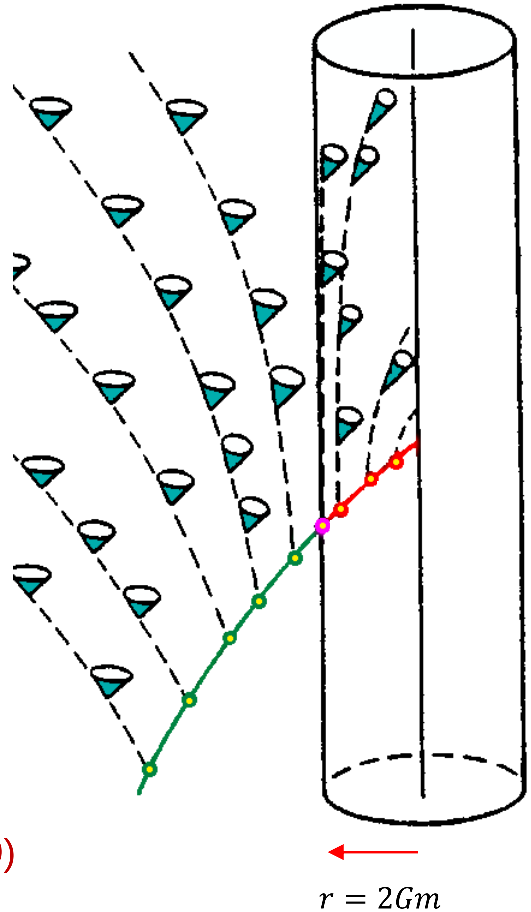
However, such "ordinary" attempts conflict w/ strange properties of black holes.

Black holes

Apparently, they are just funny classical solutions of general relativity.

Simplest solutions just carry mass ~ energy [Schwarzschild] (1916): at $c \equiv 1$

$$ds^2 = -\left(1 - \frac{2Gm}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2Gm}{r}} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$



Curved spacetime decides the causal structure:

- “lightcones”: how matters/waves can propagate.
- Big distortion of causal structure forbids particles & lights from escaping a region.
- We call its boundary surface event horizon.

- “Eternal” solutions first found in 1916.
- Physically formable by matter collapse [Oppenheimer, Snyder] (1939)

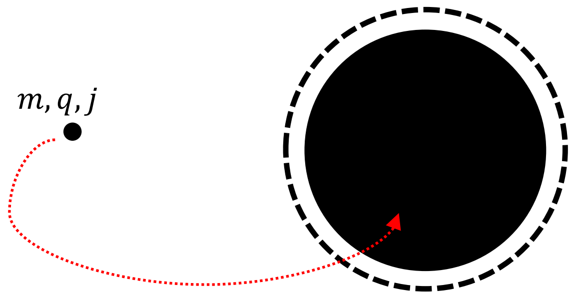
- But almost nobody trusted their relevance/importance for a long while.

Consequences of event horizon

I think black holes are disregarded mainly due to odd properties like singularities (hiding inside the even horizon), but there are more puzzling aspects.

"information loss"

- Once information falls into black holes, no obvious way to retrieve it.
- Only remaining information: Very few quantities (mass, charges, spin) encoded in the final BH solution, after swallowing the matter.



$$(M, Q, J) \rightarrow (M + m, Q + q, J + j)$$

If this is an exact property, it would deny "unitarity" of physics.

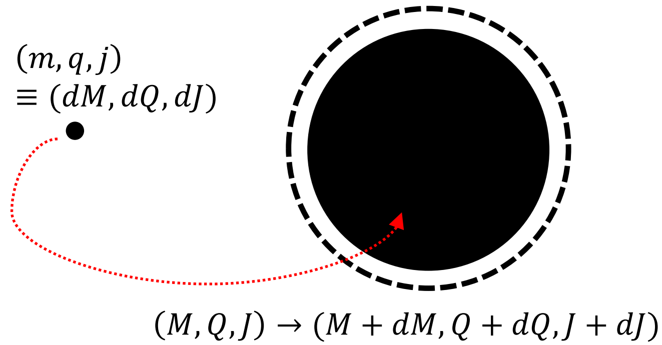
- E.g. in QM, pure states should evolve to mixed states.

$$\rho = |\Psi\rangle\langle\Psi| \ \rightarrow \ \sum_i p_i |\Psi_i\rangle\langle\Psi_i|$$

Thermodynamic behaviors

After swallowing objects, how do they “grow”?

