One-dimensional Ising Model with Long-range Interactions

In addition to the spatial dimension and the symmetry, the range of interaction plays a crucial role in determining the nature of phase transitions and the universality class of a given statistical physics model. One of the simplest testing ground where one can study the effect of the range of interaction on phase transitions is the one-dimensional (ferromagnetic) Ising model with long-range interactions. Let us consider the Ising spins $S_i = \pm 1$ at site i on a one-dimensional lattice interacting via a power-law potential. The Hamiltonian is given by

$$H = -J \sum_{i,j} \frac{1}{r_{ij}^{\alpha}} S_i S_j,$$

where $J > 0$ and r_{ij} is the distance between sites i and j.

For the usual short-ranged interaction (nearest neighbor interaction *e.g.*), one-dimensional system does not show a phase transition at any finite temperature. But in the above case, depending on the value of α, the system exhibits a phase transition at nonzero temperature. The purpose of this project is to study this model within the framework of real space renormalization group which will be discussed in the lectures.

Follow the steps in Ref. [1] and obtain the results for yourself. In contrast to the simple nearest-neighbor interaction case, one has to resort to an approximate method to obtain an RG recursion relation in this case. This is a well-known technique which is developed in Ref. [2] and discussed in many textbooks [3].

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[1]. S. A. Cannas, Physical Review B **52**, 3034 (1995).

[2]. T. Niemeijer and J. M. J. van Leeuwen, Physical Review Letters **31**, 1411 (1973).

[3] see forexample M. Kardar, *Statistical Physics of Fields* (Cambridge Univesity Press, 2007) p 108.; M. Plischke and B. Bergersen, *Equilibrium Statistical Physics* 2nd Edition (World Scientific, 1994) p. 228.