- Slowly perturb BH: behaves as if it absorbs “heat”

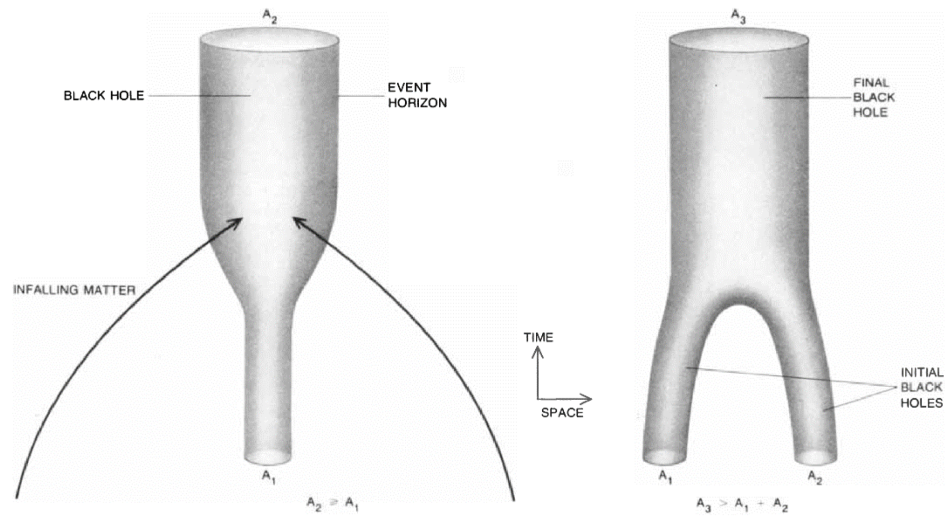


$$dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ$$

κ: surface gravity at the event horizon
A: area of the even horizon

- General time evolution of BH: obeys “area law” [Hawking] (1971)

$$\frac{dA}{dt} \geq 0$$



“Analogue thermodynamics”

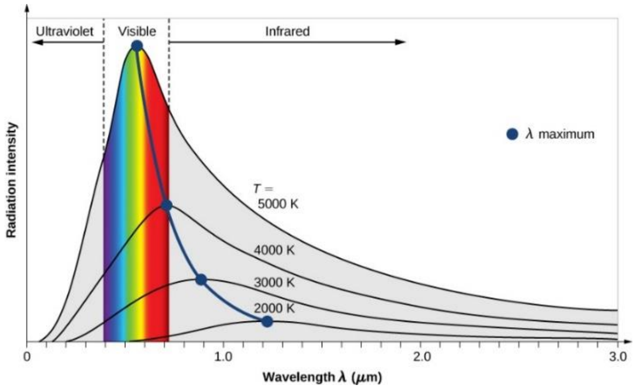
- horizon area ~ “entropy”
- surface gravity ~ “temperature” $\frac{c^2\kappa}{8\pi G} dA = TdS.$
- A and dA ~ information carried by BH & in-falling matters…? [Bekenstein]
- Proportionality constant of $S = \#A$ …? # should see $\hbar$.

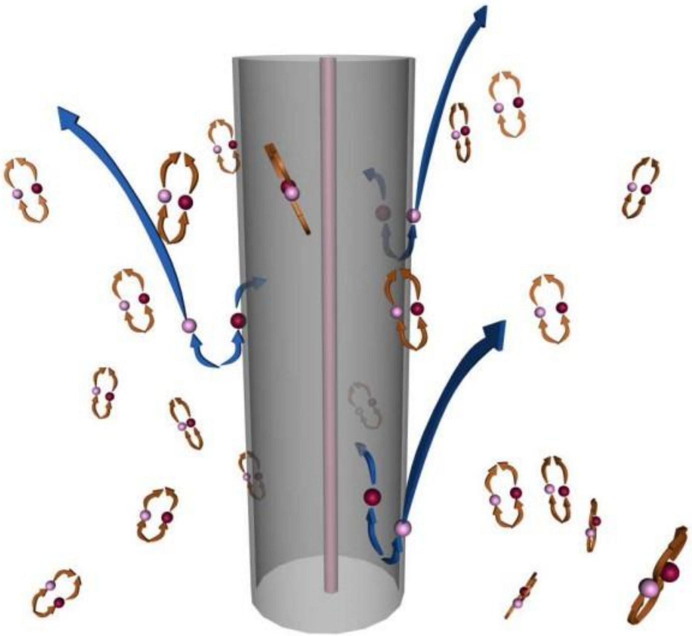
Some quantum properties

QM near BH's: Black holes emit (tiny) thermal radiations.

Hawking temperature

$$T_{\mathrm{H}} = \frac{\hbar \kappa}{2\pi k_{\mathrm{B}} c}$$





For Schwarzschild BH's: $T_{\mathrm{H}} = \dfrac{\hbar c^3}{8\pi G M k_{\mathrm{B}}} \approx 6.169 \times 10^{-8}\ \mathrm{K} \times \dfrac{M_{\odot}}{M}$

- This establishes $S_{\mathrm{BH}} = \dfrac{k_{\mathrm{B}} c^3 A}{4 G \hbar}$ Bekenstein-Hawking entropy of black holes
- Only $S_{total} = S_{BH} + S_{radiation}$ is non-decreasing. (BH's can now radiate & shrink.)

So one seeks for a true statistical interpretation of "analogue thermodynamics."

- In this idea, informal loss would simply be the usual coarse-graining loss.
- If $S_{BH} \propto$ (area) represents true information hidden inside BH, ordinary particle physics (i.e. QFT) description for gravity cannot exist. $S \propto$ (volume)

Questions

Many questions:

- Is this “thermodynamics” physical? Statistical origin? ……… (1)

  $S_{total} = S_{BH} + S_{radiation}$: $S_{radiation}$ has statistical interpretation. How about S_{BH}?

- Interior’s information on its surface: “hologram” ……… (2)

  Defies usual “particle physics like” description (local QFT)

  Are there implications of hologram to quantum gravity? If so, what?

- Are BH’s ordinary thermal systems? ……… (3)

Streamlined answers:

(1) Yes. BH area = $\log(\text{number of accessible states})\,/4G$

(2) Yes. Hologram is a universal emergent property of gravity.

(3) No. Very exotic thermal system.

I will discuss these questions in the following order: (3) → (1) → (2)

Entropy of black holes

The black holes I explained so far has negative specific heat.

- In 4 spacetime dimensions, $T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G M k_{\mathrm{B}}} \approx 6.169 \times 10^{-8}\ \mathrm{K} \times \frac{M_\odot}{M}$

Why? Entropy grows too fast in energy: $\frac{d^2 S}{dE^2} > 0$

- Schwarzschild BH in D spacetime dimension:

$$GM \sim r^{D-3}\ ,\ S \sim A/G \sim r^{D-2}/G \ \rightarrow\ S(M) \sim G^{\frac{1}{D-3}} M^{\frac{D-2}{D-3}}$$

$$\frac{dS(E)}{dE} = \frac{1}{T}\quad ,\quad \frac{d^2 S}{dE^2} = \frac{dT^{-1}}{dE} = -\frac{1}{T^2}\frac{dT}{dE} = -\frac{1}{cT^2} > 0$$

- Unstable in canonical ensemble (Cannot be in equilibrium w/ a heat bath at constant T)
- Small increase of E into the system → T decrease when $c < 0$ → further inflow of E → …

Black hole demands an unusual structure of the Hilbert space of QG.

- Entropy at high energy is very fast-growing. Is this natural?
- Or, is this possible at all…? Do we know of any systems of this sort?

Entropy of particle systems

Ideal gas of nonrelativistic particles in D spatial dimension:

$$e^{S(E,V,N)} \sim \frac{V^N}{N!(2\pi)^3} \mathrm{vol}\left[S^{DN-1}\left(\mathrm{radius}^2 = \sum_{i=1}^{N} |\vec{p}_i|^2 = 2mE\right)\right] \sim \exp\left[N \log \frac{\#V(2mE)^{D/2}}{N^{D/2+1}}\right]$$

- Since $S \propto \log E$, very mild growth: $d^2S/dE^2 < 0$.

Ideal gas of relativistic particles in D spatial dimension: massless at high E

- Violent particle creations would increase entropy at high E.
- On dimensional grounds, $$S \propto VT^D \quad , \quad E \propto VT^{D+1} \quad \rightarrow \quad S(E,V) \propto V^{\frac{1}{D+1}} E^{\frac{D}{D+1}}$$
- Since $S \propto E^{\alpha}$ with $\alpha < 1$, this is a much faster growth.
- But still not too fast: $d^2S/dE^2 \sim \alpha(\alpha-1)E^{\alpha-2} < 0$.

Marginally exotic is $S(E) \propto E$ at high E: not realized w/ ordinary particles

- Asymptotic temperature is constant: $dS/dE \equiv T_*^{-1}$.
- Starts to behave weirdly: Cannot be in equilibrium with a heat bath at $T > T_*$.
- Do we know of its physical realization?

"Exotic" entropy

String theory: Elementary (perturbative) strings exhibit $S(E) \sim E/T_H$ at high energies.

- "relativistic string" = "infinite tower of particles" from oscillation modes.
- This is called "Hagedorn growth" [Hagedorn] (1965) [Sundborg] [Atick,Witten] … (198n)

Such a fast growth, also in various variations (e.g. Kerr BH's), has motivated people to seek for connections of string theory to black hole microstates.

There are further issues:

- To reach black holes described by general relativity, one should understand interactions in detail (often including <u>strong-coupling effects</u>): Since GM is macroscopic/large for BH's.
- Still, the growth is <u>not fast enough</u> as the black hole entropy: $S \propto M^{(D-2)/(D-3)}$.

How can we do better?

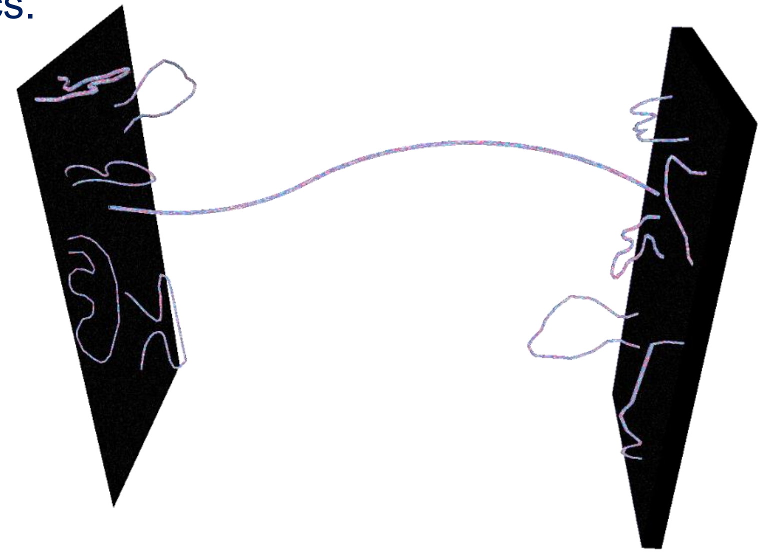
- Study "topological" or "protected" sectors in which strong-coupling calculus is more feasible.
- Extra non-perturbative degrees of freedom in string theory should play important roles.

Fast-growing entropy

Apart from elementary strings, there exist solitonic/collective/emergent objects

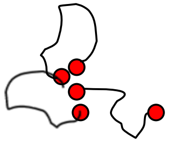
- Like magnetic monopole, vortex, … in particle physics.
- "D-branes" [Polchinski] (1995)

Joseph Polchinski
(1954 – 2018)



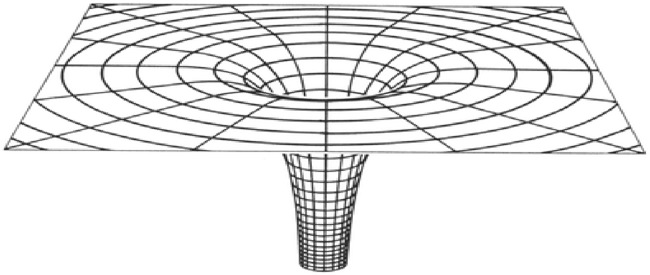
Strategy: [Strominger, Vafa] (1996) ……

- New bound states of D-branes & strings allow systems with fast-growing entropy.
- Construct approximate QM or QFT on such bound states, and do its statistical mechanics.



Do honest statistical mechanics of the quantum mechanics or QFT

strong coupling



Agrees w/ emergent gravitational picture?

Strong-coupling & extremal black holes

The 2nd issue is technical: Hard to do strong-coupling calculations.

For easier calculations, people considered "extremal" black holes

- Also called "supersymmetric" or "BPS" (Bogomolnyi-Prasad-Sommerfield) black holes
- Charged black holes [Reissner, Nordstrom]: mass bounded by charge $M \geq Q$.
- Extremal BH's have lowest mass at fixed Q: $M = Q$ and $S \sim Q^{(D-2)/(D-3)}$.

- Has some "topological properties" (like quantum hall system, topological matters…)
- Many observables are coupling independent. Strong coupling calculus doable.

Realized in certain dimensions:

- in $D = 5$: $S = 2\pi Q^{3/2}$ counted by Strominger, Vafa (1996)
- in $D = 4$: $S = 2\pi Q^{2}$ counted by Maldacena, Strominger, Witten (1997) and also by Vafa (1997)
- Very fast grow, due to non-perturbative bound states

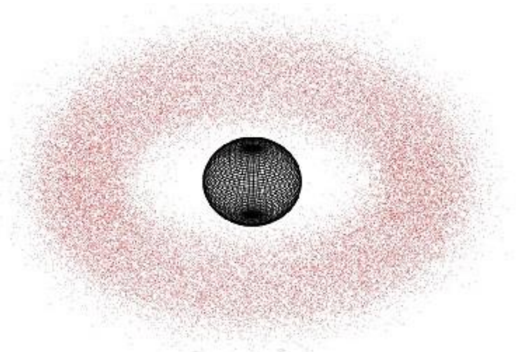
Andrew Strominger & Cumrun Vafa

Establish that BH thermodynamics is physical.

Limitations

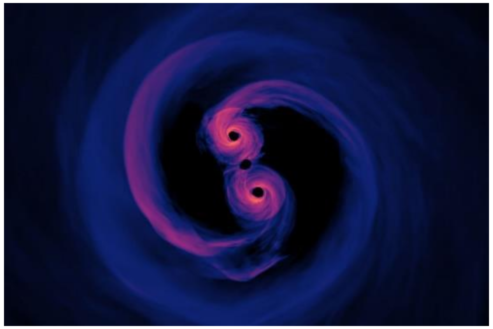
So far, never used systematic QG. This often poses serious limitations.

Distinct BH's may exist: "No-hair theorem" is often violated. Often, no uniform description.



"Hairy" black holes: matter outside BH

Black "rings" with rotation

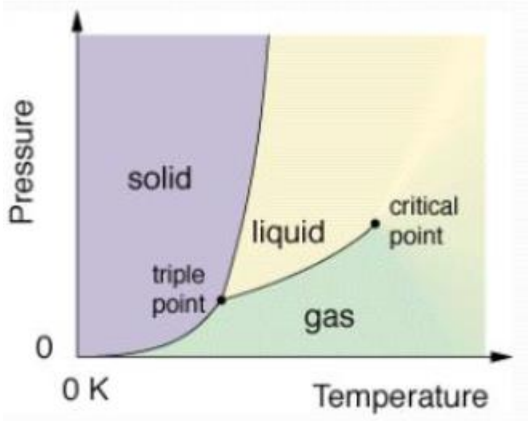


Multi-centered BH's

- Multiple large charge saddle points of thermodynamics. Dominant one decides physics.
- Can have phase transitions. But hard to compare them.

Exciting phase transitions happen in canonical ensemble.

- Like liquid-gas phase transition as T changes.
- Needs full control over QG (or reformulation) at arbitrary T.
- Hard…! (fate of strings at high T…?)

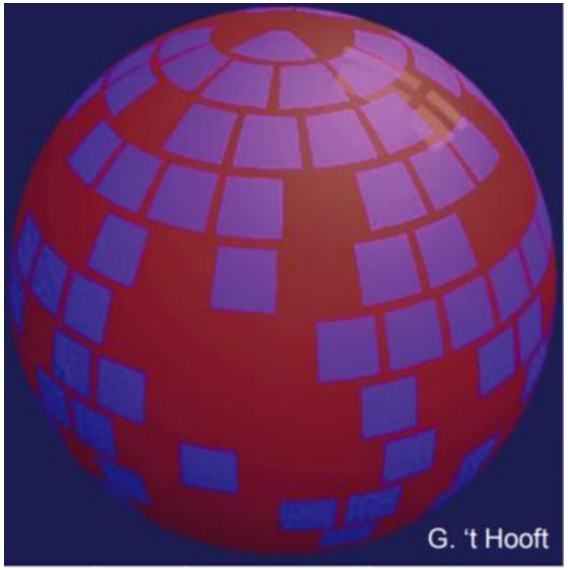


Black holes and “hologram”

We can obtain a better set-up by reconsidering the hologram of black holes.

How can the information of BH interior be encoded on its boundary?

- Just an observed emergent property in GR, but ubiquitous.
- People often dub their ignorance as “principle.”



“holographic principle” [Charles Thorne] [‘t Hooft] [Susskind]…

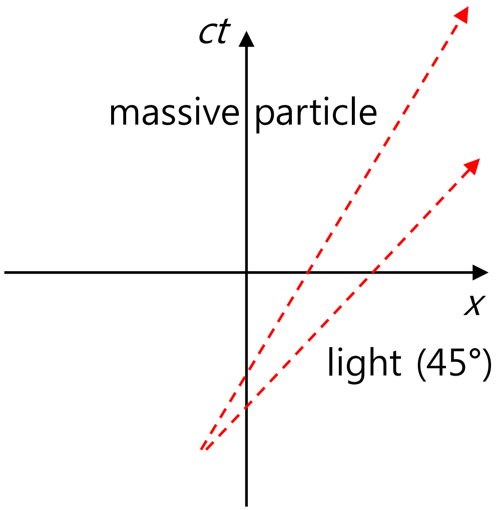
- Asserts that there may be apparently non-gravitational system at the boundary of a region, which “emergently” describes gravity inside.
- E.g. on a surface which particles/lights cannot cross (like event horizon), simple holographic description is available about the interior, like the entropy inside. [Bousso] (1999)

Although it was found as a property of BH’s, it was extended into a broader principle, based on which one can make universal studies of BH’s.

Holographic quantum gravity?

Full spacetime also has a "boundary"… at infinity.

- Can the full interior physics of quantum gravity be encoded on this boundary?
- If possible, this would be a holographic description of the whole quantum gravity.

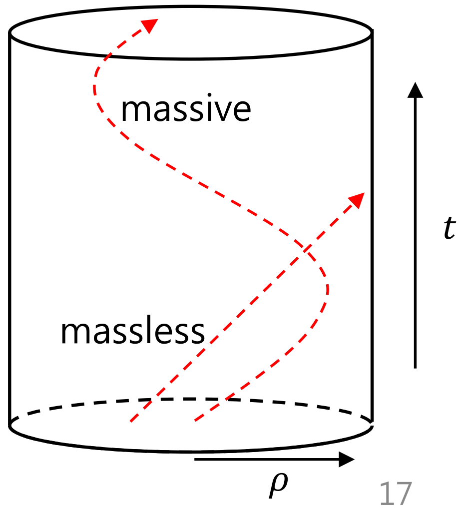


A condition for holographic quantum gravity:

- Interior information should not leak. (Recall BH event horizon)
- But particles/lights may leak: e.g. flat spacetime
- To prevent this, one needs a strong gravitational potential to trap these particles: a gravitational "box"

Anti de Sitter (AdS) spacetime: a "maximally symmetric" space

$$ds_{D+1}^2 = d\rho^2 - \cosh^2\frac{\rho}{\ell}\,dt^2 + \ell^2\sinh^2\frac{\rho}{\ell}\,ds^2(S^{D-1})$$



- Force towards the center $\rho = 0$: $\Phi \approx -g_{tt}(\rho)/2$
- Massive particles cannot escape.
- Massless particles can escape. (But that'll be OK.)

A full quantum description of gravity

Juan Maldacena

In 1997, Juan Maldacena made a proposal for QG in AdS.

- Uses the idea of hologram.
- Proposed ("derived"…?) a holographic quantum system on the boundary of AdS, which is $S^{D-1} \times R$ here.

Features:

- Lives a usual (apparently non-gravitational) quantum field theory at the boundary.
- There exist: finite # of massless fields (like gravitons)

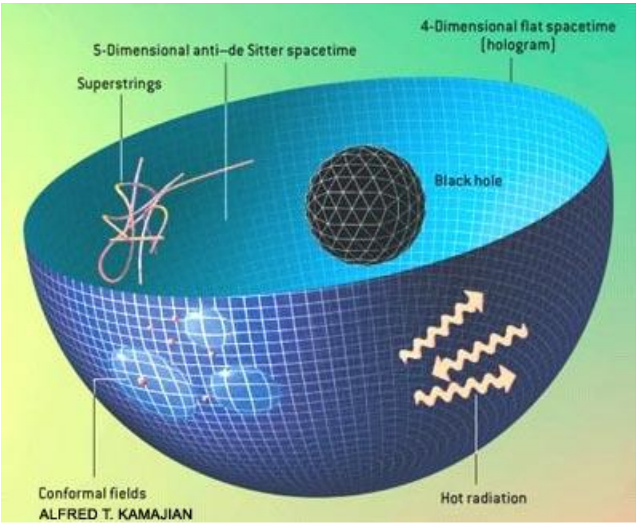
  infinite # of massive particles ~ fields (like string oscillation modes)
- The trapped massive fields are completely described holographically.
- Massless fields: need boundary conditions for their long-ranged values.

  → correspond to the parameters of boundary system (coupling constants, masses, …)

The boundary quantum system is a usual local quantum field theory.

- Since it has the so-called "conformal" symmetry, we call it CFT (conformal field theory)
- This holographic relation to gravity is often called "AdS/CFT duality"

Holographic gravity



Boundary description is very simple: QFT

- Very often Yang-Mills gauge theories, e.g. w/ SU(N)
- Has gluons & quarks: very similar to QCD

Gravity is emergent in these QFTs, at…

- strong coupling (like QCD),
- large # of gluons: $N^2 \gg 1$.

(Evades all previous no-go theorems cleverly…)

It is a system in which the whole universe behaves like a BH interior to its boundary.

Can define quantum gravity using quantum field theory.

I think it will be crowned one of the key findings of 20^{th} century physics (along w/ GR).

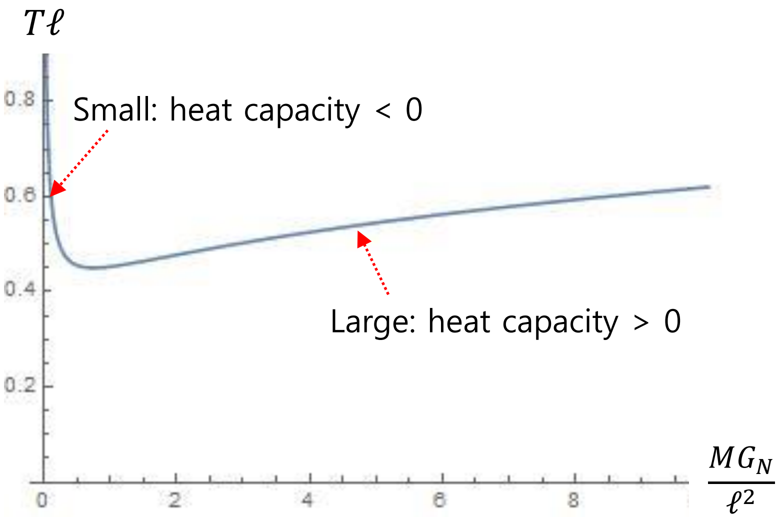
- Perfect set-up to study quantum gravity, black holes & their puzzles.
- New channel to study strongly correlated materials (e.g. table-top condensed matters).

Black holes in AdS

- Schwarzschild black holes (e.g. in AdS_5 which can be studied from QFT in $d = 3+1$):
- Small BH: Similar to BH's in flat space. Negative specific heat.
- Large BH: important in AdS thermodynamics in the canonical ensemble

$$T = \frac{r_+}{\pi \ell^2} + \frac{1}{2\pi r_+}$$

$$r_+^2 = -\frac{\ell^2}{2} + \ell\sqrt{\frac{\ell^2}{4} + \omega M} \qquad \omega \equiv \frac{16\pi G_N}{3\mathrm{vol}(S^3)}$$



- Hawking-Page phase transition: [Hawking, Page] (1983)

transition between two phases, at $T = \frac{3}{2\,\pi\ell}$ (ℓ is the radius of AdS.)

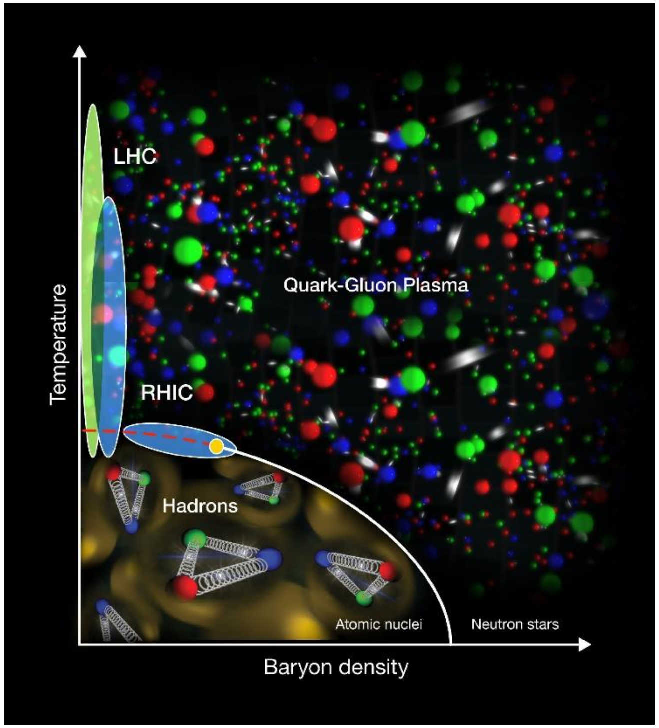
- Low T : Gas of gravitons in AdS. Free energy doesn't see Newton constant $\ell^3/G_N \sim N^2$
- High T : Large black hole phase. Sees $\mathrm{F} \propto \ell^3/\mathrm{G_N} \sim N^2$.

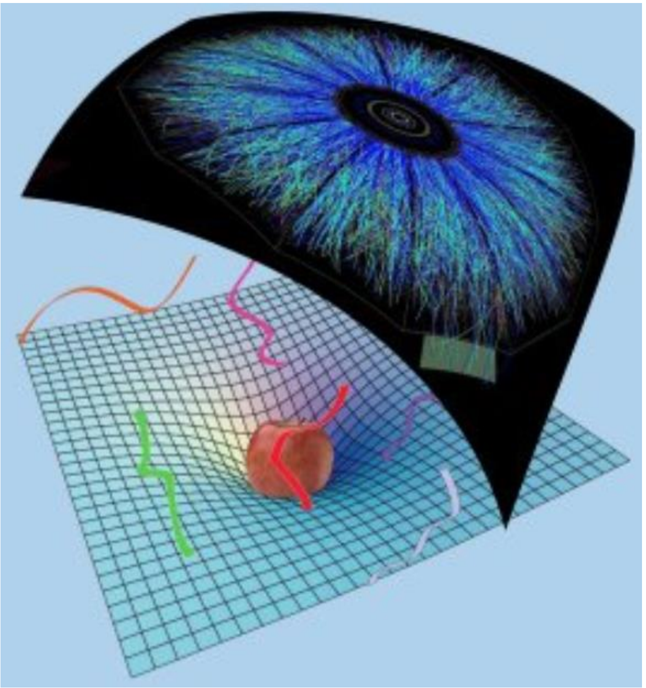
Black holes from gauge theory

A natural dual is the “confinement - deconfinement” phase transition [Witten] (1998)

- Confined phase: $F \sim O(N^0)$, glue-balls (& mesons, etc.) as gravitons.
- Deconfined phase: $F \sim O(N^2)$ from liberated gluons (& quarks)

The picture: **deconfined quark-gluon plasma (QGP) ~ black holes**



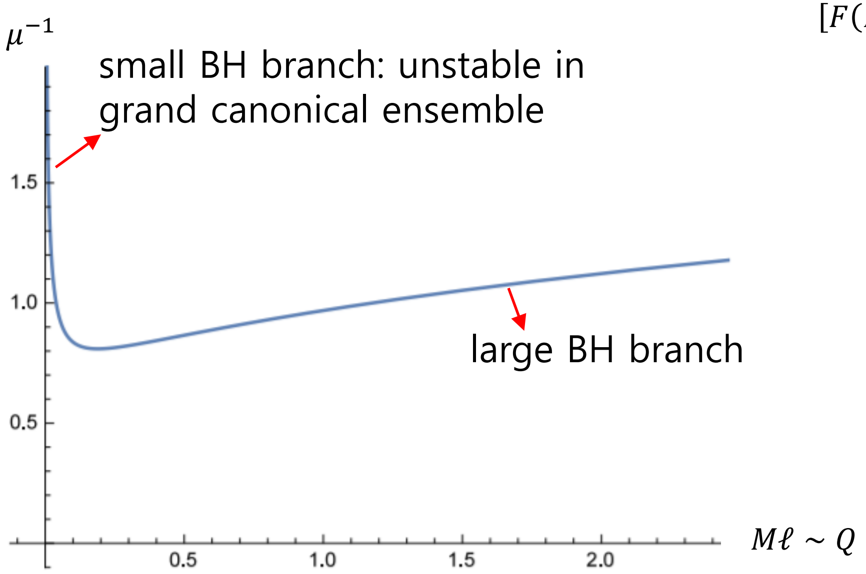


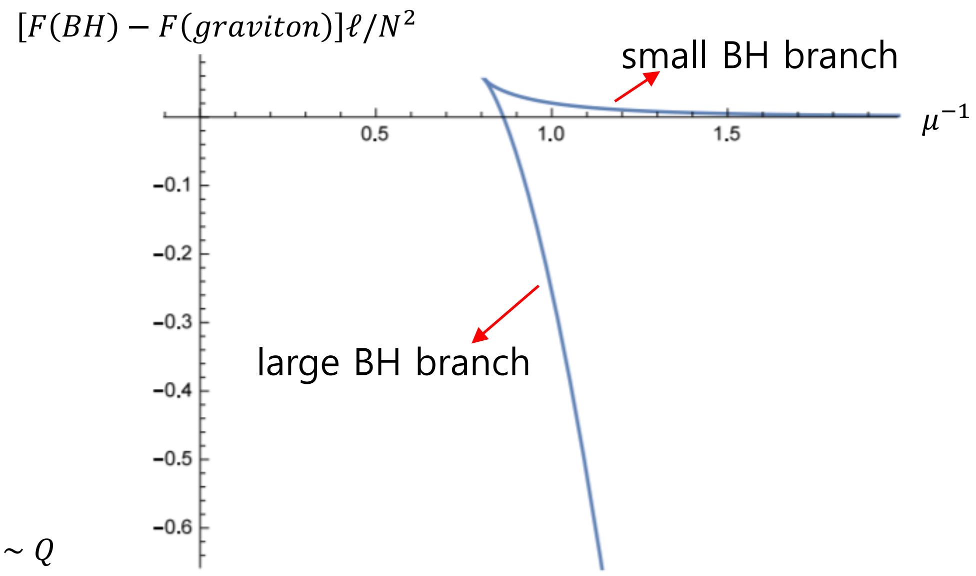
Quantitative understanding of this picture was achieved only very recently.

Deconfinement & black hole formation

- Again, we study strong-coupling QFT: focus on “tolopogical” “extremal” sector
- BHs at zero Hawking temperature but at chemical potentials μ ~ nonzero “charge density”

- Free energy $F(\mu)$ computable either from exact path integral & matrix model techniques, or using chiral anomalies of the QFT.





- <u>These are QFT plots</u>. Precisely same structure as Schwarzschild black holes.
- Explained S_{BH} precisely from the statistical mechanics of deconfined gluons.
- And we can get more from QFT: e.g. predict new black holes, etc.

Conclusion & outlook

- BH's are similar to & different from usual thermal systems.
  - "Geometric" thermodynamic behaviors have statistical origin.
  - Some black holes (those we see in the sky) have extremely fast-growing entropy.
  - I think this demands a drastic extension of gravity, something like string theory.

- Even for astronomical black holes, the entropies are macroscopic & fast-growing.
  - The matters that collapse to form black holes will presumably look ordinary.
  - Holographically screened "internal" microstates are huge. What's inside these matters?

- Studying BH & QG go side by side:
  - Many properties of BH's are intrinsic properties of QG: hologram, fast-growing entropy, …
  - Deconfined phase of dual gauge theory suggests novel high T features of QG.

- Application: Can use BH's to study strong-coupling/correlated materials.
  - "Qualitative" studies of QCD, condensed matter systems w/ large degrees of freedom…?